tionally cumbersome owing to their (inherent) iterative nature. But we should also not forget that an attempt to ignore them is not their solution.

As regards Method II, Pesaran's claim, however justified, seems to be uncalled for. We, however, take care of his objection by re-estimating K according to the Srivastava and Singh technique. It is found, although somewhat surprisingly, that the S-D NILES estimates remain nearly invariant for a rather wide range of the ratio of the variances of transitory consumption and income.

Lastly, Pesaran's point that one of the consistent estimators of K is $\overline{C}/\overline{Y}$ is well taken. While we realize that S-D ought to have mentioned it more explicitly, it may be noted that their apc in both table 1 and table 2 is the same as the ratio of these two means.

REFERENCES

Ando, A., and F. Modigliani, "The Life Cycle Hypothesis of Savings: Aggregate Implications and Tests," *American Economic Review.* 53 (1963), 55–84.

Cagan, P., "The Monetary Dynamics of Hyperinflation," in M. Friedman (ed.), *Studies in the Quantity Theory of Money* (Chicago: University of Chicago Press, 1950), 25–117.

Friedman, M. *A Study of Consumption Function* (National Bureau Economic Research, Princeton, 1957).

Hamalainen, S. L., "Savings in Finland 1948–1966," Bank of Finland Monthly Bulletin, 41 (1957), 18–23.

Holmes, J. M., "A Condition for Independence of Permanent and Transitory Components of a Series," *Journal of the American Statistical Association* 66 (1971), 13–15.

Martinelle, S., "Seminar Communication; University Institute of Uppasala (1965).

Pesaran, M. H., "An Alternative Econometric Approach to the Permanent Income Hypothesis. A Comment: An International Comparison," this REVIEW, LV (1973), 259–261.

Rao, B. B., "A Note on An Alternative Economic Approach to the Permanent Income Hypothesis: An International Comparison," this REVIEW, LV (1973), 261.

Rao, C. R., *Linear Statistical Inference and Its Applications* (New York: John Wiley and Sons, Inc. 1965).

Rao, P., and Z. Griliches, "Small-Sample Properties of Several Two-stage Regression Methods in the Context of Auto-correlated Errors," *Journal of the American Statistical Association,* 64 (1969), 253–272.

Johnston, J., *Econometric Methods* (New York: McGraw-Hill, 1963).

Koyck, L. M., *Distributed Lags and Investment Analysis* (Amsterdam: North-Holland Publishing Company, 1954).

Singh, B., and H. Drost, "An Alternative Econometric Approach to the Permanent Income Hypothesis — An International Comparison," this REVIEW, LIII, (Nov. 1971), 326–333.

Singh, B., "A Test of the Permanent Income Hypothesis — A Comment," University of Toronto, 1970 (mimeographed).

Srivastava, M. S., and B. Singh, "A Method for Estimating Linear Regression with Both Variables Subject to Errors of Measurement," University of Toronto, 1972 (mimeographed).

Tuckey, J. W., "Components in Regressions," *Biometrics,* 7 (1951), 33–82.

Walters, A. A., *An Introduction to Econometrics* (London: Macmillan Company, 1968).

Willassen, Y., "An Alternative Econometric Approach to the Permanent Income Hypothesis, An International Comparison: Some Comments," this REVIEW, this issue.

Wold, H. O. A., "A Fix-point Theorem with Econometric Background," I–II *Arkiv for Matematik,* 6 (1965).

———, "Non-linear Iterative Least Squares Procedures," in Feshchrift for F. Neyman Research Papers in *Statistics,* R. F. N. David, (ed.) (London: John Wiley and Sons 1966), 411–444.

THE AIR POLLUTION AND PROPERTY VALUE DEBATE

A. Mitchell Polinsky and Steven Shavell*

There has been a debate of sorts over what constitutes a "correct" interpretation of econometric studies of the relationship between air pollution and property values.[1] After summarizing the debate in section I, we describe in section II a model of residential location and land rent determination which takes into account the influence of air pollution.

Received for publication July 31, 1972. Revision accepted for publication May 29, 1974.

* This paper was supported by the Urban Institute under National Science Foundation Grant GS-30184. We received helpful comments from John F. Kain and two anonymous referees.

[1] For example see Anderson and Crocker (1971; 1972), Edel (1971, pp. 8–12), Freeman (1971; 1974; forthcoming), Ridker and Henning (1967), Small (1975), and the references cited therein.

The model serves as a point of departure for a critical discussion of the debate in section III. Our discussion is concerned with the positive issue of the predictability of the new equilibrium schedule of land rents resulting from a change in air quality. We demonstrate that the regression studies cannot be used for predictive purposes except to the extent that the city is "small" and there is mobility among cities, and that the assignment model cannot be used for prediction in the way that has been suggested.

I. Summary of the Debate

In a widely read paper which appeared in this *Review,* Ridker and Henning (1967) regressed median property values of census tracts in the St. Louis metropolitan area on median household income, two measures of air pollution, and other characteristics of the census tracts. After noting the statistical significance of the regression coefficient for one of the measures of air pollution (sulfation level), they stated (p. 254):

> This information can be interpreted as meaning that if the sulfation levels to which any single family dwelling unit is exposed were to drop by 0.25 mg./100 cm^2/day, the value of that property could be expected to rise by at least \$83 and more likely to \$245. Using the latter figure and assuming the sulfation levels are reduced by 0.25 mg. but in no case below 0.49 mg. (taken as the background level) the total increase in property values for the St. Louis metropolitan statistical area could be as much as \$82,790,000.[2]

Freeman (1971, p. 415), in a comment on Ridker and Henning, quoted this passage and argued

> The statement in the second sentence is invalid. R-H [Ridker and Henning] have "overinterpreted" their regression equation. This equation only purports to explain the *variation* in mean property values among observations. The air pollution coefficient can be used to predict the difference in property values between two properties within a system under *ceteris paribus* conditions, and these conditions must include no change in air quality over all other land in the system. *But the regression equation cannot be used to predict the general pattern of property values or changes in the value of any given property when the pattern of air quality over the whole urban area has changed.* (Emphasis added to last sentence.)

It is difficult to evaluate these remarks without reference to a theory which makes explicit the connection between air pollution and property values. In fact, this is the principal point of Freeman's comment (p. 415):

> The prior specification of a model . . . is necessary for the interpretation of the statistical results and serves as a logical check on the kinds of generalizations and extensions of the results which are possible . . . I am aware of no such model (of air pollution) which is capable of dealing with the problem with any degree of generality . . . What is required is a model which can be solved to yield the pattern of land rents as a function of the pattern of air quality, among other things.[3]

However, Anderson and Crocker (1972, p. 471) recently wrote in reply:

> . . . contrary to Freeman's assertion, there does exist a general equilibrium model that, given property value bid equations, can, in principle, be used to generate equilibrium property values predicated upon alternative configurations of the environment. The model to which we refer is the assignment problem model . . . as discussed in Beckmann and Koopmans (1957), Gale (1960, chapter 5), and Lind (1970).[4]

To relate our approach to that suggested by Anderson and Crocker, it is necessary to describe the assignment model and the derivation of the property value bid equations. According to the assignment model,[5] each of m parcels of land has exactly one of m households assigned to it. Each household i is willing to pay an amount π_{ij} for parcel j. If $\alpha(i)$ represents the parcel to which household i is assigned, then

$$\sum_{i=1}^{m} \pi_{i\alpha(i)} \tag{1}$$

represents the willingness to pay for all parcels. An assignment $(\alpha(i),\ i=1,\ .\ .\ .\ ,\ m)$ which maximizes (1) is said to be optimal; an optimal assignment must exist but need not be unique. A system of land rents $(r_j,\ j=1,\ .\ .\ .\ ,\ m)$ is said to sustain an assignment if each household's willingness to pay is not exceeded by the rent it actually pays and if no household would prefer to live elsewhere, that is, if

$$\pi_{i\alpha(i)} \geqq r_{\alpha(i)} \qquad (i=1,\ldots,m) \tag{2}$$

$$\pi_{i\alpha(i)} - r_{\alpha(i)} \geqq \pi_{ik} - r_k \quad (i,k=1,\ldots m). \tag{3}$$

Every optimal assignment can be sustained by some system of land rents. Such a system of rents is called a competitive equilibrium.

In this setting an improvement in air quality has been interpreted as an increase in willingness to pay (Anderson and Crocker, 1972; Lind, 1973). After

[2] Two footnotes were omitted.

[3] A footnote was omitted.

[4] For these references see Anderson and Crocker (1972, p. 473). The Lind paper has been published as Lind (1973).

[5] Our presentation is essentially that of Lind (1973).

an improvement it is assumed that household i is willing to pay some $\pi'_{ij} \geqq \pi_{ij}$ for parcel j (with strict inequality for at least one i and j). This will in general lead to a different optimal assignment $(\alpha'(i), i = 1, \ldots, m)$ and a new system of rents $(r'_j, j = 1, \ldots, m)$.

Before applying the assignment model, one must determine household willingness to pay π_{ij}.[6] Anderson and Crocker (1971, pp. 171–173) argued in an earlier paper that π_{ij} depends only on income and such characteristics of the site as air quality.[7] To show this they relied on a model of consumption in which goods are desired only for certain fundamental characteristics which they embody. Formally, Anderson and Crocker (1971, p. 172) stated the consumer's problem as

$$\begin{aligned} &\text{Maximize } U(z) \\ &\text{subject to } p'x = y \\ &\qquad\qquad z = Bx \\ &\qquad\quad z, x \geqq 0, \end{aligned} \tag{4}$$

where x is the vector of quantities of goods, p is the vector of prices of goods, y is income, z is the vector of quantities of characteristics embodied in x, and B is a matrix which transforms quantities of goods into quantities of the desired characteristics. From the first-order conditions for (4) they derived the following relationship for commodity i

$$p_i = \delta y (\partial U / \partial z)' b_i, \tag{5}$$

where

$$\delta = 1 / \sum_{j=1}^{n} (\partial U / \partial z)' b_j x_j, \tag{6}$$

and b_i is the i^{th} column of B. Anderson and Crocker then interpret p_i as willingness to pay (or property value bid), which is determined by the right-hand side of (5). They argue that, for a given consumer, variations in willingness to pay p_i for different goods are due only to variations in the consumption technology b_i for those goods, since other terms in (5) are constant. In their view this justifies the inclusion of characteristics of sites as explanatory variables in property value studies. To account also for the inclusion of income as an explanatory variable, they note (1971, p. 173) that in practice

> . . . what is observed are many submarket equilibria, where each observation expresses . . . an equilibrium relation between an individual's or a group's income and the p_i. Thus, since y in this case is variable across the sample, parameter estimates must be obtained by regressing the p_i on the b_i and the y_k . . . where k can range over individuals or groups of individuals.[8]

Based on the above reasoning they conclude (1972, p. 471) that regressions such as those of Ridker and Henning ". . . therefore explain bids for properties contingent upon bidder income and other characteristics," and that given this interpretation

> . . . the procedure for using these equations to obtain the changes in equilibrium property values ensuing from air quality changes is clear. One simply solves the assignment problem with and without the change and then compares the resulting solutions.

II. An Alternative View[9]

In order to discuss the effects of air quality changes on equilibrium property values, and to clarify the debate, it is useful to describe an alternative model. In this model a hypothetical city is inhabited by a group of individuals who for the sake of exposition are assumed to have identical tastes, represented by a utility function $U(\cdot)$, and to earn equal incomes, y.[10] Each individual must choose a location k at which to live and, given this location, the level of housing services q (which for convenience are assumed to be produced solely from land), and the level of a composite consumption good x. In making this decision he takes as given at each k the price per unit of housing services $p(k)$, the transportation cost $T(k)$ to his workplace, and the level of air quality $a(k)$. The price of the composite good is also given and independent of location; this good is used as the numeraire and its price is set at unity.

The problem of residential location for an individual may be stated as

$$\begin{aligned} &\text{Maximize}_{q,x,k} \quad U(q, x, a(k)) \\ &\text{subject to } y = p(k)q + x + T(k). \end{aligned} \tag{7}$$

An equivalent formulation of the individual's problem involves the indirect utility function, which expresses utility in terms of prices and income (and air quality in our case). At a given location k this is obtained by solving (7) for the market demand functions for housing and the composite good, and

[6] Following Anderson and Crocker we use "willingness to pay" interchangeably with "property value bids."

[7] This argument, presented below, appears to contain certain ambiguities and logical errors; these will not be discussed in the current section, which is meant only to summarize the debate.

[8] The notation in the quote has been slightly altered to correspond to that used here.

[9] In a paper which deals with related issues, the view presented below is developed in greater detail. See Polinsky and Shavell (1973).

[10] The qualitative nature of our conclusions would not be affected if there were instead an arbitrary number of classes of individuals with many individuals in each class.

then by substitution of the demand functions in $U(\cdot)$. Thus at location k the indirect utility function $V(\cdot)$ is

$$V(k) = V(p(k), y - T(k), a(k)), \tag{8}$$

and the individual's problem becomes

$$\underset{k}{\text{Maximize}}\ V(k). \tag{9}$$

If individuals are free to move from one location to another, the equilibrium pattern of property values must be such that no individual could increase his welfare by moving. Thus, in equilibrium there must exist some level of utility V^* which is independent of location, that is

$$V^* = V(p(k), y - T(k), a(k)). \tag{10}$$

Implicit in (10) is the equilibrium relationship between property value $p(k)$ on the one hand, and V^*, $T(k)$, and $a(k)$ on the other:

$$p(k) = f(V^*, y - T(k), a(k)). \tag{11}$$

As will be seen in the next section, the determination of V^* and its influence on property values is of central importance.

In (7) and (9) we assume that each individual believes that his actions alone cannot affect the schedules of housing prices, transportation cost, and air quality. Of course, these schedules may be determined by the behavior of all individuals.[11] However, since our primary interest concerns the dependence of property values on air quality, we will assume that both the $T(k)$ and $a(k)$ schedules are exogenous.

III. Application to the Debate

The air pollution and property value debate has raised the following questions (among others): Can the new property value schedule resulting from a change in air quality be predicted? Can the data required by the assignment model (willingness to pay) be obtained, and if so, is the assignment model appropriate in the present context?

A) *Prediction of the Property Value Schedule*

The criticism of Ridker and Henning may be summarized by the following statement: In order to determine the value of property at a particular location, one must know the level of air quality throughout the city and must then solve a general equilibrium model using this information. To see when this statement is correct and when it is not, consider (11). For property value at k to depend only on characteristics such as air quality of k — as Ridker and Henning implicitly assume — the equilibrium level of utility V^* must not be influenced by conditions elsewhere in the city.

One set of circumstances under which V^* would not be affected by conditions elsewhere in the city is the case of a "small" "open" city. If the city is open (i.e., there is perfect and costless migration to and from other cities), there would be a common level of utility throughout the system of cities for the same reason that there is a common level of utility within the city. Further, if the city is small (i.e., changes in it have only a negligible effect on the equilibrium level of utility in the system), then for all practical purposes V^* is exogenous. Thus, in the case of a small open city, $p(k)$ is only a function of characteristics of k. In particular, the *ceteris paribus* relationship between $p(k)$ and $a(k)$ (e.g., as revealed by a regression equation) may then be used directly to predict changes in $p(k)$ due to changes in the air quality schedule without solving any general equilibrium problem.

If migration is not perfect and costless or if the city is not small, then general improvements in air quality will usually lead to a rise in the equilibrium level of utility V^*. From (11) it can be seen that the change in $p(k)$ depends directly on the change in air quality at k and indirectly on the change in air quality elsewhere in the city through the effect on V^*. Given a general improvement in air quality, $p(k)$ would tend to rise due to the direct effect — since site k is more attractive than before — but $p(k)$ would tend to fall due to the indirect effect — since the "opportunity cost" of residing at k (i.e., V^*) has risen. Therefore, in the case of a city which is not small and open, $p(k)$ is a function of characteristics of locations elsewhere as well as at k. The *ceteris paribus* relationship between $p(k)$ and $a(k)$ may no longer be used directly to predict changes in $p(k)$ due to changes in the air quality schedule. In effect, there is now an omitted and unobservable explanatory variable, V^*, which also varies with the air quality schedule. In principle it is necessary to solve a general equilibrium problem in which both V^* and the $p(k)$ schedule would be determined endogenously.

B) *Use of the Assignment Model*

At this point one might wonder whether the assignment model can be used to derive the equilibrium property value schedule when the city is not small and open. First note, however, that the system of equilibrium land rents in the assignment model is not unique. From (2) and (3) it can be seen that a new system of land rents which differs by a constant from the existing equilibrium system

[11] For example, if air pollution depends on traffic density, $a(k)$ would be affected by the pattern of residential location.

will also sustain the equilibrium.[12] In general, an additional condition is needed to determine a unique equilibrium. One such condition is that $\pi_{ia(i)} = r_{a(i)}$, which is what Anderson and Crocker have implicitly assumed since they say that the rent observed in the market place is willingness to pay. Without this assumption $\pi_{ia(i)}$ is unobservable so that willingness to pay cannot be estimated.[13]

Because willingness to pay depends on the level of utility, precisely the same problems arise in estimating willingness to pay as in estimating property values. In urban location theory, willingness to pay for a parcel of land is usually defined as the amount a household could pay for the parcel and just maintain a specified level of utility (when income, other prices, and characteristics of the parcel are given). Our equation (11) can be reinterpreted in this way. Anderson and Crocker argued that willingness to pay π_{ij} is solely a function of the income of household i and the characteristics of parcel j, ignoring the dependence of π_{ij} on the level of utility. Although they did not seem to recognize it, the level of utility does enter (5), which they interpret as willingness to pay.[14] Therefore, for the reasons given in subsection A above, it is only proper to project the new π'_{ij} in the way suggested by Anderson and Crocker if the city in question is small and open (at least approximately).

However, when the city is small and open the assignment model is no longer appropriate. This is because the set of households to be assigned in the model remains unchanged regardless of the degree of improvement in air quality.[15] There is no scope for in or out migration to equalize the equilibrium level of utility V^* in this city with that achievable in the outside world. Therefore we conclude that regardless of whether the city is small and open, the assignment model cannot be used in the way suggested by Anderson and Crocker for studying the relationship between air pollution and property values.

IV. Conclusion

Given the generally pessimistic nature of our discussion, we agree with Freeman's concluding remark (1971, p. 416) in his comment on Ridker and Henning:

> I would suggest that further empirical studies of land values and air pollution should await the formulation of general models from which empirically testable hypotheses can be deduced. Until such models are formulated and tested, empirical land-value studies will make little or no contribution to our knowledge of the benefits of air pollution abatement.

However, we are optimistic about the possibility of developing a formal model of the kind required for proper interpretation and estimation.

REFERENCES

Anderson, R. J., Jr., and T. D. Crocker, "Air Pollution and Property Values: A Reply," this REVIEW 54 (Nov. 1972), 470–473.

———, "Air Pollution and Residential Property Values," *Urban Studies,* 8 (Oct. 1971), 171–180.

Edel, M., "Land Values and the Costs of Urban Congestion: Measurement and Distribution," *Social Science Information,* 10, no. 6 (1971), 7–36.

Freeman, A. M. III, "Air Pollution and Property Values: A Methodological Comment," this REVIEW 53 (Nov. 1971), 415–416.

———, "On Estimating Air Pollution Control Benefits from Land Value Studies," *Journal of Environmental Economics and Management,* 1 (1974), 74–83.

———, "Spatial Equilibrium, The Theory of Rents, and The Measurement of Benefits from Public Programs: A Comment," *Quarterly Journal of Economics* (forthcoming).

Lind, R. C., "Spatial Equilibrium, The Theory of Rents, and The Measurement of Benefits from Public Programs," *Quarterly Journal of Economics,* 87 (May 1973), 188–207.

Polinsky, A. M., and S. Shavell, "Amenities and Property Values in a General Equilibrium Model of an Urban Area," Working Paper no. 1207-5, The Urban Institute, Washington, D.C. (July 1973) (revised).

Ridker, R. G., and J. A. Henning, "The Determinants of Residential Property Values with Special Reference to Air Pollution," this REVIEW 49 (May 1967), 246–256.

Small, K. A., "Air Pollution and Property Values: Further Comment," this REVIEW, this issue.

[12] If each new rent equals the existing rent plus a constant h, then it must be true that $h \leqq \min_i (\pi_{ia(i)} - r_{a(i)})$; if we insist that rents are non-negative it must also be true that $h \geqq -\min_i (r_{a(i)})$.

[13] For further discussion of this point see Freeman (forthcoming).

[14] The term δ (6), which they correctly assert does not vary across commodities for a given household but to which they attach no meaning, is approximately equal to the reciprocal of total utility. To see this consider the denominator of (6). For a given commodity j, x_j is the number of units consumed of that commodity and $b_j x_j$ is the vector of the number of units consumed of each of the characteristics due to x_j. Since $(\partial U/\partial z)'$ is the vector of marginal utilities of each characteristic, $(\partial U/\partial z)' b_j x_j$ approximately equals the utility contributed by commodity j through its characteristics; summing over all commodities gives total utility.

[15] It may be possible to construct a "grand" assignment model of the entire system of cities (all parcels and all households). However, this is clearly not what Anderson and Crocker (1971; 1972) and Lind (1973) had in mind in suggesting use of the assignment model for studying rent changes in a *particular* city. Moreover, as a practical matter, the solution of such a model is virtually unobtainable.