THE EVOLUTION OF PRECEDENT

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*Presenting
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1. Introduction

In his Economic Analysis of Law (2003, 1st ed. 1973), Richard Posner raises a question: in a common law system, does judge-made law converge to efficient legal rules? Put differently, do precedents converge to fixed rules, and if so, are these rules efficient? Posner hypothesizes that common law tends toward efficiency. Rubin (1977) and Priest (1977) suggest that disputes involving inefficient legal rules are more likely to be taken to court rather than settled, leading to the replacement of such rules over time. These articles do not focus on how judges actually make decisions. In this paper, we model the evolution of precedents through a series of judicial decisions, and examine its consequences for the convergence and efficiency of legal rules.

The doctrine of *stare decisis*, of adherence to precedent, is a crucial feature of common law (e.g., Hayek 1960, Stone 1985, Posner 2003). Respect for precedents gives common law its stability and predictability. At the same time, the possibility of judges changing legal rules allows the law to evolve, to adjust to new circumstances, and therefore to become ever more efficient over time. Posner (2003) recognizes that such legal evolution is most effective when judges maximize efficiency. But even when they do not, and differ in their approaches to law, a key evolutionary argument still sees the common law as evolving toward ever better rules. “The eccentricities of judges balance one another. One judge looks at problems from the point of view of history, another from that of philosophy, another from that of social utility, one is a formalist, another a latitudinarian, one is timorous of change, another dissatisfied with the present; out of the attrition of diverse minds there is beaten something which has a constancy and uniformity and average value greater than its component elements” (Cardozo 1921, p. 177). Thus even if judges do not maximize efficiency, evolution selects better legal rules.

To assess these views of the evolution of common law, we present a new model of precedent formation by (presumably appellate) judges. Our model relies on two assumptions. First, following the legal realism literature (e.g., Cohen 1935, Frank 1930, Radin 1925, Stone 1985) and the theoretical work of Gennaioli (2004), we assume that judges hold biases favoring different types
of disputants, and that these biases vary across the population of judges. Frank (1930, p. 28) defines bias as the ideas and beliefs that come from judges’ past experiences or philosophies. For example, some judges might believe in literal interpretation of contracts, others in interpreting contracts so as to promote efficiency, and still others in interpreting contracts against the drafter (Posner 2004b). Frank (1930) and Radin (1925) go so far as to say that judges decide the cases backwards: they figure out what outcome is just from their point of view, and then find legal arguments to support their conclusions.

Many legal scholars accept the importance of judicial bias for rulings on politically sensitive issues (e.g., Pinello 1999, Rowland and Carp 1996, Revesz 1997, Sunstein, Schkade, and Ellman 2004). Judges also differ sharply in their sentencing decisions for a given crime (Partridge and Eldridge 1974). But judges may also have preferences over the outcomes of commercial disputes: they may favor the rich or the poor, the government or individuals, dog owners or bite victims. As Posner (2004a, p. 14) – echoing Frank (1930) -- writes about federal district judges: “But [deciding a particular case in a particular way might increase the judge’s utility] by advancing a political or ideological goal, economizing on the judge’s time and effort, inviting commendation from people whom the judge admires, benefiting the local community, getting the judge’s name in the newspaper, pleasing a spouse or other family member or a friend, galling a lawyer whom the judge dislikes, expressing affection for or hostility toward one of the parties – the list goes on and on.” One piece of data on the importance of judicial preferences in commercial disputes, and the consequent unpredictability of judicial decisions, is the sharp share price reactions that companies experience on the dates judges issue decisions (Haslem 2004).

Judicial bias is related to the idea that judges may be swayed by external forces, including political influence, intimidation, or bribes. This alternative assumption has been used to investigate legal systems both historically and in developing countries (Glaeser and Shleifer 2002, Glaeser, Scheinkman, and Shleifer 2003). However, to understand the evolution of common law in a developed modern economy, the assumption of judicial bias appears to be more appropriate.
Second, also following Radin (1925) and Posner (2003, 2004a), we assume that changing precedent is personally costly to judges: it requires extra investigation of facts, extra writing, extra work of persuading colleagues when judges sit in panels, extra risk of being criticized, and so on. “Judges are people and the economizing of mental effort is a characteristic of people, even if censorious persons call it by a less fine name” (Radin 1925, p. 362). The assumption that, other things equal, judges would rather not change the law implies that only the judges who disagree with the current legal rule strongly enough actually change it. Posner (2003, p. 544) sees what he calls “judicial preference for leisure” as a source of stability in the law; we revisit this issue.

Using a model relying on these two assumptions, we examine the evolution of legal rules in the case of a simple tort: a dog bites a man (e.g., Landes and Posner 1987). We consider separately two types of revision of precedents: overruling and distinguishing. By overruling we mean the discarding and replacement of a prevailing legal rule by a new one. Rubin (1977), Priest (1977), and Posner (2003) seem to have this model of precedent change in mind. By distinguishing we mean the introduction of a new legal rule that endorses the existing precedent, but adds a new material dimension to adjudication, and holds that the judicial decision must depend on both the previously established dimension and the new one. Distinguishing cases is perhaps the central mechanism, or leeway, through which the law evolves despite binding precedents (Stone 1985). But the efficiency of this process has not received much analytical attention. So we ask whether the evolution of precedents through either overruling or distinguishing leads to convergence, and if so whether convergence is to efficient legal rules.

Our argument is best illustrated with a simple example. Consider the evolution of legal rules governing the liability of an owner of a dog that bit a bystander. Suppose for concreteness that there are only two material dimensions of the dispute: the dog’s breed (a proxy for its aggressiveness), and whether the bystander provoked it. Suppose further that there are only two kinds of dogs in the world: pit bulls and golden retrievers, and two kinds of provocation: a kick or none. Suppose finally that the first best efficient liability rule calls for liability of all pit bull
owners, with or without a provocation, since pit bulls are so dangerous that their owners should efficiently guard against the risks of their biting even after a provocation. However, the first best rule calls for liability of golden retriever owners only in the event of no provocation, since golden retrievers are happy and peaceful animals. (We thank Claudia Goldin for this information.)

To illustrate our ideas, consider judges distinguishing cases. Suppose that the first case comes along, and that a man is bitten by a golden retriever with no provocation. Suppose that the issue of provocation does not even come up before the judge. However, we have a biased judge, who thinks that dogs are dangerous and unsavory pests, as are their owners, and establishes the rule that all dog owners are liable when dogs bite men. This, as we assumed, is not an efficient rule.

Suppose that, after a while, perhaps with many other cases of dog bites being adjudicated according to the established precedent of strict liability, a case comes up of a golden retriever biting a man who kicked it. Suppose, again for concreteness, that the judge who handles the case is of one of three types: anti-dog like the first judge, efficiency-oriented, or pro-dog, believing that all dogs are quiet pets and only bite men who deserve it. If the judge is anti-dog, he does nothing, and simply lets the precedent stand, without addressing the issue of provocation. In this case, this issue might perhaps be addressed by future judges. Alternatively, the same anti-dog judge can argue that he considered the provocation, but deemed it immaterial, in which case he effectively solidifies the inefficient precedent, in which owners of all dogs are liable regardless of provocation, forever.

If the judge is efficiency-oriented, he recognizes that it is a better rule to hold owners of provoked golden retrievers not liable, and so introduces provocation as a new material dimension. This judge writes that the prior court has neglected to consider that sometimes golden retrievers are provoked, in which case it is not efficient to hold their owners liable. This judge clarifies the law entirely: owners of provoked golden retrievers avoid liability, all other dog owners are liable in the event of a bite. This is the case of Posner’s efficiency-maximizing judges.

But suppose the judge is a misanthrope. He grabs the opportunity to introduce a material new dimension, and to rule that the precedent applies only to the cases of no provocation. He
accordingly revises the legal rule to say that owners of all breeds are liable in the absence of a provocation (so he respects *stare decisis*), but not liable otherwise. With this new rule, not only do the owners of provoked golden retrievers now -- efficiently -- escape liability, but the owners of provoked pit bulls -- inefficiently -- escape liability as well. In fact, the social cost of the new rule, which wrongly holds the owners of provoked pit bulls not liable, could be much greater than that of the old rule, which wrongly holds the owners of provoked golden retrievers liable. Distinguishing the case and introducing a new material dimension by a biased judge, in this instance, leads the law away from efficiency. And to the extent that *stare decisis* is respected and material dimensions are exhausted by breed and provocation, the inefficient rule is the end of the evolutionary process.

In this discussion, we have not mentioned the judge’s cost of changing the precedent relative to his benefit of doing so. Suppose that the efficiency-oriented judge does not care so much about what happens to be a small efficiency gain from eliminating the liability of the owners of provoked golden retrievers. If it is somewhat costly to change the legal rule, this judge lets the original precedent stand. In contrast, the misanthrope may really care about changing the rule relative to the cost of doing so. Now, the result is even worse than before: efficient rule changes do not take place, and only inefficient ones are implemented by extremist judges. Selection works the wrong way.

Of course, a fuller evaluation of the evolution of the precedent requires the consideration of all the different paths of change in the law, as well as a separate treatment of overruling and distinguishing. But two general principles stand out. First, legal change enables judges to reaffirm their own biases, and to undo the biases of their predecessors. Second, such change is generally implemented by the more extremist judges, whose activist zeal exceeds the personal cost of changing the law. Putting these principles to work, we find that, in general, convergence to efficient legal rules occurs under very limited circumstances. With overruling, convergence may not occur at all, and the legal rules may fluctuate between extremes. With distinguishing, convergence is more likely, and there is a presumption that legal change is on average beneficial. Even with distinguishing, there is significant path-dependence in precedent evolution: legal change
reduces welfare along some of the paths, and the likelihood that legal rules are fully efficient in the limit is low. Moreover, since extremists are more likely to change the law, polarization of judicial preferences leads to a worse performance of the system.

The next section outlines our model of legal precedent. Section 3 describes the efficient legal rules in that model. Section 4 presents a model of judicial overruling of past precedents, and describes the circumstances where this process leads to convergence and efficiency of legal rules. Sections 5 and 6 deal with the more interesting case in which judges distinguish cases and introduce new material dimensions into adjudication. Section 7 concludes the paper.

2. A Model of Legal Precedent

There are two parties, \( O \) and \( V \), and a dog. The dog bit victim \( V \), who seeks to recover damages from \( O \), the dog’s owner. The dog was not on a leash, so in order to assess \( O \)’s liability one should determine whether \( O \) breached the duty of care (in which case he is liable) or did not (in which case he is not liable).

Let \( P_{NP} \) be the probability that the dog bites \( V \) if \( O \) does not take precautions (he does not put it on a leash) and \( P_p \) the probability that the dog bites \( V \) if precautions are taken. \( O \) prefers not to take precautions because he does not want to buy a leash, dislikes limiting the dog’s freedom, or simply does not want to sweat to keep the dog quiet. Let \( C \) be the cost of precautions for \( O \). We normalize to 1 the victim’s loss from the bite.

The Hand formula holds that \( O \) has a duty of care (is liable) whenever \( P_{NP} - P_p \geq C \), i.e., when the reduction in the probability of a bite (weighted by harm, here assumed to equal 1) more than offsets the cost of precautions to \( O \). In contrast, \( O \) has no duty of care (is not liable) if \( P_{NP} - P_p < C \), because precautions cost more than they yield.

Many circumstances determine whether \( O \) was careless. The dog’s aggressiveness, the extent to which \( V \) provoked it, or the place where \( O \) and his dog walked may all influence the
probability of a bite. We assume that two empirical dimensions – the aggressiveness of the dog and $V$’s provocation – determine liability, i.e. constitute material dimensions in this legal dispute.

Variable $a \in [0,1]$ measures the dog’s aggressiveness. More aggressive dogs have larger values of $a$; a dog with $a = 0$ is extremely peaceful (a golden retriever) and less likely to bite $V$ than a dog with $a = 1$ (a pit bull). Variable $q \in [0,1]$, where $q$ stands for $V$’s quietness, measures the extent to which $V$ provoked the dog. If $q = 0$, $V$ outrageously provoked the dog; if $q = 1$, $V$ was maximally quiet. We assume that $a$ and $q$ are independently and uniformly distributed over the population of disputes. We further assume that:

$$P_{NP} - P_P = \begin{cases} \Delta P & \text{for } a + q \geq 1 \\ \Delta P & \text{for } a + q < 1 \end{cases}$$

where $\Delta P > C > \Delta P$. Thus, $O$ is optimally liable if and only if $a + q \geq 1$. Owners of violent dogs are optimally liable if $V$’s provocation was not egregious, owners of peaceful dogs may still be liable as long as $V$ has not provoked them at all ($q = 1$).

In general, the social benefit of the leash is a function $\Delta P(a, q)$ increasing in $a$ and $q$. We assume that it only depends on $a+q$, and that it “jumps” at $a+q=1$. We could allow for more general functions, but our assumptions conveniently clarify the analysis of legal change and its impact on welfare. The first restriction makes $a$ and $q$ symmetric for determining liability, which allows us to isolate the effect of legal change per se, abstracting from the specific nature of the dimension introduced into the law. The second restriction allows us to separate the probabilities of the different errors induced by a particular legal rule from their welfare cost.

A legal rule in this environment attaches a legal consequence ($O$ liable, $O$ not liable) to every possible situation, defined as a combination of $a$ and $q$. The legal rule specifies all the circumstances $(a, q)$ in which $O$ does or does not have a duty of care (i.e. when $P_{NP} - P_P$ is estimated to be greater than $C$). In other words, a legal rule puts substantive content into Hand’s formula by specifying how the incremental probability of an accident must be determined as a
function of $a$ and $q$. Different legal rules reflect different notions of how $P_{\mathcal{H}p} - P_p$ ought to be determined from the empirical attributes of a case.

We restrict the attention to “threshold rules”. A simple “threshold rule” uses only one dimension, say $a$, and specifies a threshold $A$ such that $O$ is held liable if and only if his dog is more aggressive than $A$ (i.e., $a \geq A$). A two-dimensional threshold rule -- using both $a$ and $q$ -- is defined by three thresholds $A$, $Q_0$, and $Q_1$ such that $O$ is held liable either if $a \leq A$ but $q \geq Q_0$, or if $a > A$ but $q \geq Q_1$. Figure 1 shows a generic two-dimensional threshold rule in the $(a, q)$ space:

![Figure 1](image_url)

In Figure 1, $O$ is held liable in regions denoted by $L$, but non liable in those denoted by $NL$. Relative to a one-dimensional rule, a two-dimensional rule allows for liability of owners of peaceful dogs ($a \leq A$) whom $V$ did not provoke ($q \geq Q_0$), and to hold not liable owners of aggressive dogs ($a > A$) whom $V$ provoked egregiously ($q < Q_1$). Note that one should expect $Q_0 \geq Q_1$.

By focusing on threshold rules, we rule out a perfect (or first best) rule, holding $O$ liable whenever $a + q \geq 1$. In reality, legal rules often take the form of threshold rules for reasons presumably related to enforcement costs, since they do not require judges to ascertain the exact values of $a$ and $q$, but only whether certain thresholds on each of the elements had been crossed. For instance, while under the rule of Figure 1 the knowledge that $q > Q_0$ suffices to hold $O$ liable,
this is not the case under the perfect rule, which requires a much more precise (and presumably costly) verification of the facts ($q$ and $a$).

Before calculating efficient threshold rules, we describe how judges set rules in our model of precedent. When initially no existing rule deals with dog bites, we assume that the only issue that comes up at trial is the aggressiveness of the dog. As a result, the judge adjudicating the dispute for the first time sets the legal rule by choosing the first threshold on $a$, which we call $A_1$. Owners of dogs more aggressive than $A_1$ are held liable; owners of dogs less aggressive than $A_1$ are not. This specification of judicial decisions is an intermediate way of dealing with precedents. One can alternatively assume that the first judge sets a broad precedent, in which he considers the hypothetical issue of provocation even if does not arise in the specific dispute, and maps out owner liability on the whole $(a, q)$ space. Under this specification, the law converges immediately, and we cannot talk about judges distinguishing cases; only replacing broad precedents by overruling. One can also imagine a judge setting a very narrow precedent, whereby instead of establishing an aggressiveness threshold for liability, he only makes a decision with respect to the specific breed of dog before him. In this case, there will presumably be a whole collection of narrow judicial decisions, with judges filling in gaps according to their biases, before some threshold aggressiveness level is arrived at. With such narrow precedents, legal evolution is much slower, but the issues we discuss in this paper eventually arise as well.

Once an initial precedent is set, a judge dealing with the same issue later can change the rule. We consider two different models of *stare decisis*. In the first model, which we call overruling, judges discard $A_1$ on the grounds that it was wrong and replace it with a new rule $A_2$. *Stare decisis* only binds in so far as it is costly for the judge to change the precedent. In the second model, which we call distinguishing, the second judge does not assault *stare decisis* with respect to $a$, but can still radically change the law by introducing the additional dimension $q$ into adjudication, i.e. by setting $Q_a$ and $Q_1$. By ruling that the previous precedent is incomplete and applies to only
some of the cases in the \((a, q)\) space, the judge can still radically change the law. In this model, precedent evolves through the introduction by judges of new \textit{material} dimensions \((q\) in this case) into the law. The English view of precedent contemplated only distinguishing as a source of legal change, at least until recently. In the United States, overruling coexists with distinguishing. To clarify the core properties of these two strategies of precedent change, as opposed to the judges’ choice among them, we consider the cases of overruling and distinguishing separately.

We further assume that, for both overruling or distinguishing, a judge changing the legal rule incurs a personal effort cost \(k\), regardless of how he changes the initial precedent. We take \(k\) to be a fixed cost, independent of the magnitude of precedent change. We could alternatively assume that more radical precedent changes entail higher personal costs. Some of the results of that model are different, but our broad qualitative conclusions continue to hold. We also maintain the view that \textit{stare decisis} prevents the introduction of arbitrary and irrelevant dimensions into the law.

The timeline of the model is as follows:

\(t = 0\): The first judge sets the rule by establishing the aggressiveness threshold \(A_1\). This initial precedent guides adjudication until a judge (if any) changes the rule at some \(t'\). What happens at \(t = t'\) and after depends very much on which model we are in.

Overruling: The judge changing \(A_1\) sets a new rule \(A_2\), possibly giving rise to a new round of precedent change. In this model, the issue of provocation never arises.

Distinguishing: The judge changing the rule sets two provocation thresholds \(Q_0\) and \(Q_1\). In this case, the law is permanently fixed, as there are no further material dimensions to introduce.

In Section 4, we study the judges’ objectives in changing the law, as well as their costs of doing so. But first, we investigate the efficient – welfare maximizing – rules that provide the normative benchmark for our analysis of legal change and judge made law.
3. Optimal Legal Rules

Legal rules affect social welfare – defined as the sum of $O$’s and $V$’s utility – by changing the precautions taken by dog owners. The likelihood of damages imposed on $O$ when he is found liable shapes his decision to put the dog on a leash. First best welfare, achieved under optimal precautions (i.e., $O$ puts the dog on a leash whenever $a + q \geq 1$), is equal to:

\[ W^{F.B.} = -(1/2)\Delta P - (1/2)C - P \]

In half the cases, precautions are not efficient and the parties bear the extra risk $\Delta P$ of the dog biting the man; in the other half, precautions are efficient and cost $C$ to society.

Adjudication cannot achieve such high welfare since threshold rules necessarily induce judicial errors. If $O$ is held liable but $a + q < 1$, excessive precautions are taken; if $O$ is held not liable but $a + q \geq 1$, $O$’s level of care is too low. Let $\Pr(L|NL)$ and $\Pr(NL|L)$ be the probabilities that $O$ is erroneously held liable and not liable, respectively, under a particular legal rule. The loss of social welfare relative to the first best under this rule is equal to:

\[ \Lambda = \Pr(NL|L)\Lambda^{\text{under}} + \Pr(L|NL)\Lambda^{\text{over}} \]

$\Lambda^{\text{under}} = \Delta P - C$ is the social cost of under-precautions when $O$ is mistakenly held not liable,

$\Lambda^{\text{over}} = C - \Delta P$ is the social cost of over-precautions when $O$ is erroneously held liable. In our analysis, these costs of over- and under-precautions are constant, and we focus on how different legal rules affect the likelihood of different mistakes in adjudication.

For concreteness, we assume that under-precautions are the greater evil to avoid:

**Assumption 1:** $\Lambda^{\text{over}} / \Lambda^{\text{under}} = \lambda \leq 1$.

The initial precedent consists of a threshold $A$ such that $O$ is held liable if and only if $a \geq A$.

Figure 2 represents such a rule in the $(a, q)$ space.
Under the first best, $O$ is liable above the diagonal but not below. The vertical bold line represents the threshold legal rule, $A$. This rule holds $O$ mistakenly liable in region $L|NL$ and mistakenly not liable in region $NL|L$. For a given $A$, the probabilities of these errors are given by $\Pr(L|NL) = (1/2)(1 - A)^2$ and $\Pr(NL|L) = (1/2)A^2$. The loss of social welfare for each $A$ is then:

\[
\Lambda(A) = (1/2)A^2\Lambda_{\text{under}} + (1/2)(1 - A)^2\Lambda_{\text{over}}
\]

Initial precedent $A$ triggers social loss $\Lambda(A)$ -- an average of over and under-precautions costs under the error probabilities that $A$ induces. The larger is $A$ (the more the initial rule favors $O$), the larger is the loss from under-precautions. Conversely, over-precaution costs increase as $A$ gets smaller.

The optimal initial precedent $A_L$ minimizing social losses is given by:

\[
A_L = \frac{\Lambda_{\text{over}} / \Lambda_{\text{under}}}{1 + (\Lambda_{\text{over}} / \Lambda_{\text{under}})}
\]

The larger is the relative cost of over-precautions (the larger is $(\Lambda_{\text{over}} / \Lambda_{\text{under}})$), the more lenient is the optimal rule (the larger is $A_L$).

Consider next the thresholds $A$, $Q_0$ and $Q_1$ in a two dimensional legal rule. Figure 3 illustrates the pattern of mistakes under such a rule:
Here $O$ is over-punished in region $NL|L$, with area $Pr(L|NL) = (1/2)\left[(1 - Q_0)^2 + (1 - A - Q_1)^2\right]$, and under-punished in region $NL|L$, with area $Pr(NL|L) = (1/2)\left[(A + Q_0 - 1)^2 + Q_1^2\right]$. The social loss from the use of this legal rule is given by:

$$\Lambda(A, Q_0, Q_1) = (1/2)\left[(A + Q_0 - 1)^2 + Q_1^2\right]\Lambda_{under} + (1/2)\left[(1 - Q_0)^2 + (1 - A - Q_1)^2\right]\Lambda_{over}$$

By calculating the first order conditions, one immediately finds the two-dimensional efficient thresholds $A_F$, $Q_O$ and $Q_1$ that minimize social losses

$$A_F = 1/2$$

$$Q_O = \frac{1 + 2(\Lambda_{over} / \Lambda_{under})}{2[1 + (\Lambda_{over} / \Lambda_{under})]}$$

$$Q_1 = \frac{(\Lambda_{over} / \Lambda_{under})}{2[1 + (\Lambda_{over} / \Lambda_{under})]}$$

The optimal legal rule is more lenient toward $O$, the larger is the relative cost of over-precautions (the larger is $\Lambda_{over} / \Lambda_{under}$). Interestingly, the rule induces the same ratio between different errors $Pr(NL|L)/Pr(L|NL)$ as the one-dimensional legal rule given by $A_F$.

Going back to Figure 3, the optimal rule suggests that if the cost of under-precautions is very large, $Q_O$ and $Q_1$ should be accordingly reduced so as to keep $NL|L$ -- the region where careless
owners are held not liable—small. Conversely, for larger over-precautions cost, $Q_o$ and $Q_1$ should be raised so as to reduce the size of $L|NL$, the region where $O$ is mistakenly held liable.

In the efficient two-dimensional legal rule, $A_F = 1/2$ optimizes the use of information about $a$. From Figure 3 one can see that, for given $Q_o$ and $Q_1$, when $A$ becomes larger than 1/2, uncertainty falls to the right of $A$, but increases to its left. The opposite happens when $A$ is made smaller than 1/2. These two opposite effects have a negative net impact on the precision of the rule. The imprecision of a legal rule, $\Pr(NL|L) + \Pr(L|NL)$, is increasing and convex in uncertainty. Uncertainty should then be equally shared across quadrants, i.e. $A_F = 1/2$. To summarize:

**Proposition 1:** i) The optimal one-dimensional legal rule (initial precedent) is given by

\[ A_L = \left( \Lambda_{\text{over}} / \Lambda_{\text{under}} \right) / \left[ 1 + \left( \Lambda_{\text{over}} / \Lambda_{\text{under}} \right) \right]. \]

ii) The optimal two-dimensional legal rule is given by

\[ A_F = 1/2, \quad Q_o = (1 + A_L) / 2, \quad Q_1 = A_L / 2. \]

The efficiency of a rule generally depends on two factors: its overall imprecision $\Pr(NL|L) + \Pr(L|NL)$, and the ratio of different error costs $\Lambda_{\text{over}} / \Lambda_{\text{under}}$. The optimal initial precedent and the optimal two-dimensional rule fare equally well in terms of this second factor, but the two-dimensional rule is more precise, and thus more efficient. $A_F = 1/2$ yields the full benefit of extra information. For extreme $A_F$ (1 or 0), the added dimension $q$ is worthless: a single threshold on $q$ ($Q_o$ or $Q_1$) describes liability over the entire $(a, q)$ space, just like in a one-dimensional rule.

With the results of this section in mind, we can move on to study judicial lawmaking under the two postulated forms of *stare decisis*. Our analysis is driven by two main questions. First, we ask if—consistently with the view of Cardozo (1921)—there is a tendency for the process of precedent change to converge to a decision rule limiting the impact of judicial idiosyncrasies.
Second, we scrutinize Posner’s proposition that not only do the common law rules converge, but also that they converge to the efficient ones.

### 4. How Judges Shape the Law

Like social welfare, the utility of a judge settling a dispute between $O$ and $V$ depends on the precision of the rule and on the ratio of different mistakes. However, we assume that a judge’s objective diverges from efficiency because of his bias, which reflects his preference for $V$ or $O$ and induces him to sacrifice optimal precision for a pattern of mistakes more favorable to the preferred party. Specifically, we assume that the utility of judge $j$ is given by:

\[
U_j = -\beta_{V,j} \Pr(NL|L) - \beta_{O,j} \Pr(L|NL)
\]

Judges dislike making mistakes, but they do not dislike the two types of mistakes equally. $\beta_{O,j}$ and $\beta_{V,j}$ ($\beta_{V,j}, \beta_{O,j} \geq 1$) capture the preference of judge $j$ for $O$ and $V$, respectively: the larger is $\beta_{O,j}$, the more he is eager to hold $O$ not liable, the larger is $\beta_{V,j}$, the more he is willing to hold $O$ liable.

Under the assumed utility function, judges are unhappy with any mistake they make (albeit differentially for different errors). Thus, if we did not restrict attention to threshold rules and allowed for all two-dimensional rules, even biased judges would pick the first best one (the diagonal). This judicial aversion to making mistakes leads to judicial self-restraint that is crucial for our results: even a judge heavily biased against dog owners would not introduce the most anti-owner liability rules he can if these rules lead to mistakes he can avoid, even mistakes favoring bite victims. Such preferences allow us to emphasize – in line with the legal realists – that judicial bias is more problematic in the presence of uncertainty, when judges trade off different errors. But we do not model the kind of favoritism where the judge rules against dog owners even when he knows for sure that they should not be efficiently held liable.

Importantly, our specification of judicial preferences assumes that a judge’s utility depends on the expected outcome arising from the application of a given rule, not from the resolution of a
particular case. This assumption implies, in particular, that a judge would consider replacing a legal rule he dislikes even if the outcome of the specific case before him would be the same under the old rule as under the new one. A judge cares about having a rule in place that meets his idea of justice, rather than about delivering a desired outcome in a specific dispute before him. This assumption is particularly appropriate for appellate judges, who focus on establishing legal rules rather than resolving specific disputes.

The judge’s maximand also assumes that the judge ignores the possibility that the rule he establishes will be changed in the future, and in particular does not act strategically with respect to future judges. This assumption can be relaxed, although at the cost of increased analytical complexity, and we believe our basic results would be preserved. One way to justify the present framework is by noting that precedents change relatively rarely, and therefore a judge discounting the future may not put much value on the effect of future legal change.

There is a measure one of judges, who can be of three types: a share $\alpha_n$ of judges are $\text{Unbiased}$, with preferences \((\beta_v = \Lambda_{\text{under}}, \beta_o = \lambda \Lambda_{\text{under}})\) reflecting social welfare; the rest is equally divided among $\text{Pro-O}$, with biases \((\beta_v = \Lambda_{\text{under}}, \beta_o = \pi \Lambda_{\text{under}})\) and $\text{Pro-V}$, with biases \((\beta_v = \pi \Lambda_{\text{under}}, \beta_o = \Lambda_{\text{under}})\). Parameter $\pi$ \((\pi \geq 1/\lambda)\), measures the polarization of judges’ preferences: with a larger $\pi$, the preferences of $\text{Pro-O}$ and $\text{Pro-V}$ judges are more extreme (there is more disagreement among them).

**Initial Precedent**

The first judge adjudicating a dispute between $O$ and $V$ establishes the initial precedent. We assume that, in this dispute, the issue of provocation never arises (and the judge does not entertain legal rules taking provocation into account unless that issue arises in the dispute). To resolve this dispute, the judge selects a threshold $A$ to maximize:

\[
-(1/2)\beta_v A^2 - (1/2)\beta_o (1 - A)^2
\]
\( \beta_{v,1} \) and \( \beta_{o,1} \) parameterize the bias of the initial judge. Define \( \beta_i = \beta_{o,1} / \beta_{v,1} \) as the Pro-O bias of this judge. Minimizing the objective above, we find that:

\[
A_i = \frac{\beta_i}{1 + \beta_i}
\]

The subscript indicates that \( A_i \) is the initial precedent set with Pro-O bias \( \beta_i \). The result is intuitive: the more Pro-O is the judge, the more lenient he is (the larger is \( A_i \)). \( A_i \) coincides with the efficient initial precedent \( A_e \) only if \( \beta_i = \lambda = \Lambda^{\text{over}} / \Lambda^{\text{under}} \), i.e. if the judge’s bias toward \( O \) reflects the relative social cost of over-precautions.

Under \( A_i \), social losses are given by \( \Lambda(A_i) \). Given the variety of judges’ preferences, there is no reason to presume that \( A_i \) is set efficiently, i.e. to minimize \( \Lambda(A_i) \). If the case ends up in front of a Pro-O judge (\( \beta_i > \lambda \)), too many aggressive dogs roam and bite with impunity; if instead it ends up in front of a Pro-V judge (\( \beta_i < \lambda \)) too many peaceful dogs are put on a leash.

**Overruling**

Depending on \( \beta_i \), the initial precedent may turn out to be severely inefficient. Still, this bias may be corrected through the change of precedent (Cardozo 1921). For instance, if the initial rule is very biased in one direction (say Pro-O), the successive intervention by a Pro-V judge modifies the law by tempering its initial bias with the opposite one.

Suppose that precedent \( A_i \) is in place, and judge \( j \) takes the initiative to change the law. He then sets a new threshold \( A_j \), equal to

\[
A_j = \frac{\beta_j}{1 + \beta_j}
\]

where \( \beta_j = \beta_{o,j} / \beta_{v,j} \) is the Pro-O bias of judge \( j \). To see if judge \( j \) in fact changes the law, we must consider his personal incentive to do so. If judge \( j \) abides by current precedent \( A_i \) he enjoys:
If instead judge $j$ adjudicates according to his preferred rule $A_j$, he enjoys:

$$U_j(A_j) = -(1/2)\beta_{V,j} \left[A_i^2 + \beta_j (1 - A_j)^2\right]$$

Clearly, $U_j(A_j) \geq U_j(A_i)$: by overruling precedent $A_i$, judge $j$ benefits from establishing a rule reflecting his own bias.

However, when $k > 0$, judges may be unwilling to bear the effort and other costs of legal change. Judge $j$ changes the law only if:

$$k \leq U_j(A_j) - U_j(A_i)$$

i.e. when the cost to the judge of changing the law is smaller than its benefit. Manipulating the expression above, we find that judge $j$ overrules the precedent as long as:

$$\beta_{V,j} \frac{(\beta_i - \beta_j)^2}{(1 + \beta_j)^2(1 + \beta_i)} \geq 2k$$

The smaller is the cost $k$, the greater the chance that a judge changes the law. For $k = 0$, judges always overrule precedents (and set their preferred $A$), creating expected social losses of $E_j[\Lambda(A_j)]$, where the expectation is taken over all judge types. But how do judges with a positive $k$ react to precedent? Extremist judges enjoy higher benefits from overruling the precedent, which tend to be further from their point of view, so we expect them to be more activist than unbiased judges, and the more so the more extreme their preferences (the larger is $\pi$).

The case where $\lambda = 1$ well illustrates this intuition. Now, for $k \geq 1/2$, there exist two levels of polarization $\pi_H, \pi_L$, with $\pi_H \geq \pi_L$, such that judges’ behavior can be summarized in Table 1. The boxes of the table report the circumstances when a judge $j$ changes the legal rule he inherited from $i$.

Three patterns of behavior emerge. First, judges never change the initial rule of an adjudicator of their same type. Second, Unbiased judges never change the law. Third, Pro-V and Pro-O judges change the law when their bias is sufficiently intense.
Table 1.

Since a judge changes precedent to set a rule to conform to his preferences, there is no need for him to repudiate a rule established by someone with the same views. Unbiased judges are moderate and therefore reluctant to change the law. However, as polarization gets sufficiently high, both Pro-O and Pro-V judges innovate. At intermediate levels of polarization ($\pi \in [\pi_L, \pi_H]$), Pro-O judges only overrule Pro-V ones and vice-versa. When instead polarization is high, Pro-O and Pro-V judges also change initial rules set by Unbiased judges.

The polarization of judges’ preferences ultimately determines the final configuration of judge-made law. For low polarization ($\pi < \pi_L$), precedent does not change from $A_i$. When polarization is intermediate ($\pi \in [\pi_L, \pi_H]$, precedent oscillates between Pro-O and Pro-V rules unless an Unbiased judge sets the initial rule. At high levels of polarization, precedent always oscillates between a Pro-O and a Pro-V rule. Thus, overruling can be problematic for convergence.

Little changes in the general case where $\lambda$ and $k$ are free to vary. In particular, it is always the case that biased judges overrule precedents more often than unbiased ones do. One difference for $k < \frac{1}{2}$ is that for $\pi$ large enough, unbiased judges eventually start overruling biased ones, and the law oscillates between biased and unbiased precedents. Thus the general lesson of our analysis is that convergence is only guaranteed for very high $k$’s. In that case, regardless of polarization of judicial preferences, no judge ever changes the law, which remains fixed at $A_i$. This observation suggests that, with overruling of precedents, legal rules converge only if judges are so averse to
change that they never take the pain to overrule an initial decision or if there is enough agreement among them (π is low) that their preferred rule is not so far from the current one.

The last result is at odds with the notion that precedent is a powerful mechanism to constrain judicial arbitrariness. We have shown that, when precedents can be overruled, legal unpredictability is the greatest when judges’ preferences are polarized. In a sense, a system of overruling suffers from the very same malady it seeks to cure.

What about the efficiency of legal change? Does evolution of precedent raise social welfare? In terms of efficiency, the benchmark here is \( A_e \), the optimal one-dimensional threshold rule we found in section 3. The following proposition explains when overruling leads to optimality:

**Proposition 2:** Under overruling, judge made law is optimal if and only if all judges are unbiased.

The law converges to the efficient decision rule \( A_e \) only if all judges are unbiased, i.e. if there is full agreement among them and if their views are aligned with efficiency.

When some judges are biased, there is a chance that the initial precedent is either set by a Pro-O or by a Pro-V judge. In either case, precedent does not converge to an efficient rule. The reason is that extremist judges have a greater incentive to change the law than do the unbiased ones. Thus, either the law sticks to its initial (inefficient) configuration, or extremist judges reverse the good rules introduced by the unbiased ones. In this model, legal change (which occurs when judges disagree) is detrimental to social welfare. This notion is confirmed by the following:

**Corollary 1:** Under overruling, expected social losses are (weakly) minimized for \( k = +\infty \).

In our model, when people take precautions based on the law of the moment, expected social losses are only weakly minimized for \( k = +\infty \). The worst case obtains when judges’ aversion to change \((k)\) is intermediate and only the rules of extremists survive in equilibrium as moderate judges stay passive. On the other hand, \( k = 0 \) and \( k = +\infty \) are equivalent from a welfare standpoint. In the
absence of legal change, uncertainty over the bias of the initial judge leads to social losses of 
\( E_i[A(A_i)] \), the same level prevailing for \( k = 0 \).

Despite such equivalence, we would argue that under overruling \( k = +\infty \) is preferred. Such 
values as the predictability of the law or equal treatment may render a bad but stable law preferable 
to an equally efficient on average but unpredictable law. When such criteria enter a social 
calculation (although they are absent from our formal analysis), legal stability is preferable.

Our analysis also challenges the claim that when most judges are unbiased, judicial activism 
(low \( k \)) is desirable as it allows good judges to overrule crazy decisions:

**Corollary 2:** Under overruling, the comparison of social welfare under \( k = 0 \) and \( k = +\infty \) is 
unaffected by the proportion of extremist judges in the population.

The reason for this result is that, with a low \( k \), a bad judge is just as likely to overrule a good 
precedent as he is to introduce a bad one in the first place when \( k \) is high. Yet for legal change to be 
desirable, there should be a difference between the odds of moving from a bad to a good rule, and 
those of going in reverse.

A related explanation of how judge made law evolves toward efficiency holds that 
efficiency-oriented judges are more activist (have a lower \( k \)). However, unless one is willing to 
assume that the intensity of a judge’s bias is negatively related to his own over-ruling cost \( k \), we 
should still expect the rules established by biased activists to counter those established by the 
unbiased ones, thus confirming the message of Corollary 1. Alternatively, one can imagine that, in 
a population of judges with heterogeneous overruling costs, the abler ones are characterized by 
lower \( k \). Abler judges are likely to be more ingenious, more energetic, bolder, and more likely to 
get away with innovation because of their peers’ respect for their talents. However, since there is 
no general reason to expect abler judges to be less biased, the activism of abler judges is not 
necessarily a force for efficiency.
To sum up, we have shown that if judges hold different views, overruling of precedents does not lead to the production of efficient legal rules. Under this view of legal evolution, the best possible outcome is the convergence of the law, attained with fully non-interventionist \((k = +\infty)\) judges. But such immediate convergence is to a random, not the efficient, rule. In fact, given the dismal performance of legal change under overruling, we would argue that the results in this section should be interpreted as making a case for *stare decisis*, and against judicial activism with respect to legal criteria already in place. For legal change to be beneficial, something more is needed.

5. Distinguishing

In the common law tradition, the ability of judges to distinguish cases from previous precedent serves an important constructive role. It allows new information to be considered in adjudication, and thereby enables the law to evolve, to adjust to new circumstances, and to become more precise. Such adaptability of common law has been seen by writers from Holmes (1897), to Cardozo (1921), Hayek (1960), Stone (1985), and Posner (2003) as one of the chief virtues of judge-made law. Here we study such a process of distinguishing cases from precedents, and examine its implications for the convergence and efficiency of judge-made legal rules.

*The Form of Legal Change and its Welfare Consequences*

The utility of a judge who modifies the initial precedent \(A_i\) (we call him judge 2) by introducing the dimension \(q\) into the legal rule by the choice of thresholds \(Q_0\) and \(Q_1\) is

\[
-(1/2)\beta_{y,2}\left[\left(A_i + Q_0 - 1\right)^2 + Q_1^2\right] - (1/2)\beta_{o,2}\left[(1 - Q_0)^2 + (1 - A_i - Q_1)^2\right]
\]

The first term of the expression represents the cost for judge 2 of mistakenly holding \(O\) not liable (i.e. ruling against \(V\)), while the second term is the cost for judge 2 of erroneously holding \(O\) liable. Define \(A_2 = \beta_2 / (1 + \beta_2)\). Here \(A_2\) can be interpreted as the ideal threshold on the dog’s aggressiveness that would be chosen by judge 2 if he were setting the initial precedent.
From first order conditions, we obtain

\[(19) \quad Q_{0,2}(A_i) = 1 - (1 - A_2)A_i \]

\[(20) \quad Q_{1,2}(A_i) = A_2(1 - A_i) \]

These reaction functions tell us that some re-equilibrating mechanism is indeed built into precedent, because \(Q_{0,2}\) and \(Q_{1,2}\) decrease in \(A_i\). Regardless of judge 2’s bias, a more Pro-O initial rule induces him to use dimension \(q\) relatively more in favor of \(V\). However, since the extent of the adjustment depends on the bias of the second judge, summarized by \(A_2\), we need to carefully evaluate the welfare impact of legal change through distinguishing before assessing its desirability.

The probabilities of different mistakes after precedent change are

\[(21) \quad \Pr(L|NL) = \frac{1}{2}(1 - A_2)^2\left[A_i^2 + (1 - A_i)^2\right], \]

\[(22) \quad \Pr(NL|L) = \frac{1}{2}A_2^2\left[A_i^2 + (1 - A_i)^2\right] \]

These expressions show why, contrary to the common wisdom, judges’ biases do not balance one another in judge-made law: the ratio of the two errors, \(\Pr(NL|L)/\Pr(L|NL)\), is fully determined, through \(A_2\), by the desired bias of the second judge! When judge 2 introduces \(q\) into adjudication, he discretionally sets \(Q_{0,2}\) and \(Q_{1,2}\) so as to favor the party he prefers. As a result, there is no presumption that the final configuration of the law is less biased than the initial precedent. In this sense, the eccentricities of judges do not balance one another and judge made law is not a solution to the presence of judicial bias.

In our model, judicial bias affects the efficiency of the law in two distinct ways: it sets the ratio of different errors and it affects the overall likelihood of judicial error, i.e. the law’s precision. A very Pro-O judge is willing to design a very imprecise rule in order to excuse dog owners. Due to the very discretion embodied in distinguishing cases, judge-made law cannot eliminate this first effect of judicial bias: it cannot correct the ratio of different errors.
However, distinguishing cases works through a second channel as well. The incorporation of information into legal rules via distinguishing may progressively influence successive judges so as to limit the waste of information associated with their exercise of discretion. The threshold $A_1$ set by judge 1 may limit the arbitrariness of judge 2 when introducing $q$, thus improving the precision of the law. The term $[A_1^2 + (1 - A_1)^2] \leq 1$ in $P(NL|L)$ and $P(L|NL)$ accounts for this second effect of precedent change. The tension between these two effects emerges by comparing the social loss under the initial precedent $\Lambda(A_1)$ with the social loss after $q$ is introduced.

$\Lambda(A_1, Q_{0,2}, Q_{1,2}) = \left[ A_1^2 + (1 - A_1)^2 \right] \Lambda(A_2)$

The social loss from judge 2’s revision of precedent is a product of two terms, which correspond to the two effects of precedent. The term $\Lambda(A_2)$ stands for the social loss under the hypothetical assumption that the initial rule is chosen by judge 2. This term captures the idea that by distinguishing cases judges regain their discretion. The term $[A_1^2 + (1 - A_1)^2]$ captures the fact that initial precedent influences judge 2’s optimal exercise of discretion, thereby reducing social losses. Indeed, if the initial threshold on $a, A_1$, were not binding through stare decisis, the social loss would be entirely determined by the preferences of judge 2, as reflected by the hypothetical $A_2$. We then have

**Proposition 3:** Legal change through distinguishing cases is beneficial when either of the following conditions is met

$i)$ $\Lambda(A_1) \geq \Lambda(A_2)$

$ii)$ $\Lambda(A_1) < \Lambda(A_2)$, but $[A_1^2 + (1 - A_1)^2]$ is small enough.

Condition $i)$ says that distinguishing is always beneficial when the preferences of judge 2 are more efficiency oriented than those of judge 1. Even is this is not the case, condition $ii)$ says that distinguishing may still be beneficial if the greater precision induced by the inclusion of $q$ more than offsets the loss from adversely changing the ratio of different mistakes. Put differently,
distinguishing is only harmful when two conditions hold simultaneously: judge 2’s preferences, if he was hypothetically setting the initial precedent, would yield greater social losses than those of judge 1, and also, judge 1’s preferences are sufficiently extreme that the constraints the influence of his initial decision on judge 2’s optimal choice is minimal. Or, to put this more broadly, legal change through distinguishing is most likely to be detrimental when both judge 1 and judge 2 are extremists, and when judge 2’s extremism is more detrimental to social welfare than that of judge 1.

To illustrate how distinguishing can be harmful, we make the following

**Assumption 2:** Judge 1 is Pro-V, with bias $\beta_1 < \lambda$, and judge 2 is Pro-O, with bias $\beta_2 > \lambda$.

Together with Assumption 1 (which posits that under-precautions are socially costlier than over-precautions), Assumption 2 not only tells us that $A_1 \leq A_2$ (i.e. judge 1 holds $O$ liable more often than judge 2 would like to), but also that the bias of judge 1 is more efficiency oriented than that of judge 2, so that $\Lambda(A_1) < \Lambda(A_2)$. Under Assumption 2, condition $i)$ of Proposition 3 is violated. In this case, distinguishing may be harmful to society as it just represents a way for judge 2 to excuse careless owners of very aggressive dogs, whom he is fundamentally sympathetic to, by finding $V$’s provocation. Such an excuse may be so costly to society as to undermine the desirability of legal change through distinguishing cases altogether.

But what does condition $ii)$ in Proposition 3 mean? Consider two cases. In both cases, judge 2 is extremely pro-O; in the first, case, judge 1 is extremely pro-V, and in the second, judge 1 is moderate. In the first case, judge 1 only cares about the error of excusing the owners of any dogs who should be held liable, and therefore sets $A_i = 0$. Since judge 2 only cares about incorrectly holding liable owners of dogs who efficiently should not be, his optimal choice in light of the precedent he faces is to undo the will of judge 1 completely and set $Q_0 = Q_1 = 1$. According to judge 2, any provocation, no matter how minor, eliminates dog owner’s liability. When judge 1 is
so extreme, judge 2 is both able and willing to move from the regime of strict liability to the regime of virtually no liability by distinguishing the case based on provocation.

Suppose in contrast that judge 1 is moderate, cares about both types of errors, and therefore sets \( A_1 = \frac{1}{2} \). Judge 2, who is still extremely pro-\( O \), still can set \( Q_0 = Q_1 = 1 \), but he does not want to. Why not? The reason is that he can set \( Q_0 = 1 \), and \( Q_1 = \frac{1}{2} \), and this way avoid the error of holding non-liable the owners of unprovoked vicious dogs. He still keeps the area of false liability down to zero, but because he does not like making any errors, his decision is more efficient. Judge 1’s moderation entails the relative moderation of judge 2. This discussion also shows that our assumption about judicial preferences actually matters; if judge 2 only cared about favoring dog owners without regard for making errors, he would set \( Q_0 = Q_1 = 1 \) regardless of what judge 1 did before him. This leads to:

**Corollary 3:** Judge 1’s moderation leads to judge 2’s moderation.

Formally, this result is again the consequence of the increasing and convex relationship between judicial errors and uncertainty, discussed in Section 2. \[ A_i^2 + (1 - A_i)^2 \] is minimized at \( A_i = \frac{1}{2} \), but maximized when \( A_i \) is either 0 or 1 (i.e. when judge 1 is either fully Pro-\( V \) or Pro-\( O \)).

Overall, our analysis suggests two points. First, in a system of precedent, the desirability of distinguishing and the efficiency of judge-made law depend on judicial bias, particularly on the bias of the last judge who changes legal rules. Legal precedent does not balance the different opinions of judges, and its ultimate configuration may be severely inefficient if an “anti-efficiency” judge sets it. Second, the effectiveness of precedent in constraining judges depends on its initial configuration: the more biased is the initial rule, the more likely is that the introduction of further empirical dimensions is biased as well. The precision of the law exhibits a strong path dependency.

However, we cannot properly evaluate a system of precedent before determining which judges are likely to change the law. The pathologies of judge made law we identified would
disappear if only moderate (or efficiency oriented) judges innovate. In addition, in order to assess the efficiency and convergence of judge-made legal rules we must – in line with our analysis of Section 3 – take into account the uncertainty as to which judges make the law.

*Legal Change when \( k = 0 \).*

By comparing the utility a judge \( j \) derives by abiding by judge \( i \)'s precedent \( A_i \) with the utility he obtains by introducing his preferred thresholds \( Q_{0j} \) and \( Q_{1j} \) into the law (for \( A_i \) given), we find that judge \( j \) modifies precedent when:

\[
\beta_{r_2} \geq k 
\]

By comparing the utility a judge \( j \) derives by abiding by judge \( i \)'s precedent \( A_i \) with the utility he obtains by introducing his preferred thresholds \( Q_{0j} \) and \( Q_{1j} \) into the law (for \( A_i \) given), we find that judge \( j \) modifies precedent when:

\[
\beta_{r_2} \frac{\beta_i^2 + \beta_j^2}{(1 + \beta_i)^2(1 + \beta_j)} \geq 2k
\]

If it is costless for a judge to change the law (\( k=0 \)), he always does so, for two reasons: first, just as we saw in section 3, to regain discretion in adjudication; second -- and this is special to distinguishing cases -- to improve the precision of the law. Since judges do not like making errors, they have an incentive to make the law more precise.

Suppose now that the initial precedent is \( A_i \). Then, when \( k = 0 \), all judges pursue legal change through distinguishing and expected social losses are equal to:

\[
\left[ A_i^2 + (1 - A_i)^2 \right] E_j \left[ \Lambda(A_j) \right]
\]

where the expectation is calculated across all judges. To evaluate the overall desirability of distinguishing, we need to compare \( E_j [\Lambda(A_j)] \), the expected social loss under no legal change, to the expectation of the above expression across all initial precedents. We then find that:

**Proposition 4:** If \( k = 0 \), distinguishing is on average beneficial.

Earlier, we saw that distinguishing can reduce welfare when judges biased against efficiency (in this
case Pro-O) introduce \( q \) in order to modify a precedent set by judges biased toward efficiency (in this case Pro-V). That argument suggested that legal change starting from quasi-efficient decision rules may reduce welfare, while change starting from very inefficient initial rules raises it.

Proposition 4 says that, if \( k=0 \), once we average across all the possible paths of distinguishing, legal change is on average beneficial. The net gain comes from the more accurate information (greater number of empirical dimensions) used by judges. This result stands in contrast to those in section 4, where legal change took the form of overruling of prior precedents, and, as it brought no new data to dispute resolution, in expectation did not help. Here legal change through distinguishing cases in expectation raises the efficiency of dispute resolution, and therefore helps.

Judicial Activism and Distinguishing

When it is costly for judges to modify the current precedent by distinguishing the case, equation (24) gives the condition under which judge \( j \) nonetheless amends the current precedent. This is more likely the further the preferences of \( j \) are away from those of judge \( i \). We can now characterize the activism of different adjudicators.

For \( k = +\infty \), the law does not change from initial precedent and only dimension \( a \) is used. When \( k \) is intermediate, extremist judges are again more activist than benevolent ones because they value more the discretion entailed in distinguishing cases. The case where \( k \in (1/2, +\infty) \) illustrates this intuition. Now there exist two levels of polarization \( \bar{\pi}_H, \bar{\pi}_L \), with \( \bar{\pi}_H \geq \bar{\pi}_L \), and judges rule as follows. Unbiased judges never change the law. Only extremists do so, provided their preferences are sufficiently polarized (\( \pi > \bar{\pi}_L \)). In this range, if polarization is intermediate (\( \pi \in [\bar{\pi}_L, \bar{\pi}_H] \)), Pro-O judges change Pro-V precedents and vice-versa. When polarization is extremely high (\( \pi > \bar{\pi}_H \)), both Pro-O and Pro-V judges also change precedents set by unbiased ones.

What about the final configuration of the law? When there is rough agreement among judges (\( \pi < \bar{\pi}_L \)), the law sticks to initial precedent and there is no legal change through
distinguishing. With intermediate polarization \((\pi \in [\pi_L, \pi_H])\), precedent converges to a decision rule using both \(a\) and \(q\) unless an unbiased judge sets the initial rule. At high levels of polarization \((\pi > \pi_H)\), precedent always converges to a decision rule utilizing both \(a\) and \(q\).

What about the efficiency of distinguishing? The answer depends on judges’ aversion to change and on the disagreement among them. In particular, in line with our findings of section 4, the worst situation is attained when only the extremists are active.

**Proposition 5:** Suppose \(k > 1/2\). Then, for \(\pi \leq \pi_H\) distinguishing on average improves social welfare and there exists \(\tilde{\pi} \geq \pi_H\), such that for \(\pi \geq \tilde{\pi}\), distinguishing on average reduces welfare.

When polarization is very high, the precedent set by unbiased judges is replaced, and the informational gain brought about by the introduction of \(q\) into adjudication does not compensate for this loss. This result is not surprising in light of Corollary 4, which showed how judicial bias undermines the informational gains of precedent change. Thus, when moderate judges are passive, distinguishing improves social welfare when the polarization of judicial preferences is not too high, but does not when polarization is extreme.

In summary, extremist judges are more likely to distinguish cases, and legal change is more likely when judges’ preferences are polarized. This simple proposition has two main implications. First, although we showed earlier that distinguishing could undermine the efficiency of the law, the activism of the biased – and only the biased – judges renders this possibility more concrete. Second, somewhat counter-intuitively, the problems of distinguishing become more severe when judges’ preferences are very polarized. This observation casts doubts on the ability of the system of distinguishing cases from precedents to correct the impact of judicial bias on legal rules in the areas of law where judicial preferences are highly polarized. This system suffers from the very same difficulties it is purported to cure. Perhaps this last point sheds light on the challenges of judicial law-making in politically charged cases, where judicial preferences are highly polarized, and legal
evolution itself becomes a source of unpredictability it purports to eliminate. In contrast, in the areas of law where judges share similar preferences, legal change is beneficial.

Having assessed the efficiency of legal change when judges distinguish cases, in the next section we evaluate the overall efficiency of judge-made legal rules.

6. The Properties of Judge-Made Law under Distinguishing

The first property of judge-made law we consider is convergence. From this standpoint, distinguishing is very different from overruling. Under distinguishing, the law always converges, at least if there is a finite number of empirical dimensions germane to defining a transaction and – which is essentially the same – if the nature of transactions does not change over time.

If judges are very interventionist (either because they are personally activists or because their preferences are sufficiently polarized), the law converges to a two-dimensional legal rule employing both $a$ and $q$. If instead judges are very averse to change, the law converges to a one-dimensional legal rule employing only $a$. Thus, a *stare decisis* doctrine constraining judges to modify the current precedent only by enriching the empirical content of the law is successful in making the law converge.

Still, this result does not imply that distinguishing is a powerful mechanism to constrain the arbitrariness of judges. Indeed, all judges now use the same legal rule in the long run, but the rule may be very biased because it is initially set by a biased judge. Our findings in section 5 that the desirability of legal change may be undermined by biased adjudicators already spoke to this issue.

This consideration makes it imperative to evaluate the efficiency properties of judge made-law under distinguishing. To this end, we compare the expected social losses achieved under judge made-law under the assumption that dimension $q$ is introduced ($k=0$) with those achieved under the optimal rule we analyzed in Section 3 of the paper. We then find the following result:
**Proposition 6:** Under distinguishing, judge-made law is optimal if and only if all judges are benevolent and $\Lambda^{\text{over}} = \Lambda^{\text{under}}$.

This proposition says that when judges can distinguish cases, judge-made law converges to the optimal two-dimensional rule under two conditions. First, all judges are benevolent ($\alpha_y = 1$); second, efficiency dictates that the precision of the law is maximized ($\Lambda^{\text{over}} = \Lambda^{\text{under}}$), or – to put it differently – the optimal rule is unbiased in terms of the ratio of different errors. The contribution of efficiency-seeking judges to the convergence of common law to efficiency is recognized by Posner (2003), although he does not explain just how stringent the conditions for full efficiency are.

The reason for this second condition transpires from the expression of the optimal two-dimensional rule, where $A_F = 1/2$, i.e. the threshold on aggressiveness is set independently from $\Lambda^{\text{over}} / \Lambda^{\text{under}}$, the optimal bias of the rule. We already showed in section 3 that in the optimal rule the efficient ratio between different errors (or optimal bias of the law) is set by the choice of $Q_0$ and $Q_1$, while $A_F = 1/2$ maximizes the precision of the law (i.e. minimizes $\Pr(L|NL) + \Pr(NL|L)$).

In the decentralized system of judge-made law, even with two unbiased adjudicators, the judge setting the initial precedent and the one subsequently changing it cannot get together, fix the efficient bias – implemented by the second judge with a choice of $Q_0$ and $Q_1$ – and agree on an initial (unbiased) precedent $A_i = 1/2$. The judge setting the initial precedent does not take into account the fact that, in the future, another judge may change his rule.

The only situation in which the externality from the judge setting the initial precedent on the one changing the law is absent is when the goal of maximizing the precision of the law and the goal of biasing it optimally are not divorced, i.e. when $\Lambda^{\text{over}} = \Lambda^{\text{under}}$. In this case, a benevolent judge would set $A_i = 1/2$ and, as long as all other judges have the same preferences as his own, a fully efficient rule would emerge. Because of this negative externality between the judge setting the initial precedent and the judge changing the law, full efficiency is harder to attain when judges...
distinguish cases than when they simply overrule precedents. Indeed, as Proposition 2 illustrates, in the overruling model the efficient decision rule is achieved provided all judges are benevolent, a condition that in the distinguishing model is necessary but not sufficient.

This finding should not lead one to conclude that overruling produces better legal rules. In general, distinguishing works better than overruling because the introduction of \( q \) into adjudication renders legal rules more precise. The only threat to such beneficial growth of the law is posed by the interventionism of extremist judges and the passivity of benevolent ones. It should therefore come as no surprise that:

**Corollary 4:** Under distinguishing, expected social losses are minimized for \( k = 0 \).

This result stands in stark contrast to Corollary 2, which showed that under overruling the best outcome is achieved for \( k = +\infty \). In that case, frequent legal change prevented the law from converging, leading to unpredictability.

Under distinguishing, the law always converges, so unpredictability is not relevant (at least in the long run). What is important instead is that: a) \( q \) is introduced into the law, b) it is not introduced in a systematically biased way. Both conditions are met when \( k=0 \) because judges definitely introduce \( V' \)’s provocation into the law, and not only extremists, but also unbiased judges trigger such change. Change-averse judges are very helpful when precedents can be overruled, but are a problem when cases can be distinguished from precedents. In the former case, massive legal change is detrimental, while in the latter it helps.

The different nature of legal change under the two broad views of it is reinforced by the following result:

**Corollary 5:** Under distinguishing, the smaller the proportion of extremist judges in the population, the more \( k = 0 \) is preferred to \( k = +\infty \).
This result is natural. When more judges are unbiased, legal change is beneficial and the introduction of $q$ into adjudication benefits its precision relative to the initial precedent. In contrast, when precedents are overruled, the proportion of unbiased judges does not affect the desirability of legal change because it both enhances the chance for an unbiased judge to permanently set the initial rule and the chance for him to change a bad rule. Without the net informational gain brought into the law by the introduction of $q$, legal change is not superior when judges are better.

To summarize, we have seen that when judges distinguish cases from precedents, legal change has two desirable features: it makes the law converge and it improves its precision. We also saw that under this view of *stare decisis*, judge-made legal rules are efficient only if efficiency dictates to maximize the precision of the law and every judge is aligned with this objective. We then outlined two main differences between distinguishing and overruling. First, the former system is mostly beneficial when judges are interventionist ($k = 0$), as opposed to the latter where interventionist judges only create problems. Second, we showed that only under distinguishing legal change is more beneficial when more judges are efficiency oriented.

7. Conclusion.

When does the evolution of judge-made law through precedent change lead to efficient legal rules? We addressed this question in a legal-realistic model, in which deciding judges may be both biased and averse to changing the law, but do face opportunities to either overrule the precedent or distinguish it from the case before them.

Not surprisingly, having judges interested in developing efficient legal rules, rather than pursuing more parochial objectives, generally promotes efficiency. When judges are not averse to changing rules, and can modify the precedent by adding new material dimensions into adjudication, legal change is generally beneficial, even when the rules do not converge to full efficiency.

In other circumstances, legal rules not only fail to converge to efficiency, but legal change through revision of precedents is altogether harmful. For instance, when judges overrule previous
precedents, legal rules do not even need to converge. And if implementing legal change is very costly to judges personally, then only extremists overrule previous precedents, and the law fluctuates between inefficient rules. Furthermore, when the law progresses as judges distinguish cases, the evolution of legal rules is highly path-dependent, and not always moving to efficiency. Judges shape the law to accommodate their own biases, and in particular to undo the consequences of their predecessors’ decisions they disagree with. Still, the law evolving in this fashion on average moves toward efficiency when judicial preferences are not too polarized. However, the conditions for the resting point of this process being full efficiency are implausibly stringent.

The model in this paper is a first step in the analysis of judge-made law, and omits several important aspects of legal evolution. First, unlike the previous research, we focus on decision-making by judges, and neglect the selection of disputes for judicial resolution rather than settlement. It is far from clear, however, that such selection improves the quality of law, since it may be the combination of extremist litigants and judges that leads to legal change.

Second, we have ignored the important fact that judges make decisions in panels, which could in principle moderate polarization of their views, and lead to better law. However, as shown by Revesz (1997) and Sunstein et al. (2004), panels sometimes lead to the convergence of member views to the bias of the majority, rather than to a moderate compromise. Collective decision making does not then reduce polarization, so crucial to the efficiency of legal change.

Third, we have presented an extremely limited model of judicial leeways, in which only one verifiable material dimension can be added to the judicial consideration of a dispute. In reality, there are many such dimensions and, moreover, some of them include complex issues such as causality or knowledge. According to Stone (1985), the flexibility of language offers appellate judges tremendous leeway in distinguishing cases and rewriting the law. This leeway may offer considerable benefits when the law evolves toward efficiency, but it can also slow down legal change, or turn it in bad directions, when used by judges uninterested in efficiency.
Fourth, we have focused on judicial discretion in making new laws under the assumption that the facts of the case are verifiable. However, as argued by Frank (1930, 1951), Stone (1985) and Posner (1990), judges can also manipulate their interpretation of the facts, by emphasizing some aspects of the evidence and neglecting others, thereby reaching the outcomes they desire through fact-discretion rather than changes in the law. Such fact-discretion in itself many undermine the efficiency of the law, but is also likely to slow the pace of legal change, as judges choose to “work on” the facts rather than to rewrite precedents. For this reason, fact-discretion is one of the crucial challenges in the analysis of legal evolution.

As a final note, we emphasize that ours is a theoretical analysis of the propositions that legal change in a system or precedent is beneficial, and that the law converges to efficiency. Posner’s hypotheses, however, are empirical propositions, and as such cannot be rejected by theory. We have tried to develop several testable implications of our analysis, which might make it possible to identify the areas of the law where Posner’s hypothesis is more likely to hold. These hypotheses may be easier to verify empirically than the broad propositions about the efficiency of common law.

8. Mathematical Appendix.

**Proof of Proposition 1.** The optimal one-dimensional threshold rule \( A_L \) is defined as

\[
A_L = \arg \min_{A \in [0,1]} (1/2)A^2 \Lambda_{\text{under}} + (1/2)(1-A)^2 \Lambda_{\text{over}}
\]

If \( \Lambda_{\text{over}}, \Lambda_{\text{under}} > 0 \) the objective is convex and \( A_L = (\Lambda_{\text{over}} / \Lambda_{\text{under}}) / [1 + (\Lambda_{\text{over}} / \Lambda_{\text{under}})] \) is found by solving the f.o.c. \( A_L \Lambda_{\text{under}} - (1-A_L) \Lambda_{\text{over}} = 0 \). Notice that \( A_L \in [0,1] \). The optimal two-dimensional threshold rule \( (A_F, Q_0, Q_1) \) is defined as

\[
(A_F, Q_0, Q_1) = \arg \max_{A, Q_0, Q_1 \in [0,1]} (1/2)[(A + Q_0 - 1)^2 + (Q_1)^2] \Lambda_{\text{under}} + (1/2)[(1-Q_0)^2 + (1-A-Q_1)^2] \Lambda_{\text{over}}
\]

Again, \( \Lambda_{\text{over}}, \Lambda_{\text{under}} > 0 \) ensures that the above objective is convex in \( (A, Q_0, Q_1) \) (its Hessian is positive definite). Thus, solving the first order conditions for \( (A_F, Q_0, Q_1) \)
\[
\frac{\partial \Lambda}{\partial A} = (A_F + Q_0 - 1)\Lambda_{\text{under}} - (1 - A_F - Q_1)\Lambda_{\text{over}} = 0
\]
\[
\frac{\partial \Lambda}{\partial Q_0} = (A_F + Q_0 - 1)\Lambda_{\text{under}} - (1 - Q_0)\Lambda_{\text{over}} = 0
\]
\[
\frac{\partial \Lambda}{\partial Q_1} = Q_1\Lambda_{\text{under}} - (1 - A_F - Q_1)\Lambda_{\text{over}} = 0
\]
yields \( A_F = 1/2 \), \( Q_O = (1 + A_L)/2 \), \( Q_1 = A_L/2 \). Notice that \((A_F, Q_O, Q_1) \in [0,1]^3\). ♠

**Proof of Proposition 2.** The optimal rule for judge \( j(A_j) \) is found by replacing \((\Lambda_{\text{under}}, \Lambda_{\text{over}})\) with \((\beta_{V,j}, \beta_{O,j})\) in the expression for social losses and by minimizing it accordingly. Judge \( j \) overrules \( A_i \) with \( A_j \) when \( U_j(A_j) - U_j(A_i) \geq k \). That is when

\[
f_{j,i}(\pi) = \beta_{V,j} \frac{(\beta_i - \beta_j)^2}{(1 + \beta_j)^2(1 + \beta_j)} \geq 2k
\]
Judge \( j \) never overrules \( i \) if \( \beta_i = \beta_j \). Let us look at the behavior of the three different types of judges: Benevolent \((j = b)\) \((\beta_{V,j} = \Lambda_{\text{under}}, \beta_{O,j} = \lambda\Lambda_{\text{under}})\), Pro-O \((j = o)\) \((\beta_{V,j} = \Lambda_{\text{under}}, \beta_{O,j} = \pi\Lambda_{\text{under}})\) and Pro-V \((j = v)\) \((\beta_{V,j} = \pi\Lambda_{\text{under}}, \beta_{O,j} = \Lambda_{\text{over}})\). Where \( \lambda \equiv \Lambda_{\text{over}} / \Lambda_{\text{under}} \). \( f_{j,i}(\pi) \) is the “incentive” for a judge of type \( j \) to overrule a judge of type \( i \). We have

\[
f_{b,o}(\pi) \equiv \Lambda_{\text{under}} \frac{(\pi - \lambda)^2}{(1 + \pi)^2(1 + \lambda)} ; \quad f_{b,v}(\pi) \equiv \Lambda_{\text{under}} \frac{(1 - \lambda \pi)^2}{(1 + \pi)^2(1 + \lambda)}
\]

\[
f_{o,b}(\pi) \equiv \Lambda_{\text{under}} \frac{(\lambda - \pi)^2}{(1 + \lambda)^2(1 + \pi)} ; \quad f_{o,v}(\pi) \equiv \Lambda_{\text{under}} \frac{1 - \pi^2}{(1 + \pi)^3}
\]

\[
f_{v,o}(\pi) \equiv \Lambda_{\text{under}} \frac{(\lambda \pi - 1)^2}{(1 + \lambda)^2(1 + \pi)} ; \quad f_{v,b}(\pi) \equiv \Lambda_{\text{under}} \frac{\pi^2 - 1}{(1 + \pi)^3}
\]
The functions \( f_{j,i}(\pi) \) increase in \( \pi \). If \( j \in \{o, v\} \), \( h_{v,j}(\pi) \to +\infty \) as \( \pi \to +\infty \). Also, \( f_{b,i}(\pi) \leq 1 \).

Thus, for \( k > 1/2 \) Benevolent judges never overrule precedents. Call \( \pi_{j,i} \in [1/\lambda, +\infty) \) the level of \( \pi \) (if it exists) above which \( j \) overrules \( i \). If not all judges are Benevolent, the initial precedent will be efficient only with probability \( \alpha_b < 1 \). Then, the law converges to an efficient one-dimensional rule if Benevolent judges overrule Pro-O and Pro-V ones without Pro-O and Pro-V judges overruling Benevolent ones. This can happen if there exist some \( k \) so that \( \max\{\pi_{b,o}, \pi_{b,v}\} \leq \min\{\pi_{v,b}, \pi_{o,b}\} \), i.e. if there are levels of \( \pi \) so that \( b \) overrule \( o \) and \( v \) without being in turn overruled. For this to be true it is necessary that for some \( \pi \), \( \min\{f_{b,o}(\pi), f_{b,v}(\pi)\} \geq \max\{f_{o,b}(\pi), f_{v,b}(\pi)\} \). It is easy to find that, if \( \lambda \leq 1 \), \( f_{o,b}(\pi) \geq f_{v,b}(\pi) \geq f_{b,o}(\pi) \geq f_{b,v}(\pi) \) for any \( \pi \), so \( \pi_{o,b} \leq \pi_{v,b} \leq \pi_{b,o} \leq \pi_{b,v} \). Thus, if
\( \alpha_y < 1 \) there exist no \( k < +\infty \) such that \( A_i \) converges to \( A_L \). It is easy to see that the proposition would also hold for \( \lambda > 1 \).

**Proof of Corollary 1.** At \( k = 0 \) every judge overrules a precedent not consonant with his preferences. In this case, expected social losses are \( E_i[\Lambda(A_i)] = \alpha_v \Lambda(A_v) + \alpha_y \Lambda(A_y) + \alpha_o \Lambda(A_o) \), where \( A_v \), \( A_y \), \( A_o \) are the preferred rules of Pro-V, Benevolent and Pro-O judges, respectively.

What if \( k \to +\infty \)? Proving **Proposition 1** we saw that for large \( k \) Benevolent judges stay passive. Extremists will not overrule as well, provided \( \pi \) is finite. To see what happens for large \( \pi \), suppose that \( k \) and \( \pi \) grow at the same speed (\( \lim_{k,\pi \to +\infty} k/\pi = 1 \)). Then \( \lim_{k,\pi \to +\infty} f_{o,b}(\pi)/k = \Lambda_{\text{under}} \lambda/(1+\lambda)^2 \leq 1 \) and \( \lim_{k,\pi \to +\infty} f_{o,v}(\pi)/k = \Lambda_{\text{over}} \leq 1 \). Then, since \( f_{x,y}(\pi) \geq f_{x,b}(\pi) \) \( \forall x,y \in (0,v), x \neq y \), for \( k = +\infty \) neither Pro-O nor Benevolent judges ever overrule precedent. Thus, the initial precedent will stick forever, yielding expected social losses of \( E_i[\Lambda(A_i)] \). Thus, \( k = 0 \) and \( k = +\infty \) are equally desirable from a welfare standpoint. From the proof of **Proposition 1** we know that when \( k \) is intermediate, there exist thresholds \( \pi_{o,v} \leq \pi_{o,b} \leq \pi_{v,b} \leq \pi_{b,o} \leq \pi_{b,v} \) such that for \( \pi \) above \( \pi_{o,v} \) only extremists overrule each other and social losses are still \( E_i[\Lambda(A_i)] \); as \( \pi_{v,b} \) is crossed, extremists also overrule Benevolent judges (who are still passive): now the law fluctuates between \( A_v \), \( A_o \) and social losses are larger than \( E_i[\Lambda(A_i)] \). Finally, for \( \pi \geq \pi_{b,v} \) (a possibility only if \( k < 1/2 \)) Benevolent judges also overrule extremists ones and expected social losses go back to \( E_i[\Lambda(A_i)] \).

Thus, no matter what \( \pi \) is, we can conclude that \( k = +\infty \) weakly minimizes social losses.

**Proof of Corollary 2.** It follows from the equality of social losses in \( k = 0 \) and \( k = +\infty \).

**Proof of Proposition 3.** The optimal thresholds for judge 2 after initial precedent \( A_i \) are found by replacing \( (\Lambda_{\text{under}}, \Lambda_{\text{over}}) \) with \( (\beta_{v,j}, \beta_{o,j}) \) in the expression for social losses and by minimizing it accordingly by taking \( A_i \) as given. Under the new (two-dimensional) rule, expected social losses are \( \Lambda(A_i, Q_{0,2}, Q_{1,2}) = \left[A_i^2 + (1-A_i)^2\right]\Lambda(A_2) \), as opposed to the level \( \Lambda(A_i) \) achieved under the initial precedent. Proposition 3 follows directly from the comparison of these two magnitudes.

**Proof of Corollary 3.** We say that judge 2 has more discretion under \( A'_i \) than under \( A_i \) if the impact of his bias on welfare is greater when he introduces \( q \) after \( A'_i \) than after \( A_i \). If judge 2 is
Pro-O, the impact of his bias on welfare is $\partial \Lambda(A_i, Q_{0,2}, Q_{1,2}) / \partial \beta_2 \left(- \partial \Lambda(A_i, Q_{0,2}, Q_{1,2}) / \partial \beta_2 \right)$ if he is Pro-V. Then, $\partial^2 \Lambda(A_i, Q_{0,2}, Q_{1,2}) / \partial \beta_2 \partial A_i$ tells how judge 2’s discretion varies with $A_i$. Since

$$
\frac{\partial^2 \Lambda(A_i, Q_{0,2}, Q_{1,2})}{\partial \beta_2 \partial A_i} = 2(2A_i - 1) \left[ \partial \Lambda(A_i) / \partial \beta_2 \right]
$$

We find that judge 2’s discretion is maximized at $A_i = 0$ or $A_i = 1$ (judge 1 is fully biased in either direction) and minimized at $A_i = (1/2)$. This is true both if judge 2 is Pro-V and if he is Pro-O.\▲

**Proof of Proposition 4.** If $q$ is not used in the law expected losses are $E_i[\Lambda(A_j)]$, while they are $E_i[A_i^2 + (1 - A_i)^2]E_j[\Lambda(A_j)]$ under legal change. If all judges change the law (true for $k = 0$), then $E_i[\Lambda(A_i)] = E_j[\Lambda(A_j)]$ and Proposition 4 follows from $E_i[A_i^2 + (1 - A_i)^2] \leq 1$.▲

**Proof of Proposition 5.** Judge $j$ distinguishes away precedent $A_i$ by introducing $q$ into the law when $U_j(A_i, Q_{0,j}, Q_{1,j}) - U_j(A_i) \geq k$. That is when

$$
h_{j,i}(\pi) = \beta_{V,j} \frac{\beta_i^2 + \beta_j^2}{(1 + \beta_i)^2(1 + \beta_j)} \geq 2k
$$

Judge $j$ could also distinguish if $\beta_i = \beta_j$ in order to exploit the extra precision $q$ brings into the law. Again, the evolution of the law is shaped by the behavior of different judges. Define the initial precedent set by judge $i$ as $A_i$ and $\theta_i = A_i^2 + (1 - A_i)^2$. In line with our previous analysis, define:

$$
h_{b,b}(\pi) = \Lambda_{\text{under}} \frac{2\lambda^2}{(1 + \lambda)^3}; \quad h_{b,v}(\pi) = \Lambda_{\text{under}} \frac{(1 + \lambda^2 \pi^2)}{(1 + \pi)^3(1 + \lambda)}; \quad h_{b,o}(\pi) = \Lambda_{\text{under}} \frac{(\pi^2 + \lambda^2)}{(1 + \pi)^3(1 + \lambda)};
$$

$$
h_{a,b}(\pi) = \Lambda_{\text{under}} \frac{\lambda^2 + \pi^2}{(1 + \lambda)^2(1 + \pi)}; \quad h_{a,v}(\pi) = \Lambda_{\text{under}} \frac{(1 + \pi^4)}{(1 + \pi)^3}; \quad h_{a,o}(\pi) = \Lambda_{\text{under}} \frac{2\pi^2}{(1 + \pi)^3};
$$

$$
h_{v,b}(\pi) = \Lambda_{\text{under}} \frac{1 + \lambda^2 \pi}{(1 + \lambda)^2(1 + \pi)}; \quad h_{v,v}(\pi) = \Lambda_{\text{under}} \frac{2\pi^2}{(1 + \pi)^3}; \quad h_{v,o}(\pi) = \Lambda_{\text{under}} \frac{(1 + \pi^4)}{(1 + \pi)^3};
$$

$h_{j,j}(\pi)$ ($j, i \in \{v, o, b\}$) is the incentive of judge $i$ to distinguish $A_i$ as a function of $\pi$. As, before, for $j \neq i$, $h_{j,i}(\pi)$ is increasing and if $j \in \{o, v\}, j \neq i$, $h_{j,i}(\pi) \rightarrow +\infty$ as $\pi \rightarrow +\infty$. Also, $h_{b,i}(\pi)$, $h_{v,i}(\pi)$, $h_{o,i}(\pi) \leq 1$. Thus, for $k > 1/2$ Benevolent judges are passive and extremists never distinguish their own precedents. After some tedious algebra, one can see that for $\pi \geq 1/\lambda$

$$
h_{o,v}(\pi) = h_{v,o}(\pi) \geq h_{o,b}(\pi) \geq h_{v,b}(\pi).
$$

Thus, $\pi_{o,v} = \pi_{v,o} \leq \pi_{o,b} \leq \pi_{v,b}$, where $\pi_{j,i}$ defines, for a given $k$, the level of $\pi$ above which $j$ distinguishes $A_i$. Define $\overline{\pi}_{L} = \pi_{o,v}$ and $\overline{\pi}_{H} = \pi_{o,b}$. Below $\overline{\pi}_{L}$ there
is no legal change (social welfare is unchanged). Between \(\pi_L\) and \(\pi_H\) extremists only distinguish each others’ rules. Since in \([\pi_L,\pi_H]\) Benevolent precedents are not distinguished, legal change is beneficial if

\[
\lambda[\alpha_o\theta_o + \alpha_v\theta_v - (\alpha_o + \alpha_v)\theta_o\theta_v] + (1 - \lambda)[\alpha_o A_o^2 + \alpha_v A_v^2 - \alpha_o \theta_o A_o^2 - \alpha_v \theta_v A_v^2] \geq 0
\]

it is immediate to see that since \(\alpha_v = \alpha_o\) and \(\theta_i \leq 1\), the above terms in square brackets are non-negative. Thus, legal change is beneficial for \(\pi \leq \pi_H\). For \(\pi \in (\pi_H, \pi_{v,b})\) social welfare is equal to

\[
\alpha_o \theta_o \Lambda(A_o) + \alpha_v \theta_v \Lambda(A_v) + \alpha_b \theta_b \Lambda(A_b)
\]

while for \(\pi \geq \pi_{v,b}\) social losses are equal to

\[
\alpha_o \theta_o \Lambda(A_o) + \alpha_v \theta_v \Lambda(A_v) + \alpha_b \theta_b \Lambda(A_b) + 1/2[\Lambda(A_o) + \Lambda(A_v)]
\]

Since \(\lambda < 1\), \(\Lambda(A_o) \leq \Lambda(A_v)\): social welfare is larger in \(\pi \geq \pi_{v,b}\) than in \((\pi_H, \pi_{v,b})\). To prove the existence of \(\pi \geq \pi_H\) it is enough to show that as \(\pi \to +\infty\) legal change is detrimental (although this does not prove the existence of a unique crossing point where initial social losses fall below long run ones). Since \(\theta_o = \theta_v = (1 + \pi^2)/(1 + \pi^2), \quad \Lambda(A_o) + \Lambda(A_v) = \theta_o(1 + \lambda), \quad \alpha_o = \alpha_v,\) we can write social losses in \(\pi \geq \pi_{v,b}\) as \((\alpha_o \theta_o + 0.5\alpha_b \theta_b)\theta_o(1 + \lambda). \quad \theta_o\) increases in \(\pi\), so social losses increase with \(\pi\). As \(\pi \to +\infty\) legal change is bad if \(\theta_b(1 + \lambda) \geq 2\Lambda(A_b)\), which is true because \(\lambda \leq 1\).

**Proof of Proposition 6.** The optimal two-dimensional rule is \(A_e = 1/2, \text{ } Q_o = (1 + 2\lambda)/2(1 + \lambda), \text{ } Q_l = \lambda / 2(1 + \lambda)\). To give judge-made law its best shot, suppose that \(k = 0\). By the same logic of Proposition 2, if some judges are biased, judge made law will be inefficient. If \(\alpha_b = 1\), the law converges to \(A = \lambda/(1 + \lambda), Q_o = 1 - Q_l, Q_l = \lambda/(1 + \lambda)^2\), which is efficient only insofar \(\lambda = 1\).

**Proof of Corollary 4.** By the same argument we used in Corollary 1, when \(k = +\infty\) legal change does not take place. Since \(h_{u,v}(\pi) = h_{v,o}(\pi) \geq h_{u,b}(\pi) \geq h_{v,b}(\pi)\) are the strongest incentives to change the law, at intermediate levels of \(k\) Benevolent judges may be passive and extremist judges may hamper the efficiency of legal change by heavily shaping the law. If \(k \in (1/2, +\infty)\), we saw that benevolent judges stay passive, only extremists introduce \(q\) and legal change may hamper the efficiency of the law. If \(k \in (0, 1/2]\) the same thing happens at intermediate levels of polarization, whereas for \(\pi\) very large eventually all judges (also Benevolent ones) are interventionists, which reduces the ability of extremists to bias legal change against efficiency. This last case is equivalent to \(k = 0\) where, as we established in Proposition 4, legal change is desirable. Thus, the only way to
make legal change desirable for any level of polarization is to set $k = 0$, which is by definition preferable to $k = +\infty$. ♠

**Proof of Corollary 5.** $k = 0$ is preferred over $k = +\infty$ when $E_i(\theta_i) \leq 1$. An increase in $\alpha_B$ makes $k = 0$ more desirable provided it lowers $E_i(\theta_i)$. The condition boils down to

$$\frac{d}{d\alpha_B} E_i(\theta_i) = \theta_o - \theta_b \leq 0$$

It is easy to see that for $\pi \geq 1/\lambda$ the above condition is satisfied. ♠
Bibliography.


