Eminent Domain versus Government Purchase of Land Given Imperfect Information about Owners' Valuations

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Abstract

Governments employ two basic policies for acquiring land: taking it through the exercise of their power of eminent domain, and purchasing it. The social desirability of these policies is compared in a model in which the government's information about landowners' valuations is imperfect. Under this assumption, the policy of purchase possesses the market test advantage that the government obtains land from an owner only if its offer exceeds the owner's valuation. However, the policy suffers from a drawback when the land that the government needs is owned by many parties. In that case, the government's acquisition will fail if any of the owners refuses to sell. Hence, eminent domain becomes appealing if the number of landowners is large. This conclusion holds regardless of whether the land that the government seeks is a parcel at a fixed location or instead is a contiguous parcel that may be located anywhere in a region.

1. Introduction

Governments generally enjoy the ancient right of eminent domain—the right to take land by fiat, usually on the condition that they pay fair compensation to the landowner.[1] But governments may, of course, also obtain land by purchasing it, which raises the question of why governments need the right of eminent domain for land (and not generally for goods).

In this article, I consider an aspect of that question by comparing the social desirability of eminent domain to government purchase in a simple model of land acquisition. The key feature of the model is that the government's knowledge of landowners' valuations is imperfect. Under this assumption, the policy of

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[1] On the right of eminent domain and the requirement to pay compensation in the legal system of the United States, see, for example, Dukeminier et al. (2006, chap. 12). On the history of eminent domain or its equivalents in the United States and other countries, see, for instance, Nichols (2006, sec. 1-1).

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purchase possesses the classic advantage that acquisition of land is subject to the market test—the government obtains a parcel of land only if its offer to an owner is high enough that the owner is willing to sell. In contrast, eminent domain involves no market test, which implies that the government might turn out to take a parcel of land even though the social value of the land is lower than its private value. The policy of purchase suffers from a difficulty, however, when the land that the government requires is held by multiple owners, for then the government needs all the owners to sell in order to proceed (I initially assume that the government cannot undertake its project if it fails to obtain the entire parcel that it seeks). The necessity of such unanimity in owners' decisions to sell constitutes an acute disadvantage of the policy of purchase when the number of owners grows large, for in that context the likelihood that some owner will reject any given government price offer becomes high.

Consequently, the policy of eminent domain tends to be appealing in the model when the number of owners is large, whereas the policy of purchase tends to be better when the number of owners is low. This is the major conclusion of the present article, but as its summary will make clear, the conclusion is qualified and amplified in important ways, especially in relation to whether the land the government requires is a specific parcel, only a contiguous parcel that may be located anywhere within a region, or only a parcel of given area that can be composed of many dispersed subparcels.

In Section 2, I state the main assumptions of the model. I suppose that the government places a social value on the land it needs and that the values that private owners place on the land are not observable to the government but are drawn from a known distribution. Under eminent domain, the government may take land from private owners but must pay appropriate compensation for doing so, where such compensation is interpreted as the expected private value of land.[2] Under the policy of purchase, the government makes a single price offer for the land. If the government obtains the land, it realizes the social value of the land, but the payment that the government makes (the purchase price or the compensation for a taking) is assumed to involve a positive social cost of funds (because of the implicit cost of raising funds through taxation). The government acts to maximize social welfare.

In Section 3, I address the case in which a single owner holds a specific parcel of land that the government seeks. Under the policy of eminent domain, the government will take the land if its social value minus the social cost of paying just compensation exceeds its expected private value.[3] Under the policy of purchase, the government's social-welfare-maximizing price offer is positive but less than the social value of the parcel, owing to the social cost of funds. Purchase

[2] I make the assumption that compensation must be paid because this is a legal requirement, and I discuss my specific assumption about its amount in note 12.

[3] Suppose that the government seeks an acre parcel with a social value of $1 million, that the social cost of a dollar of funds is $.10, and that the expected private value of the acre is $100,000. Then the government will take the acre, since its social value net of the social cost of paying $100,000 in fair compensation would be $990,000, which exceeds its expected private value of $100,000.

is often socially superior to eminent domain because of the market test advantage of purchase, that when the owner's value turns out to be high and government acquisition would not be socially desirable, the owner will reject the government's price offer. Still, it is possible that eminent domain would be superior to purchase because of a potential acquisition cost advantage enjoyed by the government under the former policy. Namely, under eminent domain the government acquires land at a fair compensation cost equal to its expected value to owners, whereas under the policy of purchase the government may have to (and thus will find it desirable to) offer substantially more than the expected value of the land in order to acquire the land with high probability.[4]

In Section 4, I examine the case in which multiple owners hold subparcels making up the parcel that the government wishes to obtain. In particular, I assume that there are n owners and that their values per acre are independently and identically distributed. Under eminent domain, it is optimal for the government to take the parcel (meaning take all n owners' subparcels) under the same condition as in the single-owner case. Under the policy of purchase, the government's optimal price offer is positive but is not necessarily less than the social value of the land. The explanation for this somewhat counterintuitive observation that the government's optimal price offer might exceed the social value per acre is that such a high offer may be useful in inducing every single owner to sell, as that is necessary for the government to acquire its parcel and for the social value of the parcel to be realized.[5]

With regard to the comparison of eminent domain and purchase in the multiple-owner case, eminent domain must be superior to purchase if the number of owners is sufficiently large, assuming that eminent domain leads to positive social welfare. The kernel of the argument for this conclusion (the central one of the article) is, as was mentioned, that the problem of making a price offer that all owners will accept becomes great as n grows. In particular, I show that the probability of the government's making a successful purchase tends to 0 with

[4] In the example in note 3, suppose that there are two possible owners' values: \$50,000, with probability 50 percent, and \$150,000, with probability 50 percent. Under eminent domain, the government would pay fair compensation of \$100,000. Under purchase, it can be verified that the government would choose the price offer of \$150,000 in order to obtain the land for sure. Thus, the acquisition cost—and therefore the social cost of funds—would be higher under the policy of purchase. Hence, social welfare would be lower under the policy of purchase. (In this example, note, there is no market test advantage of the policy of purchase because the distribution of owners' values is bounded below the social value. In the model, however, there always is such an advantage because the distribution of owners' values is continuous and unbounded, but the possible advantage of eminent domain still exists.)

[5] In the example in note 3, the optimal offer per acre could exceed \$1 million; for instance, it could be \$2 million per acre. If there are many owners of the parcel that the government seeks, it could be likely that one among them would place a value on land of \$2 million per acre, and to induce him to sell, the offer per acre would have to be at least \$2 million. Such a high offer might not lead to very much inefficiency because most of the other owners who would accept the offer might be likely to place values on land that are below its social value of \$1 million per acre. The optimality of offers exceeding the social value of land is not anomalous, for I show under broad conditions that such an offer must be optimal as n grows large.

n (even though the price is optimally adjusted as n increases). Moreover, assuming that eminent domain improves social welfare in expected terms, the probability that eminent domain raises social welfare in fact tends to 1 with n. Thus, when the number of owners is large, the sense in which the policy of eminent domain can be superior to the policy of purchase is strong.[6]

In Section 5, I consider an extension of the model in which the government does not require a specific parcel of land but rather any parcel in a relevant region that is of the needed size. The main point demonstrated is that the conclusion that the policy of eminent domain is superior to purchase when the number of owners is sufficiently large continues to hold. One might think otherwise because when the government can obtain the land it needs anywhere within a region, it does not require unanimous approval of a price offer from a named group of owners. But if, as is assumed in this extension, the land that the government seeks is contiguous, the government does require that some set of neighboring owners all give their approval to a price offer. This turns out to be an important enough hurdle that the policy of purchase will highly likely fail if the number of owners is sufficiently large.

In Section 6, I consider an extension of the model in which the land that the government needs can be dispersed without affecting its social value. In this case, eminent domain does not become attractive as the number of owners becomes large because the government does not need the approval even of neighboring landowners—the agreement to sell of any set of landowners of the requisite area will do.

In Section 7, I comment on two final issues: the bearing of the analysis on the use of eminent domain on behalf of private developers, and the applicability of the analysis to property other than land (why does the government not need the right of eminent domain to obtain goods?).

Previous writing on eminent domain has not, to my knowledge, addressed the notion developed here that variability in individuals' valuations of land may lead to defeat of the policy of government purchase and may justify eminent domain when the number of landowners is large.[7] A problem facing a land assembler not addressed here is that of "strategic holdout," namely, that landowners may delay and may negotiate for a high price in order to extract some of the surplus that would be gained by the assembler if it acquired their land (on this issue, see Calabresi and Melamed 1972; Menezes and Pitchford 2004; Miceli and Segerson 2007; O'Flaherty 1994). The problem of land assembly examined here is different and might be termed one of "honest holdout," as it derives from the unwillingness of owners to sell for less than the true value of

[6] As I observe, however, this conclusion depends on the assumption that the government bears a positive social cost of funds. If the cost of funds were 0, then the policy of purchase would always be superior to eminent domain (the probability of successful purchase would not tend toward 0 when government acquisition is desirable, because the optimal price offers of government would grow unboundedly with n).

[7] However, Munch (1976), a largely empirical investigation of eminent domain, includes suggestive remarks (see pp. 476–79) about the problem of purchase when landowners' values vary.

their land to them.[8] Two additional articles of note on land acquisition and eminent domain are Blume, Rubinfeld, and Shapiro (1984) and Hermalin (1995). They primarily address the effect of the government's payment of compensation under eminent domain on the private investment incentives of a single landowner; they are not concerned with comparing eminent domain to a policy of purchase and thus not with the bargaining problems that arise under purchase when there are multiple landowners. Also of note is Mailath and Postlewaite (1990), who show that when unanimous approval of a public-good project is required and money transfers involve no social cost, the probability of efficient project adoption tends to 0 as the number of individuals grows large.[9]

2. Assumptions and Framework of Analysis

The government places a social value on a specific parcel of land that it needs to acquire, such as for an airport. For concreteness, I often call the parcel an acre. Let s be the social value of the acre of land needed by the government; $s > 0$.

If the government does not acquire the full acre of land, it will obtain no value. This assumption is made to reflect situations in which it is important for the government to possess at least a threshold quantity of land (airplanes require at least a minimum amount of runway for takeoff and landing).[10]

Land is owned at the outset by private parties, called owners. An owner's value per acre of land is drawn from a distribution of possible values. Let v be the value per acre of land to an owner, where $f(v)$ is the probability density of v and $f(v) > 0$ on $[0, \infty]$ and 0 elsewhere; $F(v)$ is the distribution function of $f(v)$.

The value v is known to an owner but not observed by the government; the government knows f.

Let n be the number of owners. In the case of $n > 1$, with multiple owners (each of whom owns a different subparcel, as described in Section 4), the value per acre of the ith owner is denoted v_i, where the v_i are independently drawn from the distribution with density f. Note that this assumption is consistent with

[8] Because the government makes a single offer to owners, they refuse to sell only if the offer is less than their valuation. The problem analyzed here arises despite the government's having all the bargaining power; the problem is due to variation in owner land value and government lack of information about owner value. In contrast, the problem of strategic holdout can arise only when the government does not possess all the bargaining power, occurs even if owners are identical, would exacerbate the problem with purchase that I find, and adds to the advantages of eminent domain that I examine when the number of owners is large.

[9] Here, in contrast, when money transfers involve no social cost (that is, funds spent by government on compensation or on land purchase involve no social cost), the probability of efficient acquisition remains positive as the number of individuals grows large (because, as I mentioned in note 6, the government can and will find it desirable to increase its bids for land without bound). Only if payments by government involve positive social cost is it true that the probability of efficient acquisition tends to zero as the number of individuals grows.

[10] If I were instead to assume that the social value of land is rising and convex in the quantity of land, the qualitative results would be similar to those I find.

there being a common known value that all owners attach to each acre of land and an additional unobserved personal component.[11]

In addition, if the government acquires an owner's land and uses it to generate the social value *s*, the owner derives no benefit from *s*. The interpretation of this simplifying assumption is that a single landowner would be unlikely to capture more than a tiny fraction of the benefit from any particular public project that required his or her land.

Under the policy of eminent domain, the government may take land, and if it does, it must pay to the owner(s) fair compensation, which is assumed to be the expected value of the land per acre, namely, $E(v)$, where E stands for expectation. The motivation for this assumption is that the aspiration of the government is to make the victim of a taking whole.[12]

Under the policy of purchase, the government makes a single price offer to owners. Let x be the price offer of the government per acre.

An owner will thus accept the offer if and only if $v_i < x$.[13] That there is a single offer applying to all n owners is due to the assumption that the government cannot observe v_i.[14] I further describe the policy of purchase below.

The government bears a positive social cost per dollar of funds, which may be interpreted as being due to the administrative expense of taxation or to its distortionary effects. Let c be the social cost per dollar of government funds; $c > 0$.

Social welfare W equals the value of land (to whomever enjoys it) minus the social cost of government funds expended. The government acts to maximize social welfare by choosing between the policy of eminent domain and the policy of purchase (using each optimally). I do not investigate the choice between two more general classes of policies that the government might employ, namely, mechanisms in which (as in eminent domain) landowners' participation is not necessarily voluntary versus mechanisms in which (as in government purchase) landowners' participation is voluntary.

[11] Suppose that $v_i = k + \varepsilon_i$, where k is a common component of land value known to the government and ε_i is an unobservable personal component that is drawn independently from a distribution; then the v_i will be independently and identically distributed.

[12] The opinion in the seminal U.S. Supreme Court case on the meaning of compensation for a taking, *United States v. Miller*, 317 U.S. 369 (1943), states, "The Fifth Amendment of the Constitution provides that private property shall not be taken for public use without just compensation. Such compensation means the full and perfect equivalent in money of the property taken. The owner is to be put in as good position pecuniarily as he would have occupied if his property had not been taken." As a practical matter, compensation is usually based on market price, which presumably is below the actual value v to an owner (an owner for whom v is less than the market value would have sold to obtain the market price). But compensation sometimes includes a percentage premium over market price as an implicit corrective; see, for example, Wyman (2007). In any event, the qualitative conclusions I reach do not depend on the specific formula for compensation under eminent domain: regardless of the formula, eminent domain will be inferior to purchase when the social cost of funds is sufficiently low, and eminent domain will be superior to purchase when successful purchase is stymied by the rejection of offers, which becomes likely as the number of owners grows.

[13] If $v_i = x$, the owner will be indifferent about accepting the offer, and I adopt the convention that he will not accept it. I make similar assumptions below without further comment.

[14] I remark on the outcome if the government could observe v_i at the end of Section 4.

3. A Single Party Owns the Land Sought by the Government

Assume here that the acre needed by the government has one owner. This case is worth studying because important aspects of the comparison of eminent domain and purchase are best understood in isolation from issues concerning multiple owners.

Note that it is first best for the government to obtain the land from the owner if and only if $s > v$.

Under eminent domain, if the government takes the land, social welfare will be

$$s - cE(v), \tag{1}$$

because the government will obtain the benefit s from the land and incur the social cost $cE(v)$ from its payment of compensation of $E(v)$ for the taking. If the government does not take the land, social welfare will be

$$E(v), \tag{2}$$

since the owner will make use of the land.

Under the policy of purchase, if the government's price offer is x, social welfare will be

$$W(x) = F(x)(s - cx) + \int_x^\infty vf(v)dv, \tag{3}$$

because x will be accepted when it exceeds v and rejected when it is below v. The government's problem is to maximize $W(x)$ over $x \geqslant 0$. Let x^* denote the optimal offer.[15]

The conclusions reached about the two policies are given below.

Proposition 1.

a. Under the policy of eminent domain, land is taken if and only if its social value s is sufficiently high, that is, if and only if $s > (1 + c)E(v)$.

b. Under the policy of purchase, the optimal offer x^* is positive, so that land is purchased with positive probability, and x^* is less than the social value of land s. In particular, x^* satisfies $x = [s - cF(x)/f(x)]/(1 + c)$.

c. The social welfare comparison of the policies of eminent domain and of purchase is as follows:

- i) If the social value of land s does not exceed $(1 + c)E(v)$, so under eminent domain there would not be a taking, then purchase is superior to eminent domain.
- ii) Otherwise, either policy could be superior, and in particular, purchase is superior to eminent domain for all c sufficiently low (given s) and eminent domain is superior to purchase for all s sufficiently high (given c).

[15] The optimal offer might not be unique, but for expositional ease I will describe it as if it were.

Let me now discuss these conclusions. (The proofs of conclusions that are not shown in the text are given in the Appendix.)

With regard to eminent domain and proposition 1.*a,* it follows from expressions (1) and (2) that under eminent domain, the government will take land if and only if

$$s > (1 + c)E(v), \tag{4}$$

which is to say, when its social value exceeds the expected private value multiplied by the factor $(1 + c)$, reflecting the social cost of funds.

With regard to purchase and proposition 1.*b,* it can be shown that the problem of maximizing expression (3) over the offer x has a solution x^*. The derivative of expression (3) with respect to x is

$$W'(x) = [s - (1 + c)x]f(x) - cF(x), \tag{5}$$

which is positive at $x = 0$. Hence, x^* must be positive. The explanation is that, regardless of s, there will be some chance that v is less than s, in which case government acquisition of land would be desirable if the social cost of funds c were not positive. But no matter how high c is, if the offer x is sufficiently low, the social cost of funds will be outweighed by the social value of the land. Since x^* is positive, we know that the optimal x^* is determined by the first-order condition

$$[s - (1 + c)x]f(x) - cF(x) = 0, \tag{6}$$

so that x^* must satisfy

$$x = \frac{s - cF(x)/f(x)}{1 + c} < s. \tag{7}$$

The reason that $x^* < s$ is as follows. If $x = s$, the allocation of land would be ideal, so there would be no first-order loss in social welfare from lowering x (owners just induced to refuse the offer would obtain values only slightly below s). But there would be a first-order social savings achieved from lowering x because c is positive and the government spends less when its bid is accepted. Note too that if c is zero, equation (7) implies that $x^* = s$, and the first-best outcome would result.

To compare social welfare under the two policies, assume first that s is low enough that inequality (4) does not hold, which means that under eminent domain there will be no taking. Then, under eminent domain, social welfare will be $E(v)$. Under the policy of purchase, however, social welfare will be higher. To show this, observe that $W(0) = E(v)$, for if an offer of zero is made, no owner will accept it. But I noted above that $W'(0) > 0$. Hence, social welfare must exceed $E(v)$ at x^*. The explanation is that, under the policy of purchase, there is a positive probability that the owner's value will be lower than x^*, and land will be acquired by the government; in other words, the market test advantage sometimes results in a beneficial acquisition under the policy of purchase.

Now assume that inequality (4) holds, so that under eminent domain there will be a taking and social welfare will be $s - cE(v) > E(v)$. Let me first show that for all low c, purchase is superior to eminent domain; the underlying reason is that for low c, the market test advantage, which allows individuals with high v to retain their land, is more important than the social cost of paying for land. In particular, if c is zero, social welfare under purchase exceeds social welfare under eminent domain: under purchase x^* is s, so purchase is made if and only if $s > v$, and the first-best outcome is achieved. Under eminent domain, land is taken. Hence, social welfare is higher under purchase by the amount

$$\int_{s}^{\infty} vf(v)dv > 0. \tag{8}$$

It follows that social welfare must be higher under purchase than under eminent domain for all c sufficiently low, since social welfare under each policy is continuous in c.

Let me now explain why eminent domain might be superior to purchase. As mentioned in Section 1, the answer concerns a potential cost advantage to the government. Specifically, when the government takes land under eminent domain, it pays $E(v)$, but when the government purchases land, it pays its optimal offer price x^*, which may well exceed $E(v)$, and significantly so, because such an offer might be needed to obtain land with a high likelihood given the dispersion of the distribution of owners' values v (see note 4). This cost advantage of eminent domain becomes more pronounced as s grows.

4. Multiple Parties Own the Land Sought by the Government

Now assume that there are n individual owners of subparcels making up the acre parcel of land needed by the government. The owners' values per acre are $v_1, \ldots, v_n$, which, recall, are independent draws from the distribution F, a random sample from F. For simplicity, I suppose that each owner's subparcel is $1/n$ of the acre.[16] (Equivalently, as can be readily verified, I could suppose that each owner's subparcel is 1 acre and that the government seeks a parcel of n acres that has the social value sn.)

Note that it is first best for the government to acquire the acre if and only if $s > v_1/n + \ldots + v_n/n = (v_1 + \ldots + v_n)/n$, that is, if and only if s exceeds the sample mean.

Under eminent domain, I assume that if the government exercises its right to appropriate land, it takes the entire acre and pays each of the n owners

[16] The results obtained in this section do not depend on that assumption, but the notation would become more burdensome were individual owners' parcel sizes to vary.

$(1/n)E(v)$, as this is the expected value of each subparcel.[17] Hence, if the government takes land, its total payment to the n owners is $E(v)$. Accordingly, if the government takes the acre, social welfare is $s - cE(v)$, which is expression (1). If the government does not take the land, social welfare is $E(v_1/n + \ldots + v_n/n) = E(v)$, which is the quantity (2). Thus, social welfare when there is a taking and when there is not is the same as in the single-owner case.

Under the policy of purchase, I assume that the government's offer x holds only if all owners accept it; that is, if any owner rejects x, the government pays no one and all owners retain their land.[18] The probability that the offer of x per acre is accepted by all owners is $F(x)^n$. In this event, each owner receives $(1/n)x$, so the total payment of the government is x. Hence, if the offer x is accepted by all owners, social welfare is $s - cx$. If x is not accepted by all owners, social welfare is $v_1/n + \ldots + v_n/n$. The integral of social welfare when x is not accepted by all owners can be expressed as

$$\int_0^\infty \ldots \int_0^\infty \left[\frac{v_1}{n} + \ldots + \frac{v_n}{n}\right] f(v_1) \ldots f(v_n)dv_1 \ldots dv_n \\ - \int_0^x \ldots \int_0^x \left[\frac{v_1}{n} + \ldots + \frac{v_n}{n}\right] f(v_1) \ldots f(v_n)dv_1 \ldots dv_n. \tag{9}$$

To explain, the first term is the sum of the n owners' values of their $1/n$ acre subparcels under all possible outcomes of the v_i, and the second term is the sum of the owners' values when all owners accept x, so the difference must be the sum of owners' values when not all of them accept x. The first term in expression (9) equals $E(v)$, and the second term reduces to n similar terms, the first of which is

$$\begin{aligned} &\frac{1}{n}\int_0^x \int_0^x v_1 f(v_1) \ldots f(v_n)dv_1 \ldots dv_n \\ &= \frac{1}{n}\int_0^x v_1 f(v_1)dv_1\left[\int_0^x f(v_2)dv_2\right] \ldots \left[\int_0^x f(v_n)dv_n\right] \\ &= \frac{1}{n}F(x)^{n-1}\int_0^x v_1 f(v_1)dv. \end{aligned} \tag{10}$$

There are n such terms, adding to

$$F(x)^{n-1}\int_0^x vf(v)dv. \tag{11}$$

[17] I do not consider the exercise of eminent domain in order to take less than the entire acre because the assumption is that the social value of less than the entire acre is zero.

[18] If the government were to pay owners who accepted its bid when not all owners accepted, the analysis would not be qualitatively different, because, as will be seen, the key point of this section is that the government needs all owners to accept to obtain the social value from the land.

Hence, social welfare under purchase given x equals

$$W_n(x) = F(x)^n(s - cx) + \left[E(v) - F(x)^{n-1}\int_0^x vf(v)dv\right]. \qquad (12)$$

The government's problem is to maximize $W_n(x)$ over $x \geqslant 0$.

The results in this n-owner case are as follows, where $x^*(n)$ denotes the optimal offer given n.

Proposition 2.

a. Under the policy of eminent domain, land is taken if and only if its social value s is sufficiently high, that is, if and only if $s > (1 + c)E(v)$.

b. Under the policy of purchase,

i) the optimal offer $x^*(n)$ is positive, so that land is purchased with positive probability, and $x^*(n)$ may exceed the social value of land s. In particular, $x^*(n)$ satisfies equation (13).

ii) As n grows, the optimal offer $x^*(n)$ tends to a limit x^{**} as determined by equation (15), where $x^{**} > x^*(1)$ and where $x^{**} > s$ for all c sufficiently low. In addition, as n grows, the probability of successful purchase $F(x^*(n))^n$ tends to 0.

c. The social welfare comparison of the policies of eminent domain and purchase is as follows:

i) If the social value of land s does not exceed $(1 + c)E(v)$, so that under eminent domain there would not be a taking, then purchase is superior to eminent domain.

ii) If s exceeds $(1 + c)E(v)$, then either policy could be superior; in particular, purchase is superior to eminent domain for all c sufficiently low (given s), and eminent domain is superior for all s sufficiently high (given c).

iii) If s exceeds $(1 + c)E(v)$, then for all n sufficiently high, eminent domain is superior to purchase.

Indeed, as n grows, the probability that eminent domain is socially desirable tends to 1, whereas the probability that purchase would be successful tends to 0.

Let me explain and comment on these conclusions.[19] Proposition 2.*a* is clear, for social welfare if there is a taking, and if not, it is as discussed in Section 3. With regard to proposition 2.*b,* it can be shown that the problem of maximizing $W_n(x)$ has a solution x^*.

That x^* must be positive is essentially for the reason given with respect to proposition 1. Why, however, might x^* *exceed* the social value s of land? That is, why might it be desirable for the government to offer more than the land is worth? In the case of a single owner, it is not desirable for the government's offer x to exceed s, for such an offer could only result in a welfare-lowering

[19] Again, all claims not shown here are demonstrated in the Appendix.

purchase from an owner with a v greater than s. In the case here of multiple owners, however, an x greater than s may be needed to induce a high-v_i owner to accept, along with others whose v_i are low, and for that reason may make government acquisition of the entire acre of land socially worthwhile.[20] This intuition underlies the result in proposition 2.*c*.iii that, for all n sufficiently large, the optimal offer $x^*(n)$ must exceed s if c is not too high.

The first-order condition determining x^*, from differentiating $W_n(x)$, is

$$F(x)^{n-1}f(x)[n(s-cx)-x]-cF(x)^n-(n-1)F(x)^{n-2}f(x)\int_0^x vf(v)dv = 0. \quad (13)$$

The first term reflects the marginal expected benefit from raising x when bids are accepted. The second term is the marginal social cost due to the government's having to pay inframarginal individuals a higher amount. The third term measures the expected loss from the fact that when the marginal person is attracted to make a purchase, he brings with him $n-1$ other owners who no longer derive values from their land. Equation (13) can be rewritten as

$$(s-cx) = \frac{cF(x)}{nf(x)}+\frac{x}{n}+\frac{n-1}{nF(x)}\int_0^x vf(v)dv. \quad (14)$$

As n grows large, equation (14) tends to

$$(s-cx) = \frac{1}{F(x)}\int_0^x vf(v)dv. \quad (15)$$

This can be shown to have a unique solution x^{**} to which $x^*(n)$ tends as n grows.

That $x^{**} > x^*(1)$ is explained by the fact that as n grows, it becomes more difficult to attract all the owners to accept, because some owner is likely to have a high-value v_i.[21] In addition, the closely related intuition that I gave for why

[20] Consider the following two-owner example: v is 1 with probability .9 and 11 with probability .1, s is 10, and c is .01. There are thus three pairs of possible v_1 and v_2: (1, 1), with probability .81; (1, 11), with probability .18; and (11, 11), with probability .01. It is first best for the government to acquire the acre when the two owners' values are (1, 1) and (1, 11), for then the average value per acre is below the social value of 10; and it is first best for the owners to retain their land when their values are (11, 11), for then their average value per acre is 11, exceeding the social value. Now compare the two possible offers of 1 and 11. If $x = 1$, both owners will accept and the government will purchase the land only when the pair is (1, 1); social welfare will be $.81(10 - 1 - .01) + .18(6) + .01(11) = 8.47$. If $x = 11$, both owners will always accept; social welfare will be $10 - .01(11) = 9.89$, which is higher. The reason that social welfare rises when the offer is 11 is that an offer of 11 is necessary for a successful purchase when the pair is (1, 11) in order to attract the owner whose value per acre is 11. Further, this purchase is socially desirable because the other owner's value is low. It is true that an offer of 11 also results in a socially undesirable purchase when the owners' values are (11, 11), but this is a relatively unlikely event (its probability is .01, as opposed to the probability of .18 of (1, 11)).

[21] This intuition also suggests that $x^*(n)$ is increasing in n, but I have not been able to establish that.

an offer exceeding s is possible would seem to be more important the higher n is; this helps to explain the result that $x^{**} > s$ for all positive c sufficiently low.

That the probability of a successful purchase $F(x^*(n))^n$ tends to 0 is due to the fact that, as n grows large, the odds of some person placing a high value on his property that exceeds the government's bid rise. Of course, the optimal bid $x^*(n)$ is chosen taking into account the presence of n owners, but the problem of obtaining acceptance from all n remains. The proof follows from the point that $x^*(n)$ tends to a finite limit x^{**}, which implies that $\lim_{n\to\infty} F(x^*(n))^n = \lim_{n\to\infty} F(x^{**})^n = 0$.

Now let us turn to the social welfare comparison of eminent domain and purchase. The claims in proposition 2.*c*.i and 2.*c*.ii are explained essentially as in the single-owner case. For given n, purchase must be superior to eminent domain if under eminent domain there would not be a taking—which is when $s < (1 + c)E(v)$—for under purchase there is always some positive probability of purchase and of raising social welfare above the status quo because of the market test advantage (however attenuated this is, because of there being n owners). And if under eminent domain s is high enough that there would be a taking, either purchase or eminent domain could be superior, the latter being possible mainly because under eminent domain there is a possible cost-saving advantage relative to purchase.

The claim in proposition 2.*c*.iii, that when there would be a taking under eminent domain, eminent domain is superior to purchase for all n sufficiently high, embodies the advantage of eminent domain in the multiple-owner context. This is, in a sense, the central conclusion of the present article. The reasoning is as follows. Under eminent domain, social welfare is $s - cE(v)$, which exceeds $E(v)$ by the hypothesis that there would be a taking under eminent domain. But under the policy of purchase, we know that the probability of successful purchase tends to 0, which means that social welfare tends to $E(v)$. Hence, if n is sufficiently high, social welfare under purchase must be lower than under eminent domain.

It was also asserted in the claim that not only is eminent domain superior in expected value to purchase for n sufficiently high but also the probability that the exercise of eminent domain is socially desirable tends to 1. This follows from several observations. The law of large numbers tells us that, for any $\varepsilon > 0$, the probability that the sample mean $(v_1 + \ldots + v_n)/n$ lies within ε of $E(v)$ tends to 1 with n. Let $\varepsilon = .5[s - cE(v) - E(v)]$, which is positive by hypothesis. Hence, the probability that $s - cE(v) > (v_1 + \ldots + v_n)/n$ tends to 1 with n. But this means that the probability that a taking is socially desirable tends to 1 with n.

The preceding point and the point that $F(x^*(n))^n$ tends to 0 with n mean that, as n grows large, the likelihood that purchase will result in acquisition of land goes to 0, whereas the likelihood that a taking would be desirable goes to 1. Therefore, the sense in which eminent domain is superior to purchase is, as stated in Section 1, very strong.

Last, let me remark on the importance of three assumptions to the results in proposition 2. One assumption is that $c > 0$. This assumption was needed for the limiting results of propositions 2.*b*.ii and 2.*c*.iii. That is because, if $c = 0$, the $x^*(n)$ do not tend to a positive limit; they rise unboundedly if $s > E(v)$.[22] In effect, there is no problem with offers failing when n grows because the government can raise its bid x without incurring any social cost. For that reason, when $c = 0$, purchase is superior to eminent domain.[23]

The second assumption is that the distribution of v is unbounded. If v were bounded by b, then $x^*(n)$ might equal b for n sufficiently high, so that the probability of a successful purchase might be one. However, were that so, eminent domain would still be superior to purchase, for eminent domain would involve a lower cost to the government ($E(v)$ rather than b).

The third assumption is that v is not observable to the government. If the v_i were observable, then purchase and eminent domain would be equivalent. Under purchase, the government would know v_i and offer only that to owner *i;* and under eminent domain, the government would be required to pay v_i to owner *i.*

5. The Land Parcel Sought by the Government Can Be Located Anywhere

In the above analysis, the land that the government sought was a specific parcel. Here, however, I assume that the land sought by the government can be located anywhere in a relevant region. Then the government does not need any particular group of landowners to unanimously accept its price offer—the consent of any group of landowners of the kind of parcel it seeks will do. Hence, one might believe that eminent domain is not needed when the number of landowners grows, but this intuition is incorrect.

To develop this point, I assume here that the government needs to obtain a square parcel, with a side of k, where $0 < k \leqslant 1$, and the square parcel can be located anywhere in the square acre of area 1. If the government makes a price offer and it obtains acceptances for more than the square parcel of side k that it needs, I assume that it buys only what it requires (choosing any square of

[22] If $c = 0$, then it can be shown that a solution $x^*(n)$ exists. To do so, it can be verified that the derivative of social welfare, the left side of equation (13), is negative for x exceeding sn. Hence, one may restrict attention to x in $[0, sn]$, a compact set, implying that $x^*(n)$ exists. To prove that the $x^*(n)$ rises unboundedly with n, it suffices to show that, for any $b > 0$, the derivative of social welfare must be positive for all x in $[0, b]$ if n is sufficiently large. Now when $c = 0$, the derivative of social welfare is, from equation (13), $F(x)^{n-2} f(x) [F(x)(ns - x) - (n-1)\int_0^x vf(v)dv]$. This integral is less than $F(x)E(v)$, so the integral exceeds $F(x)^{n-2} f(x)[F(x)(ns - x) - (n-1)F(x)\mathrm{E}(v)] = F(x)^{n-2} f(x)[F(x)(ns - x) - (n-1)F(x)E(v)] = F(x)^{n-1} f(x)[ns - x - (n-1)E(v)]$. But $ns - x - (n-1)E(v) = (n-1)[s - E(v)] + s - x$. Because $s > E(v)$, it is clear that if $x \leqslant b$, $(n-1)[s - E(v)] + s - x$ must be positive for all n sufficiently high, which implies that $x^*(n)$ must exceed b for all n sufficiently high.

[23] That purchase is superior to eminent domain follows from the fact that a price offer of ns results in higher social welfare than under eminent domain. To show this, note that if ns is the offer, then the government does not acquire its parcel if and only if there is a landowner for whom the value exceeds ns. But when that is so, social welfare would fall because of government acquisition, since the landowner's value alone exceeds $(1/n)(ns) = s$. Hence, social welfare under eminent domain must be lower than social welfare if the offer is ns.

side k from the accepted parcels). Otherwise, I retain the assumptions from above. I restrict attention here to proving the limiting result from proposition 2 (although most of the other results from above carry over as well).[24] The conclusion to be proved is this:

Proposition 3. Suppose that the government seeks a square parcel of side k, where $1 \geqslant k > 0$, where the square can be located anywhere in the unit acre and oriented in any direction. Then proposition 2.*c*.iii holds. Namely, if s exceeds $(1 + c)E(v)$ (which implies that a taking would raise social welfare), then for all numbers n of landowners sufficiently high, eminent domain is superior to purchase. Indeed, the probability that eminent domain is socially desirable tends to 1, whereas the probability that purchase would be successful tends to 0.

Note that this proposition is quite general, as it implies that if the government seeks a parcel that has any minimum contiguous part, the government will tend to fail under the policy of purchase and must use eminent domain if the number of landowners is sufficiently large. This is because if the parcel sought by the government is required to have any contiguous part of minimum size, it must contain a square of at least k for some positive values of k.

The strategy of the proof presented in the Appendix is to demonstrate that for the government to be able to purchase a square of side k is equivalent to its obtaining a type of run of successes in Bernoulli trials, where the probability of success in each trial is the probability that its offer of x is accepted by an owner. The reason that purchasing a square is equivalent to achieving a run of successes, rather than only a number of possibly separated successes, is in essence that the owners of the square are neighboring, which means that they will occur in a row in an ordering of the landowners. Furthermore, the length of the run of successes must be such as to cover a certain proportion $q > 0$ of all landowners, for the square of side k covers a proportion (k^2) of the region of area 1.

From the foregoing argument, we know that for the government to acquire the square it needs, it must achieve a run of successes in n trials of length at least qn. (Note that this is a generalization of the fact that, in Section 4, where the government had to acquire a specific parcel, it had to achieve a run of n successes in n trials.) But the probability of such a run tends to 0 as n becomes large. For instance, consider the probability of a run of heads in coin tosses of length at least 10 percent of all the tosses (q is .1). If the number of tosses is not large, say, 20, then the probability of the run would be high, for the run of heads need only be two (that is, $.1 \times 20$) in length. If, however, the number of tosses is 100,000, the run of heads would have to be 10,000 in length, an extremely unlikely event. I verify in the Appendix that the probability of a run of successes

[24] It is clear that proposition 1 and propositions 2.*a* and 2.*c*.i–ii would hold by the logic of arguments already made. Proposition 2.*b* would hold in modified form (equations [13] and [15] would be altered).

of length at least qn tends to 0 as n grows, as long as $q > 0$, where $1 > p > 0$ is the probability of success in a single trial.

Because the probability of a run of successes of the required nature tends to 0 with n, the probability that the policy of purchase results in acquisition of the square of land tends to 0 as n increases, whereas eminent domain by definition will succeed and is socially desirable, because of the hypothesis that $s > (1 + c)E(v)$. Moreover, the law of large numbers implies, as in proposition 2, that the probability that a taking is socially desirable approaches 1 as n grows.

6. The Land Parcel Sought by the Government Can Be Dispersed

Suppose here, unlike in previous sections, that the land that the government needs to acquire can be dispersed: the government seeks a parcel of area a, where $0 < a < 1$ and where the social value of the land is as no matter how the area a is composed. Otherwise, I maintain the assumptions from above.

Intuition suggests that if the number of landowners n grows, eminent domain will not become appealing because the government will not have to obtain agreements to sell from any specific group of landowners or from any neighboring group of them. Indeed, the law of large numbers suggests that as the number of landowners grows, the actual distribution of their values will approach the true distribution F, so if the government bids x, it will obtain approximately the fraction $F(x)$ of land from those owners placing the lowest values on their land. Hence, the policy of purchase should be superior to eminent domain if the area a sought is sufficiently small, allowing the government to make its purchase at a price less than $E(v)$.

To confirm these intuitions, let me now describe social welfare under the policy of purchase and under eminent domain and then compare the policies.

Under the policy of purchase, if the percentage of acceptances of a government price offer is at least a, the government can undertake its project because of the assumption that the government can make use of any subparcels composing a land area of a. Define $v(a)$ as the v such that $F(v) = a$. It is shown in the Appendix that a sequence of price offers $v(n)$ exceeding $v(a)$ can be constructed such that the $v(n)$ tend toward $v(a)$ with n; the probability of successful acquisition of area a tends to 1 with n, and social welfare under the price offer sequence tends with n to the level of social welfare that the government would obtain if it offered the price $v(a)$ and precisely the fraction a of landowners accepted, namely,

$$[sa - cav(a)] + \int_{v(a)}^{\infty} vf(v)dv = a[s - cv(a)] + \int_{v(a)}^{\infty} vf(v)dv. \qquad (16)$$

If the government does not purchase area a, social welfare is $E(v) = \int_0^\infty vf(v)dv$.

Hence, as long as

$$s - cv(a) > E[v|v < v(a)], \tag{17}$$

it is better in the limit for the government to purchase a than not to do so. Notice that inequality (17) states that the net social return per acre exceeds the expected return conditional on v being less than $v(a)$, for only owners with valuations below $v(a)$ accept the government price.

Under eminent domain, a taking results in social welfare (regardless of n) of

$$[sa - caE(v)] + (1 - a)E(v) = a[s - cE(v)] + (1 - a)E(v). \tag{18}$$

Hence, it is better for the government to take land than not provided that

$$s - cE(v) > E(v). \tag{19}$$

To compare purchase when n grows to eminent domain, consider the difference in social welfare under the policies assuming that the government acquires land under each. It will be optimal for the government to acquire land under each policy if inequalities (17) and (19) hold. Observe that inequality (19) implies inequality (17) if $v(a) < E(v)$, which is to say, if $a < F(E(v))$. Now, subtracting inequality (18) from equation (16), we obtain

$$ac[E(v) - v(a)] + (1 - a)\{E[v|v \geq v(a)] - E(v)\}. \tag{20}$$

The first term is an advantage of purchase as long as $a < F(E(v))$, when a can be purchased at a price per acre less than $E(v)$. The second term is always an advantage of purchase, because under the policy of purchase the remaining land area $(1 - a)$ is the highest valued land, whereas under eminent domain it is only average-value land (because the land that is taken is of average value). Consequently, we have the following:

Proposition 4. Suppose that the government seeks a parcel of area a, where $1 > a > 0$ and where the social value of the area sa does not depend on its location or how dispersed the area is. Then proposition 2.*c*.iii does not hold. In particular, if s exceeds $(1 + c)E(v)$ (which implies that a taking would raise social welfare), it is not true that for n sufficiently high, eminent domain is necessarily superior to purchase. Indeed, if $a < F(E(v))$, purchase is superior to eminent domain for all n sufficiently high.

Note that in this section, if the government makes offers of (slightly more than) $v(a)$, it will obtain area a with a probability approaching 1 as the number of landowners grows, by the law of large numbers. Thus, for example, if a were 20 percent of the land area, the government would be able to acquire that area with a price of about $v(.2)$ with a probability tending to 1 as n grows. In contrast, in Section 5, even if the goal of the government were to acquire a much smaller square parcel, say a parcel with an area of only .1 percent of the land area, the government would fail to acquire that relatively tiny parcel with a probability approaching 1 as n grows. The resolution of these apparently conflicting outcomes is that, as n grows, the success of the government in acquiring 20 percent of the land area must come about through the purchase of many separated parcels.

7. Concluding Comments

7.1. Use of Eminent Domain by Private Developers

The main point analyzed in this article would seem to apply to private developers of land as well. That is, it might be socially desirable to allow developers to borrow the power of eminent domain when they have to assemble large tracts of land held by many owners because they would confront a problem in acquiring the required subparcels through purchase.[25] In reality, the law sometimes does allow private parties to enjoy the right of eminent domain in roughly these circumstances.[26]

7.2. Applicability of the Analysis to Property Other Than Land

The analysis in this article has been interpreted as applying to land, but in what respects does it apply more generally? The single-owner analysis of Section 3 applies to any type of property. Hence, the point made there that purchase is an advantageous policy of government unless the social value of the property is very high in relation to the distribution of private values pertains to property in general and has a rough consistency with the fact that the law gives the government the right to seize property other than land in certain emergency situations.[27]

The analysis of the multiple-owner case, however, is special to land, or at least best fits land. In particular, the main assumption of Section 4—that the government needs all of the subparcels making up a specific parcel of land (or the assumption of Section 5 that the government needs contiguous parcels) in order to obtain the social benefit from the parcel—does not seem natural to make for most other kinds of property. Suppose that the government wants a material, like steel. It would be unlikely that the government's benefit would be enjoyed mainly only if it obtained at least a threshold quantity of steel and also (and

[25] Were one to analyze this situation, one would have to take into account two differences from the situation that I considered. First, the private cost of spending a dollar for the developer equals a dollar, instead of the presumably smaller social cost of raising a dollar of funds through taxation. This difference between the private and social cost of funds would seem to lead the developer to offer less than the government for land and thus would apparently exacerbate the problem of failure to obtain land through purchase. Second, the social cost of funds for the developer is zero, since the funds are not raised through taxation, which also suggests that the circumstances under which acquisition of land by the developer would be desirable would be broader than those under acquisition by the government.

[26] Notably, the law has granted railroads, utility companies, and, sometimes, private developers of substantial parcels who have a public purpose the right to take land via eminent domain. See, for example, the description given by Kelly (2006).

[27] This has occurred during wars and other armed conflicts, natural disasters, fires, and various crises. See, for example, Scheiber (2003).

more importantly) that there would be many owners of that quantity of steel, all of whom would have to sell in order for the government to succeed. Hence, the central point of this article, concerning the need for eminent domain in the multiple-owner context, seems to be relevant mainly to land.

Appendix

Additional Proofs

A1. Proposition 1

Proof of Proposition 1.*b*

The only claim that needs to be shown is that there exists a solution to the government's problem of choosing a price offer $x \geqslant 0$ to maximize social welfare $W(x)$. To prove the existence of a solution, it suffices to demonstrate that there exists a t such that if $x > t$, then $W(x) < W(0)$. For then we know that to maximize $W(x)$, we can restrict attention to x in $[0, t]$, and since $W(x)$ is continuous in x, it must have a maximum in a closed interval such as $[0, t]$. Now let $t = s/c$. Then if $x > t$, social welfare must be negative whenever x is accepted, because the social cost of the purchase will exceed $c(s/c) = s$. Hence, $W(x) < E(v) = W(0)$.

Proof of Proposition 1.*c*.ii

The claim that needs to be shown is that for all s sufficiently high, eminent domain is superior to purchase. This is proved in five steps. Denote the optimal x^* given s by $x^*(s)$.

Step 1. The value of $x^*(s)$ grows unboundedly with s; that is, for any $y > 0$, $x^*(s) > y$ for all s sufficiently high: Equation (7), determining $x^*(s)$, may be rewritten as

$$x = \frac{s}{1+c} - \left(\frac{c}{1+c}\right)\left(\frac{F(x)}{f(x)}\right). \tag{A1}$$

Let $m(y)$ be the maximum of the term $[c/(1+c)][F(x)/f(x)]$ over x in $[0, y]$. I claim that if $s > [y + m(y)](1+c)$, then $x^*(s) > y$. If this is not true, then $x^*(s) \leq y$ and equation (A1) must then be satisfied at some x in $[0, y]$. But if x is in $[0, y]$, the right side of equation (A1) must exceed

$$\frac{[y + m(y)](1+c)}{1+c} - m(y) = y + m(y) - m(y) = y, \tag{A2}$$

which implies that equation (A1) cannot be satisfied at x in $[0, y]$, which is a contradiction.

Step 2. The probability of purchase tends to 1 as s grows; that is, $\lim_{s\to\infty} F(x^*(s)) = 1$: This follows immediately from step 1.

Step 3. For any $y > 0$, first-best social welfare W^* exceeds social welfare under purchase $W(x^*(s))$ by at least cy for s sufficiently high: Social welfare under purchase is obviously lower than W^* by at least the expected social cost of purchase, $F(x^*(s))x^*(s)c$. By step 1, there exists a threshold t_1 such that if $s > t_1$, then $x^*(s) > 2y$ (let $2y$ play the role of y in step 1). By step 2, there exists a threshold t_2 such that if $s > t_2$, then $F(x^*(s)) > .5$. Let $t = \max(t_1, t_2)$. Then if $s > t$, we know that $F(x^*(s))\,x^*(s)c > .5(2y)c = cy$, so W^* exceeds $W(x^*(s))$ by at least cy.

Step 4. First-best social welfare W^* minus social welfare under eminent domain tends to $cE(v)$ as s grows: For $s > (1 + c)E(v)$, we know that a taking is desirable under eminent domain, and let us assume that s is this high. Then W^* minus welfare under eminent domain is

$$\int_s^\infty (v - s)f(v)dv + cE(v), \tag{A3}$$

which tends to $cE(v)$ as s grows.

Step 5. If s is sufficiently high, eminent domain is superior to purchase: This follows from steps 3 and 4. In particular, choose $y = 2E(v)$. Then by step 3, W^* minus welfare under purchase is at least $2cE(v)$ for all s sufficiently large. By step 4, for any positive ε, W^* minus welfare under eminent domain is within ε of $cE(v)$ for all s sufficiently large. Hence, choosing ε to be small, it follows that for all s high enough, welfare under eminent domain must exceed welfare under purchase.

A2. Proposition 2

Proof of Proposition 2.*b*.i

I first need to show that a solution to the problem of choosing the optimal offer exists. This follows from the argument of proposition 1 that in the problem with one owner an optimal x exists, since again it is clear that the optimal x cannot exceed s/c.

I next must show that $x^*(n) > 0$. This follows because social welfare is higher for any positive $x < s/(1 + c)$ than for $x = 0$. If $x < s/(1 + c)$, then $x < s - cx$. This implies that if n individuals accept x, social welfare must rise ($s - cx$ is social welfare, whereas if individuals keep their land, social welfare is bounded by x). Since the probability of acceptance of an offer is positive for any positive x, social welfare must be higher for any positive $x < s/(1 + c)$ than for $x = 0$.

Proof of Proposition 2.*b*.ii

I first want to show that equation (15) has a unique solution x^{**}. This is clear, since at $x = 0$, $(s - cx)$ is s and the right side of equation (15) is zero and since $(s - cx)$ is decreasing in x and negative for x sufficiently high, whereas the right side is positive and increasing in x.

I next want to show that $\lim_{n\to\infty} x^*(n) = x^{**}$.[28] Let us rewrite equations (14) and (15) as

$$0 = \frac{cF(x)}{nf(x)} + \frac{x}{n} + \frac{n-1}{nF(x)}\int_0^x vf(v)dv - (s - cx) \tag{A4}$$

and

$$0 = \frac{1}{F(x)}\int_0^x vf(v)dc - (s - cx). \tag{A5}$$

Observe first that the right side of equation (A4) must converge uniformly to the right side of equation (A5) over any closed interval $[0, t]$. That is, for any $\varepsilon > 0$, if n is sufficiently high, the right side of equation (A4) will be within ε of the right side of equation (A5) for all x in $[0, t]$.[29] Let t be s/c, so that the right side of equation (A4) must converge uniformly to the right side of equation (A5) in $[0, s/c]$, and recall from the proof of proposition 2.*b*.i that $x^*(n)$ must be in $[0, s/c]$. Now to establish the claim, it must be shown that for any small $\delta > 0$, $x^*(n)$ will be within δ of x^{**} for all n sufficiently large. We know that over the interval $[0, x^{**} - \delta]$, the minimum distance from zero of the right side of equation (A5) is positive, say, b_1, since equation (A5) has a unique solution, and similarly over the interval $[x^{**} + \delta, s/c]$, the minimum distance of the right side from zero is positive, say b_2. Let b be $\min(b_1, b_2)$, so that the distance of equation (A5) from zero over the two intervals is at least b. Since the right side of equation (A4) converges uniformly to the right side of equation (A5), if we choose $\varepsilon = b/2$, we know that for n sufficiently large, the right side of equation (A4) must be within $b/2$ of equation (A5) in $[0, s/c]$. But this implies that the right side of equation (A4) cannot equal zero in $[0, x^{**} - \delta]$ or in $[x^{**} + \delta, s/c]$. Hence, the right side of equation (A4) can equal zero only within δ of x^{**}, which is to say, $x^*(n)$ must be within δ of x^{**}.

Now I will demonstrate that $x^{**} > x^*(1)$. To do this, it suffices to show that the left side of equation (15), $(s - cx)$, exceeds the right side when evaluated at $x^*(1)$, since, as noted, the left side is decreasing in x and the right side is increasing in x. Now from equation (6), which determines $x^*(1)$, we have

$$s - cx = \frac{cF(x)}{f(x)} + x. \tag{A6}$$

[28] If $x^*(n)$ is not unique, choose any optimal $x^*(n)$ as the element in the sequence of $x^*(n)$.

[29] The difference between the right sides of equations (A4) and (A5) is $cF(x)/[nf(x)] + x/n - \int_0^x vf(v)dv/[nF(x)] = cF(x)/[nf(x)] + x/n - E[v | v \le x]/n$, which is of the form $g(x)/n$, where $g(x)$ is continuous. Since $|g(x)|$ has a maximum M over $[0, t]$, the distance between the right sides of equations (A4) and (A5) on $[0, t]$ is bounded by M/n. Hence, if $n > M/\varepsilon$, the distance is bounded by $M/(M/\varepsilon) = \varepsilon$.

But

$$\frac{cF(x)}{f(x)} + x > x = \frac{xF(x)}{F(x)} > \int_0^x \frac{vf(v)dv}{F(x)}, \tag{A7}$$

which is the right side of equation (15).

I next show that $x^{**} > s$ for c sufficiently low. To prove this, it suffices to demonstrate that the left side of equation (15) exceeds the right side when evaluated at $x = s$ for c sufficiently low. We know that the right side of equation (15) at s is less than s, and thus the left side $s - cs$ must exceed the right side for all c sufficiently low.

Finally, that the probability of successful purchase $F(x^*(n))^n$ tends to 0 is clear since $x^*(n)$ tends to a limit x^{**} and $F(x^{**}) < 1$.

Proof of Proposition 2.*c*.i

That purchase is superior to eminent domain if $s \leq (1 + c)E(v)$ follows from the result in proposition 2.*b*.i that $x^*(n) > 0$, for then $W_n(x^*(n)) > W_n(0) = E(v)$, which is social welfare under eminent domain given the hypothesis about s.

Proof of Proposition 2.*c*.ii

To show that if $s > (1 + c)E(v)$, purchase is superior to eminent domain for c sufficiently low, it suffices to demonstrate that purchase is superior to eminent domain when $c = 0$, since $W_n(x^*(n))$ is continuous in c. When $c = 0$, we have (see equation [13])

$$W_n'(x) = F(x)^{n-2}f(x)\left[F(x)(ns - x) - (n-1)\int_0^x vf(v)dv\right], \tag{A8}$$

from which it is clear that $W_n(x)$ is decreasing for $x \geqslant ns$. Also, when $c = 0$,

$$\lim_{x\to\infty} W_n(x) = \lim_{x\to\infty}\left[F(x)^n s + E(v) - F(x)^{n-1}\int_0^x vf(v)dv\right] = s. \tag{A9}$$

Hence, $W_n(ns) > s$, and thus $W_n(x^*(n)) > s$. But s is social welfare under eminent domain. In other words, social welfare is higher under purchase than under eminent domain when $c = 0$.

To prove that eminent domain is superior to purchase for all s sufficiently high, I proceed in five steps; the proof is analogous to that in proposition 1.

Step 1. The value of $x^*(s)$ grows unboundedly with s; that is, for any $y > 0$, $x^*(s) > y$ for s sufficiently high: Choose some positive s, say s_o. Now to prove that $x^*(s) > y$ for s sufficiently high, it suffices to prove only that $x^*(s)$ cannot lie in $[x^*(s_o), y]$ for s sufficiently high. This will suffice because we already know that $x^*(s) > x^*(s_o)$ for $s > s_o$, since $x^*(s)$ is increasing in s.[30] To prove that $x^*(s)$ cannot be in $[x^*(s_o), y]$ for s sufficiently high, we show that equation (13) cannot be satisfied in $[x^*(s_o), y]$ if s is sufficiently high. To do this, we demonstrate that

$$F(x)^{n-1}f(x)[n(s - cx) - x] - cF(x)^n - (n - 1)F(x)^{n-2}f(x)\int_0^x vf(v)dv > 0 \quad \text{(A10)}$$

for x in $[x^*(s_o), y]$ if s is sufficiently high. This is true since the first term of equation (A10) grows unboundedly with s. In particular, let m be the minimum of $F(x)^{n-1}f(x)$ over $[x^*(s_o), y]$. Thus, for x in $[x^*(s_o), y]$, the first term is at least $m[n(s - cy) - y]$, which grows unboundedly with s. The other terms do not depend on s so must be exceeded by the first term for s sufficiently high.

Step 2. The probability of purchase tends to 1 as s grows; that is, $\lim_{s\to\infty}F(x^*(s))^n = 1$: This follows from step 1.

Step 3. For any positive y, first-best social welfare W^* exceeds social welfare under purchase by at least cy for all s sufficiently high: Social welfare under purchase is lower than W^* by at least the expected social cost of purchase, $F(x^*(s))^n cx^*(s)$. By step 1, there exists a threshold t_1 such that if $s > t_1$, then $x^*(s) > 2y$. By step 2, there exists a threshold t_2 such that if $s > t_2$, then $F(x^*(s))^n > .5$. Let $t = \max(t_1, t_2)$. Then if $s > t$, we know that $F(x^*(s))^n cx^*(s) > .5c(2y) = cy$, so W^* exceeds social welfare under purchase by at least cy.

Step 4. First-best social welfare minus social welfare under eminent domain tends to $cE(v)$ as s grows: For all s above $(1 + c)E(v)$, we know that a taking is desirable under eminent domain, and let us assume that s is this high. Then W^* minus welfare under eminent domain is

$$\int_0^\infty \cdots \int_0^\infty \int_{s-(v_2+\cdots+v_n)}^\infty \left[\left(\frac{v_1}{n} + \cdots + \frac{v_n}{n}\right) - s\right] \times f(v_1)f(v_2)\cdots f(v_n)dv_1dv_2\cdots dv_n + cE(v). \quad \text{(A11)}$$

The first term is the integral of the social loss when there is a socially undesirable taking, that is, when $v_1/n + \cdots + v_n/n \geqslant s$. This term tends to zero as s grows. Hence, W^* minus welfare under eminent domain tends to the cost of compensation $cE(v)$ as s grows.

Step 5. If s is sufficiently high, eminent domain is superior to purchase: This follows from steps 3 and 4. In particular, choose $y = 2E(v)$. Then by step 3, W^* minus social welfare under purchase is at least $2cE(v)$ for all s sufficiently large. By step 4, for any positive ε, W^* minus welfare under eminent domain is within ε of $cE(v)$ for all s sufficiently large. Hence, choosing ε to be small, it follows that for all s high enough, welfare under eminent domain must exceed welfare under purchase.

[30] We know that $x^{*\prime}(s) > 0$ because the partial derivative of the left side of equation (13) with respect to s is positive.

Proof of Proposition 2.*c*.iii

I need to prove that if s exceeds $(1 + c)E(v)$, then for all n sufficiently high, eminent domain is superior to purchase. Social welfare under purchase is

$$W_n(x^*(n)) = F(x^*(n))^n[s - cx^*(n)] + E(v) - F(x^*(n))^{n-1}\int_0^{x^*(n)} vf(v)dv. \quad \text{(A12)}$$

Since $x^*(n)$ tends to x^{**} as n grows by proposition 2.*b*, $W_n(x^*(n))$ tends to $E(v)$. However, social welfare under eminent domain is $s - cE(v) > E(v)$. Hence, if n is sufficiently high, social welfare under eminent domain exceeds social welfare under purchase.

A3. Proof of Proposition 3

The sequence of increasing numbers of owners of land will for convenience be taken to be a sequence of successively finer partitions of the unit acre constructed as follows. The first partition in the sequence is obtained by dividing the unit square in half by a line going down its middle. Thus, the first partition has two partition elements (two subparcels held by different owners), each a rectangle that is $\frac{1}{2}$ unit in width and 1 unit in length. The second partition is obtained by taking each of the two rectangles in the first partition and dividing it into two squares by drawing a line horizontally across it. Thus, the second partition has four elements, each a square with side $\frac{1}{2}$ unit. Continuing in this way, I obtain partitions that are successively finer, where the following is true. The nth partition, has 2^n partition elements, each with area $\frac{1}{2}^n$. In every odd-numbered partition, the elements are rectangles with sides of length $\frac{1}{2}^{(n+1)/2}$ and $\frac{1}{2}^{(n-1)/2}$, and in every even-numbered partition, the elements are squares with sides of length $\frac{1}{2}^{n/2}$.

The proof consists of five steps.

Step 1. Let n be the number of repeated independent trials, in each of which there are two possible outcomes, success or failure, and in which the probability of success in a trial is p, where $1 > p > 0$. Let q be a proportion, where $1 > q > 0$. Let $P(n)$ be the probability of a run of consecutive successes of at least qn in length. Then $\lim_{n\to\infty} P(n) = 0$: Denote by $i(qn)$ the lowest integer greater than or equal to qn, so that the run of successes must be at least $i(qn)$ in length. The probability that there is a run of $i(qn)$ successes beginning in the first trial and allowing for any outcomes (including a longer run) otherwise is $p^{i(qn)}$. Similarly, the probability that there is a run of $i(qn)$ successes stretching from the second trial up to trial $i(qn) + 1$, and allowing for any outcomes otherwise (including a longer run beginning earlier or ending later), is also $p^{i(qn)}$. Because there are $n - i(qn) + 1$ such originating points for runs of at least length $i(qn)$, the probability $P(n)$ of a run of at least length $i(qn)$ must be less than $[n - i(qn) + 1]p^{i(qn)}$, for the $n - i(qn) + 1$ events under discussion include not only all the ways in which there could be a run of length at least $i(qn)$ but also the events are not mutually exclusive. (For example, a run of length $2i(qn)$ beginning at trial 1 would be included in the first event, the second, . . . , as well as the $i(qn)^{\text{th}}$.) Now let $T(n) = (n - i(qn) + 1)p^{i(qn)}$. Then

$$\frac{T(n+1)}{T(n)} = \left(\frac{n + 1 - i(q(n+1)) + 1}{n - i(qn) + 1}\right)\left(\frac{p^{i(q(n+1))}}{p^{i(qn)}}\right). \tag{A13}$$

It is clear that the first factor on the right side tends to 1 as n grows. Observe that $i(q(n+1))$ equals either $i(qn)$ or $i(qn) + 1$. Hence, the second factor on the right is either 1 or p, where p must occur with regularity, since $i(q(n+1))$ will be 1 higher than $i(qn)$ with a frequency of about $1/q$. This implies that, for high n, the sequence of ratios $T(n+1)/T(n)$ is approximately a sequence of ones and p's, where p occurs with a constant frequency. It follows that $T(n)$ tends to 0 with n and thus that $P(n)$, being less than $T(n)$, tends to 0 with n.

Step 2. For the government to purchase a square parcel of side k when it makes an offer of x to owners in the nth partition, it must achieve a run of successes of at least $q2^n$ in length in 2^n independent trials, where the probability of success in a trial is $F(x)$ and where $q > 0$ and is independent of n: Suppose that the acre is partitioned into squares with sides of no more than $k/4$. Then it is clear from geometry that if the government acquires a square of side k, it must entirely contain some square of side at most $k/4$. Let n_o be the first even-numbered partition with squares with sides less than or equal to $k/4$. Thus, if the government acquires its parcel, some square in the n_o partition must be entirely contained in that parcel.

In each partition n beyond the n_oth, for each square in the n_o partition, number consecutively all owners who have land in that square. For instance, in partition $n_o + 4$, each of the squares in the n_o partition will be divided into 16 smaller squares. The owners of these 16 squares making up a square of the n_o partition are to be numbered consecutively. (The numbering of owners whose land falls in different squares of the n_o partition does not matter.)

We know, therefore, that, for any partition n beyond the n_oth, if the government acquires its square parcel of side k by making a price offer of x, the government must have succeeded in acquiring a run of subparcels covering an entire square of the n_o partition, where the probability of success for each parcel is $F(x)$. Since such a square in the n_o partition has area $(\frac{1}{2})^{n_o}$, the run must cover that area, so the length of the run must be $(\frac{1}{2})^{n_o}2^n$, which is to say that $q = (\frac{1}{2})^{n_o}$.

Step 3. The optimal price offer for any partition n, denoted $x^*(n)$, of the government must be below s/c: If the offer is s/c or above, then social welfare must be zero or below if land is acquired, as the social cost of making payment is at least $c(s/c) = s$. Thus, such an offer cannot be optimal.

Step 4. The probability that the government succeeds in obtaining its parcel when it makes optimal price offers $x^*(n)$ tends to 0 as the number n of partitions and of owners grows large: Let $Q(n)$ be the probability that the government succeeds in obtaining its parcel with respect to partition n of the acre when it makes the optimal offer $x^*(n)$. Let $P(n)$ be the probability that the government succeeds in obtaining its parcel with respect to partition n of the acre if it makes the offer s/c. By step 3, $x^*(n) < s/c$, so that $Q(n) < P(n)$. By steps 1 and 2, $\lim_{n\to\infty} P(n) = 0$. Since $Q(n) < P(n)$, $\lim_{n\to\infty} Q(n) = 0$.

Step 5. The claimed results now follow for the reasons that proposition 2.*c*.iii does. Namely, since the probability $Q(n)$ that the government obtains its parcel tends to 0, social welfare under purchase tends to $E(v)$. But since social welfare under eminent domain is $s - cE(v) > E(v)$, if the partition n is sufficiently high, social welfare under eminent domain exceeds social welfare under purchase. Also, the reason that the probability that government acquisition of a parcel of the type it seeks is socially desirable goes to 1 with n is essentially that, as discussed after proposition 2, the sample mean must tend in probability to $E(v)$ by the law of large numbers.

A4. Proof of Proposition 4

This is self-evident from the text except for the construction of the sequence of price offers $v(n)$, where n is the number of landowners. To construct the price offers, first define for each positive integer i

$$[sa - ca(v(a) + 1/i)] + \int_{v(a)+1/i}^{\infty} vf(v)dv. \qquad \text{(A14)}$$

This expression equals social welfare if area a is purchased at a price of $v(a) + 1/i$. Let us show that if the price $v(a) + 1/i$ is offered, there is an $n(i)$ such that for all $n \geqslant n(i)$ social welfare must be within $.3/i$ of expression (A14). By the law of large numbers, the probability that the fraction of acceptances of the price offer will be within any named distance ε of $F(v(a) + 1/i)$ tends to 1 with n. Choose ε to be .5 of $F(v(a) + 1/i) - a$, so that if the fraction of acceptances is within ε of $F(v(a) + 1/i)$, the fraction will exceed a and the government will succeed in its acquisition. Hence, for any $p < 1$, if n is sufficiently high, the probability will be at least p that the government will succeed, which means that expression (A14) will give social welfare. It follows that, by choosing p sufficiently close to 1, social welfare must be within $.3/i$ of (A14). Let $n(i)$ be the n such that if n is at least this high, p is such that social welfare must be within $.3/i$ of expression (A14).

Now define the sequence of prices $v(n)$. Let the prices be fixed at $v(a) + 10$ for all n until $n(1)$ is reached. At $n(1)$ let the price change to $v(a) + 1$ and continue with this price until $n(1) + n(2)$ is reached. At $n(1) + n(2)$, let the price change to $v(a) + \frac{1}{2}$ and continue with this price until $n(1) + n(2) + n(3)$ is reached. At this point, let the price change to $v(a) + \frac{1}{3}$ and so on. Notice therefore that when the price is $v(a) + 1/i$, it must be that n exceeds $n(i)$, so that social welfare must be within $.3/i$ of expression (A14).

By construction, we can say the following about the sequence of prices $v(n)$.

First, the prices are nonincreasing, exceed $v(a)$, and tend to $v(a)$ in the limit (because $1/i$ tends to 0).

Second, the limit of social welfare under the sequence is the quantity in equation (16). To demonstrate this, consider any $d > 0$. I wish to show that for all n sufficiently high, social welfare must be within d of the quantity in equation (16). It is clear from expression (A14) that if i is sufficiently high, then expression (A14) will be within $d/2$ of the quantity in equation (16) (because expression [A14] tends to the quantity in equation [16] as i increases). We also know that if i is sufficiently high, social welfare, if the price in the sequence is $v(a) + 1/i$, will be within $d/2$ of expression (A14) (because social welfare is within $.3/i$ of expression [A14]). Hence, if n is sufficiently high, social welfare must be within $d/2 + d/2$ or d of the quantity in equation (16).

Third, the probability of government success in acquiring a using the sequence of prices tends to 1. This must be true, for if it were not, then the limit of social welfare could not be the quantity in equation (16).

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