ISSN 1045-6333

HARVARD

John M. Olin Center for Law, Economics, and Business

THE DISADVANTAGES OF AGGREGATE DEDUCTIBLES

Alma Cohen

Discussion Paper No. 367

6/2002

Harvard Law School
Cambridge, MA 02138

The Disadvantages of Aggregate Deductibles

Alma Cohen•

Abstract

This paper analyzes the choice of deductible in insurance contracts that insure against a risk that, as is common, might materialize more than once during the life of the policy. As was established by Arrow (1963), from the perspective of risk-bearing costs, the optimal contract is one that uses an aggregate deductible that applies to the aggregate losses incurred over the life of the policy. Aggregate deductibles, however, are uncommon in practice. This paper identifies two disadvantages that aggregate deductibles have. Aggregate deductibles are shown to produce higher expected verification costs and moral hazard costs than contracts that apply a per-loss deductible to each loss that occurs. I further show that each of these disadvantages can make an aggregate deductible contact overall undesirable. Using data from the automobile insurance market, I estimate that the verification costs disadvantage that aggregate deductibles have in this market is by itself sufficient to make them inferior, for plausible levels of policyholders' risk-aversion, to the per-loss deductibles that are actually used. The results of the analysis can help explain the rare use of aggregate deductibles and, in addition, might explain why umbrella policies that cover all types of losses are rarely used.

Keywords: insurance, deductible, moral hazard, verification.
JEL classification: D89, G22.

• Post-doctoral fellow, National Bureau of Economic Research; John M. Olin Research Fellow in Law, Economics and Business (acohen@kuznets.harvard.edu). This paper is based on a chapter of my Ph.D. dissertation at Harvard University. I am grateful to Lucian Bebchuk, Baruch Berliner, Gary Chamberlain, David Cutler, Caroline Hoxby, Yehuda Kahane, Jack Porter, Zvika Safra, Harris Schlesinger, Steven Shavell, and Richard Zeckhauser for valuable comments. I gratefully acknowledge the financial support of the NBER and the John M. Olin Center.

1 Introduction

One of the basic questions in the design of insurance contracts concerns the choice of deductibles. In most lines of insurance, losses of the type covered by the insurance policy can occur more than once during the life of a policy.[1] This raises the question, which will be my focus here, whether deductibles should be applied to the aggregate losses during the life of the policy.

Under a policy with an aggregate deductible, a deductible will be applied to the aggregate losses of the policyholder during the period covered by the policy; the insurer will cover the excess of these aggregate losses over the specified aggregate deductible. From the perspective of reducing risk-bearing costs, well-known results in the economics of insurance (starting with the classic paper by Arrow (1963)) imply that an aggregate deductible would reduce the risk-bearing costs of the policyholder most effectively. To reduce their risk-bearing costs, policyholders would prefer to concentrate dollars of insurance coverage in states of the world in which aggregate losses are high. A policy with an aggregate deductible indeed would channel all insurance payments to those states of nature in which aggregate losses will be sufficiently high.

As will be discussed later, there are some cases in which aggregate deductibles are used, notably reinsurance contracts and some types of health insurance.[2] However, in most lines of insurance, aggregate deductibles are not used. Rather, the policies that are commonly used have a per-loss deductible under which the insurer will apply a deductible to *each* loss during the period of the policy. Per-loss deductibles are generally used in policies purchased by individuals, such as policies for automobile insurance, fire and homeowner insurance, and boat-owner insurance.[3] Per-loss deductibles also dominate in the various lines of commercial property insurance and commercial liability insurance.[4]

Given the optimality of aggregate deductibles in terms of risk-bearing costs, why are such deductibles uncommon in practice? The model developed in this paper identifies and analyzes two factors that can help explaining this observed pattern. In particular, it is shown that, compared with per-loss deductibles, aggregate deductibles might well increase the expected verification costs of the

[1] In automobile insurance, for example, there are many cases in which policyholders have two or more claims during the year covered by the standard policy. See, e.g., Cohen (2001)

[2] See, e.g., Phifer (1996), Casse (1997).

[3] See Alliance of American Insurers (1998). This policy kit contains the insurance contracts and forms that are commonly used by insurers in each line of insurance.

insurer and the severity of the moral hazard problem and thus the expected value of total losses. Each of these advantages might be sufficient under some identified circumstances to make aggregate deductibles sub-optimal and, in particular, inferior to per-loss deductibles.

The first disadvantage of aggregate deductibles that I analyze concerns verification costs. These are the costs that the insurer will have to bear upon the submission of a claim to verify (i) whether a damage of the type covered by the policy indeed occurred, and, if so, (ii) the magnitude of the damage. Because of the potential severity of the problem of false claims, it is generally recognized, as well as documented, that insurers must often incur substantial costs to verify submitted claims.[5] While the theoretical literature has conducted some analysis of how administrative costs affect the design of insurance contracts (see, e.g., Bond and Crocker (1991), Huberman et al (1983), and Kaplow (1994)), such analysis generally either assumed that administrative costs are a function of the actuarial value of the policy or assumed that only one loss can occur over the life of the policy (or, equivalently, that there will be a fixed administrative cost if there are any losses during the policy). In contrast, I will consider here the case in which more than one loss can occur during the life of the policy and in which administrative costs will have to be incurred for each claim.

In this setting of multiple potential claims, I show that an aggregate deductible will tend to have a verification costs disadvantage in the common case in which the distribution function of the damages in the event of a loss is declining so that small losses are more likely than large ones. I first analyze the case in which losses must be verified shortly after they are reported. For example, in automobile insurance or homeowners insurance (where aggregate deductibles are indeed not used), reports of a loss by policyholders need to be verified before the damage is fixed, as it would be difficult to assess what the damage was after it is fixed. Under a policy with an aggregate deductible, in the early stages of the period covered by the policy, policyholders will have an incentive to file claims also when losses are small to prepare for the possibility that the aggregate deductible threshold will be reached at later stages of the policy period. Furthermore, whenever the aggregate deductible is reached, the policyholder will

---

[4] See Alliance of American Insurers (1998).

[5] See, e.g., Bond and Crocker (1997), Caron and Dionne (1997), Crocker and Morgan (1998), Crocker and Tennyson (1999), Cummins and Tennyson (1996), Picard (1996), and Smith and Wright (1992). For a comprehensive report on the vast literature on insurance fraud and verification, see Derrig (2001).

subsequently have an incentive to file claims even for very small losses (such as a trivial scratch to their car or trivial water damage to their home).

In contrast, if a deductible is applied to each loss, policyholders will never file claims, and no verification costs will be incurred, in connection with small losses. Indeed, I show that the expected verification costs under any given contract with an aggregate deductible is higher than the expected verification costs under a contract with a per-loss deductible that provides the same actuarial value (i.e. offers the same expected insurance payments).

I also examine the case in which verification of losses can be deferred until the end of the period covered by the policy, and I identify also for this case circumstances under which an aggregate deductible policy will produce higher expected verification costs than a per loss deductible policy that has the same actuarial value. I further show that the identified potential disadvantage of aggregate deductibles can outweigh their risk-bearing advantage and make them overall inferior to some per-loss deductibles.

I then proceed to examine whether such verification-based inferiority of aggregate deductible is likely to arise under plausible circumstances by examining the conditions for it using actual data from the automobile insurance market. Estimating the distribution of losses and expected verification costs under both the per-loss deductible contracts that are used and under aggregate deductible contracts with the same actuarial value, I find that the per-loss deductible contracts are superior for policyholders with plausible degrees of risk-aversion.

The second potential disadvantage of aggregate deductibles that I identify concerns moral hazard. Much research has been done on how deductibles affect moral hazard.[6] The standard setup in this literature is one in which only one claim might arise during the period covered by a policy, and the policyholder's level of precautions affects the probability that such a claim will arise. However, the literature has not analyzed the issue on which my analysis focuses, namely how the choice of deductible affects precautions during the life of a policy when more than one loss might occur.

Under a policy with a per-loss deductible, the policyholder will always have some incentive to take precautions throughout the period of the policy, even after one or more losses have occurred. In contrast, under a policy with an aggregate deductible, in the event that the aggregate deductible threshold is reached at any point in time during the life of the policy, the policyholder will subsequently have

[6] Pioneering studies on this subject include Pauly (1968), Spence and Zeckhauser (1971), Shavell (1979). For an excellent recent survey, see Winter (2000).

no incentive to take any precautions whatsoever (as subsequent losses will be fully covered). It is shown that, if the marginal contribution of precautions declines at a sufficiently high rate with their level, which implies that it is relatively important to have the policyholder always take minimal precautions, aggregate deductible policies lead to a higher expected number of losses. Furthermore, it is shown that this disadvantage of aggregate deductibles might make them inferior to per-loss deductibles.

To illustrate the moral hazard problem, consider insurance coverage for the loss of valuables on trips away from home. Because precautions by a policyholder are hard to verify, concerns about moral hazard arise. Under a policy with a per-loss deductible, a policyholder whose watch was lost will continue to have an incentive to avoid another loss of another valuable later on. In contrast, under a policy with an aggregate deductible, once a golden watch is lost (assuming the value of the watch exceeds the aggregate deductible), the policyholder will have subsequently no incentive to take any precautions to avoid other losses of valuables.

To look at another example where moral hazard might be important, consider coverage under health insurance policies for doctor office visits. Such contracts often provide only partial coverage for such expenses. When this is done, what is used is commonly not an aggregate deductible for such expenses but rather a deductible (a "co-insurance" payment) that is applied to each doctor visit. Whereas an aggregate deductible would better protect a policyholder against a negative health shock that increases the number of genuinely needed doctor office visits, such a deductible would introduce the possibility that, once it is reached, the policyholder will face no out-of-pocket costs whatsoever from additional doctor visits.

The analysis of this paper can also contribute to explaining another puzzling feature of insurance practices -- the rare use of "umbrella" policies insuring policyholders against all the categories of risks that they face. Insurers generally sell separately policies for different categories of risk (say, automobile insurance, home insurance, etc.) rather than an "umbrella" policy covering all types of losses. This is the case even though many insurers offer a full line of policies for the different categories of losses and many individuals buy all their insurance coverage (but through a number of separate policies) from the same insurer. From the perspective of risk-bearing costs, such policies appear attractive. The established results in the literature following Arrow (1963) imply that from the perspective of risk-bearing costs it would be optimal to set a limit on the aggregate loss that an individual would have to bear. For this reason, the rare use of umbrella policies has been viewed as a puzzling feature of the insurance landscape (see, e.g., Eeckhoudt and Gollier (1999), Gollier and Schlesinger (1995, 1996)).

The analysis of this paper, however, implies that considerations of verification costs and moral hazard might well make it undesirable to use an umbrella policy that covers the excess of aggregate losses from all categories over an aggregate deductible. Of course, individuals could be offered an umbrella policy with per-loss deductibles applied to each loss, with possibly different per-loss deductibles applied to losses of different types. But such a structure would eliminate the reason why an umbrella policy seems to begin with attractive, which is the desire to cap the policyholder's aggregate losses from all sources. Assuming that per-loss deductibles are to be applied, and given that in such a case it might be optimal to set a different level of per-loss deductible to different categories of losses, there will be no benefit from having all categories of risk bundled in one policy rather than simply having separate policies for different categories of risks.

It should be noted that I do not attempt to analyze in this paper the optimal design of contacts that apply a deductible to each loss. What I wish to show is that verification costs and moral hazard costs can make aggregate deductible inferior to some alternative contracts. To this end, I will show that these problems can make aggregate deductible inferior to one simple type of per-loss deductible that is commonly used in practice – a uniform per-loss deductible under which the same deductible level is applied to each and every loss occurring during the life of the policy. Whether and to what extent insurers could do better than uniform per-loss deductibles is beyond the scope of the analysis. The demonstration of how uniform per-loss deductible dominates aggregate deductible in certain plausible circumstances is sufficient to explain on why aggregate deductibles are not used in such circumstances.

The remainder of this paper is organized as follows. Section 2 presents the framework of analysis. Section 3 analyzes the verification costs disadvantage of aggregate deductibles, and section 4 analyzes their moral hazard disadvantage. Finally, Section 5 makes concluding remarks on how the results of the analysis can explain insurance practice.

2 Framework of Analysis

2.1 Basic Setup

The setup of the analysis is as follows. An agent (the "policyholder") buys from an insurance company (the "insurer") an insurance policy against losses of a certain kind in a certain period. The policyholder is risk-averse and has a utility function U, where $U' > 0$ and $U'' > 0$.

I shall assume that the period covered by the policy is made of two sub-periods, 1 and 2, in each of which a loss might occur. As will be clear from the analysis, it can be adjusted to apply to the case in which there are many such sub-periods. I shall denote by $x_i \geq 0$ the loss (if any) in period i (i=1,2).

I assume that the realizations of x_1 and x_2 are distributed independently of each other.[7] To abstract initially from considerations of moral hazard, I will assume that the probability of a loss in each period is the same and is exogenously given and unaffected by the design of the insurance policy $\Pr(x_1 > 0) = \Pr(x_2 > 0) = p$. Section 4 will drop this assumption and focus on moral hazard considerations.

In each of the two periods, if a loss occurs, the value of the loss will be distributed with a density function $f(\cdot)$, and a cumulative distribution function $F(\cdot)$. The density function $f(\cdot)$ is assumed to be continuous and positive in $(0, \bar{x})$ and zero outside it. For illustration purposes I shall throughout consider the common case in which the loss is distributed with an exponential distribution. I shall denote by $\mu = E(x \mid x > 0)$ the expected value of the loss in the event that a loss occurs. Thus, the expected value of the policyholder's loss is $p\mu$ in each period, and the expected value of the policyholder's aggregate losses in the two periods is $2p\mu$.

I will also assume that insurers are risk-neutral and operate in a competitive market. Consequently, the price of any given insurance contract offered in the market (the premium) will be equal to the insurer's expected cost. Because of the presence of the competition, the contract that will be used is such that, subject to the policyholder's paying a premium equal to the insurer's expected costs, will maximize the policyholder's expected utility.

[7] I also assume that x_1 and x_2 are independently distributed of whatever other sources exist for fluctuations in the policyholder's wealth. If x_1 and x_2 are correlated with such sources, then the optimal design of the insurance policy will have to take into account "portfolio" consideration. See Mayers and Smith (1983) and Doherty and Schlesinger (1983).

The expected cost of a policy to the insurer will be equal to the expected insurance payments to the policyholder -- that is, actuarial value of the policy -- plus the insurer's expected administrative costs. Because of the presence of administrative costs, the price of an insurance policy will always exceeds its actuarial value. Initially, to abstract from the effect of the type of deductible on verification costs, it will be assumed, as in Arrow (1963) and the literature following it, that expected administrative costs are a function of the policy's actuarial value. This assumption, which rules out the plausible possibility that there are some fixed per claim administrative costs, will be dropped in section 3.

2.2 Aggregate deductible contracts

Under an insurance policy with an aggregate deductible, the policyholder will get from the insurer the excess of the total losses occurring during the policy period over a specified aggregate deductible *D*, where $0 < D < 2\bar{x}$. Thus, under this policy, the total insurance payments will be

$$I_A(x_1, x_2, D) = Max(x_1 + x_2 - D, 0).$$

Accordingly, the actuarial value of the policy with an aggregate deductible *D* is

$$\begin{aligned}
(1)\quad & E(I_A(x_1, x_2, D)) = \Pr(x_1 + x_2 > D)E(x_1 + x_2 - D \mid x_1 + x_2 > D) = \\
&= p(1-p)\int_D^\infty (x_1 - D) f(x_1)dx_1 + (1-p)p\int_D^\infty (x_2 - D) f(x_2)dx_2 + \\
&+ p^2\left\{\int_0^D \int_{D-x_2}^\infty (x_1 + x_2 - D) f(x_1) f(x_2)dx_1dx_2 + \int_D^\infty \int_0^\infty (x_1 + x_2 - D) f(x_1) f(x_2)dx_1dx_2\right\}.
\end{aligned}$$

Denote by $C_A(D)$ the expected administrative costs to the insurer of a policy with an aggregate deductible, the premium to the policyholder will be:

$$P_A(D) = E_A(D) + C_A(D),$$

and the expected utility under this contract will be:

$$(2)\quad U_A(D) = EU(W - P_A(D) - \min(x_1 + x_2, D)).$$

2.3. The Risk-Bearing Advantage of Aggregate Deductibles

Lemma 1: If *(i) the expected administrative costs are a function only of the expected insurance payments, and (ii) the probability of an accident in each of the two period is not affected by the choice of policy, then, among the set of all possible policies with a given actuarial value, the expected utility of the policyholder will be highest under the policy with an aggregate deductible.*

Remark: This Lemma follows from the classic result of Arrow (1963).[8] Arrow established that, under the above conditions, a policyholder's expected utility will be maximized under a policy guaranteeing that the policyholder's final wealth will not fall below a certain threshold in any state of nature. The intuition behind this result is that, assuming that any dollar of actuarial value has the same cost (in terms of the additional premium charged) to the policyholder, it would never be optimal to have the policyholder receive an insurance payment in a state of the world if the policyholder's final wealth in another state of the world will be lower. For if that were the case, the policyholder's expected utility would be raised by having dollars of insurance payments moved to the state of the world in which final wealth is lower (and thus marginal utility of money higher).

In the setting under consideration, an aggregate deductible is the only one that never spends dollars of insurance payments when there are states of the world in which the policyholder's final wealth is lower. In contrast, the disadvantage of a per-loss deductible is that it spends dollars of actuarial value in cases in which a loss occurs only in one sub-period, while leaving the policyholder with a lower final wealth in those states of the world in which such a loss occurs in both sub-periods of the policy.

2.4 Per-Loss Deductibles

In the following sections of the paper I will show that dropping the two assumptions in the statement of Lemma 1 introduces considerations that might well make it undesirable to use aggregate deductibles. In particular, I will show that such considerations can make aggregate deducible inferior to per-loss contracts that apply some positive deductible to each and every loss occurring during the life of the policy. There are different ways in which such per-loss contract can be fashioned, and I will not attempt to rank them. Rather, to show

[8] Arrow's result was subsequently extended by many papers (see, e.g., Raviv (1979), Karni (1992), and Gollier and Schlesinger (1996)).

that verification and moral hazard considerations can make aggregate deductible contracts inferior to some per-loss contract, I will focus on comparing an aggregate deductible contract to a simple, and commonly used in practice, form of a per loss contracts – a *uniform* per-loss deductible policy under which the *same* deductible is applied to the loss if any occurring in each period. Below I will refer to such a conventional contract as a contract with a per-loss deductible.

Accordingly, under an insurance policy with a per-loss deductible, the policyholder will get from the insurer, for any loss that occurs, the excess (if any) of the loss over a specified deductible level d, where $0 < d < \bar{x}$. Thus, under this policy, the total insurance payments during the life of the policy (if any) will be

$$I_L(x_1, x_2, d) = Max(x_1 - d, 0) + Max(x_2 - d, 0).$$

Accordingly, the actuarial value of this policy (that is, the expected insurance payments under the policy) will be:

$$(3)\ E(I_L(x_1, x_2, d)) = \Pr(x_1 > d) \cdot E(x_1 - d \mid x_1 > d) + \Pr(x_2 > d) \cdot E(x_2 - d \mid x_2 > d) =$$
$$= p\int_d^\infty (x_1 - d) f(x_1) dx_1 + p\int_d^\infty (x_2 - d) f(x_2) dx_2 .$$

Denoting by $C_L(d)$ the expected administrative costs to the insurer of a policy with a per-loss deductible, the premium charged to the policyholder will be:

$$P_L(d) = E_L(d) + C_L(d)$$

and the policyholder's expected utility under this per-loss deductible contract will be:

$$(4)\ U_L(d) = EU(W - P_L(d) - \min(x_1, d) - \min(x_2, d)).$$

It is important to observe that, for any given contract with an aggregate deductible, there exists a contract with a per loss deductible that has the same actuarial value. The following result establishes this and characterizes this actuarially equivalent contract.

Lemma 2: *For any policy with an aggregate deductible D, there exists a (unique) policy with a per-loss deductible d that has the same actuarial value, with the level of d satisfying* $0.5D < d < D$.

Proof: Let us first show that for $d = 0.5D$ the value of (1) is smaller than the value of (3). To see this, let us show that, if $d = 0.5D$, then any case in which the policy with an aggregate deductible *D* will call for payment, the policy with a per-loss deductible will also call for the same or higher payment. If $x_1 + x_2 > D$, so that payment will be made under the aggregate deductible policy, then there will be two cases. Suppose first that both losses x_1, x_2 exceed $0.5D$, *i.e., d*. In this case, the per-loss deductible policy will call for a payment $(x_1-d)+(x_2-d)= x_1+x_2-2d$, which is the same as under the aggregate deductible policy.

The second case is that in which one of the losses exceeds *d* and the other does not, and suppose without loss of generality that $x_1 > d$ and $x_2 < d$. In this case the payment under the per-loss policy will be higher than $(x_1-d)+(x_2-d)= x_1+x_2-2d$, which will be the payment under the aggregate deductible policy.

Next let us show that, for $d = D$, the value of (1) is higher than the value of (3). To see this, suppose that $d = D$. Under this assumption, in any case in which the per-loss deductible policy will produce a payment, the aggregate deductible policy will produce the same or higher. If $x_1 + x_2 > d$, then there are two cases. Suppose first that both losses x_1, x_2 exceed *d*. In this case, the per-loss deductible policy will call for a payment $(x_1-d)+(x_2-d)= x_1+x_2-2d$, which is smaller than the aggregate deductible policy. The second case is that in which one of the losses exceeds *d* and the other does not, and suppose without loss of generality that $x_1 > d$ and $x_2 < d$. In this case, the payment under the per-loss deductible policy will be lower than $x_1 + x_2 - d$, which will be the payment under the aggregate deductible policy.

Having seen that the value of (1) exceeds the value of (3) for $d = 0.5D$, and that the value of (1) is lower than the value of (3) for $d = D$, and since (1) is a continuous and increasing function of *d*, we get that for the values of (3) and (1) to equal, *d* must be between *0.5D* and *D*. Q.E.D

3 Verification Costs

Thus far I have assumed that the size of administrative costs is completely determined by the actuarial value of the policy and is thus unaffected by the type of deductible used. I now turn to examine how the size of expected administrative costs might depend on the type of deductible used. In particular, I will focus on per-claim administrative costs that the insurer will have to bear to verify for each claim the occurrence of a loss and its magnitude. Following Gollier (1987), I will assume that there is a fixed verification cost that will have to be incurred for each claim if the insurer wishes to establish the existence and magnitude of the loss. I also will assume without loss of generality that these per claim costs are the only administrative costs that the insurer might have to bear; the results would be the same if one were to assume that these per claims costs came on top of some other administrative costs that were a function of the actuarial value of the policy (and thus the same for any two policies with the same actuarial value).

The analysis below will initially focus on the common case in which verification costs must be incurred shortly upon the occurrence of the loss and cannot be deferred to the end of the policy period. In the case of a damage suffered by an automobile, for example, it would be normally difficult, if not impossible, to verify a claim after the damage to the automobile is repaired. For this reason, all verification and assessment must be made right after the reporting of a loss and only then the policyholder is allowed to fix the damage. Section D extends the analysis to cases in which verification costs can be deferred until the end of the policy period.

3.1 The Disadvantage of Aggregate Deductibles

Let us consider the expected verification costs under an aggregate deductible policy in the case in which verification cannot be deferred to the end of the policy period. Under an aggregate deductible *D*, the policyholder will submit any claim that will occur in the first period.[9] Reporting each loss occurring in the first period has a positive expected value even if the loss is small, because even a small loss will be ultimately covered in the event that the aggregate deductible will be reached in the second period. In the second period, of course, reporting will occur only if the aggregate deductible is indeed reached. Accordingly, the expected value of verification costs will be:

$$(5)\quad C_A(D) = c\Pr(x_1 > 0) + c\Pr\big((x_1 + x_2 > D) \cap (x_2 > 0)\big).$$

[9] Not necessarily true if the policyholder also bears cost in submitting a claim. In this case submitting a claim below the level of the deductible that occur in the first period is a gamble to the policyholder.

Let us now consider the expected verification costs under a policy with a per-loss deductible d. Under this policy, since the policyholder will submit claims only if doing so will produce a positive gain, the policyholder will report in each period only losses that exceed the per-loss deductible d. Accordingly, the expected verification costs will be:

$$(6)\ C_L(d) = c\Pr(x_1 > d) + c\Pr(x_2 > d).$$

In comparing the expected verification costs under an aggregate deductible policy and under a per-loss policy that has the same actuarial value, I am focusing on the common case in which the density function $f(y)$ is declining -- that is, the case in which, when a loss happens, small losses are more likely than large losses. For this common case, the following sharp result can be established:

Proposition 1: *If the density function of the value of the loss, when a loss occurs, is decreasing, then any given policy with an aggregate deductible will involve higher expected verification costs than a policy with a per-loss deductible that has the same actuarial value.*

Remark: The intuition behind this result is as follows. Under the policy with a per-loss deductible, the policyholder will never report small losses but will report only those losses that exceed the per-loss deductible level d. In contrast, under the policy with an aggregate deductible, the policyholder will submit any claim in the event of a loss in the first period, because verification costs by the insurance company must be incurred shortly after the loss occurs. Furthermore, the policyholder might report small losses even in the second period (if the aggregate deductible was reached in the first period). When small losses are more likely than large ones, the fact that the aggregate policy induces a significant reporting of such losses is what makes this policy more costly in terms of verification costs.

Proof: Let us assume that d and D are such that $E_A(D) = E_L(d)$, and let us show that $C_A(D) > C_L(d)$. Since $C_A(x)$ declines when x increases (see (6)), and since $D < 2d$, $C_A(D)$ must be smaller than $C_A(2d)$. Thus, we have:

$$(7)\ C_A(D) - C_L(d) > C_A(2d) - C_L(d) =$$

$$= c(\Pr(x_1 > 0) + \Pr((x_1 + x_2 > 2d) \cap (x_2 > 0))) - c(\Pr(x_1 > d) + \Pr(x_2 > d)) =$$

$$= c\left\{\int_0^\infty f(x)dx + \int_0^{2d} f(x_2) \int_{2d-x_2}^\infty f(x_1)dx_1dx_2 + \int_{2d}^\infty f(x_2)dx_2 \int_0^\infty f(x_1)dx_1\right\} -$$

$$- c\left\{\int_d^\infty f(x)dx) + \int_d^\infty f(x)dx\right\}.$$

Thus,

$$(8)\ C_A(D) - C_L(d) > c\left\{\int_0^d f(x)dx - \int_d^{2d} f(x)dx\right\} + c\int_0^{2d} f(x_2) \int_{2d-x_2}^\infty f(x_1)dx_1dx_2 .[10]$$

And since each of the two terms on the right-hand size of (8) must be positive, $C_A(D)$ must exceed $C_L(d)$ and the proposition is proved. Q.E.D

3.2 The Possible Overall Inferiority of an Aggregate Deductible

Proposition 2: *For any given policy with an aggregate deductible, the expected utility of the policyholder will be larger under a policy with a per-loss deductible and the same actuarial value if the cost c of verifying a claim is sufficiently large.*

Remark: The intuition for this result is as follows. The expected administrative costs of the policy with an aggregate deductible are higher than those of the per-loss deductible policy by an amount that increases without bound in *c*. Since these higher administrative costs are borne by the policyholder, a sufficiently large *c* will erode any given advantage that the aggregate deductible policy has in terms of risk-bearing costs.

Proof: From the proof of proposition 1, it follows that $C_A(D) - C_L(d)$ is greater than the second term on the right-hand side of (8), which increases linearly and without bound with *c*. It follows that, if *c* is increased sufficiently, $C_A(D) - C_L(d)$ can be made higher than any given value. It follows, specifically, that *c* can be set so that $C_A(D) - C_L(d)$ will exceed $2d - D$. In this case, the additional price required for the aggregate deductible policy *D* will be such that the policyholder's

---

[10] Since the density function is a decreasing function, $\int_0^d f(x)dx - \int_d^{2d} f(x)dx$ must be positive.

final wealth will be lower under the aggregate deductible policy than under the per-loss deductible policy in every state of nature (i.e., any realization of x_1 and x_2). Q.E.D

3.3 Illustration and Assessment Using Automobile Insurance Data

Below I use insurance data to get a sense of the magnitude of the verification costs disadvantage of aggregate deductibles and of whether it could make them inferior to a per loss deductible. To this end, I use data from the automobile insurance market to examine whether it is plausible to expect that the magnitude of the verification disadvantage is sufficient to make aggregate deductibles overall inferior for risk-averse policyholders.

I use data that I received from an insurance company that operates in the market for automobile insurance in Israel. The data contains information about 216,524 policies, and about 111,138 different policyholders, for the years 1995-1999. As is conventional in this market, all the policies sold by the insurer use a per-loss deductible. The data includes for each policy all the information known by the insurer about the policyholder and about the claim history of the policy, including the policyholder's demographic characteristics, the characteristics of the policyholder's car, the insurer's information about the policyholder's past driving experience, the level of the deductible, and each of the claims reported and the amount paid by the insurer.[11]

A study of this data indicates that the distribution of the damages in the event of an accident can be reasonably well approximated by an exponential distribution. Assuming an exponential distribution, the expected verification costs under a policy with a per-loss deductible d can be calculated to be:

$$(9)\ \ C_L(d) = 2cp \cdot e^{\frac{-d}{\mu}} ,$$

where d is the level of deductible and μ is the mean of the damage distribution. I used for c the estimate of the per-claim administrative costs given by the company, and I calculated the expected verification costs of average policy in my data.

Given the exponential distribution of damage, it can be shown that, for any given per-loss policy d, the aggregate deductible policy D that has the same actuarial value must satisfy:

---

[11] The data is described in fuller detail in Cohen (2001)).

$$(10)\quad e^{\frac{D-d}{\mu}} = 1 + \frac{pD}{2\mu}\ .$$

The expected verification costs under this policy are given by:

$$C_A(D) = cp + cp(1-p)e^{\frac{-D}{\mu}} + cp^2 e^{\frac{-D}{\mu}}(\frac{D}{\mu}+1)\,,$$

or, after rearrangement,

$$(11)\quad C_A(D) = cp + cpe^{\frac{-D}{\mu}} + cp^2 e^{\frac{-D}{\mu}}\frac{D}{\mu}\ .$$

After some algebraic rearrangement, the ratio of expected verification costs under the two policies is:

$$(12)\quad \frac{C_L(d)}{C_A(D)} = \frac{e^{\frac{-d}{\mu}} + e^{\frac{-D}{\mu}}\left(1 + p\frac{D}{2\mu}\right)}{1 + e^{-\frac{D}{\mu}}\left(1 + p\frac{D}{\mu}\right)}.$$

I calculate that, for the average policy, a move to a policy with an aggregate deductible and the same actuarial value will increase expected verification costs by about one third.

I now proceed to examine how likely it is that this verification costs disadvantage of aggregated deductibles would by itself outweigh the risk-bearing considerations and make the per-loss deductibles used superior to aggregate deductibles. In doing so I will assume below that the policyholders have a constant-absolute risk-aversion utility function (see Bell and Fishburn (2000)) of $-e^{-ax}$. With this added assumption, it can be shown that the expected utility of a policyholder under a per-loss deductible policy will be higher than under the aggregate deductible policy that has the same actuarial value if

$$E\left(-e^{-a(W-P-C_L(d)-\min(x_1,d)-\min(x_2,d))}\right) > E\left(-e^{-a(W-P-C_A(D)-\min(x_1+x_2,D))}\right),$$

which, after rearrangement, yields

$$\text{(13)}\quad c > \frac{\ln \dfrac{E\left(e^{a\min(x_1+x_2,D)}\right)}{E\left(e^{a\min(x_1,d)+a\min(x_2,d)}\right)}}{\left(ap\left(1+e^{\frac{-D}{\mu}}+pe^{\frac{-D}{\mu}}\dfrac{D}{\mu}\right)-a2pe^{-\frac{d}{\mu}}\right)} \quad .$$

For the average policy in my data, this condition (13) is satisfied as long as the risk-aversion coefficient *a* is less than about 1. Since the coefficient of absolute risk aversion is sensitive to the monetary units used, let us examine this threshold in light of the monetary units used for the above calculations. It can be calculated, given the units used, that a level above 1 for the coefficient *a* would imply that the policyholder would prefer paying 10 times the level of expected damages in terms of a premium over bear the actual level of damages. This appears to be a very high if not implausible degree of risk-aversion. Therefore, it seems likely that the per-loss deductible policies used by the considered insurance company were indeed superior to the (actuarially equivalent) aggregate deductible policies for the vast majority of the policyholders.

3.4 Verification at the End of the Policy Period

Let us suppose that the occurrence and magnitude of a loss do not need to be verified right away but can be also verified at the end of the policy period. In such a case, the insurer can defer verifying a first-period loss until the end of the policy's second period. This deferral might reduce the expected verification costs of an aggregate deductible policy. Under such a policy, such deferral would enable avoiding verification costs for small first-period losses when the second period does not bring total losses to the aggregate deductible threshold. That is, there will be verification costs incurred in connection with a loss smaller than *D* in first-period only if there is a loss in the second period and $x_1 + x_2 > D$.

Thus, under a policy with an aggregate deductible *D*, the insurer will expend resources to verify a first-period loss only if $x_1 > 0$ (there is a loss) *and* $x_1 + x_2 > D$. Thus, the expected verification costs will be reduced compared with (5) to the level of:

$$\text{(14)}\quad c\Pr\left(\left(x_1 > D\right)\cap\left(x_2 = 0\right)\right) + c\Pr\left(\left(x_2 > D\right)\cap\left(x_1 = 0\right)\right) +$$

$$+ 2c\Pr\left(\left(x_1 + x_2 > D\right)\cap\left(x_1 > 0\right)\cap\left(x_2 > 0\right)\right).$$

The question that arises now is how the possibility of verification at the end of the policy period affects the comparison between the two types of deductibles in terms of verification costs. For a preliminary exploration of this extension, let us focus on the exponential distribution case. For this standard case, the proposition below indicates that, even if verification can be deferred to the end of the policy period, an aggregate deductible policy will still have a disadvantage in terms of expected verification costs.[12]

Proposition 3: *In the exponential distribution case, if verification costs can be deferred to the end of the period, then the expected verification costs of a policy with an aggregate deductible will always be higher than those of a policy with a per-loss deductible that has the same actuarial value.*

Proof: In the case under consideration, the expected verification costs under a policy with a per-loss deductible *d* are:

$$(15)\ \ C_L(d) = 2cp \cdot e^{\frac{-d}{\mu}} \left\{ (1-p) + p(1-e^{\frac{-d}{\mu}}) \right\} + 2cp^2 e^{\frac{-2d}{\mu}}$$

And the expected verification costs under a policy with an aggregate deductible *D* are:

$$(16)\ \ C_A(D) = 2cp(1-p)e^{\frac{-D}{\mu}} + 2cp^2 e^{\frac{-D}{\mu}} (\frac{D}{\mu}+1)\,.$$

Comparing (15) with (16) and rearranging terms, I find that $C_L(d) < C_A(D)$ if an only if

$$e^{\frac{-d}{\mu}} < e^{\frac{-D}{\mu}} (1 + \frac{pD}{\mu})$$

Using the relationship between *d* and *D* that must be satisfied for actuarial equivalence (see (10)), and rearranging terms, we get that $C_L(d)$ is always smaller than $C_A(D)$, which complete the proof. Q.E.D.

[12] I conjecture that this result holds generally for distribution functions that are continuous and declining. Proving this conjecture is left for future research.

4 Moral Hazard

Thus far I have put aside the question of moral hazard by assuming that the choice of deductible does not affect the probability of a loss in each period. The moral hazard issue is of course important and has been the subject of a large literature (see Winter (2000) for a survey)). I now turn to examine the effects of the type of deductible used on the severity of the moral hazard problem.

To do this, I will drop the assumption that *p*, the probability of loss in any given period, is exogenously given. Instead, I will allow *p* to be a function of *e*, the policyholder's precautions (measured in the monetary units) in the relevant period. Precautions reduce the probability of a loss: $p'(e) < 0$. It will be assumed, as is conventional, that $p''(e) > 0$. I will denote by $p_{\max} = p(0)$ the probability of a loss in the absence of any precautions, and by $p_{\min} = \lim_{e \to \infty} p(e)$ the level of *p*, (whether 0 or some positive number above 0), which represents the lower bound below which *p* cannot be reduced however large the level of precautions. Finally, to focus below on the potential moral hazard disadvantage of aggregate deductibles, I will put aside the verification costs disadvantage by assuming that there are no fixed per claim verification costs and that all administrative costs are a function of the actuarial value of the policy.

4.1 The Disadvantage of Aggregate Deductibles

I will now assess the levels of precautions and the expected number of losses under a policy with an aggregate deductible *D*. My analysis below will focus on the case, which might well be common, in which the marginal contribution of precautions declines quickly. This is the case in which the most important contribution of the policyholder's precautions comes already with a relatively small level of precautions or, in other words, the case in which a zero level of precautions is especially detrimental. (For example, in the case of policy covering the loss of valuable in trips away from home, leaving valuables unattended in the public domain (which can be avoided at little cost) would have an especially detrimental effect on the likelihood of a loss.)

As the proposition below establishes, comparing any given contract with an aggregate deductible *D* and a given contract with a per loss deductible *d*, the former will produce a higher expected number of accidents if the marginal contribution of precautions declines quickly enough.

Proposition 4: *For any given policy with an aggregate deductible D and any given policy with a per-loss deductible* $d < D$ *, if* $-[p''(e)/p'(e)]$ *is sufficiently large throughout, then the expected number of losses under the former policy will be higher than the expected number of losses under the latter policy.*[13]

Remark: The intuition behind this result, which is proved in the Appendix, is as follows. Under the policy with an aggregate deductible *D*, the policyholder might end up taking no precautions whatsoever in the second period. This will occur if the aggregate deductible threshold is reached in the first period, in which case the policyholder will face in the second period a full insurance coverage for any loss that might occur. When it is important to have the policyholder always take at least some minimal precautions, a significant cost of an aggregate deductible policy will arise from the fact that it might lead to a situation in which the policyholder will take no precautions in the second period. In contrast, under the policy with a per-loss deductible *d*, however small *d* might be, the policyholder will never face in the second period a complete coverage for losses and therefore will always take some precautions in both periods.

4.2 The Possible Overall Inferiority of an Aggregate Deductible

As the next result indicates the identified disadvantage of aggregate deductibles can make them inferior to some contract with a per-loss deductible.

Proposition 5: *For any given policy with an aggregate deductible D, the expected utility of the policyholder will be lower under it than under some policy with a per-loss deductible if* $-[p''(e)/p'(e)]$ *is sufficiently large.*

Remark: The intuition behind this result, which is proved in the Appendix, is as follows. In terms of moral hazard, the preceding proposition 4 implies that, for any given policy with an aggregate deductible *D* and *any* given policy with a per-loss deductible $d < D$, the former will result in a smaller expected number of losses if the marginal contribution of precautions declines sufficiently quickly. Note that this implies that this superiority of a per-loss deductible policy in terms of moral hazard will hold, if the marginal contribution of precautions declines sufficiently quickly, even for a per-loss deductible policy with a very small *d*. In

[13] For example, a functional form that would satisfy such a condition is:
$p(e) = \exp^{-\lambda e}(p_{\max} - p_{\min}) + p_{\min}$, where λ is sufficiently large.

such a case, with a sufficiently small level of d, the per-loss deductible policy will be superior to the aggregate deductible policy also in terms of the policyholder's risk-bearing costs, because a policy with a sufficiently small per-loss deductible d will leave the policyholder with less risk than would be left by the policy with the given aggregate deductible D.

5 Concluding Remarks

Classical results, which were established for conditions that do not include verification and moral hazard costs, indicate that aggregate deductibles are optimal. This paper has identified and analyzed two disadvantages of aggregate deductibles that arise from the presence of per claim verification costs and moral hazard. Such deductibles might well do worse on both counts than (commonly used) contracts with a per loss deductible. First, under some very plausible conditions, an aggregate deductible will involve higher expected verification costs than the per-loss contract that has the same actuarial value. Second, whenever it is important to have the policyholders maintain a positive level of precautions, an aggregate deductible would produce higher moral hazard costs than some contract(s) with a per loss deductible. Each of these disadvantages might be sufficient, in a range of plausible circumstances, for making any given aggregate deductible contract inferior to a certain contract with a per-loss deductibles.

These results can help explain the dominant use of per-loss deductibles. In many of the types of insurance that use per-loss deductibles, one or both of the identified disadvantages of aggregate deductibles appears potentially significant. For example, the verification costs consideration might well be significant when losses must be verified without much delay, when losses of small magnitude are frequent, and when expenditures on verification are meaningful relative to the amounts at stake. These conditions seem to be all present for the types of insurance commonly purchased by individuals -- such as policies for automobile insurance, homeowner insurance, and so forth. Indeed, estimates derived from auto insurance data indicate that the verification costs consideration is by itself likely to make aggregate deductibles undesirable for most individuals.

The moral hazard consideration also might well be significant for some common categories of insurance. This consideration might be important when it is desirable to rule out the emergence of situations during the life of the policy in which the cost of additional loss (and thus the incentive to take precautions) would be zero.

Consistent with the analysis of this paper, the instances in which aggregate deductibles are used appear to be ones in which the identified disadvantages are relatively less significant. The context in which aggregate deductibles play the greatest role appears to be that of reinsurance. Reinsurance policies often take the form of "stop-loss" policies, under which the policy covers fully the excess of the primary insurer's aggregate losses over a certain deductible. In the context of reinsurance, however, the problems of verification costs and moral hazard do not appear to be substantial. Verification costs appear small relative to the stakes involved. Furthermore, the most important precautions for the primary insurer to take are those taking place before losses start to occur – precautions in the selection of customers and the pricing of policies sold to them. Thus, the moral hazard disadvantage of aggregate deductibles also might be insubstantial in the reinsurance context.

Finally, the advantages of using per-loss deductibles can help explain why economic agents often purchase two or more separate policies for the different categories of risks they face rather than an umbrella policy covering all these categories of losses. Because the potential risk-bearing benefits of umbrella policies require the use of aggregate deductibles, the identified disadvantages of such deductibles might make umbrella policies unattractive. Thus, the analysis can help explain the observed patterns of insurance contracts also in this respect.

Whereas the analysis has shown how aggregate deductible contracts are dominated in a range of circumstances by contracts that apply the same deductible to each loss that occurs, contracts which are in fact in widespread use, it has left open the question of whether it is possible in general to improve upon the latter. In particular, one could consider contracts under which a deductible applies to each loss but in which the level of this deductible is influenced by the total losses of any that has accumulated. How such contracts will compare with the standard per loss contracts that I have considered in terms of verification costs and moral hazard is a worthwhile question for future research.

Appendix

Proof of Proposition 4:

Let us first consider the expected number of losses under the policy with an aggregate deductible *D*. Under this policy, the probability of a loss in the first period will be at least p_{min}. In the second period, the probability of a loss will be always at least p_{min}. Furthermore, there is a probability of at least $p_{\min}(1-F(D))$ that the aggregate deductible will be reached in the first period -- in which case the policyholder will take no precautions in the second period and the probability of a loss will be consequently at least p_{max}. Therefore, the probability of a loss in the second period will be at least:

$$p_{\min} + (p_{\max} - p_{\min})(1-F(D)).$$

Thus, denoting by $N_A(D)$ the expected number of losses under the policy with an aggregate deductible *D*, this expected number of losses will satisfy

(A1) $N_A(D) \geq 2p_{\min} + (p_{\max} - p_{\min})p_{\min}(1-F(D)).$

Turning to the policy with a per-loss deductible *d*, it can be shown that under this policy the policyholder will always use a positive level of precautions in each of the two periods. Let us denote by $e_L^*(d)$ the minimal level of precautions that the policyholder might use under this policy (such a level can be shown to exist for any given *d*. The expected number of losses that occur under the policy with the per-loss deductible *d* will satisfy:

(A2) $N_L(d) < 2p(e_L^*(d)).$

Using (A1) and (A2), a sufficient condition for $N_L(d)$ to be small than $N_A(D)$ is that

$$2p[e_L^*(d)] < 2p_{\min} + (p_{\max} - p_{\min})p_{\min}(1-F(D))$$

or that

(A3) $p(e_L^*(d)) - p_{\min} < \frac{1}{2}(p_{\max} - p_{\min})p_{\min}(1-F(D)).$

Let us now show that if -[$p''(e)/p'(e)$] is sufficiently large, the above condition (A3) will hold. In particular, let us suppose that -[$p''(e)/p'(e)$] >K throughout, and let us show that if K is sufficiently large, then condition (A3) must hold. To see this note that

$$\text{(A4)}\quad p\left[e_L^*(d)\right]-p_{\min}=-\int_{e_L^*(d)}^{\infty}p'(x)dx<\int_{e_L^*(d)}^{\infty}\frac{p''(x)}{K}dx=$$

$$=\frac{1}{K}\left\{p'(\infty)-p'\left(e_L^*(d)\right)\right\}<\frac{1}{K}\frac{1}{\int_0^{\infty}\min(x,d)f(x)dx}.$$

Thus, (A3) will hold if

$$\text{(A5)}\quad \frac{1}{K}\frac{1}{\int_0^{\infty}\min(x,d)f(x)dx}<\frac{1}{2}\left(p_{\max}-p_{\min}\right)p_{\min}\left(1-F(D)\right).$$

Equation (A5) will hold for a sufficiently large K, which completes our proof. Q.E.D.

Proof of Proposition 5:

For any given policy with an aggregate deductible D, let us denote (as before) the expected number of accidents by $N_A(D)$, and the expected utility of the policyholder by $EU_A(D)$. The policyholder will bear the full cost of the expected losses (either directly or indirectly through the premium). Because the policyholder will be left with some risk, the policyholder will bear some risk-bearing costs denoting by $R_A(D)$. Specifically, we can state the level of the policyholder's expected utility as equal to:

$$\text{(A6)}\quad EU_A(D)=U\left(W-N_A(D)\mu-E\left(e_1(D)\right)-E\left(e_2(D)\right)-R_A(D)\right).$$

Similarly, for any given policy with a per-loss deductible d, denoting by $R_L(d)$ the risk-bearing costs left by the policy (and, as before the expected number of accidents by $N_L(d)$, the level of the policyholder's expected utility can be stated as:

$$\text{(A7)}\quad EU_L(d)=U\left(W-N_L(d)\mu-E\left(e_1(d)\right)-E\left(e_2(d)\right)-R_L(d)\right).$$

If $-[p''(e)/p'(e)]$ is sufficiently large, then, by proposition $N_L(d)$ is smaller than $N_A(D)$. Furthermore as $-[p''(e)/p'(e)]$ grows, the expected expenditure on precautions go to zero. It follows that if $-[p''(e)/p'(e)]$ is large enough, the expected final wealth is higher under the per-loss deductible contract than under the aggregate deductible contract. Furthermore, d can be always set sufficiently small to make $R_L(d)$ as small as designed (since the final wealth under the per-loss deductible policy is never lower than the expected final wealth by more than 2d. If follows that, if $-[p''(e)/p'(e)]$ is sufficiently large, there exists a per-loss deductible policy under which the value of (A7) will be higher that the value of (A6). Q.E.D.

References

Alliance of American Insurers (1998), The Policy Kit.

Arrow, K. (1963), "Uncertainty and the Welfare Economics of Medical Care, " *American Economic Review*, Vol. 53, pp. 941-973.

Bell, David and Peter Fishburn (2000), Utility Functions for Wealth," *Journal of Risk and Uncertainty*, Vol. 20, pp. 5-44.

Bond, Eric W., and Keith, J. Crocker (1997), "Hardball and the Soft Touch: The Economics of Optimal Insurance Contracts with Costly State Verification and Endogenous Monitoring Costs," *Journal of Public Economics*, Vol. 63, No. 2, pp. 239-264.

Caron, Louis, and Georges Dionne (1997), "Insurance Fraud Estimation: More Evidence from Quebec Automobile Insurance Industry," *Assurance*, Vol. 64 pp.567-578.

Casse, R. Michael, et al. (1997), *Reinsurance Practices*. American Institute.

Cohen, Alma (2001), "Asymmetric Information and Learning in the Automobile Insurance Market, Working Paper, Harvard University, available at http://papers.ssrn.com/ abstract_id=289966.

Crocker, Keith J. and John Morgan (1998), "Is Honesty the Best Policy? Curtailing Insurance Fraud Through Optimal Incentive Contracts," *Journal of Political Economy*, Vol. 106, No. 2, pp. 355-375.

Crocker, Keith J., and Sharon Tennyson (1999), "Costly State Falsification or Verification? Theory and Evidence from Bodily Injury Liability Claims," *in Automobile Insurance: Road Safety, New Drivers, Risks, Insurance Fraud, and Regulation*, (G. Dionne and C. Laberge-Nadeau. eds.), Kluwer Academic Press, Boston, pp. 119-130.

Cummins, J. David, and Sharon, Tennyson (1996), "Moral Hazard in Insurance Claiming: Evidence from Automobile Insurance," *Journal of Risk and Uncertainty*, Vol. 12, No. 1, pp. 29-50.

Derrig, Richard A. (2001), "Insurance Fraud Research Register: Eleventh Report," published by the Insurance Fraud Bureau of Massachusetts.

Doherty, Neil, and Harris Schlesinger (1983), “Optimal Insurance in Incomplete Markets,” *Journal of Political Economy*, Vol. 01, pp. 1045-54.

Eeckhoudt, Louis, and Christian Gollier (1999), "The Insurance of Lower Probability Events," *The Journal of Risk and Insurance*, Vol. 66, No. 1, pp. 17-28.

Gollier, Christian (1987), “Pareto-Optimal Risk Sharing with Fixed Costs Per Claim,” *Scandinavian Actuarial Journal*, pp. 62-73.

Gollier, Christian and Schlesinger, Harris (1996), "Arrow's Theorem on the Optimality of Deductibles: A Stochastic Dominance Approach," *Economic Theory*, Vol. 7, No. 2, pp. 359-363.

Gollier, Christian and Schlesinger, Harris (1995), "Second-best Insurance Contract Design in an Incomplete Market,” *Scandinavian Journal of Economics*, Vol. 97, pp. 123-135.

Huberman, G., D. Mayers, and C. Smith, Jr. (1983), “Optimal Insurance Policy Indemnity Schedules, “ *Bell Journal of Economics*, pp. 415-426.

Kaplow, Louis (1994), “Optimal Insurance Contracts when Establishing the Amount of Losses is Costly,” *Geneva Papers on Risk and Insurance*, Vol. 19, pp. 139-152.

Karny, Edy (1992), “ Optimal Insurance: A Nonexpected Utility Analysis,” in Dionne, G. (ed.), *Contributions to Insurance Economics*.

Mayers, D., and C. Smith (1983), “The Interdependence of Individual Portfolio Decisions and the Demand for Insurance,” *Journal of Political Economy*, Vol. 91, pp.304-11.

Paulsen, Jostein (1995), "Optimal Per Claim Deductible in Insurance with the Possibility of Risky Investment," *Insurance Mathematics & Economics*, Vol. 17, No. 2, pp. 133-147.

Pauly, M. (1968), “The Economics of Moral Hazard: Comment,” *American Economic Review*, Vol. 58, pp. 531-536.

Phifer, Ross (1996), *Reinsurance Fundamentals: Treaty and Facultative*. John Wiley & Sons.

Picard, P. (1996), "Auditing Claims in Insurance Markets with Fraud: The Credibility Issue", *Journal of Public Economics*, Vol. 63, No. 1, pp. 27-56.

Raviv, Arthur (1979), “The Design of an Optimal Insurance policy,” *American Economic Review*, pp. 84-96.

Shavell, Steven (1979), “On Moral Hazard and Insurance,” *Quarterly Journal of Economics*, pp. 541-562.

Smith, Ed, and Wright R. (1992), “Why is Automobile Insurance in Philadelphia so Damn Expensive,” *American Economic Review*, Vol. 82, pp. 756-772.

Spence, A. Michael and Richard Zeckhauser (1971), “Insurance, Information and Individual Action,” *American Economic review*, Vol. 61, pp. 380-387.

Winter, Ralph (2000), “Optimal Insurance under Moral Hazard,” in Handbook of Insurance (G. Dionnes, editor).