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Professors Louis Kaplow and Oliver Hart

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**“Hold-up, Asset Ownership, and Reference Point”**

Oliver Hart

## **Hold-up, Asset Ownership, and Reference Points**

by

Oliver Hart\*

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## Abstract

We study two parties who desire a smooth trading relationship under conditions of value and cost uncertainty. A contract fixing price works well in normal times since there is nothing to argue about. However, when value or cost is exceptional, one party will deviate from the contract and hold up the other party, causing deadweight losses as parties withhold cooperation. We show that allocating asset ownership and indexing contracts can reduce the incentives to engage in hold up. In contrast to much of the literature, the driving force in our model is payoff uncertainty rather than noncontractible investments.

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## 1. Introduction

This paper reexamines some of the themes of the incomplete contracts literature – in particular, the hold-up problem and asset ownership – through a new theoretical lens, the idea that contracts serve as reference points (see Hart and Moore (2008)). We consider a buyer and seller who are involved in a (long-term) economic relationship where the buyer’s value and seller’s cost are initially uncertain. For the relationship to work out the parties need to cooperate in ways that cannot be specified in an initial contract. Suppose that the parties write a rigid contract that fixes price. Such a contract works well in “normal” times because there is nothing to argue about: we assume that in the absence of argument the parties are willing to cooperate. However, if value or cost falls outside the normal range, one party will have an incentive to threaten to withhold cooperation unless the contract is renegotiated, i.e., the party will engage in hold-up. For example, if value is exceptionally high the seller will hold up the buyer to get a higher price, while if cost is exceptionally low the buyer will hold up the seller to get a lower price. We suppose that hold-up transforms a friendly relationship into a hostile one. The consequence is that the parties withhold cooperation: they operate within the letter rather than the spirit of their (renegotiated) contract, causing deadweight losses. However, even a hostile relationship is assumed to create more surplus than no trade, and so, if value or cost has moved sufficiently far outside the normal range, hold-up will occur.

We show that there is a range of prices  $[p_L, p_H]$ , which depends on the state of the world, such that, if the long-term contract price  $p$  ( chosen before the state of the world is known ) lies in this range, hold-up is avoided; while if  $p$  lies outside this range hold up occurs. One can think of  $[p_L, p_H]$  as the “self-enforcing” range (see, e.g., Klein (1996) and Masten(1988)). In standard hold-up models this range is degenerate:  $p_L = p_H$ . In the current model it is nondegenerate

because the deadweight costs of hold-up introduce a friction. If there is no ex-ante uncertainty it is easy to find a  $p$  that lies in  $[p_L, p_H]$  (exactly where it lies does not matter since lump sum transfers can be used to redistribute surplus ). However, if the state of the world is uncertain, it may be impossible to find a price  $p$  that lies in  $[p_L, p_H]$  with probability 1. Under these conditions hold-up will sometimes occur.

We establish two results. First, not surprisingly, indexation can improve matters. If a verifiable signal  $\sigma$  can be found that is correlated with the state of the world, then indexing the contract price to  $\sigma$  makes it easier to ensure that  $p$  lies in  $[p_L, p_H]$ . Second, an appropriate allocation of asset ownership can help. Ceteris paribus, allocating an asset to the buyer is good to the extent that this increases the correlation between the buyer's outside option and his value from trade. This improves matters because, when value is exceptionally high, the buyer's outside option will also be high, which reduces the seller's ability to hold up the buyer. To put it a bit more formally, the  $[p_L, p_H]$  range becomes less sensitive to the buyer's value. However, there is a countervailing effect on the seller: taking an asset away from the seller makes it easier for the buyer to hold up the seller. Thus there is a trade-off. One feature of our approach is that, in contrast to much of the literature, it focuses on ex post rather than ex ante inefficiencies. Indeed (noncontractible) relationship-specific investments play no role.<sup>1</sup>

Our model resonates with a number of papers in the empirical/applied contracting literature. This literature studies contractual relationships between buyers and sellers of petroleum coke, and of trucking services, and between producers of coal and electricity, and of aluminum and copper wire, among other things; see Goetz and Scott (1983), Goldberg (1985), Goldberg and Erickson (1987), Joskow ( 1985), Klein ( 1996), Masten (1988), Lafontaine and

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<sup>1</sup> Lafontaine and Masten (2002) and Masten (2007) argue that in the trucking industry relationship-specific investments are relatively unimportant and long-term contracts are written to save on ex post negotiation or bargaining costs. The approach adopted here is consistent with this view.

Masten (2002), and Masten (2007). A theme of this literature is that trading relationships become problematic in situations where the gains from trade are unevenly divided under an existing contract. This will lead one party to engage in costly opportunistic or noncooperative behavior in order to wriggle out of, or renegotiate, the contract. To quote Klein(1996, p.444 ): “Hold-ups occur when market conditions change sufficiently to place the relationship outside the self-enforcing range.”<sup>2</sup> However, the existing literature does not explain why the parties cannot negotiate around the costs of opportunism. A benefit of our approach is that we are able to formalize hold-up costs by introducing behavioral features. In our model, hold-up sours a trading relationship, causing deadweight losses from a reduction in cooperation. Moreover, the parties cannot bargain around these losses.

It is useful to compare the current paper with the existing literature on asset ownership and vertical integration. According to transaction cost economics (see, e.g., Williamson (1971) and Klein et al. (1978)), contracts between independent parties are problematic because, given contractual incompleteness, deadweight losses will occur as parties haggle over the ex post division of the quasi-rents. A key factor in determining vertical integration decisions is the size of these quasi-rents. According to property rights theory (see, e.g., Grossman and Hart (1986) and Hart and Moore (1990)), parties will bargain around the deadweight losses from haggling, but ex ante investments will be distorted. A key factor determining vertical integration is the marginal product of quasi-rents with respect to (noncontractible) ex ante investments. This paper emphasizes a third factor: the variability of quasi-rents with respect to the state of the world; that

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<sup>2</sup> Examples of opportunistic behavior include breaching the contract, performing according to the literal terms of the contract, exploiting ambiguous terms, contesting facts, or threatening to do these things. See Goetz and Scott (1983, pp.977 ,982) and Goldberg (2006, pp.328-329). Also see Klein (1996), and Goldberg (2006, Chapter 20) for discussions of the *Alcoa v. Essex* case.

is, payoff uncertainty. We return to the implications of our paper for the theory of vertical integration in the conclusions.

Our paper is also connected to the relational contracts literature. Although our theory does not rely on a repeated relationship, there is something akin to the notion of trust or good will in the model, in the sense that cooperation is destroyed if hold-up occurs. In a way our model can be seen as a reduced form of a relational contracting model. Note, however, that, in contrast to the leading formal models of relational contracts and asset ownership ( see , e.g., Baker et al. ( 2002 ) and Halonen ( 2002 )), we emphasize ex post inefficiency of trade rather than ex ante inefficiency of investments.

While this paper is stimulated by Hart and Moore (2008) (particularly Section III of that paper), there are several important differences. First, Hart and Moore (2008) is concerned with the trade-off between flexible and rigid contracts. For the most part we focus on rigid contracts (although we show in the appendix that all our results generalize to flexible contracts). Second, Hart and Moore (2008) ignores assets and does not allow hold-up. Parties quit in states of the world where their payoff is negative, but in the leading version of the model there is no renegotiation and so the parties never threaten to quit in order to get a better deal. We believe that the case of hold-up is interesting in its own right, but we have another reason for introducing it. In any model of asset ownership it is plausible that the parties can buy and sell assets if the relationship breaks down, i.e., some renegotiation is possible. But if this is so then a party can extract surplus from the other party by quitting and retrading assets. In other words some form of hold-up is possible. In this sense we believe that the model of this paper, in which renegotiation is allowed, is a natural extension of Hart and Moore (2008) to study asset ownership. In the

conclusions we will return to this issue, and briefly discuss what happens when asset retrading is not possible.

The paper is organized as follows. In Section 2 we lay out the basic model and in Section 3 we introduce assets. Section 4 contains extensions and concluding remarks. The Appendix studies a more general class of contracts and contains proofs of propositions.

## 2. The Model

We consider a buyer B and a seller S who are engaged in a long-term relationship. The parties meet at date 0 and can trade a widget at date 1. There is uncertainty at date 0, but this is resolved shortly before date 1, at date 1-, say. There is symmetric information throughout and the parties are risk neutral and face no wealth constraints. Each party has an outside option that he (or she) earns if trade does not occur. Let  $v, c$  denote B's value and S's cost if trade proceeds smoothly (i.e., the parties cooperate at date 1), and let  $r_b, r_s$  denote B and S's outside options. The random variables  $v, c, r_b, r_s$  are uncertain at date 0, and, although they are supposed to be observable to both parties at date 1- once they are realized, they are not verifiable.

We follow Hart and Moore (2008) in supposing that for the gains from trade to be fully realized each party must take a number of "helpful" or "cooperative" actions at date 1. These actions cannot be specified in a date 0 contract because they are too complicated to describe in advance. When the uncertainty is resolved at date 1-, some of these actions become describable and so can be contracted on, while others are never contractible.<sup>3</sup> Thus some modification or

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<sup>3</sup> An example of an ex post contractible helpful action would be where one party allows the other party to modify the characteristics of the good to be traded, e.g., the delivery time. An example of a noncontractible (or less contractible) helpful action would be where one party provides useful information to the other party, consults with them before making a decision, or responds to their phone calls or emails expeditiously. For evidence that the same contractible action can lead to different performance outcomes (depending on ownership structure), suggesting that noncontractible actions are important, see Januszewski Forbes and Lederman (2007).

renegotiation of the contract is possible at date 1-. We assume that all the helpful actions are chosen simultaneously by B and S at date 1. See Figure 1 for a time line.

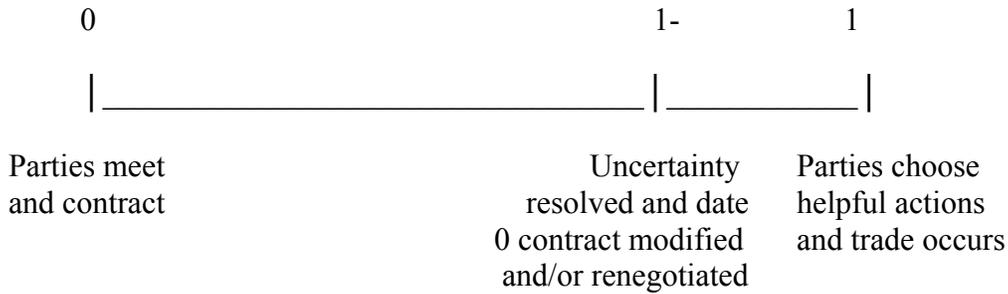


Figure 1

We make the following assumptions:

- (A1) If at date 1 all helpful actions are taken, the value of the widget to B is  $v$  and the cost to S is  $c$ , where  $v > c$ . Hence net surplus =  $v - c$  in this case.
- (A2) If at date 1 all the contractible, but none of the noncontractible, helpful actions are taken, the value of the widget to B is  $v - \frac{1}{2} \lambda (v - c)$  and the cost to S is  $c + \frac{1}{2} \lambda (v - c)$ , where  $0 < \lambda < 1$ . Hence net surplus =  $(1 - \lambda)(v - c)$  in this case.
- (A3) If at date 1 none of the helpful actions (contractible or otherwise) is taken, B's value is very low (approximately,  $-\infty$ ) and S's cost is very high (approximately,  $+\infty$ ). In this case each party walks away from the contract (neither party has an incentive to enforce it) and no trade occurs; that is, the parties earn their outside options. (We normalize the no trade price to be zero.)

Note that the import of (A2) is that withholding noncontractible helpful actions moves  $v$  and  $c$  in the direction of  $\frac{1}{2}(v + c)$ .

What determines whether a party is helpful? As in Hart and Moore (2008), we suppose that being helpful does not cost significantly more than not being helpful: either it costs slightly more or it costs slightly less, that is, a party may actually enjoy being helpful. To simplify matters, we assume that a party is completely indifferent between being helpful and not.

Given this indifference we take the view that a party will be willing to be helpful if (and only if) he is “well treated” by the other party.<sup>4</sup> Importantly, as in Hart and Moore (2008), a party is “well treated” if he receives what he feels entitled to under the contract. In particular, the date 0 contract is a reference point for entitlements: among other things, it is regarded as fair, e.g., because it is negotiated under competitive conditions. Implicitly we are assuming a more competitive situation at date 0 than at date 1-; this could be because the parties make (unmodeled, contractible) relationship-specific investments after date 0.

It is useful to begin with the case where the parties write a “simple” contract at date 0 that specifies a single trading price  $p$  (recall that the no trade price is zero). Consider what happens at date 1-, once the uncertainty is resolved. Each party has a choice. He can stick to the contract. Or he can try to force the other party to renegotiate the contract – we interpret this as “hold-up.”

Let’s consider the scenario where the parties stick to the contract. Under these conditions, each party feels well-treated by the other party since he is getting exactly what the contract said he would. Thus each party is willing to be helpful, and all cooperative actions are undertaken. The buyer and seller’s payoff are, respectively,

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<sup>4</sup> There is a large amount of empirical and experimental evidence in support of reciprocal behavior. See the references in Hart and Moore (2008). Works that are particularly relevant for our context include Bewley (1999) and Kube, Maréchal and Puppe (2007).

$$(1) U_b = v - p,$$

$$(2) U_s = p - c.$$

In the second scenario, one party engages in hold-up: that is, he tries to force the other party to renegotiate the contract. He does this by threatening not to undertake any helpful actions unless he receives a sidepayment. We assume that such behavior is viewed as a hostile act by the victim— it is a breach of the spirit of the date 0 contract – and leads, in the first instance, to the end of cooperation. The result is a Nash equilibrium where neither party cooperates. This yields the no-trade outcome described in (A3), with payoffs  $r_b$ ,  $r_s$  for B, S, respectively.

However, renegotiation is possible.<sup>5</sup> Even if the relationship is soured the parties can and will agree to undertake the helpful contractible actions at date 1. At the same time, it is supposed that neither party will provide noncontractible cooperation (again this is a Nash equilibrium). In effect the parties have a cold but correct relationship.<sup>6</sup> Renegotiation therefore yields surplus  $(1 - \lambda)(v - c)$  by (A2). Thus if

$$(A4) \quad (1 - \lambda)(v - c) > r_b + r_s,$$

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<sup>5</sup> In fact we suppose that the parties cannot commit ex ante not to renegotiate. In Section 4 we discuss whether such a commitment would be desirable.

<sup>6</sup> Note that we suppose that the parties cannot negotiate around this coldness. One could imagine that the buyer, anticipating that the seller is about to hold him up, would make a sidepayment to the seller to deflect the hold-up and preserve the relationship. We take the view that, given that there is a perceived threat in the background, the relationship is poisoned nonetheless. For some evidence supporting this view, and suggesting that even big players act in this way, see New York Times, August 27, 2007, p.A1 (reporting on the fact that the renegotiation of a large buy-out deal of a Home Depot business, triggered by the 2007 subprime mortgage crisis, created considerable bad feeling).

the parties will renegotiate away from the no-trade outcome. We will assume that (A4) holds in what follows, but we will discuss in Section 4 what happens if (A4) is relaxed.

In summary, hold-up leads to a loss of surplus of  $\lambda(v-c)$ .

We suppose a 50:50 split of the surplus from renegotiation, so after hold-up the parties' payoffs are

$$(3) U_b = r_b + \frac{1}{2} G,$$

$$(4) U_s = r_s + \frac{1}{2} G,$$

where

$$(5) G = (1 - \lambda) (v - c) - r_b - r_s.$$

It is easy to determine when hold-up occurs. Define  $p_L$  to be the price  $p$  such that S is indifferent between receiving  $p$  and holding B up, and  $p_H$  to be the price such that B is indifferent between paying  $p$  and holding S up. Then from (1) – (4)

$$(6) p_L - c = r_s + \frac{1}{2} G,$$

$$(7) v - p_H = r_b + \frac{1}{2} G,$$

and so

$$(8) p_L = c + r_s + \frac{1}{2} G,$$

$$(9) p_H = v - r_b - \frac{1}{2} G.$$

Note that

$$(10) p_H - p_L = \lambda (v-c) > 0.$$

(10) reflects the fact that there is some friction in the renegotiation process. If hold-up did not lead to the souring of the relationship,  $\lambda$  would be zero, and  $p_H = p_L$ . However, since hold-up causes some dissipation of surplus,  $p_H > p_L$ : the price at which B is just ready to hold up S is strictly greater than the price at which S is just ready to hold up B.

Since S is indifferent between holding B up and not at  $p = p_L$ , S will strictly prefer to hold up B when  $p < p_L$ . Similarly, B will strictly prefer to hold up S when  $p > p_H$ . Thus hold-up is avoided if and only if

$$(11) p_L < p < p_H.$$

Note that  $p_H, p_L$  varies with the state of the world, whereas  $p$  is chosen ex ante. The situation is illustrated in Figure 2. The interval  $[p_L, p_H]$  can be interpreted as the “self-enforcing” contractual range (see Klein(1996) ). Masten (1988) derives a similar “self-enforcing” price range under the assumption that there is an exogenous cost of hold-up.



$$(15) \quad p_L(\omega) = \frac{1}{2} [\alpha_s + \gamma_s \eta - \alpha_b - \varphi - \gamma_b \varepsilon + ((1 - \lambda) - \beta_b) v + ((1 + \lambda) - \beta_s) c],$$

$$(16) \quad p_H(\omega) = \frac{1}{2} [\alpha_s + \gamma_s \eta - \alpha_b - \varphi - \gamma_b \varepsilon + ((1 + \lambda) - \beta_b) v + ((1 - \lambda) - \beta_s) c].$$

Clearly  $p_L, p_H$  are monotonic in  $v, c$ . Since hold-up occurs when  $p_L(\omega) > p$  or  $p_H(\omega) < p$ , that is, when  $p_L(\omega)$  is high or  $p_H(\omega)$  is low, it follows that hold-up occurs, *ceteris paribus*, if  $v$  is exceptionally high or low or  $c$  is exceptionally high or low. This is intuitive: if  $v$  is high,  $S$  does well in the renegotiation process since there is a lot of surplus on the table ( even taking into account that a fraction  $\lambda$  gets lost ), and so  $S$  has an incentive to hold  $B$  up; similarly, if  $c$  is low,  $B$  does well in the renegotiation process and so  $B$  has an incentive to hold  $S$  up. In addition it is clear from (15) – (16) that the effect of exceptional values of  $v, c$  is less pronounced if  $\beta_b$  and  $\beta_s$  are large since  $p_L, p_H$  are less sensitive to  $v, c$  under these conditions.

We turn now to an optimal simple contract. Since date 0 lump-sum transfers can be used to allocate surplus, an optimal contract maximizes expected net surplus. Thus an optimal simple contract solves:

$$(17) \quad \left\{ \begin{array}{l} \text{Max} \\ p \end{array} \left\{ \begin{array}{l} \int (v - c) dF(\omega) + \int (1 - \lambda)(v - c) dF(\omega) \\ p_L(\omega) \leq p \leq p_H(\omega) \\ p < p_L(\omega) \\ \text{or } p > p_H(\omega) \end{array} \right. \right\},$$

where  $F$  is the distribution function of  $\omega$ .

It is clear that the first-best can be achieved in the case of certainty: just pick any price  $p$  in the interval  $[p_L(\omega_0), p_H(\omega_0)]$ , where  $\omega_0$  is the state of the world. However, the first-best

typically cannot be achieved under uncertainty since it is not generally possible to find a single price that lies in the intersection of a number of different  $[p_L(\omega), p_H(\omega)]$  intervals.

Our analysis so far has a shortcoming. Suppose that the parties write a simple contract at date 0. Then, as we have observed, with uncertainty it is very likely that  $p$  will lie outside the  $[p_L(\omega), p_H(\omega)]$  range for some  $\omega$ , and so one party will hold up the other to get a better price. Why don't the parties anticipate this and build the renegotiated price into the original contract? For example, the initial contract could state that price will normally be 10, but can rise to 20 in exceptional circumstances. Or the contract might give one party the right to choose price from a menu of prices.

To deal with (some of) these possibilities, in the Appendix we broaden the analysis to allow the parties to specify a range of possible trading prices  $[p, \bar{p}]$  in their date 0 contract. In each state of the world the parties negotiate about which price to pick in the range. As long as they stay in the range hold-up is avoided. Thus the benefit of a large price range is that it reduces hold-up. However, as in Hart and Moore (2008) a large price range has a cost. We suppose that at date 1- each party may feel entitled to a different price in the range. Not getting what you are entitled to leads to aggrievement and “shading”, i.e., the partial withholding of cooperation. Note that without the partial withholding of cooperation the first-best could be achieved with a price range  $p = -\infty, \bar{p} = \infty$ : since no restrictions are put on price this is equivalent to postponing contracting until date 1-. Recall that there are no ex ante (noncontractible) investments in our model, and information is symmetric, and so the usual arguments for long-term contracting do not apply.

However, if aggrievement and shading exist, then a very large price range is suboptimal since it leads to too little cooperation in “normal” states of the world. Thus a limited price range

is optimal, a fixed price contract being an extreme example of this. In the appendix we show that all the results of this and the next section generalize to the case of price ranges. We also show that introducing price ranges can elucidate Goldberg and Erickson's (1987) observation that parties write shorter-term contracts in a more volatile environment.

We close this section by noting that our model is consistent with Goldberg and Erickson's (1987) and Joskow's (1985) findings that price indexation is a common feature of contracts between suppliers and purchasers of petroleum coke, and between coal mines and electricity generating plants. Although risk aversion is a possible explanation, these authors argue that it is more likely that price indexation is used to reduce opportunistic behavior. Our model is consistent with this. By indexing the trading price  $p$  to a verifiable signal  $\sigma$  that is correlated with  $\omega$ , the parties can more easily ensure that  $p \in [p_L(\omega), p_H(\omega)]$ . This reduces hold-up and increases efficiency.

### 3. Asset Ownership

In this section we explore the idea that asset ownership can improve the parties' trading relationship. We take a simple view of asset ownership. Asset ownership matters because it determines which assets each party can walk away with if trade does not occur.<sup>7</sup> This in turn affects parties' outside options and their incentives to engage in hold-up.

Denote by  $A$  the set of all assets at B and S's disposal; we assume  $A$  is fixed and finite.<sup>8</sup> Let  $A_b$  be the set of assets B owns and  $A_s$  the set of assets S owns. We suppose

$$(18) \quad A_b \cap A_s = \emptyset, A_b \cup A_s \subseteq A.$$

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<sup>7</sup> As in Hart and Moore (1990). See also Grossman and Hart (1986).

<sup>8</sup> We suppose that the assets in  $A$  are already specialized, and so cannot be bought or sold on the open market.

The first part of (18) says that B and S can't walk away with the same asset. The inclusion in the second part reflects the possibility that if an asset is jointly owned neither party can walk away with it: that is, joint ownership gives each party a veto right on its use.<sup>9</sup>

We now suppose that the coefficients  $\alpha_b, \beta_b, \gamma_b, \alpha_s, \beta_s, \gamma_s$  depend on asset ownership. In particular,  $\alpha_b = \alpha_b(A_b), \beta_b = \beta_b(A_b), \gamma_b = \gamma_b(A_b), \alpha_s = \alpha_s(A_s), \beta_s = \beta_s(A_s), \gamma_s = \gamma_s(A_s)$ . We also make assumptions similar to those in the property rights literature (see Grossman and Hart (1986) and Hart and Moore (1990)) about how these coefficients vary with asset ownership. In particular, we assume that owning more assets increases the marginal payoffs of  $r_b, r_s$ , with respect to  $v$  and  $c$ . That is,

(19)  $\beta_b$  is nondecreasing in  $A_b$ ,

(20)  $\beta_s$  is nondecreasing in  $A_s$ .

Of course, one might also suppose that a party's total payoff increases in the assets he or she owns ( $\alpha_b, \alpha_s$  increasing in  $A_b, A_s$ ), but we will not need this in what follows. We assume that (14) and (A4) hold for all ownership structures.

We suppose that assets can be traded at date 0. Thus a contract is now a 3-tuple  $(A_b, A_s, p)$ , specifying an asset ownership allocation  $(A_b, A_s)$  and a price  $p$ , where  $A_b, A_s$  satisfy (18). An optimal contract solves:

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<sup>9</sup> We confine attention to simple ownership structures. It would not be difficult to generalize the analysis to allow for options to own, etc.

$$(21) \quad \text{Max}_{(A_b, A_s, p)} \left\{ \begin{array}{l} \int (v - c) dF(\omega) + \int (1 - \lambda)(v - c) dF(\omega) \\ p_L(\omega; A_b, A_s) \leq p \leq p_H(\omega; A_b, A_s) \quad \begin{array}{l} p < p_L(\omega; A_b, A_s) \\ \text{or } p > p_H(\omega; A_b, A_s) \end{array} \end{array} \right\},$$

where  $p_L$ ,  $p_H$  are now indexed by the asset ownership allocation  $(A_b, A_s)$ , as well as by  $\omega$ .

We begin our analysis of asset ownership by considering what happens if, *ceteris paribus*, assets are transferred at date 0 from S to B. Then, given (19) – (20),  $\beta_b$  rises and  $\beta_s$  falls. As is clear from (15) – (16), this makes  $p_L$  and  $p_H$  less sensitive to  $v$  than before since

$$(22) \quad \frac{\partial p_L}{\partial v} = \frac{1}{2} ((1 - \lambda) - \beta_b),$$

$$(23) \quad \frac{\partial p_H}{\partial v} = \frac{1}{2} ((1 + \lambda) - \beta_b),$$

and these both decrease. On the other hand,  $p_L(\omega)$  and  $p_H(\omega)$  become more sensitive to  $c$  since

$$(24) \quad \frac{\partial p_L}{\partial c} = \frac{1}{2} ((1 + \lambda) - \beta_s),$$

$$(25) \quad \frac{\partial p_H}{\partial c} = \frac{1}{2} ((1 - \lambda) - \beta_s),$$

and these both increase.

Intuitively, a reduction in sensitivity of  $p_L$ ,  $p_H$  is good since, if the interval

$[p_L, p_H]$  does not vary much, it is easier to find a price  $p$  that lies in  $[p_L, p_H]$  for many  $\omega$ . That is, hold-up is less likely. This suggests that it is optimal for B to own all the assets if only  $v$  varies, since this minimizes the sensitivity of  $p_L$  and  $p_H$  with respect to the state of the world; while, it is optimal for S to own all the assets if only  $c$  varies. Proposition 1 confirms this.

Proposition 1. (1) Suppose that  $\varphi = \varepsilon = \eta \equiv 0$ , and  $c \equiv c_0$  where  $c_0$  is a constant. Then there exists an optimal contract in which B owns all the assets, i.e.,  $A_b = A, A_s = \phi$ .

(2) Suppose that  $\varphi = \varepsilon = \eta \equiv 0$  and  $v \equiv v_0$ , where  $v_0$  is a constant. Then there exists an optimal contract in which S owns all the assets, i.e.,  $A_s = A, A_b = \phi$ .

In the Appendix we prove a more general version of Proposition 1, and of all the other propositions in this section. In these more general versions, a range of possible trading prices is allowed in the date 0 contract.

Note that it would not be difficult to establish uniqueness in Proposition 1 under slightly stronger stochastic assumptions.

Proposition 1 is reminiscent of the result in the property rights literature that one party should own all the assets if his investment is important. Here the conclusion is that one party should own all the assets if his payoff is uncertain. Proposition 1 is also similar to results that have been obtained by Simon (1951) and Wernerfelt (1997), among others, showing that the party with a more variable payoff should be the boss.

Of course, in general, both  $v$  and  $c$  vary. Proposition 1 is not very helpful here since it tells us only when one party should own everything. However, some progress can be made if we introduce the idea of an idiosyncratic asset.

Define an asset to be idiosyncratic to B if B's owning it increases the sensitivity of  $r_b$  to  $v$  and S's not owning it has no effect on the sensitivity of  $r_s$  to  $c$ . Define an asset to be idiosyncratic to S similarly.

**Definition** (i) Asset  $a$  is idiosyncratic to B if  $\beta_b(A_b \cup \{a\}) > \beta_b(A_b)$  for all  $A_b \subseteq A$ ,  $A_b \cap \{a\} = \emptyset$ , and  $\beta_s(A_s \cup \{a\}) = \beta_s(A_s)$  for all  $A_s \subseteq A$ .

(ii) Asset  $a$  is idiosyncratic to S if  $\beta_s(A_s \cup \{a\}) > \beta_s(A_s)$  for all  $A_s \subseteq A$ ,  $A_s \cap \{a\} = \emptyset$ , and  $\beta_b(A_b \cup \{a\}) = \beta_b(A_b)$  for all  $A_b \subseteq A$ .

Note that one reason an asset may be idiosyncratic to a party is if that party has human capital that is complementary to the asset, e.g., he is the only person who knows how to operate it. Under these conditions taking away the asset from the other party is unlikely to affect the sensitivity of that party's outside option to the state of the world.

Allocating an asset to the party to whom it is idiosyncratic would seem desirable since it reduces payoff variability. However, it turns out that to establish this one must make strong assumptions about the stochastic structure. In the next proposition we suppose that with high probability  $v$ ,  $c$  take on "normal" values  $v = v_0$ ,  $c = c_0$ , while with small probability  $v$ ,  $c$  can each take on an "exceptional" value. Since exceptional values are unusual, we ignore the possibility that  $v$  and  $c$  can take on an exceptional value at the same time. We also suppose that there is a small amount of exogenous noise through the random variable  $\phi$ , but we set  $\varepsilon = \eta = 0$ .

**Proposition 2.** Assume that  $\varepsilon = \eta = 0$  and  $\phi$  is uniformly distributed on  $[-k, k]$ . Suppose that with probability  $0 < \pi < 1$  event 1 occurs:  $v = v_0$ ,  $c = c_0$ ; with probability  $(1 - \pi) \alpha_v$  event 2 occurs:  $c = c_0$ ,  $v$  has support  $[v_L, v_H]$ , where  $v_L \leq v_0$  and  $v_H \geq \frac{-\beta_b(A)v_0 + (1 + \lambda)v_0 - 2\lambda c_0}{1 - \lambda - \beta_b(A)}$ ; with probability

$(1 - \pi) \alpha_c$  event 3 occurs:  $v = v_0$ ,  $c$  has support  $[c_L, c_H]$ , where  $c_H \geq c_0$  and  $c_L \leq$

$\frac{-\beta_s(A)c_0 - 2\lambda v_0 + (1 + \lambda)c_0}{1 - \lambda - \beta_s(A)}$ . Here  $\alpha_v > 0$ ,  $\alpha_c > 0$ ,  $\alpha_v + \alpha_c = 1$ ,  $k > 0$ , and  $\varphi$  is independent of  $v$  and  $c$  in

events 2 and 3 respectively. Then for small enough  $k$  the following is true: if  $\pi$  is close to 1 it is uniquely optimal for B to own asset  $a$  if  $a$  is idiosyncratic to B and for S to own asset  $a$  if  $a$  is idiosyncratic to S.

Proof. See Appendix

It is useful to understand why Proposition 2 requires such strong assumptions about the probability distribution of  $v$  and  $c$ . The reason is the following. Let  $p$  be the optimal price for the general case where  $v, c$  are uncertain. Suppose that we transfer an asset that is idiosyncratic to B from S to B. (In what follows we suppress assets in the notation.) We know that this will reduce the variability of  $p_L(\omega), p_H(\omega)$  with respect to  $v$ . But this might reduce the probability that  $p$  lies in  $[p_L(\omega), p_H(\omega)]$  if, say,  $p \in [p_L(\omega_1), p_H(\omega_1)]$ ,  $p \notin [p_L(\omega_2), p_H(\omega_2)]$  and  $[p_L(\omega_1), p_H(\omega_1)]$  moves closer to  $[p_L(\omega_2), p_H(\omega_2)]$ . The stochastic structure in Proposition 2 avoids this kind of situation.

A simple application of Proposition 2 is to the case of strictly complementary assets. Suppose assets  $a_1$  and  $a_2$  are strictly complementary, in the sense that they are valuable only when used together. Then  $a_2$  by itself is of no use to S, while  $a_1$  and  $a_2$  together may be very useful to B. Assume B owns  $a_1$ . Then we can define a new economy in which  $a_1$  is inalienable, i.e., B always owns  $a_1$ , and the effective set of (alienable) assets is  $A \setminus \{a_1\}$ . For this economy,  $a_2$  is idiosyncratic to B in the sense of Definition (i). Hence, according to Proposition 2, it is better for B to own  $a_1$  and  $a_2$ . The same argument shows that if S owns  $a_1$ , it is better for S to own both. The conclusion is that strictly complementary assets should be owned together (by B or S –

without further information we cannot say which). A similar argument shows that joint ownership is suboptimal under the conditions of Proposition 2.

Of course, these results are very reminiscent of those obtained in the property rights literature (see particularly Hart and Moore (1990)). However, the driving force is different: uncertainty rather than ex ante investments.

So far we have emphasized the idea that ownership of an asset is good for one party because it reduces the variability of their payoff relative to their outside option. However, it is possible that asset ownership increases the variability of outside options relative to inside values. Under these conditions it may be better to take assets away from people. The next proposition describes a situation where joint ownership is optimal. In this proposition inside values  $v$  and  $c$  are constant while outside options are stochastic; in particular,  $\phi$  and  $\varepsilon$  or  $\eta$  vary.

Proposition 3. Assume  $\gamma_b, \gamma_s$  are strictly increasing in  $A_b, A_s$  respectively, and  $\phi$  is uniformly distributed on  $[-k, k]$ . Suppose that with probability  $0 < \pi < 1$  event 1 occurs:  $v = v_0, c = c_0, \varepsilon = 0, \eta = 0$ ; with probability  $(1 - \pi) \alpha_\varepsilon$  event 2 occurs:  $v = v_0, c = c_0, \eta = 0, \varepsilon$  has support  $[\varepsilon_L, \varepsilon_H]$ , where  $\varepsilon_L \leq \frac{2\lambda(c_0 - v_0)}{\gamma_b(\phi)}$  and  $\varepsilon_H > 0$ ; with probability  $(1 - \pi) \alpha_\eta$  event 3 occurs:  $v = v_0, c = c_0, \varepsilon = 0, \eta$  has support  $[\eta_L, \eta_H]$ , where  $\eta_L \leq \frac{2\lambda(c_0 - v_0)}{\gamma_s(\phi)}$  and  $\eta_H > 0$ . Here  $\alpha_\varepsilon \geq 0, \alpha_\eta \geq 0, \alpha_\varepsilon + \alpha_\eta = 1, k > 0$ , and  $\phi$  is independent of  $\varepsilon$  and  $\eta$  in events 2 and 3 respectively. Then for small enough  $k$  the following is true: if  $\pi$  is close to 1 it is uniquely optimal for all assets to be jointly owned by B and S.

Proof. See Appendix

Again similar results have been obtained in the property rights literature in situations where ex ante investments are used for “rent-seeking” purposes (see, e.g., Halonen (2002) and Rajan and Zingales (1998)).

#### 4. Conclusions

In this paper we have studied a buyer and seller who have a long-term relationship in an uncertain environment. A fixed price contract works well in normal times because there is nothing to argue about. However, if value or cost is exceptionally high or low, one party will have an incentive to hold up the other, which sours the relationship and leads to the end of cooperation. We have shown that contract indexation can help by making it more likely that price remains within the “self-enforcing” range. In addition a judicious allocation of asset ownership increases efficiency to the extent that it raises the correlation between parties’ outside options and their value inside the relationship; this reduces the likelihood of hold-up.

It is worth noting that in our model assets are equivalent to outside options. In practice there are other ways a party can guard against hold-up than through owning assets. For example, a buyer can choose a flexible technology so that it is easier to switch suppliers, or form a relationship with more than one seller at date 0, so that it can play them off against each other. Our analysis throws light on the desirability of these kinds of strategies as well.

We have argued that our analysis can help to understand the empirical contracting literature. Several papers in this literature emphasize the idea that parties face problems in their trading relationship when the gains from trade become unevenly divided under an existing contract. However, there is no developed theory of why such a situation leads to inefficiency. We are able to provide a theory by appealing to behavioral ideas. When one party is doing badly he

or she will hold up the other party, leading to hostility and deadweight losses from the end of cooperation. Moreover, the parties cannot bargain around these costs.

Our paper also throws light on the vertical integration literature. Our model, where costly hold-up typically occurs with positive probability under uncertainty, seems relevant for understanding the costs of nonintegration, and why vertical integration—interpreted as a transfer of assets—can improve matters. Furthermore we identify different factors from the literature in being important: payoff uncertainty rather than the size of payoffs (as in the transaction cost literature) or the sensitivity of payoffs to investments (as in the property rights literature). It is less clear that our model can be applied directly to understand the costs of integration. Suppose that the buyer, say, purchases all the assets of the seller, i.e., the seller becomes an employee. According to our model, in a state of the world where the seller's cost is low, i.e., her profit inside the relationship is high, her outside option will be lower than before given that she has no assets. This gives the buyer the chance to hold her up. However, it is a bit of a stretch to argue that an employee's effort costs—as opposed to a seller's production costs—vary so much exogenously that hold-up by a boss is a serious problem in practice. It seems more plausible that the seller's costs vary for endogenous reasons: an employee, fearing hold-up by a boss, is less likely to make an ex ante investment in cost reduction than an independent contractor.

In future work it would be desirable for these reasons as well as others to introduce (noncontractible) ex ante investments into the model. One can obtain some idea of how this would work as follows. Take the model of Sections 2 and 3. Suppose that it is more likely that the buyer's value  $v$  will be high if he has made a prior investment, and that it is more likely that the seller's cost will be low if she has made a prior investment. Then there may be no single price such that for all realizations of  $v$  and  $c$  hold-up is avoided. Not only will there be ex post

inefficiency, but also ex-ante investments will be distorted. Asset ownership helps not only to reduce ex post inefficiency but to improve investment incentives. The buyer will invest more when he owns more assets because he faces a lower threat of hold-up when  $v$  is high, and the seller will invest more when she owns more assets because she faces a lower threat of hold-up when  $c$  is low.

Of course, this sounds like the standard story in the property rights literature. The difference is that in that literature it is unclear why a fixed price contract makes sense in the first place. Why can't the parties do better with a flexible price contract or a "clever" mechanism? The benefit of the present approach is that, according to the extension of the model described in the Appendix, a flexible price contract or mechanism is costly: it causes argument and aggravation, leading to deadweight losses from "shading". Thus while a mechanism might be used it won't be eliminate the role of asset ownership.

There are several assumptions of our analysis that could usefully be relaxed. First, we have assumed that, even though hold-up causes deadweight losses, the parties prefer to continue trading with each other after hold-up rather than go their separate ways (assumption (A4)). Relaxing this assumption opens up interesting new possibilities. Two cases need to be distinguished. In the first, although the trading relationship is terminated after hold-up, the parties can costlessly renegotiate asset ownership (this is similar to the assumption made in Baker et al. (2002)). Under these conditions, the surplus obtained after hold-up is independent of the date 0 assignment of assets, which is the critical feature of (A4). Thus our results are unlikely to change significantly. The second case is where there are frictions in the asset renegotiation process (perhaps because it is difficult even to trade assets after a relationship is poisoned), in which case the date 0 ownership structure will affect ex post surplus. Under these conditions the

initial assignment of assets matters not only in determining when hold-up occurs, but also in affecting ex post surplus when it does. This opens the door to a richer theory of ownership.<sup>10</sup>

It would also be worth relaxing our assumption that hold-up sours the parties' relationship by the same amount regardless of the reasons for it. Consider two cases of seller hold-up. In one the seller holds the buyer up because the buyer's value has increased. In the other the seller holds up the buyer because the seller's cost has increased. Even though price rises in both cases it is arguable that the seller's behavior is less opportunistic in the second case, and so the buyer will be less angry, perhaps because without the price increase the seller's profits would be negative and she would quit. We have taken the contrary view that the buyer will view the cases as the same on the grounds that if the seller wanted the price to rise under some conditions she should have built this into the initial contract (see the model of the Appendix). However, it seems useful to develop a model where the two cases are treated differently.

In most models of incomplete contracts, if the parties can commit not to renegotiate their contract, the first-best is achieved. This is not true here. In the absence of renegotiation hold-up will never occur since it is impossible to change the terms of trade. However, the parties will have an incentive to quit, by refusing to cooperate, whenever their outside option exceeds their payoff inside the relationship. In particular, the buyer will quit when  $v - p < r_b$  and the seller will quit when  $p - c < r_s$ . (This is similar to the case analyzed in Hart and Moore (2008).) There will still be inefficiency—in fact in some cases it will be greater. Asset ownership will matter although the effects will be different. In fact the model is likely to resemble the second case

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<sup>10</sup> For a start in this direction, see Hart and Moore (2007).

discussed above in which ( A4) doesn't hold, since, as in that case, asset ownership will affect ex post surplus in the event that the relationship breaks down.<sup>11</sup>

An obvious question to ask is: to what kind of firm does the model apply? One point of view is that the feelings of hostility and aggrievement that we have emphasized are plausible in the case of small owner-managed firms, but less so in the case of large corporations. We are not sure that this is entirely true. Large corporations are run by individuals who have big egos and presumably therefore can have strong emotions. In fact the Home Depot case, described in footnote 6, where contract renegotiation caused coldness, is one involving a large company. Still it is undoubtedly the case that the effects we describe are likely to be different in a company where many decisions are delegated. In fact one interesting trade-off is that it might be good for a subordinate to make contract decisions given that he does not care about hold-up ( e.g., because he is on low-powered incentives ), and hence this will reduce ex post deadweight losses, but it might be bad because he will negotiate an unfavorable deal in the first place.

Of course, the question of how decisions are delegated in large firms is a much more general one. Most models to date in the incomplete contracts literature, including this one and the companion model in Hart and Moore (2008), involve only two parties. Generalizing the model to allow for multiple parties, with an eye to studying the internal organization of firms, is an important and challenging goal for future research.

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<sup>11</sup> An interesting possibility is that, rather than committing not to renegotiate, the parties might try to constrain the renegotiation process in their date 0 contract. For example, they could put in place an information revelation game to be played if the relationship breaks down. Note, however, that one point of view is that incorporating a renegotiation process in the initial contract enlarges the set of feasible outcomes and hence increases parties' feelings of entitlement in the event that the relationship does not break down. This raises aggrievement and shading costs, as in the model of the Appendix.

## Appendix

We first present a discussion of price ranges. We then prove Propositions 1-3.

As we noted in Section 2, anticipating the possibility of hold-up, the parties can build some price flexibility into their contract. Following Hart and Moore (2008), we therefore now allow the parties to specify a range of possible trading prices  $[\underline{p}, \bar{p}]$  in their date 0 contract; the idea is that the parties will agree on a price in  $[\underline{p}, \bar{p}]$  once the uncertainty is resolved at date 1-

Suppose that the parties pick the range  $[\underline{p}, \bar{p}]$ . What happens at date 1-, once  $\omega$  is realized? There are two cases. Define

$$H(\omega, \underline{p}, \bar{p}; A_b, A_s) = [p_L(\omega; A_b, A_s), p_H(\omega; A_b, A_s)] \cap [\underline{p}, \bar{p}].$$

If  $H \neq \emptyset$  the parties can avoid hold up by choosing a price in  $H$ : such a price is consistent with the date 0 contract and gives neither party an incentive to hold up the other. However, if  $H = \emptyset$  hold-up cannot be avoided.

Start with the first case. Even though no hold-up occurs, there will be disagreement about the appropriate outcome within the contract. In the spirit of Hart and Moore (2008), we suppose that each party is aggrieved to the extent that he does not receive what he feels entitled to, and will respond by shading, i.e., cutting back on helpful actions. We assume that each party feels entitled to the best outcome permitted by the contract. However, each party recognizes that he faces the feasibility constraint that the other party can trigger hold-up, i.e., B doesn't expect to pay less than  $p_L$  or  $S$  to receive more than  $p_H$ . In other words B feels entitled to the lowest price in  $H$ ,  $\text{Max}(p_L, \underline{p})$ , and S feels entitled to the highest price in  $H$ ,  $\text{Min}(p_H, \bar{p})$ . Note that the assumption that entitlements are constrained by what is feasible simplifies the analysis, but is not crucial.

To simplify matters we assume that the parties split the difference and set

$$(26) \hat{p} = \frac{1}{2} \{ \text{Max} (p_L (\omega; A_b, A_s), \underline{p}) + \text{Min} (p_H (\omega; A_b, A_s), \bar{p}) \}.$$

As part of this deal the parties agree to undertake the contractible helpful actions. However, each party cuts back on the noncontractible helpful actions, in proportion to his aggrievement. B is aggrieved by

$$(27) a_b = \hat{p} - \text{Max} (p_L (\omega; A_b, A_s), \underline{p}),$$

and shades to the point where S's payoff falls by  $\theta a_b$ . S is aggrieved by

$$(28) a_s = \text{Min} (p_H (\omega; A_b, A_s), \bar{p}) - \hat{p},$$

and shades to the point where B's payoff falls by  $\theta a_s$ . The parameter  $\theta$  is taken to be exogenous and the same for B and S, and  $0 < \theta \leq 1$ .

Thus in Case 1, where  $H \neq \phi$ , net surplus is

$$(29) W_1 (\omega, \underline{p}, \bar{p}; A_b, A_s) = v - c - \theta(a_b + a_s) = v - c - \theta \{ \text{Min} (p_H (\omega; A_b, A_s), \bar{p}) - \text{Max} (p_L (\omega; A_b, A_s), \underline{p}) \}.$$

In contrast, in Case 2, where  $H = \phi$ , hold-up occurs, followed by renegotiation, and net surplus is given by

$$(30) \quad W_2(\omega) = (1 - \lambda)(v - c).$$

Note that since hold-up leads to the withdrawal of all noncontractible helpful actions, while shading leads to the withdrawal of only some of them, there is an implicit constraint that total shading costs cannot exceed total hold-up costs, i.e.,

$$(31) \quad W_1(\omega, \underline{p}, \bar{p}; A_b, A_s) \geq W_2(\omega).$$

Fortunately, this constraint is automatically satisfied, given  $0 < \theta \leq 1$ , since

$$(32) \quad \begin{aligned} \text{Min } (p_H(\omega; A_b, A_s), \bar{p}) - \text{Max } (p_L(\omega; A_b, A_s), \underline{p}) \\ \leq p_H(\omega; A_b, A_s) - p_L(\omega; A_b, A_s) \\ = \lambda(v - c), \end{aligned}$$

where we are using (10). In other words, however large the price range  $[\underline{p}, \bar{p}]$  is, net surplus is higher if hold-up is avoided than if it occurs.<sup>12</sup>

An optimal contract maximizes expected net surplus. Thus an optimal contract solves:

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<sup>12</sup> In fact (A2) implies the further constraint that each party's shading costs cannot exceed  $\frac{1}{2}\lambda(v - c)$ . This is automatically satisfied given (26).

$$(33) \quad \text{Max}_{(A_b, A_s, \underline{p}, \bar{p})} \left\{ \begin{array}{l} \int W_1(\omega, \underline{p}, \bar{p}; A_b, A_s) dF(\omega) + \int W_2(\omega) dF(\omega) \\ H(\omega, \underline{p}, \bar{p}; A_b, A_s) \neq \emptyset \quad H(\omega, \underline{p}, \bar{p}; A_b, A_s) = \emptyset \end{array} \right\},$$

where  $F$  is the distribution function of  $\omega$ . (We assume that  $F$  has bounded support.)<sup>13</sup> The trade-off is the following. As  $\underline{p}$  falls or  $\bar{p}$  rises, the set  $H$  becomes larger and so hold-up is less likely. This is good given that hold-up reduces surplus, i.e.,  $W_1 \geq W_2$ . However, shading represented by  $\theta \{\text{Min}(p_H, \bar{p}) - \text{Max}(p_L, \underline{p})\}$  rises, which means that surplus in the absence of hold-up falls, i.e.,  $W_1$  is lower. This is bad.

It is useful to analyze the optimal contract in some simple cases. We have already seen in Section 2 that a simple contract can achieve the first-best if there is no uncertainty. It turns out that a non-simple contract can achieve the first-best if there are just two states:  $\omega = \omega_1$  or  $\omega_2$ . (In what follows we suppress assets.) To see why, note that there are two possibilities. Either  $[p_L(\omega_1), p_H(\omega_1)] \cap [p_L(\omega_2), p_H(\omega_2)]$  is non-empty or it is empty. In the first case, choose any price  $\hat{p}$  in the intersection and set  $\underline{p} = \hat{p} = \bar{p}$ . In the second case, suppose without loss of generality that

$$p_L(\omega_1) < p_H(\omega_1) < p_L(\omega_2) < p_H(\omega_2).$$

Then set  $\underline{p} = p_H(\omega_1)$ ,  $\bar{p} = p_L(\omega_2)$ . In state  $\omega_1$ ,  $p = p_H(\omega_1)$  and in state  $\omega_2$ ,  $p = p_L(\omega_2)$ . Hold-up is avoided and there is no aggrievement since the set  $H$  is a singleton in both states.

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<sup>13</sup> It is easy to show that an optimal contract exists since the objective function is upper semicontinuous in  $[\underline{p}, \bar{p}]$ .

Once there are three states it is easy to show that the first-best typically cannot be achieved even with a non-simple contract.

One obvious question that can be asked is: Can the parties do better than specify a price range? For example, would it help for them to play a message game that reveals the state of the world? We refer the reader to Hart and Moore (2008), who show that at least under some assumptions the answer is no.

A price range has several interpretations. One is the following. Suppose date 1 trade is not instantaneous: rather the parties trade over a period of time, e.g., between dates 1 and 2. (For simplicity, suppose uncertainty is still resolved at date 1-.) Then the parties might agree at date 0 on the length of their contract. For example, they might agree on a trading price that will operate for a fraction  $\tau$  of the period between dates 1 and 2. The parties recognize that when they get to date 1-, they will renegotiate the price for the remaining fraction  $(1 - \tau)$  of the period. A small value of  $\tau$  then corresponds to a flexible contract -- there is a wide range of possible (average) prices over the period from an ex ante perspective-- while a large value of  $\tau$  corresponds to a rigid contract.

This interpretation can throw light on Goldberg and Erickson (1987). Goldberg and Erickson find that parties in the petroleum coke industry tend to write shorter-term contracts in a more volatile environment. The parallel result in our model is that an increase in uncertainty leads to a larger price range  $[\underline{p}, \bar{p}]$ . Although further assumptions would be required to prove a general result along these lines, the result is at least true at the extremes. With no uncertainty the optimal contract is a single price  $p \in [p_L(\omega; A_b, A_s), p_H(\omega; A_b, A_s)]$ ; while with sufficiently large uncertainty the optimal contract will be a nondegenerate price interval since the chance that a

single price will lie in  $[p_L(\omega; A_b, A_s), p_H(\omega; A_b, A_s)]$  becomes vanishingly small. Thus in a broad sense our model seems consistent with Goldberg and Erickson (1987).

Proof of Proposition 1. We prove (1). Let  $(A_b, A_s, [\underline{p}, \bar{p}])$  be an optimal contract. The proof proceeds in two steps. We first replace  $[\underline{p}, \bar{p}]$  by another price interval, and show that shading costs fall (weakly). We then allocate all the assets to B and make another change in the price interval, and show that shading costs fall again and that the hold-up region becomes (weakly) smaller. Thus the new contract in which B owns everything must also be optimal.

Index the state by  $v$ . Let  $\underline{v}$  be the smallest value of  $v$ , and  $\bar{v}$  the largest of  $v$ , in the support of  $F$  such that no hold-up occurs under contract  $(A_b, A_s, [\underline{p}, \bar{p}])$ . Then

$$(34) \quad [p_L(v), p_H(v)] \cap [\underline{p}, \bar{p}] \neq \emptyset$$

for  $v = \underline{v}$  and  $v = \bar{v}$ . Since  $p_L(v), p_H(v)$  are increasing in  $v$ , (34) must also hold for  $\underline{v} \leq v \leq \bar{v}$ , i.e., hold-up does not occur for intermediate  $v$ 's. Note that (34) implies that  $p_H(\underline{v}) \geq \underline{p}$ ,  $p_L(\bar{v}) \leq \bar{p}$ .

Now define a new price interval  $[\underline{p}', \bar{p}']$ , where

$$\underline{p}' = p_H(\underline{v}) \text{ and } \bar{p}' = \text{Max}(p_L(\bar{v}), p_H(\underline{v})).$$

Clearly  $\underline{p}' \geq \underline{p}$ . Also either  $\bar{p}' \leq \bar{p}$  or  $\bar{p}' = \underline{p}'$ . In the first case the new price interval is a subset of the previous price interval. In the second case it is a singleton. In both cases

$$[p_L(v), p_H(v)] \cap [p', \bar{p}'] \neq \emptyset$$

for  $\underline{v} \leq v \leq \bar{v}$ . Hence the new price interval avoids hold-up for  $\underline{v} \leq v \leq \bar{v}$  just like the old one.

In addition aggrivement and shading costs are lower under the new price interval given that either the new price interval is a subset of the previous price interval or it is a singleton (in which case shading costs are zero).

Now assign all the assets to B, i.e., set  $A_b = A$ ,  $A_s = \emptyset$ . Call this the new ownership structure. Define a new price interval  $[p'', \bar{p}'']$ , given by

$$(35) \quad p'' = p_H^N(\underline{v}), \quad \bar{p}'' = \text{Max}(p_L^N(\bar{v}), p_H^N(\underline{v})),$$

where  $p_L^N, p_H^N$  represent the values of  $p_L, p_H$  under the new ownership structure. The price interval  $[p'', \bar{p}'']$  avoids hold-up under the new ownership structure when  $\underline{v} \leq v \leq \bar{v}$ .

We show next that shading costs are lower for each  $\underline{v} \leq v \leq \bar{v}$  under the new ownership structure and price interval  $[p'', \bar{p}'']$  than under the old ownership structure and  $[p', \bar{p}']$  (which in turn are lower than those under the old ownership structure and  $[p, \bar{p}]$ ). That is, we demonstrate that

$$(36) \quad \begin{aligned} & \text{Min}(p_H^N(v), \bar{p}'') - \text{Max}(p_L^N(v), p'') \\ & \leq \text{Min}(p_H(v), \bar{p}') - \text{Max}(p_L(v), p'). \end{aligned}$$

There are several cases to consider. Note first that if  $\underline{p}', \bar{p}' = p_H(\underline{v}) \geq p_L(\bar{v})$ , i.e., the right-hand side (RHS) of (36) is zero, then

$$\begin{aligned}
p_H^N(\underline{v}) - p_L^N(\bar{v}) &= - (p_H^N(\bar{v}) - p_H^N(\underline{v})) \\
&\quad + p_H^N(\bar{v}) - p_L^N(\bar{v}) \\
&\geq - (p_H(\bar{v}) - p_H(\underline{v})) \\
&\quad + p_H(\bar{v}) - p_L(\bar{v}) \\
&= p_H(\underline{v}) - p_L(\bar{v}) \\
&\geq 0,
\end{aligned}$$

where we are using the fact that  $p_H^N(\bar{v}) - p_H^N(\underline{v}) \leq p_H(\bar{v}) - p_H(\underline{v})$  since  $\frac{\partial p_H}{\partial v}$  falls the more assets B owns (by (23)), and  $p_H^N(\bar{v}) - p_L^N(\bar{v}) = p_H(\bar{v}) - p_L(\bar{v})$ , i.e.,  $p_H - p_L$  is independent of the ownership structure (see (10)). Hence  $p_H^N(\underline{v}) \geq p_L^N(\bar{v})$ . It follows from (35) that  $\underline{p}'' = \bar{p}'' = p_H^N(\underline{v})$  and so the left-hand side (LHS) of (36) is zero. Therefore (36) holds.

Consider next the case where  $\underline{p}' = p_H(\underline{v}) < \bar{p}' = p_L(\bar{v})$ . If  $\underline{p}'' = \bar{p}'' = p_H^N(\underline{v}) \geq p_L^N(\bar{v})$ , (36) again holds. So suppose  $\underline{p}'' = p_H^N(\underline{v}) < p_L^N(\bar{v}) = \bar{p}''$ . We must show that

$$\begin{aligned}
(37) \quad & \text{Min}(p_H^N(\underline{v}), p_L^N(\bar{v})) - \text{Max}(p_L^N(\underline{v}), p_H^N(\bar{v})) \\
& \leq \text{Min}(p_H(\underline{v}), p_L(\bar{v})) - \text{Max}(p_L(\underline{v}), p_H(\bar{v})).
\end{aligned}$$

We can rewrite (37) as

$$(38) \quad \text{Min} \{p_H^N(v) - p_H^N(\underline{v}), p_L^N(\bar{v}) - p_H^N(\underline{v}), p_H^N(v) - p_L^N(v), p_L^N(\bar{v}) - p_L^N(v)\} \leq \text{Min} \{p_H(v) - p_H(\underline{v}), p_L(\bar{v}) - p_H(\underline{v}), p_H(v) - p_L(v), p_L(\bar{v}) - p_L(v)\}.$$

To establish (38) one shows that each component in the min formula on the LHS of (38) is no greater than the corresponding component on the RHS of (38). This follows from the fact that

$$\frac{\partial p_H}{\partial v}, \frac{\partial p_L}{\partial v} \text{ are nonincreasing in the assets that B owns, and that } p_H - p_L \text{ is independent of}$$

ownership structure for a given  $v$ . Hence (37) holds and so does (36).

In summary, the new ownership structure (in which B owns all the assets) and price range  $[p'', \bar{p}'']$  yields (weakly) lower shading costs than the original ownership structure and price range  $[p, \bar{p}]$ . Also the hold-up region is no larger (hold-up does not occur for  $\underline{v} \leq v \leq \bar{v}$ ). This shows that allocating all the assets to B is optimal. Q.E.D.

Proof of Proposition 2. Suppose  $a$  is idiosyncratic to B. We show that B should own  $a$ . The proof is by contradiction. If the proposition is false, then, however small  $k$  is, we can construct a sequence of optimal contracts  $(A_{br}, A_{sr}, \underline{p}_r, \bar{p}_r)$  such that  $a \in A_{sr}$  for all  $r$ , i.e., S owns asset  $a$ , and  $\pi_r \rightarrow 1$  as  $r \rightarrow \infty$ . Without loss of generality (wlog) suppose that  $A_{br} \rightarrow A_b(k)$ ,  $A_{sr} \rightarrow A_s(k)$ ,  $\underline{p}_r \rightarrow \underline{p}(k)$ ,  $\bar{p}_r \rightarrow \bar{p}(k)$ . Then  $A_b(k)$ ,  $A_s(k)$ ,  $\underline{p}(k)$ ,  $\bar{p}(k)$  must be optimal for the case where event 1 occurs with probability 1. For small  $k$  the first-best can be achieved (exactly) in event 1 since there is almost no uncertainty. A necessary condition for this is that there is a single trading price  $p(k)$  in the limit, i.e.,  $\underline{p}(k) = p(k) = \bar{p}(k)$  (so that shading costs are zero), and

$$(39) \quad p_L(\omega, A_b(k), A_s(k)) \leq \underline{p}(k) = p(k) = \bar{p}(k) \leq p_H(\omega, A_b(k), A_s(k))$$

for all  $-k \leq \varphi \leq k$ , where  $\omega = (v_0, c_0, \varphi)$  and we now suppress  $\varepsilon = 0, \eta = 0$ . Here  $p_L, p_H$  are as in (15) – (16) and are indexed by the limiting ownership structure.

Consider a new sequence of contracts  $(A_{br'}, A_{sr'}, \underline{p}_r', \bar{p}_r')$ , where the only difference between  $A_{br'}, A_{sr}'$  and  $A_{br}, A_{sr}$  is that asset  $a$  is transferred to B, and

$$\begin{aligned}
(40) \quad \underline{p}_r' - \underline{p}_r &= \bar{p}_r' - \bar{p}_r \\
&= \frac{1}{2} [\alpha_s(A_{sr} \setminus \{a\}) - \alpha_s(A_{sr}) \\
&\quad + \alpha_b(A_{br}) - \alpha_b(A_{br} \cup \{a\}) \\
&\quad + \beta_b(A_{br}) v_0 - \beta_b(A_{br} \cup \{a\}) v_0] \\
&= \Delta_r(k).
\end{aligned}$$

In other words we adjust  $\underline{p}_r, \bar{p}_r$  by an amount equal to the change  $\Delta p_L, \Delta p_H$  in  $p_L, p_H$  that occurs as a result of the shift in ownership structure, where  $\Delta p_L, \Delta p_H$  are evaluated at  $v = v_0$ . Note that, given the assumption that  $a$  is idiosyncratic to B,  $\Delta p_L, \Delta p_H$  depend on  $v$ , but not on  $c$  (or  $\varphi$ ).

What happens to expected net surplus as a result of this change? Expected net surplus is a weighted average of surplus in the three events 1, 2, 3. Given that  $\underline{p}_r, \bar{p}_r, p_{Lr}, p_{Hr}$  all shift by  $\Delta_r$  when  $v = v_0$ , nothing changes in events 1 and 3 for all  $r$ . That is, for each state  $\omega$ , hold-up occurs if and only if it did before, and the level of shading costs if hold-up doesn't occur remains constant. Thus net surplus is unchanged in events 1 and 3.

Since the new contract cannot deliver higher expected net surplus than the original contract, given that the original contract is optimal, it follows that net surplus must be (weakly) lower in event 2. Let  $r \rightarrow \infty$ . Wlog  $(A_{br'}, A_{sr'}, \underline{p}_r', \bar{p}_r') \rightarrow (A_b'(k), A_s'(k), \underline{p}'(k), \bar{p}'(k))$  and

$\Delta_r(k) \rightarrow \Delta(k)$ . Given (39) – (40), we must have

$$(41) \quad p_L(\omega, A_b'(k), A_s'(k)) \leq \underline{p}'(k) = p'(k) = \bar{p}'(k) \leq p_H(\omega, A_b'(k), A_s'(k))$$

for all  $-k \leq \varphi \leq k$ , where  $\omega = (v_0, c_0, \varphi)$ . By the above arguments the contract  $(\underline{p}'(k), \bar{p}'(k), A_b'(k), A_s'(k))$  delivers surplus no higher in event 2 than the contract  $(\underline{p}(k), \bar{p}(k), A_b(k), A_s(k))$ .

We show that this conclusion is false. Since the primed and unprimed contracts both have a single trading price ( $p'(k), p(k)$ , respectively), shading costs are zero in both contracts. We demonstrate that there is less hold-up in the primed contract. Since we are in event 2 index the state by  $(v, \varphi)$ . Then, from (40),

$$(42) \quad \begin{aligned} p'(k) - p(k) &= p_L((v_0, \varphi), A_b'(k), A_s'(k)) - p_L((v_0, \varphi), A_b(k), A_s(k)) \\ &= p_H((v_0, \varphi), A_b'(k), A_s'(k)) - p_H((v_0, \varphi), A_b(k), A_s(k)) \\ &= \Delta(k) \end{aligned}$$

for all  $\varphi$ . Now hold-up occurs in the primed contract in state  $(v, \varphi)$  if and only if either  $p'(k) < p_L((v, \varphi), A_b'(k), A_s'(k))$  or  $p'(k) > p_H((v, \varphi), A_b'(k), A_s'(k))$ . Consider the first. Given (41) and the fact that  $p_L$  is increasing in  $v$ ,  $p'(k) < p_L((v, \varphi), A_b'(k), A_s'(k))$  only if  $v > v_0$ . But, if  $v > v_0$ ,

$$\begin{aligned}
(43) \quad & p_L((v, \varphi), A_b'(k), A_s'(k)) - p_L((v_0, \varphi), A_b'(k), A_s'(k)) \\
&= \frac{1}{2} [(1 - \lambda) - \beta_b (A_b \cup \{a\})] (v - v_0) \\
&< \frac{1}{2} [(1 - \lambda) - \beta_b (A_b)] (v - v_0) \\
&= p_L((v, \varphi), A_b(k), A_s(k)) \\
&\quad - p_L((v_0, \varphi), A_b(k), A_s(k)),
\end{aligned}$$

since  $a$  is idiosyncratic to  $B$ . From (42) – (43), we may conclude that  $p'(k) < p_L((v, \varphi), A_b'(k), A_s'(k)) \Rightarrow p(k) < p_L((v, \varphi), A_b(k), A_s(k))$ , i.e., hold-up occurs in the unprimed contract if it occurs in the primed contract. A similar argument shows that  $p'(k) > p_H((v, \varphi), A_b'(k), A_s'(k)) \Rightarrow p(k) > p_H((v, \varphi), A_b(k), A_s(k))$ . Putting the two arguments together, we may conclude that hold-up costs are weakly lower in the primed contract than the unprimed one. In fact, they are strictly lower: this follows from the assumption about the support of  $v$  in Proposition 2, which ensures that  $p_L((v, \varphi), A_b(k), A_s(k)) > p_H((v_0, \varphi), A_b(k), A_s(k))$  for large  $v$  and  $\varphi$  close to zero (i.e., hold-up does occur sometimes), but not for  $v$  close to  $v_0$  (i.e., hold-up does not always occur). Contradiction. Q.E.D.

Proof of Proposition 3. We sketch the proof since the argument is very similar to that of Proposition 3. Suppose joint ownership is not optimal. For small  $k$  choose a sequence of optimal contracts as  $\pi \rightarrow 1$ . The limiting contract is optimal for event 1. Hence (39) is satisfied. Consider a new sequence of contracts where all assets are jointly owned and  $\underline{p}_r, \bar{p}_r$  are adjusted to reflect the new ownership structure, i.e.,

$$\begin{aligned}
\mathbf{p}_r' - \mathbf{p}_r &= \bar{\mathbf{p}}_r' - \bar{\mathbf{p}}_r \\
&= \frac{1}{2} [\alpha_s(\phi) - \alpha_s(A_{sr}) - \alpha_b(\phi) + \alpha_b(A_{br}) \\
&\quad - \beta_s(\phi) c_0 + \beta_s(A_{sr}) c_0 - \beta_b(\phi) v_0 + \beta_b(A_{br}) v_0].
\end{aligned}$$

Then surplus does not change in event 1. Since the initial contract is optimal surplus must weakly fall in events 2 or 3. Wlog suppose it falls in event 2. Take limits as  $r \rightarrow \infty$ . The limiting joint ownership contract has the property that  $p_L, p_H$  vary less with  $\varepsilon$  than under the original contract. But this makes hold-up less likely. Hence the joint ownership contract creates higher net surplus. Contradiction.

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