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**“Collateral Damage? Derivatives Trading and Empty Voting”**

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# Collateral Damage? Derivatives Trading and Empty Voting

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*[Preliminary and incomplete]*

## Abstract

This paper analyzes a model of parallel trading of voting securities and derivatives with perfectly negatively correlated cash-flows, such as bonds and credit default swaps. In the presence of noise from liquidity traders, a hedge fund can sometimes profitably acquire a voting block of securities over-hedged by an even larger block of derivatives, such that its incentives are to vote for decisions that *reduce* the value of the security ("empty voting"). At other times, the hedge fund votes to increase the value of the security. The resulting uncertainty and the hedge fund's private information with respect to its own voting plans enable the hedge fund to reap trading gains. As collateral damage, social value is destroyed each time the hedge fund votes to minimize the value of the security. Derivative market participants can protect against this strategy by including position limits and payout information pooling in their derivative contracts.

In recent years, commentators have voiced grave concerns about "empty voting" enabled by derivatives (especially Hu and Black 2006, 2007, 2008a, 2008b). The fear is that certain owners of voting securities such as bonds or shares will use derivatives to hedge more than all the cash-flow attached to the securities. As a consequence, like an over-insured home-owner might prefer to see his house burn down, these owner-voters might favor an outcome that *reduces* the value of their security. Thus derivatives might set incentives for value-destroying voting behavior. This concern has been particularly strong with respect to credit default swaps (CDS) and bonds. Referring to empty voting, George Soros (2010) has called CDS a "license to kill," and fellow fund manager David Einhorn has called CDS "inherently unsafe" (Sender 2009).

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Not all agree, however, that derivatives, and CDS in particular, pose such risks. The International Swaps and Derivatives Association (ISDA) claims that the fears are not justified with respect to CDS (Mengle 2009, pp. 10-11). A would-be empty voter, so the argument goes, would have to pay a high CDS price reflecting the increased default probability of the bond. Given this high price, the strategy would no longer be profitable.

This paper provides a model to assess such claims, and shows that fears of a destructive effect of a parallel derivatives market on corporate voting are indeed justified. To do so, it embeds into a standard trading model (Kyle 1984; Kyle and Vila 1991) parallel trading of (a) voting securities and (b) derivatives with exactly opposite cash-flows and no voting rights. (For concreteness, one may think of the security as a bond and the derivative as a credit default swap.) In the model, a large trader – a "hedge fund" – acquires a block of voting securities. In parallel, the fund also acquires a position in derivatives that is smaller or larger than its securities position, as the case may be. When the hedge fund's derivative position is larger than its securities position (i.e., the hedge fund is "net short"), it maximizes its payoff by voting to *reduce* the value of the security. Rational market participants ("market makers") anticipate the hedge fund's general strategy. But in any given case, they do not know the size of the hedge fund's derivative position because the hedge fund's trades are confounded with those of liquidity traders with varying exogenous demand for derivatives. Liquidity traders thus provide camouflage for the hedge fund, and prices of securities and derivatives reflect an average of the high and low payoff of the security. The hedge fund gains by buying securities relatively cheaply (and possibly selling derivatives expensively) when it votes to maximize the securities' value, and it gains by buying derivatives relatively cheaply when it votes to minimize the securities' value. That is, the hedge fund gains by trading on the uncertainty that its own voting behavior generates. The costs are borne by the liquidity traders. As collateral damage, social value is destroyed each time the hedge fund votes to minimize the value of a security.

The model shows that the problem of value-destroying votes is more severe in more "liquid" derivatives markets, that is, markets in which trades by liquidity traders are subject to greater fluctuations and trading costs are lower. This is a cause for concern given derivatives' markets explosive growth over last decade or so. At the same time, the model also shows that the major costs of the empty-voting problem are internalized by the participants of the derivatives market, notably liquidity traders. These participants therefore have the (almost) correct incentives to adopt a contractual solution to the problem, which the paper outlines.<sup>1</sup> The paper also briefly reviews legal doctrines that may stand in way of value-destroying empty voting, but it argues that the current law does not protect against all such voting.

Absent legal or contractual constraints, the market's ability or inability to infer the hedge fund's behavior from observed trades is the key determinant of

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<sup>1</sup>The incentives for self-regulation are slightly too weak because derivative market participants do not internalize the negative effects on other security holders.

profitability of the hedge fund’s strategy. On the one hand, if the market could perfectly observe the hedge fund’s positions, it could perfectly predict the voting outcome (since only a hedge fund with more derivatives than securities would rationally vote to reduce the value of the securities). In that case, however, the hedge fund would have to pay for the derivative exactly what the derivative contract is going to pay out, and so the hedge fund could not make any profits. On the other hand, if the market could not learn about the hedge fund’s position under any circumstances, then the hedge fund could take very large positions without affecting the price and make very large profits – by going short derivatives if the market price presumes (falsely, given the hedge fund’s intervention) that the value-reducing decision will be adopted, and by going long derivatives in the opposite case. In reality, and in the model, large trades will affect the price of both securities and derivatives. The trades reveal information, in this case about the hedge fund’s position. This information, however, is partially hidden by the noise from liquidity trades. The problem for market makers, both in the real world and in the model, is what inferences to draw from the imperfectly observed information.

The emphasis on trading and market makers’ inference problem sets the present model apart from Bolton and Oehmke (2010). Bolton and Oehmke also model the interaction of derivatives and control rights. Concretely, they analyze the effect of CDS availability on renegotiation in an incomplete contracting model of debt with strategic default. In their model, creditors’ ability to hedge their exposure to the debtor with CDS contracts increases creditors’ bargaining power in renegotiation. This reduces the incidence of strategic default and therefore has the beneficial effect of increasing the debt capacity of the firm; at the same time, overinsurance may lead to an inefficiently high frequency of bankruptcy. Crucially, Bolton and Oehmke (2010) assume that the CDS sellers observe the exact position of the buyer-creditor, who therefore never gains from dealing in CDS as such. By contrast, the present paper builds on the assumption that derivative (CDS) sellers know neither how many securities (bonds) the hedge fund holds, nor how many derivatives (CDS contracts) the hedge fund ends up holding. The latter assumption appears justified because derivatives (CDS) are available from multiple sellers who do not know how many contracts have been sold by the others, and to whom.

The idea that a hedge fund can use its empty voting power to create uncertainty about the security payoff and profit on a net long or a net short position is also present in Brav and Matthews (forthcoming). In their model, however, the only traded assets are shares, and the only way to create a short position is by shorting the shares. To retain voting power while being short the shares, i.e., to build an *empty* voting position, the hedge fund acquires naked votes from other shareholders through the share lending market. Brav and Matthews assume that the hedge fund can do so for free up to a certain amount, and not at all beyond that amount.<sup>2</sup> This assumption does not work outside the share

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<sup>2</sup>Cf. Christoffersen et al. (2007), who document that the average vote does indeed sell for a price of zero in the share lending market.

lending market, however, because the hedge fund will have to pay for votes bundled with a cash-flow into a security. Moreover, trading in derivatives presents additional profit opportunities for the hedge fund.

The treatment of derivatives and empty voting in this paper is entirely theoretical. Commentators generally agree that no presently available data either proves or disproves that the issues considered here are a serious problem, and that such data would be difficult to come by (Hu and Black 2008a, 2008b; Mengle 2009, p. 9; Brav and Matthews, forthcoming, mscr. p. 34). Hu and Black (2006, 2007, 2008a, 2008b) present at least anecdotal evidence, however, of the problem occurring in individual instances. Given the explosive growth of derivatives markets over the last decade or so, it is also possible that the problem was not serious in the past but may become so in the future. If this were so, the normative recommendations of this paper could be seen as forestalling a potential problem from turning into a real one.

The rest of this paper is structured as follows. Section I sets up the model. Section II explains the equilibrium concept. Section III solves for the equilibrium. Section IV presents comparative statics. Section V discusses contractual remedies that parties to derivative contracts might adopt to solve the problem identified in this paper; it also addresses existing legal limits. Section VI concludes.

## 1 Model Setup

This section introduces the setup of the model: the two types of traded assets (securities and derivatives), the three types of market participants (hedge fund, liquidity traders, and market makers), and trading including information. It concludes with some remarks on this setup.

The basic setup – and the equilibrium concept discussed in the next section – is similar to Kyle and Vila (1991), who model the trading of a raider who has the power to declare a value-increasing takeover. In contrast to Kyle and Vila (1991), however, the present model considers the parallel trading of two correlated assets, in which the acquisition of a sufficient amount of one of them is required for the power to declare the "takeover" in the first place.

### 1.1 Traded assets

There are two traded<sup>3</sup> assets with perfectly negatively correlated payoffs: securities, which will throughout be denoted by the letter  $X$ , and derivatives, which will throughout be denoted by the letter  $Y$ . If the security pays  $v$ , the derivative pays  $1 - v$ . Consequently, the derivative can be interpreted as an insurance claim on the security. In particular, if the security were a bond, the derivative

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<sup>3</sup>"Trading" does not need to be understood literally in this model. In particular, it is possible that the derivative is a contract that is sold over the counter. What matters is that there be an active market for the contract in which various parties can act as sellers or buyers, which is true for many derivative markets.

could be a credit default swap; if the security were a share, the derivative could be a total equity return swap.

The security payoff  $v$  depends on a binary choice between two actions, which is determined by a vote of the security holders. For example, if the security is a bond, the choice could be whether or not to agree to a proposed restructuring; if the security is a share, it could be whether or not to agree to a merger. Normalize the payoff when the "right" decision is taken to  $v = 1$ , and when the "wrong" decision is taken to  $v = 0$ .

Each security provides one vote; derivatives do not provide any votes.

Naturally, a rational, informed, and *unhedged* security holder would always vote for the "right" decision in order to receive  $v = 1$ . As will be discussed in the next subsection and the concluding remarks on the model setup, this is indeed what all security holders are assumed to do. The one exception is the hedge fund if and because the hedge fund owns more derivatives than securities, i.e., if the hedge fund is over-hedged and thus has an "empty" voting position. The hedge fund's ability to block the "right" decision depends on whether the hedge fund's security-holding is above some voting threshold. The voting threshold is assumed to be a random variable uniformly distributed on the unit interval. The randomness captures unexpected variation in voter participation, uncertainty arising from legal concerns, different formal thresholds for different types of decisions, etc. The voting threshold is assumed to be independent of other exogenous variables in the model, and it will be independent of any trading activity since it will be only revealed after all trading occurs.

The number of securities is normalized to one (of which infinitesimal divisions are traded). Short-selling is allowed for both derivatives and securities.

## 1.2 Market Participants

There are three types of risk-neutral market participants: liquidity traders, one hedge fund, and competitive market makers.

### 1.2.1 Liquidity traders

The liquidity traders do not act strategically. They exogenously trade quantities  $\tilde{x}$  and  $\tilde{y}$  of securities and derivatives, respectively, where positive numbers indicate that the liquidity traders are selling, and negative numbers indicate that they are buying. These trades are not sensitive to price, and the source of these trades is not modelled. To motivate these trades and their price insensitivity, one may think of large institutional investors and their regulatory constraints. For example, certain pension funds might be forced to sell bonds following a credit downgrade of the borrower. Similarly, financial institutions might be forced to purchase credit default swaps on certain bonds they hold. Or one may think of mutual funds having to liquidate part of their portfolio to meet redemption requests.

Liquidity traders' supply of derivatives,  $\tilde{y}$ , is stochastic (keeping in mind that the "supply" can be negative). With probability  $\lambda$ , the supply is high ( $\tilde{y} = \bar{y} -$

state  $h$ ), while with probability  $(1 - \lambda)$ , supply is low ( $\tilde{y} = \underline{y}$  - state  $l$ ). Define the difference between these supply realizations as  $\delta \equiv \bar{y} - \underline{y} > 0$ . For simplicity, liquidity traders' supply of securities,  $\tilde{x}$ , is assumed to be constant.

### 1.2.2 Hedge fund

The hedge fund does act strategically. Initially, the hedge fund does not hold any securities or derivatives. It purchases quantities  $x$  and  $y$  of securities and derivatives, respectively, taking into account the effect of its trades on the price (as explained below), its own voting power, and its own voting incentives. As explained in the previous subsection, holding  $x \geq 0$  securities gives the hedge fund the voting power to implement the "wrong" decision with probability  $x$  (assuming that it is not possible to own more than all of the outstanding securities<sup>4</sup>). Of course, the hedge fund will only have an incentive to use this power if  $y > x$ . The hedge fund incurs a financing cost  $\frac{c}{2}(x^2 + y^2)$ .

### 1.2.3 Market makers

The market makers absorb any excess supply or demand  $(\hat{x}, \hat{y}) = (\tilde{x} - x, \tilde{y} - y)$ . Since they are risk-neutral and in perfect competition with one another, they purchase or sell these quantities at prices that equal expected value, as explained in more detail below. Market makers have rational expectations, i.e. they observe the net supply of securities  $(\hat{x}, \hat{y})$ , and update their beliefs about  $\tilde{y}$ .

## 1.3 Trading and Information

Trading proceeds as follows.

1. Liquidity traders exogenously supply  $(\tilde{x}, \tilde{y})$ , where  $\tilde{y}$  is stochastic as described in the previous subsection.
2. *The hedge fund observes  $(\tilde{x}, \tilde{y})$  before submitting its own market order  $(x, y)$ .*
3. The market makers observe only *net* market supply  $(\hat{x}, \hat{y}) = (\tilde{x} - x, \tilde{y} - y)$ . Based on this observation, they update their beliefs about the expected value of the securities and derivatives, as explained in more detail below. At these values (prices), they fill all net orders  $(\hat{x}, \hat{y})$ .

The assumption that only the hedge fund observes liquidity traders' orders is crucial, but it need not be interpreted literally. A more liberal and realistic interpretation is that both the hedge fund and the market makers observe net market supply, but since only the hedge fund knows its own orders, only the hedge fund is able to back out liquidity traders' orders. In a fuller, dynamic model, the hedge fund could gradually adjust its trades to the available supply.

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<sup>4</sup>Otherwise, the hedge fund's probability of winning the vote would be  $\min\{x, 1\}$ . In principle, it is conceivable to allow  $x > 1$  if short-selling increases the amount of "securities" available in the market. To simplify matters, it is not allowed in this paper.

## 1.4 Remarks on the model setup

The model assumes that the hedge fund is able to acquire any amount  $x$  of securities that it desires. In particular, this ability does not depend on the amount  $\tilde{x}$  supplied by liquidity traders. In reality, it may often be difficult or impossible to acquire large blocks of shares or bonds. There are, however, many situations in which exogenous sales of securities  $\tilde{x}$  are large, and the reader may restrict the applicability of the model to such situations. For example, many institutional investors sell all their holdings of a bond if the bond's credit rating drops below a certain threshold. Moreover, in the model, an upper bound on the amount  $x$  of securities that the hedge fund can acquire would not change the hedge fund's strategy, and the only change from the results presented below would be that the hedge fund might have to settle for the upper bound rather than its preferred, higher position (i.e., one would observe corner solutions).

Relatedly, the assumption that large purchases have no price impact beyond the probability update by the market makers is not literally true. To go back to the acquisition of securities, it would presumably become harder and harder to find additional securities as the hedge fund's position grows, and this would be reflected in higher trading costs for larger positions. Mathematically, however, the assumption of a financing cost for the hedge fund has the same affect as assuming increasing trading costs for larger blocks, so that nothing substantive seems to hinge on the assumption of constant prices conditional on the updated probability.

Finally, it is a strong assumption that only the one hedge fund is ready to buy large stakes, and to consider over-hedging its securities position and to vote the securities for the "wrong" decision. This excludes, first, that any of the other market participants in the model, namely individual market makers and liquidity traders, who must hold the remaining supply of securities, would ever hold more derivatives than securities, or if they did, that they nevertheless voted for the "right" decision. One justification for this could be institutional, namely that reputational concerns or sheer apathy prevent market makers and liquidity traders to vote for the "wrong" decision, or to over-hedge their securities position in the first place.

Second, the above assumptions rule out strategic competition with a second large player. For example, one can imagine a second hedge fund trying to share the spoils, or to buy up enough of the security at a low price to prevent the first hedge fund from ever winning a vote for the "wrong" decision. One way to justify this restriction is to assume genuinely private information on behalf of the hedge fund, in particular about liquidity trader's trades. From a practical point of view, adding another strategic player would complicate the model but not eliminate the underlying economic problem. For example, even if the security were trading at deep discount because of hedge fund's presence, another large player could not necessarily profitably intervene by buying up the entire supply of securities if and because that second large player incurs similar financing cost as the first hedge fund. Moreover, even if the second large player could profitably do this, then in expectation the price of the security would re-adjust

to 1, so that the strategy would end up being not profitable after all.

## 2 Equilibrium Concept

The above assumptions on individual behavior allow summarizing the strategic interaction in two simple equilibrium conditions (Kyle 1984; Kyle and Vila 1991).

First, the assumption that liquidity traders' trades are exogenous means that liquidity traders decisions need not be considered at all.

Second, the assumption that market-makers act as competitive price takers and absorb or supply any demanded quantities  $(\hat{x}, \hat{y})$  means that their behavior can be summarized by the price function. More specifically, prices are entirely pinned down by market-makers' rational expectations of the security and derivative payoffs. Moreover, there is effectively only one price function because the payoffs of derivatives and securities are perfectly negatively correlated. Let  $P_x(\hat{x}, \hat{y})$  be the price of securities and  $P_y(\hat{x}, \hat{y})$  be the price of derivatives. Furthermore, let  $\theta(\hat{x}, \hat{y}) : \mathbb{R}^2 \rightarrow [0, 1]$  describe the correctly inferred probability that the security will pay zero conditional on observed net supply of securities and derivatives,  $(\hat{x}, \hat{y})$ . Then market-makers' equilibrium behavior is fully captured by the following condition:

*Efficient markets:* for some  $\theta(\cdot, \cdot)$ ,  $P_x(\hat{x}, \hat{y}) = 1 - \theta(\hat{x}, \hat{y})$  and  $P_y(\hat{x}, \hat{y}) = \theta(\hat{x}, \hat{y}) = 1 - P_x(\hat{x}, \hat{y})$ .

Finally, observe that at the voting stage, the hedge fund's behavior is pinned down by its holdings of securities and derivatives. If the hedge fund holds more securities than derivatives ( $x > y$ ), it will vote for the "right" decision; otherwise, it will vote for the "wrong" decision (with mixing possible only if  $x = y$ ). Consequently, the hedge fund only has two true choice variables, namely its trades  $x$  and  $y$ . Let  $\Pi_i(x, y)$  be the hedge fund's profit in state  $i \in \{h, l\}$ , and  $(x_i, y_i)$  the hedge fund's (possibly mixed) strategy in those states. Then the hedge fund's equilibrium strategy is captured by the following condition:

*Profit maximization:* for every  $i \in \{h, l\}$ ,  $(x_i, y_i)$  maximizes  $\Pi_i(x_i, y_i)$ .

In choosing its profit-maximizing positions  $(x, y)$ , the hedge fund will of course take into account the effect of its trades on prices, i.e., on the probability inference of the market makers,  $\theta(\hat{x}, \hat{y})$ . In that sense, the efficient market condition implies an inverse demand curve against which the hedge fund's profit maximization operates.

Constructing an equilibrium principally requires the definition of some inference function  $\theta(\cdot, \cdot)$  for which both conditions can hold simultaneously. In principle,  $\theta(\cdot, \cdot)$  equals the probability that the hedge fund is able to implement the "wrong" voting decision, provided that it would want to do so in the first place. That is, in principle,  $\theta(\hat{x}, \hat{y}) = \mathbb{E}[\max\{0, \min\{x, 1\}\} \cdot 1_{\{y > x\}} | \hat{x}, \hat{y}]$ . Note, however, that  $\theta(\hat{x}, \hat{y})$  is objectively defined only at points  $(\hat{x}, \hat{y})$  that are actually observed in equilibrium. At other points,  $\theta(\hat{x}, \hat{y})$  is derived from subjective off-equilibrium beliefs of market-makers.

### 3 Equilibrium

This paper focuses on a full pooling equilibrium, that is, an equilibrium in which the observed net market supply  $(\hat{x}^*, \hat{y}^*)$  is identical in both states of the world ( $h$  and  $l$ ), i.e., the hedge fund fully hides behind the noise. When liquidity traders are large sellers of derivatives (or equivalently, small buyers), the hedge fund acquires a large amount of derivatives, votes for the "wrong" decision, and, if successful, gains on its derivative position. When liquidity traders are small sellers of derivatives (or equivalently, large buyers), the hedge fund acquires few derivatives or even sells them, votes for the "right" voting decision, and gains on its holdings of the security.

To find the equilibrium, the paper first conjectures that it exists, and then proceeds to verify all the equilibrium conditions. Some of the more technical conditions are relegated to the appendix, including a discussion of the exogenous parameter space for which the equilibrium exists. It can be shown that the equilibrium considered here is unique for sufficiently low values of  $c$ , i.e., for sufficiently low financing costs for the hedge fund.<sup>5</sup>

#### 3.1 Direct implications of the pooling conjecture

Certain relationships between exogenous and endogenous variables follow directly from the conjecture that observable net market supply  $(\hat{x}^*, \hat{y}^*)$  is identical in both states, and yet the hedge fund is able to affect the vote differently in the two states.

First, the hedge fund must always buy the same amount of securities,  $x^*$ .

Second, the hedge fund must buy differential quantities of derivatives  $y_l < y_h$  that perfectly offset the difference between  $\bar{y}$  and  $\underline{y}$ . That is, it must be that  $\hat{y}^* = \bar{y} - y_h = \underline{y} - y_l$ , implying  $y_h - y_l = \bar{y} - \underline{y} = \delta$ .

<sup>5</sup>In the alternative, separating equilibrium, the hedge fund's holdings are fully revealed, and hence the hedge fund earns zero profit. To "enforce" this equilibrium, there must be a function  $\theta(\cdot, \cdot)$  such that the hedge fund cannot deviate to a trade with positive profits in either state. Let  $x = \bar{x} - \hat{x}$  and  $y = \bar{y} - \hat{y}$ . Then to hold the hedge fund to zero profits, it would have to be the case that for all  $(\hat{x}, \hat{y})$ ,

$$\begin{aligned}\Pi_h &= (y + \delta - x) [x - \theta(\hat{x}, \hat{y})] - \frac{c}{2} [x^2 + (y + \delta)^2] \leq 0 \\ \Pi_l &= (x - y) \theta(\hat{x}, \hat{y}) - \frac{c}{2} [x^2 + y^2] \leq 0.\end{aligned}$$

Combining these two inequalities, one can obtain the condition

$$2 \left( x - y - \frac{\delta}{2} \right) \theta(\hat{x}, \hat{y}) \leq c \left[ x^2 + y^2 + y\delta + \frac{\delta^2}{2} \right] + x(x - y - \delta).$$

But then for any  $(\hat{x}, \hat{y})$  such that  $\delta > x - y > \frac{\delta}{2}$  and  $x, y > 0$ , it would follow that for  $c$  small enough,

$$\theta(\hat{x}, \hat{y}) \leq \frac{c \left[ x^2 + y^2 + y\delta + \frac{\delta^2}{2} \right] + x(x - y - \delta)}{2 \left( x - y - \frac{\delta}{2} \right)} < 0,$$

which is inconsistent with the definition of a probability function.

Third, to create different security and derivative payoffs in the two states and hence profit opportunities for the hedge fund, it must be that the probability of the "wrong" decision being adopted differs between the two states. One condition for this is  $y_l < x^* < y_h$ , so that the hedge fund will vote differently in the two states. The other condition is that  $x^* > 0$  so that the hedge fund can actually influence the voting outcome.

Finally, since pooling means that the market makers cannot tell the two states apart, it must be that  $\theta(\hat{x}^*, \hat{y}^*) = \lambda x^*$ .<sup>6</sup> Together with the efficient market condition, this implies  $P_y(\hat{x}^*, \hat{y}^*) = 1 - P_x(\hat{x}^*, \hat{y}^*) = \theta(\hat{x}^*, \hat{y}^*) = \lambda x^*$ .

### 3.2 First-order conditions

The profit maximization condition requires that the hedge fund cannot do better in either state by adjusting its trades. Assuming that the profit function is differentiable, this requires that the first-order conditions for an optimum are satisfied. That the profit function is indeed differentiable can be proved along the lines of Kyle and Vila (1991).<sup>7</sup>

Given the results of the previous sub-section, around the equilibrium for states  $h$  and  $l$ , the profit function can be written (where  $F(x)$  is the probability that the hedge fund can get the "wrong" decision implemented, given its security holdings  $x$ ):

$$\begin{aligned}\Pi_h(x, y) &= x[1 - F(x)] + yF(x) - xP_x(\hat{x}, \hat{y}) - yP_y(\hat{x}, \hat{y}) - \frac{c}{2}(x^2 + y^2) \\ &= (y - x)[x - \theta(\hat{x}, \hat{y})] - \frac{c}{2}(x^2 + y^2) \\ \Pi_l(x, y) &= x - xP_x(\hat{x}, \hat{y}) - yP_y(\hat{x}, \hat{y}) - \frac{c}{2}(x^2 + y^2) \\ &= (x - y)\theta(\hat{x}, \hat{y}) - \frac{c}{2}(x^2 + y^2)\end{aligned}$$

Denoting by subscripts  $i$  the derivative of a function with respect to its  $i$ th argument, the first-order conditions are:

$$\begin{aligned}\frac{\partial \Pi_h}{\partial x}(x^*) &= (y_h - x^*)[1 + \theta_1(\hat{x}^*, \hat{y}^*)] - [x^* - \theta(\hat{x}^*, \hat{y}^*)] - cx^* = 0, \\ \frac{\partial \Pi_h}{\partial y}(y_h) &= (y_h - x^*)\theta_2(\hat{x}^*, \hat{y}^*) + [x^* - \theta(\hat{x}^*, \hat{y}^*)] - cy_h = 0, \\ \frac{\partial \Pi_l}{\partial x}(x^*) &= -(x^* - y_l)\theta_1(\hat{x}^*, \hat{y}^*) + \theta(\hat{x}^*, \hat{y}^*) - cx^* = 0, \\ \frac{\partial \Pi_l}{\partial y}(y_l) &= -(x^* - y_l)\theta_2(\hat{x}^*, \hat{y}^*) - \theta(\hat{x}^*, \hat{y}^*) - cy_l = 0.\end{aligned}$$

<sup>6</sup> Recall that we have assumed that it is not possible to own more than all of the outstanding securities, i.e.,  $x \leq 1$ .

<sup>7</sup>[...]

### 3.3 Equilibrium values

Combining the first-order conditions and the direct implications of the pooling conjecture yields the following equilibrium value for  $x^*$ :

$$\begin{aligned} x^* &= \max \left\{ \frac{\delta(\lambda + c)[1 - \lambda - c]}{2c}, 1 \right\}, \\ y_l &= x^* - \delta(\lambda + c), \\ y_h &= x^* + \delta(1 - \lambda - c), \\ \theta_2(\hat{x}^*, \hat{y}^*) &= c - \frac{x^*}{\delta} \end{aligned}$$

The upper bound on  $x^*$  comes from the assumption that it is not possible to own more than all the outstanding securities, whose supply is normalized to one. Assuming that we are not at the corner solution  $x^* = 1$ , the solutions for the other endogenous variables can be written

$$\begin{aligned} y_l &= \frac{\delta(\lambda + c)[1 - \lambda - 3c]}{2c}, \\ y_h &= \frac{\delta(\lambda + 3c)[1 - \lambda - c]}{2c}, \end{aligned}$$

and the impact of trades on the probability inference function  $\theta(\cdot, \cdot)$  at the equilibrium point is

$$\begin{aligned} \theta_1(\hat{x}^*, \hat{y}^*) &= c + \frac{x^*}{\delta} - (1 - \lambda) = \frac{(\lambda - c)[1 - \lambda - c]}{2c}, \\ \theta_2(\hat{x}^*, \hat{y}^*) &= c - \frac{x^*}{\delta} = c - \frac{(\lambda + c)[1 - \lambda - c]}{2c}. \end{aligned}$$

### 3.4 Other conditions

To verify that the hedge fund's trades are indeed profit maximizing, it remains to verify the second-order conditions, and that the hedge fund's equilibrium profits are positive in both states (since the hedge fund always has the option not to trade). As already mentioned, it is also necessary to verify the assumption made above that  $x^* \in (0, 1)$  and that no security holdings outside this interval yields higher profits. Finally, since the derivation above assumed differentiability of  $\theta(\cdot, \cdot)$ , it must be checked that the hedge fund cannot earn higher profits at points where  $\theta(\cdot, \cdot)$  is not differentiable, i.e., where  $\theta$  equals either one or zero. Verifying these conditions is relegated to the appendix, where it is shown that all conditions hold subject to certain restrictions on the parameter space.

## 4 Comparative Statics

The primary variable of interest is the probability that value-destroying decisions will be adopted, that is

$$\lambda x^* = \lambda \max \left\{ \frac{\delta (\lambda + c) [1 - \lambda - c]}{2c}, 1 \right\}.$$

At the internal solution, this probability is increasing in  $\delta$ , the fluctuation in liquidity traders' derivative trades, and decreasing in  $c$ , the hedge fund's financing cost. The probability is increasing in  $\delta$  because greater fluctuation in liquidity trades provides more camouflage to the hedge fund. This enables the hedge fund to take greater net positions, with concomitantly greater profits conditional on obtaining the desired voting result. This in turn makes it more attractive for the hedge fund to acquire a more powerful voting position. As the noise  $\delta$  goes to zero, so does the hedge fund's voting position and hence the probability of value-destroying decisions.

The probability is decreasing in  $c$  because greater financing costs make it more costly for the hedge fund to acquire large voting blocks and derivative positions. This is perhaps most clear in the expressions for the equilibrium holdings of derivatives, which are centered around  $x^*$  but shifted downwards by an amount  $\delta c$ . As shown in the appendix, the hedge fund does not even make positive profits in both states of the world if the trading cost parameter  $c$  is greater than  $\sqrt{1/8}$ ; in that case, full pooling is no longer a sustainable equilibrium.<sup>8</sup>

The slope of the inference and hence of the price function with respect to the observed net supply of securities,  $\theta_1$ , reflects the ambiguous nature of increasing the hedge fund's security holdings. On the one hand, greater security holdings make it less attractive for the hedge fund to vote for the "wrong" decision. On the other hand, greater security holdings also make it easier for the hedge fund to get the "wrong" decision adopted, if it wants to. Reflecting this dual character of increasing the hedge fund's security position, the inferred probability of the "wrong" decision being adopted can either rise or fall if the hedge fund increases its security holdings, depending on the relative size of  $\lambda$  and  $c$ . If  $\lambda > c$ ,  $\theta_1(\hat{x}^*, \hat{y}^*) > 0$ , i.e., when the hedge fund buys more securities, the inferred probability of a "wrong" vote goes down. By contrast, if  $\lambda < c$ ,  $\theta_1(\hat{x}^*, \hat{y}^*) < 0$ , i.e., the inferred probability of a "wrong" vote goes up when the hedge fund buys more securities.

## 5 Contractual Remedies

[this section is particularly incomplete]

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<sup>8</sup>Equilibrium behavior for  $c > \sqrt{1/8}$  remains to be verified. It is probably not an equilibrium for the hedge fund never to do any trades because at an initial trade of zero, the hedge fund's marginal cost of trading is zero. So it is likely that there will always be some positive probability of the hedge fund acquiring some securities to attempt to implement the negative voting outcome.

The discussion thus far has not considered legal or contractual limitations on the "empty voting" strategy pursued by the hedge fund. This section briefly outlines two of the main legal constraints before putting forward a contractual solution, which the participants of the derivative market should find in their self-interest to adopt.

## 5.1 Legal limits

On the legal side, the hedge fund might find its behavior incriminated under different doctrines in the debt and equity case. In the equity case, holding a sufficiently large block of voting shares would qualify the hedge fund as a controlling shareholder. Controlling shareholders, however, arguably have a fiduciary duty not to exercise their voting power against the benefit of the corporation in furtherance of some private benefits, here the payoff on the derivatives.

In the debt case, creditor votes in bankruptcy can be disallowed if they are not cast in "good faith," and casting the votes "empty" with the intention of limiting recovery by creditors would arguably qualify as not being in "good faith."

In each case, however, application of these doctrines requires that the emptiness of the hedge fund's voting position be known by fellow shareholders, creditors, or the judge, as the case may be. At present, this is not necessarily guaranteed by existing disclosure obligations (Hu and Black ...). Moreover, limits on "empty voting" do not appear to apply to creditors outside of bankruptcy.

## 5.2 Contractual position limits and information pooling

Participants in the derivative market have the possibility and the incentive, however, to adopt a contractual solution to the "empty voting" problem.

In the model, the hedge fund's gains translate directly into losses for liquidity traders in the derivative market. When liquidity traders buy many derivatives, the hedge fund buys few, votes the "right" way, and thereby decreases the payoff of the derivatives. When liquidity traders sell many derivatives, the hedge fund buys many, votes the "wrong" way, and thereby increases the payoff of the derivatives. Liquidity traders should therefore prefer – and competitive market makers would be forced to provide – derivative contracts that exclude the possibility of "empty voting."

At present, certain derivative contracts, in particular CDS, already contain clauses that purport to force the contract owner to exercise any voting rights in the corresponding security with a view to maximizing the value of that security. As Henry Hu and Bernie Black note, however, these clauses are hardly enforceable because the required information is not generally available (Hu and Black 2008a, p. 733; 2008b, pp. 682-3). It would appear difficult and cumbersome to collect voting information on a frequent basis.

A more easily implementable protective mechanism would be to limit the amount of hedging that users of derivatives are allowed to do, i.e., to insert position limits into derivatives contracts. This would render the "empty voting"

strategy relatively unattractive to hedge fund's who need to be able to over-hedge a sizeable position of voting securities to make the strategy profitable. To be sure, position limits could be circumvented by buying derivatives from multiple sellers. Sellers could reserve the contractual right, however, to pool information at the pay-out stage to ensure that position limits were respected. The information could be pooled in the hands of a third party subject to strict confidentiality provisions. The third party would collect pay-out information from all derivative counterparties and only release any information if it found that individual recipients had exceeded their position limits.

## 6 Conclusion

Commentators have been worrying about the voting incentives of a security holder who is "over-hedged," i.e., who holds more off-setting derivatives than voting securities. This paper provides the first rigorous demonstration that building such an "empty voting position" can be a profitable strategy for a hedge fund if securities and derivatives trade in parallel noisy markets. The strategy will be all the more profitable, and value-destroying votes all the more likely, as trading costs decrease and market liquidity increases, as has generally been the trend in derivatives markets over the last decade. This paper therefore adds reason to take the "empty voting" problem seriously.

At the same time, the model shows that the hedge fund's gains come at the expense of liquidity traders in the derivative markets, with losses to security holders merely arising as collateral damage. Hence liquidity traders in the derivative markets have the right incentives to push for a contractual solution of the "empty voting problem" discussed in this paper. In particular, they may insist on using contracts with position limits.

## Appendix – Additional Equilibrium Conditions

This appendix verifies remaining conditions for the full pooling equilibrium considered in the main text.

### 7 $x^* \in (0, 1)$

The derivation of the hedge fund's profit maximizing trades assumed that the hedge fund's probability of winning the vote around the equilibrium point equals  $x$ , and that the derivative of the probability function is 1. That is, the derivation of optimal trades only considered  $x \in (0, 1)$ . It needs to be verified, first, that the optimum trade actually falls into this interval, and, second, that no  $x$  outside this interval yields higher profits. For

$$x^* = \frac{\delta(\lambda + c)[1 - \lambda - c]}{2c} \in (0, 1)$$

to hold true requires the following restrictions on the exogenous parameters:

$$\begin{aligned} & \delta(\lambda + c)[1 - \lambda - c] > 0 \\ \iff & 1 - \lambda > c. \\ & \delta(\lambda + c)[1 - \lambda - c] < 2c \\ \iff & \delta < \frac{2c}{(\lambda + c)[1 - \lambda - c]}. \end{aligned}$$

Next, check that no  $x'$  outside  $(0, 1)$  would yield higher profits for the hedge fund, given  $\theta(\hat{x}, \hat{y})$ . First,  $\Pi_l$  does not depend on  $F(x)$ , so the global maximization of  $\Pi_l$  has the same conditions as the restricted optimization above, with the same outcome. Second,  $\Pi_h(x') < \Pi_h(x^*)$  for any  $x' \geq 1$  since we found that  $x^* \in (0, 1)$  maximizes  $\Pi_h$  even under the assumption that  $F(x) = x$  throughout, while in reality  $F(x) = 1$  for all  $x' \geq 1$ , and  $\Pi_h$  is increasing in  $F(x)$ . Third, for  $x \leq 0$ ,  $F(x) = 0$ , so market makers know that  $v = 1$  for sure, i.e.,  $\theta(\hat{x} - x, \cdot) = 0$ , and hence

$$\begin{aligned} \Pi_h(x, y) |_{x \leq 0} &= (x - y)\theta(\hat{x}, \hat{y}) - \frac{c}{2}(x^2 + y^2) \\ &= 0 - \frac{c}{2}(x^2 + y^2) \\ &\leq 0. \end{aligned}$$

So the hedge fund would be earning non-positive profits, and this deviation would not be profitable.

## 8 Positive profits

The hedge fund always has the option not to trade and earn zero profits. Thus full pooling can only be an equilibrium if the hedge fund makes positive profits

in both states. Using the equilibrium values derived in the main text, we must have

$$\begin{aligned}
\Pi_h(x^*, y_h) &= (y_h - x^*) [x^* - \lambda x^*] - \frac{c}{2} (x^{*2} + y_h^2) \\
&= \delta^2 (1 - \lambda - c)^2 \frac{2(\lambda + c) [1 - \lambda - c] - 2c^2 - (\lambda + c)^2}{4c} \\
&\geq 0 \\
&\iff 2(\lambda + c) [1 - \lambda - c] - 2c^2 \geq (\lambda + c)^2.
\end{aligned}$$

We also must have

$$\begin{aligned}
\Pi_l(x^*, y_l) &= (x^* - y_l) \lambda x^* - \frac{c}{2} (x^{*2} + y_l^2) \\
&= \delta^2 (\lambda + c)^2 \frac{2(\lambda + c) [1 - \lambda - c] - 2c^2 - [1 - \lambda - c]^2}{4c} \\
&\geq 0 \\
&\iff 2(\lambda + c) [1 - \lambda - c] - 2c^2 \geq [1 - (\lambda + c)]^2.
\end{aligned}$$

The positive profits condition can thus be stated

$$\begin{aligned}
&(\lambda + c) [1 - (\lambda + c)] \geq c^2 + \frac{1}{2} \max \left\{ (\lambda + c)^2, [1 - (\lambda + c)]^2 \right\} \\
\iff c &\in \left[ 0, \sqrt{\frac{1}{8}} \right] \wedge \lambda \in \left[ \frac{2}{3} - c - \frac{1}{3} \sqrt{1 - 6c^2}, \frac{1}{3} - c + \frac{1}{3} \sqrt{1 - 6c^2} \right]
\end{aligned}$$

The upper bound for  $\lambda$  implies

$$1 - \lambda \geq c + \frac{2}{3} - \frac{1}{3} \sqrt{1 - 6c^2} \geq c + \frac{1}{3} > c,$$

i.e., it contains the condition  $1 - \lambda > c$  required for  $x^* \in (0, 1)$ .

## 9 Second-order conditions

To ensure that we are at an optimum, we need to check the second derivatives. To facilitate this, we restrict the search to linear functions  $\theta(\cdot, \cdot)$ , with slopes  $\theta_1$  and  $\theta_2$  as given in the previous subsection. As pointed out in the main text,  $\theta(\cdot, \cdot)$  is objectively defined only at the point  $\theta(\hat{x}^*, \hat{y}^*)$ , so there are potentially many different functions  $\theta(\cdot, \cdot)$  that can sustain the equilibrium. Therefore, to the extent that the second-order conditions for linear  $\theta(\cdot, \cdot)$  considered here yield restrictions on the parameters, it is possible that these restrictions would not apply if a broader class of functions were considered.

Under the linearity assumption, the second derivatives are:

$$\begin{aligned}
\frac{\partial^2 \Pi_h}{\partial x^2} &= -2 - 2\theta_1 - c \\
\frac{\partial^2 \Pi_h}{\partial y^2} &= 2\theta_2 - c \\
\frac{\partial^2 \Pi_h}{\partial y \partial x} &= \theta_1 - \theta_2 + 1 \\
\frac{\partial^2 \Pi_l}{\partial x^2} &= -2\theta_1 - c \\
\frac{\partial^2 \Pi_l}{\partial y^2} &= 2\theta_2 - c \\
\frac{\partial^2 \Pi_l}{\partial y \partial x} &= \theta_1 - \theta_2
\end{aligned}$$

The conditions for an optimum are thus<sup>9</sup>

$$\begin{aligned}
-2\theta_1 - c &< 0 \\
(-2\theta_1 - c)(2\theta_2 - c) - (\theta_1 - \theta_2)^2 &> 0
\end{aligned}$$

Using the results from the first-order conditions, the first of these conditions requires

$$\begin{aligned}
-2\theta_1 - c &= -2 \left[ c + \frac{x^*}{\delta} - (1 - \lambda) \right] - c \\
&= \frac{-2c^2 + c - \lambda[1 - \lambda]}{c} \\
&< 0 \\
\Rightarrow c(1 - 2c) &< \lambda[1 - \lambda] \\
\Rightarrow \lambda &\in \frac{1}{2} \pm \frac{1}{2} \sqrt{8c^2 - 4c + 1}
\end{aligned}$$

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<sup>9</sup>The condition

$$-2 - 2\theta_1 - c < 0$$

is implied by

$$-2\theta_1 - c < 0.$$

The condition

$$2\theta_2 - c < 0$$

is implied by the first condition and

$$(-2\theta_1 - c)(2\theta_2 - c) - (\theta_1 - \theta_2)^2 > 0.$$

Finally, the condition

$$(-2 - 2\theta_1 - c)(2\theta_2 - c) - (\theta_1 - \theta_2)^2 > 0$$

is implied by the preceding condition.

The second condition requires

$$\begin{aligned}
& \left[ 2\frac{x^*}{\delta} - 2(1-\lambda) + 3c \right] \left( 2\frac{x^*}{\delta} - c \right) - \left[ 2\frac{x^*}{\delta} - (1-\lambda) \right]^2 \\
&= 2(\lambda+c)[1-\lambda-c] + c[2(1-\lambda)-3c] - (1-\lambda)^2 \\
&= -3\lambda^2 - 5c^2 - 6\lambda c + 4\lambda + 4c - 1 \\
&> 0 \\
\Rightarrow c &\in \left[ 0, \sqrt{\frac{1}{6}} \right) \wedge \lambda \in \frac{2}{3} - c \pm \frac{1}{3}\sqrt{1-6c^2},
\end{aligned}$$

which is guaranteed by the (stricter) positive profit condition.

As can be checked algebraically, the positive profit condition also implies the upper bound for  $\lambda$  from the first condition, but not the lower bound for  $c > 2/7$ .

## 10 Corner solutions [NEED TO CHECK]

The first-order approach assumed that  $\theta(\cdot, \cdot)$  is differentiable in both arguments. This, however, can only be true on a limited interval of  $(\hat{x}, \hat{y})$  since  $\theta(\cdot, \cdot) \in [0, 1]$ .

...

## 11 Summary of parameter restrictions

In sum, we have the following parameter restrictions from the requirement that  $x^* \in (0, 1)$  and that profits be positive in both states:

$$\begin{aligned}
c &\in \left[ 0, \sqrt{\frac{1}{8}} \right] \\
\lambda &\in \left[ \frac{2}{3} - c - \frac{1}{3}\sqrt{1-6c^2}, \frac{1}{3} - c + \frac{1}{3}\sqrt{1-6c^2} \right] \\
\delta &< \frac{2c}{(\lambda+c)[1-\lambda-c]}.
\end{aligned}$$

For a linear function  $\theta$  and  $c > 2/7$ , the second-order conditions also require

$$\lambda > \frac{1}{2} - \frac{1}{2}\sqrt{8c^2 - 4c + 1}.$$

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