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AMENITIES AND PROPERTY VALUES IN A MODEL OF AN URBAN AREA*

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The dependence of property values on a schedule of 'amenities' is examined in the case of a 'small' and 'open' city and in the case of a 'closed' city. Questions concerned with the predictability and interpretation of changes in equilibrium property values following an improvement in amenities are considered in these cases. The problem of identifying the implicit demand for amenities from a single equilibrium rent schedule is also addressed.

1. Introduction

Theoretical and empirical studies of the effect of location-dependent amenities (such as air and noise pollution and local public services) on urban property values have raised the following questions:[1]

(1) How is land rent at a given location affected by the level of amenities at that location and by amenities elsewhere in a city?
(2) How would one predict the new rent schedule (and therefore the change in aggregate property values) resulting from a change in the amenity schedule?
(3) If there is an improvement in the amenity schedule, does the change in aggregate property values correspond to willingness to pay?
(4) What can be learned about the underlying demand for amenities from a single equilibrium rent schedule?

For example, the usual practice of regressing property value at a particular location only on variables describing that location implicitly raises the first question and assumes that the pattern of property values does not depend on the

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[1]See, for example, Edel (1971), Freeman (1971), Kain and Quigley (1970), Lind (1973), Mohring (1961), Oates (1969), Paul (1971), Ridker and Henning (1967), and Strotz (1968).

supply of, and demand for, land throughout the city. The other three questions have arisen in studies which predict the change in property values that would result from public improvements such as a new airport, pollution abatement, or urban renewal. The interest in these predictions is due in part to the frequently made assumption that property value changes represent willingness to pay.[2]

In section 2 below the traditional model of residential location in an urban area is modified to include an amenity schedule. In sections 3 and 4, the questions are answered (in somewhat different order) in each of two versions of the model: one in which the urban area is 'small' and there is household mobility both within and among different urban areas (referred to as the 'small–open' model), and one in which household mobility is restricted to a given urban area (the 'closed' model). In section 5 the paper is summarized and two concluding remarks are made.

2. Residential location and amenities

Consider a simple model of residential location in which the following hold:

(1) The city is circular and is built on a featureless plain around a predetermined central business district (CBD).
(2) Each resident is identical and works the same number of hours at a fixed wage in the CBD, to which he makes a fixed number of trips per unit time.
(3) Transportation is instantaneous but incurs costs which are an increasing function of distance from the CBD.
(4) Housing is a homogeneous commodity composed of land and structure in a fixed proportion.
(5) The city is in long-run equilibrium.
(6) Residents rent their dwellings from an absentee landlord.
(7) The level of amenities at a given location – exogenously determined and provided at no cost – enters (positively) only into the utility functions of individuals residing at that location.

The restrictiveness of most of these assumptions is not necessary but facilitates the analysis.[3]

The residential location decision for a representative individual may be stated formally as

$$\max_{x,q,k} U(x, q, a(k)) \quad \text{subject to} \quad y = x+p(k)q+T(k), \tag{1}$$

[2]For a more detailed discussion of the debate over some of these questions as they relate to air pollution, see Polinsky and Shavell (1975).

[3]For example, in section 5 below, the assumption of identical individuals is relaxed with little change in the results. See also footnote 7 below. Given the assumptions about housing production and ownership, no distinction will be made between 'housing' and 'land', or among housing 'prices', 'rents', and 'values'.

where

x = consumption of a private good (used as the numeraire with a price set at unity),
q = consumption of housing,
k = distance from the CBD,
$a(k)$ = index of amenities,[4]
y = income,
$p(k)$ = price per unit of housing,
$T(k)$ = transportation cost.

Rather than work with the (direct) utility function U, one can use the indirect utility function V,[5] in which a household's utility can be expressed as a function of prices at a particular location, income net of transportation costs from that location, and amenities at that location:

$$V(k) = V(p(k), y-T(k), a(k)), \tag{2}$$

where (with subscripts denoting partial derivatives)

$V_1 < 0$ because an increase in land rents decreases utility, given income and amenities;
$V_2 > 0$ because an increase in net income increases utility, given land rents and amenities;
$V_3 > 0$ because an increase in amenities increases utility given income and other prices.

3. Property values in a 'small-open' city

In equilibrium, land prices display a pattern such that none of the identical individuals could increase their utility by changing residence. That is, each individual enjoys a common level of utility, V^*, which is independent of his location:

$$V^* = V(p(k), y-T(k), a(k)). \tag{3}$$

Adjustment in land rents is the mechanism by which utility is equalized over space. If location k_1 is more attractive than location k_2 – considering the rent,

[4]Although $a(k)$ is, in general, a vector of characteristics, for purposes of exposition it will be assumed to be a scalar.

[5]This function embodies the information in the utility function relevant to the market behavior of the consumer and is therefore just as basic a starting point as is the utility function. To derive the indirect utility function from the utility function, one solves for the market demand functions from the standard maximization problem (with fixed income and prices) and then substitutes the demand functions for the commodity arguments in the utility function. The usefulness of the indirect function for analyzing residential location models was first emphasized by Solow (1973).

transportation cost, and amenities at both places – then rents at k_1 are bid up and rents at k_2 fall until k_1 and k_2 become equally desirable. This process occurs throughout the city, generating the equilibrium rent schedule. Differentiating (3) with respect to k and then solving for $p'(k)$, the slope of the rent schedule, one obtains

$$p'(k) = \frac{V_2(k)}{V_1(k)}T'(k) - \frac{V_3(k)a'(k)}{V_1(k)}. \tag{4}$$

The first term is negative (since $V_2 > 0$, $T' > 0$, $V_1 < 0$) and shows at what rate the rent schedule must decline, as one moves further from the CBD, in order to keep land attractive enough to compensate for increasing transportation costs. The last term has the same sign as $a'(k)$ (since $V_3 > 0$) and reflects the change in amenities and their effect on individual utility. If $a'(k)$ is sufficiently large, the rent schedule will have a positive slope over some range. However, one normally observes a declining rent schedule, the condition for which is

$$V_2 T' > V_3 a'. \tag{5}$$

Since V_2 is the marginal utility of income and V_3 is the marginal utility of amenities, eq. (5) states that the rent schedule must fall as long as a small movement away from the CBD results in a greater utility loss from higher transportation expenses than it does in a gain from improved amenities.

Transportation costs are assumed to become so high at some distance from the CBD that city residents are no longer able to outbid alternative users of land (usually presumed to be agricultural). At the boundary the bids of residential users will equal the bids of agricultural users,

$$p(k_o) = p_o, \tag{6}$$

where k_o is the boundary and p_o is the fixed price per unit of agricultural land. From (6) one can determine the height of the schedule at the boundary. However, this information is not sufficient to determine the size of the city. There are many rent schedules which satisfy (4) and (6), but which correspond to different common levels of utility and which have different boundaries. The higher the level of utility, the lower the rent schedule and the smaller the boundary.

To fix the level of utility, the urban area is assumed to be small and open. Because the area is open – there is perfect migration between it and other areas – there will be a common level of utility throughout the system. Because the city is small, this level of utility may be treated as exogenous. The V^* in (3) will be interpreted as this common level of utility. Therefore, the city will expand to the point at which those living at the boundary, k_o, just achieve utility V^* from their net income given rent p_o and amenities $a(k_o)$,

$$V^* = V(p_o, y-T(k_o), a(k_o)). \tag{7}$$

The remaining equilibrium condition is that in each ring the supply of land and the demand for land are equal. The total supply of land in the ring $[k+\mathrm{d}k]$ is $2\pi k\mathrm{d}k$. The total demand for land in the ring is $q(k)n(k)\mathrm{d}k$, where $n(k)$ is residential density at distance k and $q(k)$ is the per capita demand for land at k. The demand for land can be derived from the indirect utility function,[6]

$$q(k) = -\frac{\partial V/\partial p}{\partial V/\partial y} \equiv -\frac{V_1(p(k), y-T(k), a(k))}{V_2(p(k), y-T(k), a(k))}. \tag{8}$$

The density function is then determined by

$$n(k) = 2\pi k/q(k) = -2\pi k V_2/V_1, \tag{9}$$

and total population is obtained by integrating (9) from 0 to k_o. From (3) one can see that an upward shift in the amenity schedule must be accompanied by an upward shift of the rent schedule if utility is to be held constant at V^*. Thus, improvements in amenities will cause the city to expand. However, density and population could decrease, for example, if the increase in amenities strongly increases the demand for space.

The crucial relationship to be noted is $V^* = V(p(k), y-T(k), a(k))$, which shows directly that in a small–open city the rent at any location depends only on the level of amenities at that location.[7] This is so because in equilibrium a parcel of land in the city must yield the *fixed* level of utility V^*. It is fixed since the city is such a small part of the world that a change in amenities there has only a negligible effect on the equalized (by migration) utility prevailing in the system. Hence, given an individual's income, transportation costs from the parcel to the CBD, and the level of amenities at the parcel, there is only one level of rent that will result in utility V^*. Stated differently, the forces of demand and supply throughout the system so dominate those in a small–open city that supply of land there has no effect on the system's supply of land and therefore no effect on the rent earned by a parcel of land with given characteristics.

In the absence of a real market for amenities, can one determine preferences for amenities from information contained in an equilibrium rent schedule? This may be possible if sufficient assumptions are made about the structure of

[6]This formula is not difficult to prove; see, for example, Lau (1969). However, it may be explained heuristically as follows: $-(\partial V/\partial p)$ = marginal utility gained with \$1 fall in rent $\doteq$ (number of units of land rented) $\times$ (marginal utility of a dollar) = (number of units of land rented) $\times (\partial V/\partial y)$. Rearranging terms gives (number of units of land rented) $= -(\partial V/\partial p)/(\partial V/\partial y)$.

[7]It is clear from the locational equilibrium condition (3) that if the amenity schedule were endogenous, the statement would still be valid. However, if the wage rate or the transportation cost schedule were endogenous, then amenities at locations other than k may affect y or $T(k)$, and therefore $p(k)$, even in an open city.

preferences. For example, consider the case of the Cobb–Douglas utility function

$$U(x, q, a(k)) = Ax^{\alpha}q^{\beta}a(k)^{\delta}, \tag{10}$$

where A is a positive constant and α, β, δ are positive constants less than unity. Without loss of generality, let $\alpha+\beta = 1$ by a suitable normalization. From (10) the demand functions for the private good and housing are

$$x(k) = \alpha[y-T(k)], \tag{11}$$

$$q(k) = \beta[y-T(k)]/p(k). \tag{12}$$

Substituting (11) and (12) into (10) gives the indirect utility function

$$V(k) = C[y-T(k)]p(k)^{-\beta}a(k)^{\delta}, \tag{13}$$

where $C = A\alpha^{\alpha}\beta^{\beta}$. Setting $V(k)$ in (13) equal to the exogenous level of utility V^*, one can solve for the equilibrium rent schedule:

$$p(k) = (C/V^*)^{1/\beta}[y-T(k)]^{1/\beta}a(k)^{\delta/\beta}. \tag{14}$$

Eq. (14) demonstrates that the rent at distance k is determined by transport costs and amenities only at distance k.

In answering the question of identification, it is useful to convert (14) to a linear relationship by taking logs,

$$p_k = b'_0+b'_1y_k+b'_2a_k, \tag{15}$$

where

$$\begin{aligned} p_k &= \log p(k), \\ y_k &= \log\,[y-T(k)], \\ a_k &= \log a(k), \\ b'_0 &= (1/\beta)\log\,(C/V^*), \\ b'_1 &= 1/\beta, \\ b'_2 &= \delta/\beta. \end{aligned}$$

Although (15) is nothing more than a rewriting of the condition for locational equilibrium, it may be considered as a hypothetical regression equation. From this viewpoint, the coefficients b'_1 and b'_2 together allow identification of β and δ. Since $\alpha+\beta = 1$, α is also determined. This may seem somewhat surprising since it is often true that equilibrium data cannot be used to identify either the demand or the supply equation. What permits identification of the demand for amenities even in the absence of a market is that land is differentiated across the urban area by amenities. After adjusting for varying transportation costs, remaining differences in the price of land can therefore be imputed to variations in amenities.

Assuming a small-open urban area, a cross-section regression which estimates the relationship between rents and amenities implicit in the condition for locational equilibrium (3) can be used to predict the new rent gradient resulting from an amenity improvement. (In computing the change in aggregate property values, one should include newly incorporated fringe areas.) This is obviously true for the Cobb–Douglas example since (15) is equivalent to (13), which uniquely defines the relationship between rents and amenities.

What is the relationship between the change in aggregate property values and willingness to pay for the amenity improvement? In the small-open model, renters are neither better nor worse off than before the improvement. Their willingness to pay is therefore zero. The profits of the absentee landlord, however, have increased by the change in aggregate land values. A lump-sum tax of this amount would leave him just as well off. In this sense, the change in aggregate property values corresponds to the total willingness to pay on behalf of all parties.

4. Property values in a 'closed' city

In this section a version of the model is discussed which may be considered short-run since it does not allow migration to or from the city or any new construction beyond the city's present boundary. The city's land area is fixed at πk_0^2 and its population is held at N.[8] However, it is still assumed that individuals can change location within the city at no cost. The locational equilibrium condition, analoguous to (3), is

$$V^{**} = V(p(k), y-T(k), a(k)), \qquad 0 \leqq k \leqq k_o, \tag{16}$$

for some V^{**} to be determined. The condition that demand and supply of land are equal at each location implies a density schedule (9) which, when integrated over the fixed radius of the city, must result in the population N,

$$\int_0^{k_o} n(k)\,\mathrm{d}k = -2\pi \int_0^{k_o} k(V_2/V_1)\,\mathrm{d}k = N. \tag{17}$$

For each V^{**}, condition (16) determines a possible equilibrium rent schedule, and therefore a demand for land and a residential density, at each location. Only one residential density schedule, and therefore only one V^{**}, satisfies (17).

[8]An alternative characterization of a closed city would be to fix only the population, letting the boundary of the city adjust. The answers to the four questions in section 1 are the same in either version.

The equilibrium level of utility V^{**} is now endogenous since the city is isolated from the rest of the system. An improvement in amenities may therefore lead to an increase or a decrease in V^{**} (although the presumption would be for V^{**} to increase).[9] It is clear from (16) that, in contrast to the case of the small-open city, rent at any location k will depend both on $a(k)$ and on amenities elsewhere (through their effect on V^{**}). Moreover, the change in aggregate land values no longer corresponds to willingness to pay for improvements. In a small-open city, consumers as renters are willing to pay nothing and the absentee landlord is willing to pay the change in aggregate land values. In a closed city, the absentee landlord is still willing to pay the change in aggregate land values, but since consumers may be better off (or worse off), they would be willing to pay (or would have to be compensated by) an amount which would return them to their original level of utility. If the utility level rises (falls), the change in aggregate land values will understate (overstate) benefits.

It is even possible to construct an example in which amenities improve everywhere in the closed city, utility rises, but rents do not change anywhere. Suppose that each resident has the Cobb–Douglas indirect utility function (13) and that amenities everywhere double. From (14) (with V^{**} instead of V^{*}) it is easily verified that in the new equilibrium the ratio of rents at any two locations must be the same, and therefore that all rents rise or fall together. If rents rise, the demand for land falls (12) and residential density increases everywhere, violating (17). Similar reasoning holds for a decline in rents.

In general, the change in the rent at a particular location k may be decomposed into an indirect component – associated with changes in amenities elsewhere in the city – and a direct component – associated with the change in amenities at k. To see this, suppose that amenities improve everywhere but at k. Then assuming V^{**} rises, $p(k)$ falls (16) because locations elsewhere have become relatively more desirable. Now let amenities improve at k too. This will cause a rise in $p(k)$ which might be termed the direct effect since it is due only to the change in $a(k)$ and not to a change in V^{**}. (In the previous example, rents did not change because the two effects exactly offset each other.)

To illustrate the decomposition of rents in a closed city, let amenities at each location depend positively on a shift parameter λ: $a_\lambda \equiv \partial a(k, \lambda)/\partial\lambda > 0$, for all k. Since V^{**} is, in general, a function of the entire amenity schedule, it is a function of λ. Assuming that each resident has the Cobb–Douglas indirect utility function (13), an increase in amenities everywhere will raise V^{**}, so that

[9]Let amenities at each location be increasing in a shift parameter λ and differentiate (17) with respect to λ:

$$\int n(V^{**}, \lambda) = N \therefore \frac{\mathrm{d}V^{**}}{\mathrm{d}\lambda} = -\int \frac{\partial n}{\partial \lambda} \Big/ \int \frac{\partial n}{\partial V^{**}} \gtreqless 0$$

as $\partial n/\partial\lambda \gtreqless 0$ since $\partial n/\partial V^{**} < 0$ (if housing is not a Giffen good). $\partial n/\partial\lambda$ may be positive or negative depending on the form of the utility function. A sufficient condition for $\partial n/\partial\lambda$ to be positive, and therefore for utility to rise, is that the utility function is separable.

$V^{**}_{\lambda} \equiv \mathrm{d}V^{**}/\mathrm{d}\lambda > 0$.[10] Taking the log of (14) (with V^{**} instead of V^{*}) and differentiating with respect to λ, it can be shown that

$$\mathrm{d}p(k)/\mathrm{d}\lambda \gtreqless 0 \qquad \text{as} \qquad \delta E_{a\lambda} \gtreqless E_{V^{**}\lambda}, \tag{18}$$

where $E_{a\lambda} \equiv a_{\lambda}\lambda/a$ and $E_{V^{**}\lambda} \equiv V^{**}_{\lambda}\lambda/V^{**}$. In other words, the rent at location k will rise if the weighted (by δ) elasticity of amenities with respect to λ is greater than the elasticity of utility with respect to λ. The direct effect corresponds to $E_{a\lambda}$ – rents are more likely to rise the more important amenities are to the consumer (as indicated by δ) and the more sensitive amenities are to the parameter λ. The indirect effect corresponds to $E_{V^{**}\lambda}$ – rents are more likely to fall the more sensitive the common level of utility is to the entire amenity schedule. Holding δ and $E_{a\lambda}$ constant, as $E_{V^{**}\lambda}$ becomes larger, the new common level of utility becomes larger. But to achieve this higher V^{**}, rents will have to fall (or rise less than otherwise) in order to maintain the higher V^{**} given the new amenity schedule. The result of these two forces working in opposite directions determines whether rents at any particular location rise or fall. (In the open model, $E_{V^{**}\lambda} = 0$ and condition (18) reduces to '$\mathrm{d}p(k)/\mathrm{d}\lambda > 0$ if $E_{a\lambda} > 0$', which is always true since $E_{a\lambda} \equiv a_{\lambda}\lambda/a$ and $a_{\lambda} > 0$.)

Because amenities throughout a closed city affect rents at a given location, it is not generally correct to predict the new property value schedule in the city on the basis of a cross-section regression. This may be demonstrated in the Cobb–Douglas example by expressing the rent schedule (14) (with V^{**} instead of V^{*}) in log form,

$$p_k = b''_0 + b''_1 y_k + b''_2 a_k, \tag{19}$$

where

$$b''_0 = 1/\beta \log C/V^{**},$$
$$b''_1 = 1/\beta,$$
$$b''_2 = \delta/\beta,$$

and where all else is defined as in (15). The term b''_0 is no longer a true constant since a change in the amenity schedule will change V^{**}. Although b''_2 still measures the direct effect of amenities on property values, there is now the indirect effect through the 'constant' term b''_0.[11]

The problem of identifying the demand for amenities in a closed city is equivalent to that in an open city. If one observes an urban area in equilibrium, there is no way to tell (without knowing if there is a price differential at the

[10] This follows for the Cobb–Douglas utility function since it is separable. See footnote 9 above.

[11] Even in the closed model, however, b_2'' can be interpreted as the effect on property values of a marginal increase in amenities at a particular location, ceteris paribus (including amenities everywhere else).

boundary) whether the city is open or closed.[12] Therefore, the information embodied in an equilibrium rent schedule of a closed city should be the same as that of an open city. For example, one sees from (19) that α, β, and δ are again identified.

5. Summary and concluding remarks

The following conclusions have been suggested by the discussion of a simple model of amenities and property values in an urban area:

(1) In one limiting case – when the city is small and open – property values at any location depend only on amenities (and other relevant variables) at that location. In the other limiting case – when the city is closed – property values at any location depend on amenities throughout the city.

(2) The validity of using cross-section regression results to predict property value adjustments in response to changes in the amenity schedule depends on the degree of mobility within and among cities. In the small–open model, cross-section regression results may be used to predict a new rent schedule. But in the closed model, the results cannot be used in a direct way to predict the overall pattern of property value changes. It is then necessary to solve a more complicated general equilibrium model.

(3) Changes in aggregate land values do not correspond to willingness to pay except in the case of the small–open model. To the extent that migration is imperfect, the residents of a city in which amenities have been improved may be better off or worse off relative to households in other cities. The value placed on this utility difference will not be reflected in rents, and therefore the change in aggregate land values may understate or overstate total willingness to pay.

(4) Regardless of the degree of intercity mobility, enough information may be contained in a single equilibrium rent schedule to deduce the demand for amenities. In the Cobb–Douglas example, the ratio between the coefficients with respect to (the log of) amenities and to (the log of) income net of transportation costs identifies the amenity exponent in the utility function.

The introduction of many classes of households would not change these conclusions in an essential way, although it would complicate the model. There would then be a separate locational equilibrium condition, analogous to (3), for each class and, in general, one would estimate separate regression equations. In the case of a small–open city, the equilibrium level of utility of each class would be determined exogenously, whereas in a closed city these levels would be determined endogenously. The analysis of the questions would remain unchanged except that one would have to take into account the possibility that the class occupying any particular location might change.

[12]If the city were characterized by a fixed population and a variable boundary, there would not even be a price differential at the boundary.

The conclusions of this paper may be applied to a 'small' neighborhood in a single 'large' urban area. If there is perfect mobility throughout the urban area, then amenity changes in the neighborhood can be analyzed as in the small-open city model. Similarly, to the extent that movement to or from the neighborhood is limited, the closed model applies.

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