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PIGOUVIAN TAXATION WITH ADMINISTRATIVE COSTS*

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This paper examines how the optimal Pigouvian tax should be adjusted to reflect administrative costs. Several cases are examined, depending on whether the administrative costs are fixed per firm taxed or are a function of the amount of tax collected, and on whether such costs are borne by the government or by the taxed firm. In some cases the presence of administrative costs leads to an optimal tax greater than the external cost, while in other cases it leads to a tax less than the external cost.

1. Introduction

When a Pigouvian tax is used to control an externality-generating activity, administrative costs are generally incurred. These costs might be borne by the government or by the taxed party — assumed for concreteness to be a firm. They include the cost of monitoring the externality-generating activity, the time spent completing forms, and the expense of resolving disputes over tax liability. Some administrative costs, such as those associated with the processing of forms, depend on the number of firms taxed but not on the tax revenue collected; these will be referred to as 'fixed' per firm. Other administrative costs, such as the expense of resolving disputes, may depend on the amount of tax collected (e.g. legal expenditures may rise with the size of the dispute); these will be referred to as 'variable' per firm.

This paper will examine how the optimal Pigouvian tax should be adjusted to reflect administrative costs when these costs are fixed or variable and

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when they are borne by the government or the taxed firms.[1] As is well known, in the absence of administrative costs, the tax should equal the external cost. It is shown here that in the presence of fixed administrative costs borne by the government, the Pigouvian tax should exceed the external cost. This is because raising the tax above the external cost reduces the number of firms that engage in the activity and thereby saves administrative costs. However, raising the tax causes those firms which do participate in the activity to choose too low a level of activity; the administrative cost of taxing these firms is 'sunk', so, with respect to them, the closer the tax is to the external cost, the better. This limits the extent to which the tax should be raised.

If, on the other hand, the fixed administrative costs are borne by the taxed firms, the tax should equal the external cost. In this case, it is not desirable to raise the tax in order to reduce the number of firms which engage in the activity because their bearing of the administrative costs already accomplishes this to an appropriate extent. By setting the tax equal to the external cost, those firms which do engage in the activity are induced to choose the correct level of activity.

When the administrative costs are variable and borne by the government, the optimal tax could be above or below the external cost. For example, if the activity levels of those firms which engage in the activity are not very responsive to changes in the tax, then lowering the tax reduces tax revenue and thereby saves administrative cost. This savings may make it desirable to lower the tax. Similarly, if firms are very responsive to the tax, it may be optimal to raise the tax above the external cost.

Finally, if the variable administrative costs are borne by the taxed firms, the optimal tax is below the external cost. This will be explained below.

The results in the four cases analyzed are summarized in table 1.

2. The model

The model analyzed here has the following features. Risk-neutral firms derive benefits from engaging in an activity which imposes a constant external harm per unit of activity. Each firm is described by its schedule of

[1]The only other analytical studies of administrative costs and taxation have been in the context of 'optimal taxation', in which the problem is typically to choose excise or income taxes to raise a fixed amount of revenue with least deadweight burden (rather than, as here, to internalize an externality). In this context, Heller and Shell (1974) characterize administrative costs in a very general way and examine whether, given such costs, it is desirable for production to be efficient. Yitzhaki (1979) considers the tradeoff between higher administrative costs and lower deadweight burden as the number of taxed commodites increases. Stern (1982) compares optimal income taxation to lump-sum taxation in which the taxing authority makes errors in classifying individuals. Also, Atkinson and Stiglitz (1980, pp. 359–360, 455) have noted the desirability of incorporating administrative costs in the analysis of taxation.

Table 1

The optimal Pigouvian tax.[a]

		Type of cost	
		Fixed per firm	Variable per firm
Who bears cost:	Government	$t>e$	$t\gtreqless e$
	Firms	$t=e$	$t<e$

[a]Here t and e are, respectively, the tax and externality cost per unit of activity.

benefits from participating in the activity at different levels. The government imposes a tax per unit of activity. A firm's problem is to choose a level of activity to maximize its profits — its benefits less its tax payments and less any administrative costs it bears. One possibility is that the firm chooses not to participate in the activity at all. The government's problem is to choose the tax rate to maximize social welfare subject to the constraint that firms maximize profits. Social welfare equals the benefits to those firms which engage in the activity less the external harm they impose and less administrative costs, whether borne by the firms or the government.[2]

The following notation will be used:

x = activity level of a firm ($x \geqq 0$),
λ = parameter defining the benefit schedule of a firm ($0 \leqq \lambda \leqq 1$),
$f(\lambda)$ = density of λ-type firms ($f(\lambda)>0$, $\int f(\lambda)\,\mathrm{d}\lambda = 1$),
$b(x,\lambda)$ = benefit to a λ-type firm from engaging in the activity at level x ($b_x>0$, $b_{xx}<0$, $b_\lambda>0$, $b_{x\lambda}>0$, $b(0,\lambda)=0$),
e = external harm per unit of activity,
t = tax per unit of activity.

Thus, each firm's problem is:

$$\max_x b(x,\lambda) - tx - (\text{administrative costs}), \tag{1}$$

where the administrative costs are only a possibility and may be of the fixed or variable kind.[3] Let $x(\lambda,t)$ be the activity level chosen by a λ-type firm given a tax t; if $x(\lambda,t)=0$, this means that the firm does not participate in the activity at all. Note that a higher λ corresponds to a higher benefit schedule; thus, for any tax t, there will be a critical value of the benefit parameter, denoted $\lambda(t)$, such that firms with λ below $\lambda(t)$ will choose not to engage in

[2]Tax revenues collected are assumed to be distributed in a lump-sum way and thus do not affect social welfare.

[3]It will be assumed that a solution to (1) exists. Uniqueness is guaranteed (since $b_{xx}<0$) when the firm does not bear any administrative costs or, when the firm does bear such costs, if they are fixed; when the firm bears variable administrative costs, uniqueness is assumed.

the activity at all, and firms with λ excceding $\lambda(t)$ will engage in the activity at a positive level.

Given the above, social welfare W may be written as

$$W(t)=\int_{\lambda(t)}^{1} b(x(\lambda,t),\lambda)f(\lambda)\,\mathrm{d}\lambda-e\int_{\lambda(t)}^{1} x(\lambda,t)f(\lambda)\,\mathrm{d}\lambda$$
$$-(\text{administrative costs}). \tag{2}$$

The first term represents the benefits to those firms which engage in the activity and the second term represents the external harm they impose. The government's problem is to choose t to maximize W.[4]

3. No administrative costs

For purposes of comparison, it will be useful to verify that if there are no administrative costs, then setting the tax equal to the external harm is optimal. To see this, maximize social welfare (2) assuming that there are no administrative costs. The first-order condition is

$$W'(t)=-\lambda'(t)b(x(\lambda(t),t),\lambda(t))f(\lambda(t))+\int_{\lambda(t)}^{1} b_x(x(\lambda,t),\lambda)x_t(\lambda,t)f(\lambda)\,\mathrm{d}\lambda$$
$$+e\lambda'(t)x(\lambda(t),t)f(\lambda(t))-e\int_{\lambda(t)}^{1} x_t(\lambda,t)\,f(\lambda)\,\mathrm{d}\lambda=0. \tag{3}$$

This can be simplified by noting that firms which are just indifferent between participating and not choose an activity level of zero: $x(\lambda(t),t)=0$.[5] Also, since each firm which does participate chooses its level of activity so as to maximize its benefits net of taxes, $b(x,t)-tx$, it follows that $b_x(x(\lambda,t),\lambda)=t$. Hence (3) reduces to

$$W'(t)=(t-e)\int_{\lambda(t)}^{1} x_t(\lambda,t)f(\lambda)\,\mathrm{d}\lambda=0, \tag{4}$$

which implies that $t=e$ (since $x_t(\lambda,t)<0$).[6]

[4]It will be assumed that a unique interior solution to this problem exists.

[5]Since these firms are just indifferent between participating and not, their profits must be zero. Now if it were strictly optimal for these firms to participate at a positive level, then their profits would have to be greater than they would be at a zero level of activity. But their profits at a zero level of activity are zero.

[6]This result can be derived in a simpler, more direct way. The first-best solution — what the government would order if it had complete control — is to have each firm choose a level of activity x which maximizes $b(x,\lambda)-ex$. Since the firm's problem is to maximize $b(x,\lambda)-tx$, setting $t=e$ will induce the firm to choose the first-best level of activity. This result was derived in the text by maximizing the social welfare function because later results require use of this method.

4. Fixed administrative costs per firm

Now suppose that there is a fixed administrative cost per firm taxed, represented by a. The administrative cost term in the social welfare function (2) is then

$$a \int_{\lambda(t)}^{1} f(\lambda)\,\mathrm{d}\lambda,$$

and consequently, the first-order condition determining the optimal tax is

$$\begin{aligned} W'(t) = & -\lambda'(t) b(x(\lambda(t), t), \lambda(t)) f(\lambda(t)) \\ & + \int_{\lambda(t)}^{1} b_x(x(\lambda, t), \lambda) x_t(\lambda, t) f(\lambda)\,\mathrm{d}\lambda \\ & + e\lambda'(t) x(\lambda(t), t) f(\lambda(t)) \\ & - e \int_{\lambda(t)}^{1} x_t(\lambda, t) f(\lambda)\,\mathrm{d}\lambda + a\lambda'(t) f(\lambda(t)) = 0. \end{aligned} \tag{5}$$

As noted in the introduction, the administrative costs might be borne by the government or by the firms themselves.

4.1. Borne by the government

In this case, since (from the previous section) $x(\lambda(t), t) = 0$ and $b_x(x(\lambda, t), \lambda) = t$, (5) reduces to

$$t = e + \left[\lambda'(t) f(\lambda(t)) \Big/ - \int_{\lambda(t)}^{1} x_t(\lambda, t) f(\lambda)\,\mathrm{d}\lambda \right] a. \tag{6}$$

The term in brackets is positive since $\lambda'(t) > 0$ (the higher the tax, the higher the minimum λ required for firms to be just willing to engage in the activity), since $f(\lambda(t)) > 0$ (there are some firms which are just indifferent between engaging in the activity and not), and since $x_t(\lambda, t) < 0$ (the higher the tax, the lower the level of activity chosen). Thus, the optimal tax exceeds the external cost.

In order to understand this result consider two extreme cases. First, suppose that all firms strictly preferred to engage in the activity or, equivalently, all firms had a λ exceeding $\lambda(t)$. Then $f(\lambda(t))$ would be zero, so that, from (6), the optimal tax would equal the external cost. This is true for the following reason. The administrative costs would be a sunk cost for society. Consequently, all that would matter would be that firms choose the appropriate activity level, and to achieve this the tax should equal the external cost.

Second, suppose that if a firm participates in the activity, it does so at a fixed level — say a unit level of activity — and that the benefits to a λ-type firm from participation are λ. Then, of course, there could not be any effect of the tax on the activity level; the tax would affect only the decision whether to engage in the activity. Since by participating in the activity the firm would impose on society the external cost plus the administrative costs, the optimal tax would equal the sum of those two costs.[7]

These two cases illustrate respectively the effect of the tax on the activity levels of firms and the effect of the tax on the number of firms engaging in the activity, and hence on the magnitude of administrative costs. As seen, the first effect tends to make the optimal tax close to the external cost and the second to raise it above the external cost.[8] However, it should be explained why the second effect is always strong enough to raise the optimal tax above the external cost. If the tax were equal to the external cost, then the activity levels of firms which choose to participate in the activity would be correct. Thus, the effect on social welfare of a marginal change in their activity levels would be zero. However, if the tax were equal to the external cost, the effect on social welfare of a marginal reduction in the number of firms that participate in the activity would be positive (this is the second effect). Hence, a small increase in the tax above the external cost would increase social welfare.

4.2. Borne by firms

In this case, for a firm to be indifferent between participating and not, it must be true that

$$b(x(\lambda(t),t),\lambda(t)) - tx(\lambda(t),t) - a = 0.$$

Thus, since $b_x(x(\lambda,t),\lambda) = t$, (5) implies that $t = e$. As noted in the introduction, because firms bear the administrative cost, they make the correct decision whether to participate in the activity; and by setting the tax equal to the external cost, those firms which do participate are induced to choose the correct level of activity.

5. Variable administrative costs per firm

In this section, suppose that the administrative cost per firm taxed depends

[7]This result can easily be seen to follow from (2) with $b(x(\lambda,t),t) \equiv \lambda$ and $x(\lambda,t) \equiv 1$.

[8]The fact that in (6) a higher $a, f(\lambda(t))$, or $\lambda'(t)$ tends to raise t reflects the second effect.

on the amount of the tax. Specifically, let $a(tx)$ be the administrative cost when the tax collected is tx, and assume that $a(0)=0$[9] and that $a'>0$. Therefore, the administrative cost term in the social welfare function (2) is

$$\int_{\lambda(t)}^{1} a(tx(\lambda,t))f(\lambda)\,\mathrm{d}\lambda,$$

so that the first-order condition determining the optimal tax is

$$\begin{aligned} W'(t) = &-\lambda'(t)[b(x(\lambda(t),t),\lambda(t)) - ex(\lambda(t),t) \\ &- a(tx(\lambda(t),t))]f(\lambda(t)) \\ &+ \int_{\lambda(t)}^{1} [b_x x_t - ex_t - (x+tx_t)a']f(\lambda)\,\mathrm{d}\lambda = 0, \end{aligned} \tag{7}$$

where the arguments in the integrand of the last term have been omitted.

5.1. Borne by the government

As before, $x(\lambda(t),t)=0$ and $b_x(x(\lambda,t),\lambda)=t$. Therefore, (7) reduces to

$$t = e + \left[\int_{\lambda(t)}^{1} (x+tx_t)a'f(\lambda)\,\mathrm{d}\lambda \Big/ \int_{\lambda(t)}^{1} x_t f(\lambda)\,\mathrm{d}\lambda\right]. \tag{8}$$

Observe that $\int (x+tx_t)a'f(\lambda)\,\mathrm{d}\lambda$ is the impact on administrative costs of a marginal increase in the tax rate. This consists of a positive direct effect due to the increase in the tax given firms' existing levels of activity ($\int xa'f(\lambda)\,\mathrm{d}\lambda$) and a negative indirect effect due to reductions in firms' levels of activity ($\int tx_t a'f(\lambda)\,\mathrm{d}\lambda$). Either effect may dominate. Thus, the optimal tax could be above or below the external cost (or equal it).[10]

To better understand this result, suppose the tax were equal to the external cost. Then firms will be led to consider the effect of their behavior on the external costs (and, of course, on their own benefits), but not on administrative costs. Therefore, it will be beneficial to change the tax to reduce the unaccounted for administrative costs. This may require raising or lowering the tax for the reasons noted in the previous paragraph. (And note also that for a small change in the tax, the resulting effect on social welfare

[9]If $a(0)$ were positive, then there would be some fixed administrative costs per firm. The assumption that $a(0)=0$ is made in order to isolate the effect of variable administrative costs on the optimal tax.

[10]The optimal tax would equal the external cost, for example, if for each firm there was no change in tax revenue collected as a result of a marginal increase in the tax, i.e. $x+tx_t=0$.

due to changes in firms' benefits and their external costs would be zero since, with respect to these considerations, their activity levels would have been correctly chosen.)

5.2. Borne by firms

Since $x(\lambda(t),t)=0$ and now $b_x(x(\lambda,t))=t+a'(tx(\lambda,t))t$, (7) reduces to

$$t=e+\left[\int_{\lambda(t)}^{1} xa'f(\lambda)\,\mathrm{d}\lambda \Big/ \int_{\lambda(t)}^{1} x_t f(\lambda)\,\mathrm{d}\lambda\right]. \tag{9}$$

Note that $\int xa'f(\lambda)\,\mathrm{d}\lambda$ is the effect on administrative costs of a marginal increase in the tax rate *given* firms' existing levels of activity. Since this effect is positive, the optimal tax is unambiguously less than the external cost (since $x_t<0$).

The explanation of this result is as follows. Suppose the tax were equal to the external cost. As in the previous case, firms will be led to consider the effect of their behavior on the external cost and their own benefits. But since they now bear the administrative costs, they will also be led to consider the indirect effect of changes in their activity levels on the administrative costs. However, firms cannot consider the direct effect of changes in the tax on administrative costs. By lowering the tax, administrative costs are reduced at the existing levels of activity. (And observe that the resulting effects on social welfare due to changes in firms' benefits, the external costs, and the administrative costs would be zero since, with respect to these considerations, their activity levels were correct.)

6. Numerical examples

To illustrate the effect of administrative costs on the optimal Pigouvian tax, some examples have been computed using specific functional forms. In each case the external cost is $5.00 per unit of activity and the parameter defining the benefit schedule, λ, is assumed to be uniformly distributed between 0 and 1. The examples differ in terms of who bears the administrative cost and in terms of the benefit schedules.

Table 2 shows how the optimal tax rises when there are fixed administrative costs borne by the government. For administrative costs of $50 per firm, the optimal tax is nearly twice the magnitude of the external cost.

Table 3 illustrates how the optimal tax is affected when there are variable administrative costs. As demonstrated above, if these costs are borne by the government, the optimal tax might be higher or lower than the external cost. Both cases are shown in table 3. For administrative costs of $0.50 for every

Table 2

Fixed administrative cost borne by the government.

Fixed administrative cost ($)	Optimal tax ($)[a]
0	5.00
1	5.05
5	5.26
10	5.56
15	5.88
20	6.25
25	6.67
30	7.15
35	7.70
40	8.34
45	9.10
50	9.88

[a] $b(x, \lambda) = 100\lambda(1 - e^{-x})$.

Table 3

Variable administrative cost.

Variable administrative cost ($)	Borne by the government		Borne by the firm
	Optimal tax ($)[a]	Optimal tax ($)[b]	Optimal tax ($)[c]
0.00	5.00	5.00	5.00
0.05	5.13	3.86	4.88
0.10	5.26	3.11	4.76
0.15	5.41	2.57	4.65
0.20	5.56	2.18	4.55
0.25	5.72	1.89	4.44
0.30	5.88	1.66	4.35
0.35	6.06	1.47	4.26
0.40	6.25	1.32	4.17
0.45	6.45	1.20	4.08
0.50	6.67	1.09	4.00

[a] $b(x, \lambda) = 100\lambda[(x+1)^{1/2} - 1]$.
[b] $b(x, \lambda) = 100\lambda(1 - e^{-x})$.
[c] $b(x, \lambda) = 100\lambda[(x+1)^{1/2} - 1]$.

dollar collected in taxes, the optimal tax rises 33 percent above the external cost in one case and falls 78 percent in the other. Table 3 also shows how the optimal tax falls when the variable administrative costs are borne by the taxed firm. For administrative costs of $0.50 per dollar collected, the optimal tax falls 20 percent below the external cost.

References

Atkinson, A.B. and J.E. Stiglitz, 1980, Lectures on public economics (McGraw-Hill, New York).

Heller, W.P. and K. Shell, 1974, On optimal taxation with costly administration, American Economic Review 64, 338–345.

Stern, N., 1982, Optimum taxation with errors in administration, Journal of Public Economics 17, 181–211.

Yitzhaki, S., 1979, A note on optimal taxation and administrative costs, American Economic Review 69, 475–480.