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CONSUMER MISPERCEPTION IN A HOTELLING MODEL:
WITH AND WITHOUT PRICE DISCRIMINATION

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# Consumer Misperception in a Hotelling Model: With and Without Price Discrimination

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Abstract

This paper studies the implications of consumer misperception in a market for a (horizontally) differentiated product. Two distinct type of misperceptions are considered: (i) a common misperception that leads consumers to similarly overestimate the benefit from both firms' products; and (ii) a relative misperception that leads consumers to overestimate the relative benefit of one firm's product as compared to the product offered by its competitor. The paper analyzes the implications of misperception for social welfare and consumer surplus. In particular, the effects of price discrimination are considered, for each type of misperception.

Keywords: Horizontal product differentiation, Hotelling model, price discrimination, consumer misperception.

JEL Classification: D43, K21, L13, L40

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## 1. Introduction

Using the classic Hotelling (1929) model, this paper studies the implications of consumer misperception in a market for a (horizontally) differentiated product. Two firms, located at the two ends of a Hotelling line, offer otherwise identical products to consumers who are Uniformly distributed along the line. I distinguish between two types of misperception: (i) a common misperception that leads consumers to similarly overestimate the benefit from both firms' products; and (ii) a relative misperception that leads consumers to overestimate the relative benefit of one firm's product as compared to the product offered by its competitor.[1] For each type of misperception, I consider both a setup where each firm sets a uniform price for all customers (the no price discrimination, or NPD, case) and a setup where the firms can engage in price discrimination and set a personalized price for each consumer (the price discrimination, or PD, case). (In the absence of misperception, uniform price models have been studied by Hotelling 1929, Hurter and Lowe 1976a,b, d'Aspremont et al. 1979, Moorthy 1988; and price discrimination models have been studied by Hoover 1936/7, Hurter and Lederer 1985, Armstrong & Vickers 2001.)

Starting with the no misperception benchmark, I replicate standard results about the effects of price discrimination: When product benefits are low, consumers located around the center of the Hotelling line will be priced out in an NPD regime (Moorthy 1988, 153). PD increases efficiency, as firms charge lower prices for these marginal consumers. On the other hand, PD reduces the consumer surplus, as firms extract the full surplus from each customer (Armstrong and Vickers 2001, 594). When product benefits are high, the market is covered even in the NPD case, and so PD does not affect efficiency. Here, PD increases the consumer surplus, as it allows for more intense competition between the two firms (see Section 3 below).

These results must be reconsidered, when consumer misperception is prevalent. I begin with common misperception. Consider the NPD case. When product benefits are low, overestimation of value increases efficiency by correcting the inadequate quantity problem, as long as the level of misperception is not too high. For both low and high product benefits, misperception reduces consumer surplus by allowing sellers to raise prices. Next consider the PD case. When product benefits are low, overestimation of value leads to excessive purchases and thus reduces efficiency. As in the NPD case, for both low and high product benefits, misperception reduces consumer surplus by allowing sellers to raise prices. But in the PD case the problem is worse, since consumers are incurring an actual loss.

A comparison between the NPD and PD cases reveals the effects of PD when consumers suffer from common misperception: When product benefits are low, overestimation of value reduces the efficiency advantage that PD had in the absence of misperception. Indeed, beyond a threshold level of misperception, NPD is more efficient. In terms of consumer surplus, NPD induces better outcomes for lower levels of misperception, whereas PD is better when misperception levels are

[1] Consider health club membership. A common misperception would result in an overestimation of the benefit from health club membership (e.g., if a consumer thinks that she will attend twice a week, when in fact she will attend once a month); this overestimation has an identical effect on the perceived benefit from membership in the two competing health clubs. A relative misperception would result in an overestimation of the relative advantage of one health club over the other, e.g., if the consumer overestimates the quality of service at one of the two competing health clubs.

high. When product benefits are high, the market is covered with both NPD and PD, and so PD does not affect efficiency. PD induces greater consumer surplus, because it allows for more intense competition.

The effects of relative misperception are quite different. Consider the NPD case. For both low and high product benefits, misperception reduces social welfare and harms consumers. When product benefits are low and the market is not covered, relative misperception leads to insufficient purchases from one firm and to excessive purchases from the other firm. When product benefits are high and the market is covered, relative misperception inefficiently shifts demand from one firm to the other. The effects of relative misperception are similarly negative in the PD case. The misperception reduces social welfare and harms consumers.

A comparison between the NPD and PD cases reveals the effects of PD when consumers suffer from relative misperception: In terms of social welfare, while misperception is harmful in both the NPD and PD cases, the distortions are greater with PD. When product benefits are low, the benchmark, no-misperception advantage of PD dominates, as long as the level of misperception is not too great. When the level of misperception is higher, NPD is more efficient. When product benefits are high, NPD is more efficient for any level of misperception. Moving on to consumer surplus: When product benefits are low, NPD induces greater consumer surplus. When product benefits are high, NPD induces greater consumer surplus when the level of misperception is high, and PD induces greater consumer surplus when the level of misperception is low. To sum up, relative misperceptions reduce the efficiency benefits of PD and increase NPD's advantage in terms of consumer surplus.

The analysis in this paper relates to several strands in the industrial organization (IO) literature. Most important, I rely on the large literature on horizontally differentiated products, with and without price discrimination,[2] and extend this literature by considering the effects of consumer misperception.

Several papers study the effects of advertising in a Hotelling framework or, more generally, in the context of horizontal product differentiation (for a survey see Bagwell, 2007). Most of these papers do not study misperception, but offer related analysis. Some of the papers assume that advertising expands demand (see, e.g., Dixit and Norman 1978), which relates to common misperception, while others assume that advertising only shifts demand between the two firms (see, e.g., Bloch and Manceau 1999), which relates to relative misperception. My analysis more closely relates to models that view advertising as conveying information, and less to models that view advertising as changing preferences. Spiegler (2014) considers firms' framing choices, which clearly covers the notion of misperception. And Locati (2014) expressly studies misperception, assuming that consumers initially misperceive their location on the Hotelling line (a type of relative misperception) and that firms can correct this misperception through advertising. These papers

[2] This literature originates with Hotelling (1929), an NPD model that focuses on firms' location decisions, followed by d'Aspremont et al. (1979). Subsequent PD models where location is endogenous include Hurter and Lederer (1985), Lederer and Hurter (1986), Moorthy (1988) and MacLeod et al. (1988). Contributions where firms' locations are fixed, include the PD models in Hoover (1937), Armstrong and Vickers (2001); and the NPD models in Hurter and Lowe (1976a,b).

focus on firms' prices and profits and on whether advertising increases or decreases profits; not on social welfare or consumer surplus (Bloch and Manceau 1999, Spiegler 2014). And most of them assume that the market is covered (Bloch and Manceau 1999, Spiegler 2014). Finally, these papers do not consider the effects of price discrimination.[3]

More generally, this paper contributes to the behavioral IO literature, which incorporates bounded rationality and prevalent consumer misperceptions into the analysis of firms and markets. See Ellison (2006) and Armstrong (2008) for literature reviews and Spiegler 2011 for a textbook treatment. This literature has explored the implications of different types of consumer misperception in different contexts, but has not yet focused on horizontally differentiated products (with the exception of Spiegler 2014). I also note the important early contribution by Poilnsky and Rogerson (1983) who compare different products liability regimes when consumers suffer from misperception (specifically, when consumers underestimate losses from defective products).

Finally, this paper engages with the literature on price discrimination and competition policy. The standard analysis highlights the potential benefits of price discrimination and cautions against policy interventions that would prevent or limit price discrimination. See, e.g., Carlton and Israel (2009). Consumer misperception qualifies this conventional wisdom. The analysis in this paper identifies circumstances, where price discrimination reduces efficiency and harms consumers, but also circumstances where price discrimination can be beneficial. A more nuanced policy response may be justified.

The remainder of this paper is organized as follows. Section 2 lays out the framework of analysis and characterizes the first-best outcome. Section 3 presents the no-misperception benchmark. Section 4 studies the common misperception case. Section 5 studies the relative misperception case. Section 6 offers concluding remarks, discussing normative and policy implications and listing possible extensions. Proofs are relegated to an [online?] appendix.

## 2. Framework of Analysis and the First-Best Benchmark

### 2.1 Framework of Analysis

Two firms, F1 and F2, are located at opposite ends of a Hotelling line. The distance between the two firms is 1 unit. Both firms produce and sell an identical good. The cost, to both firms, of producing one unit of the good is c, which we normalize to zero. [Consider $c > 0$ in an extension?] There is a unit mass of consumers, and they are uniformly distributed along the line between F1 and F2. Consumer x is located x units from F1 and 1-x units from F2. (Following the standard interpretation of the Hotelling model, this "distance" can represent any difference between F1's product and F2's product, not only geographical distance.) See Figure 1 below.

[3] Some papers consider the welfare effects of informative advertising. See, e.g., Grossman and Shapiro (1984), Rogerson (1988), Bester and Petrakis (1995).

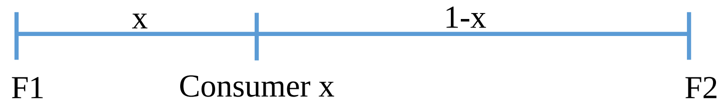


Figure 1: Hotelling Framework

I study two cases: (1) No Price Discrimination (NPD) – F1 sets a single price, $p_1$, and F2 sets a single price, $p_2$. (2) Perfect Price Discrimination (which I will call "price discrimination" or "PD" for short) – F1 and F2 can charge Consumer x a personalized price; F1 charges $p_1(x)$ and F2 charges $p_2(x)$.

Each consumer buys at most 1 unit of the good and enjoys a benefit of $v > 0$, if she buys the good. If Consumer x buys from F1, she enjoys a payoff of: $v - x - p_1$; if the consumer buys from F2, she enjoys a payoff of: $v - (1 - x) - p_2$.

My goal is to study how misperception changes the welfare effects of price discrimination. I consider two distinct types of misperception. I begin with a "common misperception," namely, misperception that affects both firms equally. In particular, let $\hat{v} = v + \delta$ denote the perceived value. I focus on demand-inflating misperceptions, i.e., $\delta > 0$.

I also consider a "relative misperception," namely, misperception that affects the relative benefit, to the consumer, from choosing one firm over the other. In particular, let $\hat{x} = x + \delta$ denote the perceived location of Consumer $x$ on the Hotelling line. Without loss of generality, we focus on the case where $\delta > 0$, namely, where Consumer x overestimates the advantages of F2, as compared to F1. (More precisely, let $\hat{x} = min\{x + \delta, 1\}$, to stay within the confines of the Hotelling line.)

## 2.2 First Best

As a benchmark, I characterize the first-best outcome. Consumer x should buy from F1 iff (i) $x \leq \frac{1}{2}$, and (ii) $v - x \geq 0$ or $x \leq v$. Consumer x should buy from F2 iff (i) $x > \frac{1}{2}$, and (ii) $v - (1 - x) \geq 0$ or $x \geq 1 - v$. Consumer x should not buy at all iff (i) $x \leq \frac{1}{2}$ and $x > v$, or (ii) $x > \frac{1}{2}$ and $x < 1 - v$.

The first-best outcome is characterized in Lemma 1.

*Lemma 1 (First Best):*

*(a) When $v \geq \frac{1}{2}$, all consumers should buy the product; consumers with $x \leq \frac{1}{2}$ should buy from F1, and consumers with $x > \frac{1}{2}$ should buy from F2.*

*(b) When $v < \frac{1}{2}$, consumers with $x \leq v$ should buy from F1, consumers with $x \geq 1 - v$ should buy from F2, and consumers with $x \in (v, 1 - v)$ should not buy at all.*

## 3. No Misperception

I begin, in Section 3, with an analysis of the no misperception case, as a benchmark for evaluating the effects of misperception – common misperception in Section 4 and relative misperception in Section 5. The No Price Discrimination (NPD) case is analyzed in Subsection 3.1. The Price Discrimination (PD) case is analyzed in Subsection 3.2. And the two cases are compared in Subsection 3.3.

### 3.1 No Price Discrimination

Let $x_1(p_1; p_2)$ denote the demand for F1's product, as a function of the price that F1 charges, $p_1$, and the price that F2 charges, $p_2$. This means that all consumers with $x \leq x_1(p_1; p_2)$ will buy from F1. F1 solves:

(1) $max\langle p_1 \cdot x_1(p_1; p_2)\rangle$
s.t.

(1a) IC: $v - x_1 - p_1 \geq v - (1 - x_1) - p_2$ or $p_1 \leq p_2 + (1 - 2x_1)$
(1b) IR: $v - x_1 - p_1 \geq 0$ or $p_1 \leq v - x_1$
(1c) $p_1 > 0$

Similarly, let $x_2(p_2; p_1)$ denote the demand for F2's product, as a function of the price that F2 charges, $p_2$, and the price that F1 charges, $p_1$. This means that all consumers with $x \geq x_2(p_2; p_1)$ will buy from F2. F2 solves:

(2) $max\langle p_2 \cdot \left(1 - x_2(p_2; p_1)\right)\rangle$
s.t.

(2a) IC: $p_1 > p_2 + (1 - 2x_2)$ or $p_2 < p_1 - (1 - 2x_2)$
(2b) IR: $v - (1 - x_2) - p_2 \geq 0$ or $p_2 \leq v - (1 - x_2)$
(2c) $p_2 > 0$

The market equilibrium is characterized in the following lemma.

*Lemma 2 (No Misperception; No Price Discrimination): With no misperception and no price discrimination –*
*(a) For $v \leq 1$,*

*(a.1) Prices are: $p_1 = p_2 = \frac{1}{2}v$.*

*(a.2) The market is not covered. Consumers with $x \in \left(\frac{1}{2}v, 1 - \frac{1}{2}v\right)$ do not buy the good. Social welfare is: $\int_0^{\frac{1}{2}v}(v - x)dx + \int_{1-\frac{1}{2}v}^{1}\left(v - (1 - x)\right)dx = \frac{3}{4}v^2$.*

*(a.3) Consumer x enjoys a payoff of:* $\frac{1}{2}v - x$ *for* $x \le \frac{1}{2}v$, $\frac{1}{2}v - (1-x)$ *for* $x \ge 1 - \frac{1}{2}v$*, and zero for* $x \in \left(\frac{1}{2}v, 1 - \frac{1}{2}v\right)$*. Consumer surplus is:* $\int_0^{\frac{1}{2}v}\left(\frac{1}{2}v - x\right)dx + \int_{1-\frac{1}{2}v}^{1}\left(\frac{1}{2}v - (1-x)\right)dx = \frac{1}{4}v^2$.

*(b) For* $v \in \left(1, \frac{3}{2}\right)$,

*(b.1) Prices are:* $p_1 = p_2 = v - \frac{1}{2}$.

*(b.2) The market is covered. Social welfare is:* $\int_0^{\frac{1}{2}}(v-x)dx + \int_{\frac{1}{2}}^{1}\big(v-(1-x)\big)dx = v - \frac{1}{4}$.

*(b.3) Consumer x enjoys a payoff of:* $\frac{1}{2} - x$ *for* $x \le \frac{1}{2}$*, and* $\frac{1}{2} - (1-x)$ *for* $x > \frac{1}{2}$*. Consumer surplus is:* $\int_0^{\frac{1}{2}}\left(\frac{1}{2} - x\right)dx + \int_{\frac{1}{2}}^{1}\left(\frac{1}{2} - (1-x)\right)dx = \frac{1}{4}$.

*(c) For* $v \ge \frac{3}{2}$,

*(c.1) Prices are:* $p_1 = p_2 = 1$.

*(c.2) The market is covered. Social welfare is:* $\int_0^{\frac{1}{2}}(v-x)dx + \int_{\frac{1}{2}}^{1}\big(v-(1-x)\big)dx = v - \frac{1}{4}$.

*(c.3) Consumer x enjoys a payoff of:* $v - 1 - x$ *for* $x \le \frac{1}{2}$*, and* $v - 1 - (1-x)$ *for* $x > \frac{1}{2}$*. Consumer surplus:* $\int_0^{\frac{1}{2}}(v-1-x)dx + \int_{\frac{1}{2}}^{1}\big(v-1-(1-x)\big)dx = v - \frac{5}{4}$.

## 3.2 Price Discrimination

When price discrimination is possible, the firms' optimization problems change. For each consumer, i.e., for each *x*, F1 solves:

(3) $\max_{p_1(x)}\langle p_1(x)\rangle$

s.t.

(3a) IC: $v - x - p_1(x) \ge v - (1-x) - p_2(x)$ or $p_1(x) \le p_2(x) + (1-2x)$

(3b) IR: $v - x - p_1(x) \ge 0$ or $p_1(x) \le v - x$

(3c) $p_1(x) > 0$

And, for each consumer, i.e., for each *x*, F2 solves:

(4) $\max_{p_2(x)}\langle p_2(x)\rangle$

s.t.

(4a) IC: $v - x - p_1(x) < v - (1-x) - p_2(x)$ or $p_1(x) > p_2(x) + (1-2x)$

(4b) IR: $v - (1-x) - p_2(x) \ge 0$ or $p_2(x) \le v - (1-x)$

(4c) $p_2(x) > 0$

The market equilibrium is characterized in the following lemma.

*Lemma 3 (No Misperception; Price Discrimination): With no misperception and price discrimination –*

*(a) For* $v \leq \frac{1}{2}$,

*(a.1) Prices are:* $p_1(x) = v - x$ *and* $p_2(x) = 0$ *for* $x \leq v$*; and* $p_2(x) = v - (1 - x)$ *and* $p_1(x) = 0$ *for* $x \geq 1 - v$.

*(a.2) The market is not covered. Consumers with* $x \in (v, 1 - v)$ *do not buy the good. Social welfare is:* $\int_0^v (v - x)dx + \int_{1-v}^1 \big(v - (1 - x)\big)dx = v^2$.

*(a.3) Consumer x enjoys a payoff of zero for all* $x \in [0,1]$. *Consumer surplus is zero.*

*(b) For* $v \in \left(\frac{1}{2}, 1\right)$,

*(b.1) Prices are: For* $x < 1 - v$, $p_1(x) = v - x$ *and* $p_2(x) = 0$*; for* $x \in \left[1 - v, \frac{1}{2}\right]$, $p_1(x) = 1 - 2x$ *and* $p_2(x) = 0$*; for* $x \in (\frac{1}{2}, v]$, $p_2(x) = -(1 - 2x)$ *and* $p_1(x) = 0$*; and for* $x > v$, $p_2(x) = v - (1 - x)$ *and* $p_1(x) = 0$.

*(b.2) The market is covered. Social welfare is:* $\int_0^{\frac{1}{2}} (v - x)dx + \int_{\frac{1}{2}}^1 \big(v - (1 - x)\big)dx = v - \frac{1}{4}$.

*(b.3) Consumer x enjoys a payoff of* $v - (1 - x) \geq 0$ *for* $x \in \left[1 - v, \frac{1}{2}\right]$*;* $v - x \geq 0$ *for* $x \in (\frac{1}{2}, v]$*; and zero for* $x < 1 - v$ *and* $x > v$. *Consumer surplus is:* $\int_{1-v}^{\frac{1}{2}} (v - (1 - 2x) - x)dx + \int_{\frac{1}{2}}^v \big(v + (1 - 2x) - (1 - x)\big)dx = \left(v - \frac{1}{2}\right)^2$.

*(c) For* $v \geq 1$,

*(c.1) Prices are:* $p_1(x) = 1 - 2x$ *and* $p_2(x) = 0$ *for* $x \leq \frac{1}{2}$*; and* $p_2(x) = -(1 - 2x)$ *and* $p_1(x) = 0$ *for* $x > \frac{1}{2}$.

*(c.2) The market is covered. Social welfare is:* $\int_0^{\frac{1}{2}} (v - x)dx + \int_{\frac{1}{2}}^1 \big(v - (1 - x)\big)dx = v - \frac{1}{4}$.

*(c.3) Consumer x enjoys a payoff of* $v - (1 - x) \geq 0$ *for* $x \leq \frac{1}{2}$*; and* $v - x \geq 0$ *for* $x > \frac{1}{2}$. *Consumer surplus is:* $\int_0^{\frac{1}{2}} (v - (1 - 2x) - x)dx + \int_{\frac{1}{2}}^1 \big(v + (1 - 2x) - (1 - x)\big)dx = v - \frac{3}{4}$.

### 3.3 Comparison

Collecting the results from the preceding analysis, we compare NPD and PD in terms of social welfare (Table 1) and consumer surplus (Table 2)

| | No Price Discrimination (NPD) | Price Discrimination (PD) | Comparison |
|---|---|---|---|
| $v \le \frac{1}{2}$ | $\frac{3}{4}v^2$ | $v^2$ | PD is better |
| $v \in \left(\frac{1}{2}, 1\right)$ | $\frac{3}{4}v^2$ | $v - \frac{1}{4}$ | PD is better |
| $v = 1$ | $\frac{3}{4}v^2$ | $v - \frac{1}{4}$ | Same |
| $v \in \left(1, \frac{3}{2}\right)$ | $v - \frac{1}{4}$ | $v - \frac{1}{4}$ | Same |
| $v \ge \frac{3}{2}$ | $v - \frac{1}{4}$ | $v - \frac{1}{4}$ | Same |

Table 1: Comparing Social Welfare with NPD and PD, without Misperception

| | No Price Discrimination (NPD) | Price Discrimination (PD) | Comparison |
|---|---|---|---|
| $v \le \frac{1}{2}$ | $\frac{1}{4}v^2$ | zero | NPD is better |
| $v \in \left(\frac{1}{2}, 1\right)$ | $\frac{1}{4}v^2$ | $\left(v - \frac{1}{2}\right)^2$ | NPD is better |
| $v = 1$ | $\frac{1}{4}v^2$ | $v - \frac{3}{4}$ | Same |
| $v \in \left(1, \frac{3}{2}\right)$ | $\frac{1}{4}$ | $v - \frac{3}{4}$ | PD is better |
| $v \ge \frac{3}{2}$ | $v - \frac{5}{4}$ | $v - \frac{3}{4}$ | PD is better |

Table 2: Comparing Consumer Surplus with NPD and PD, without Misperception

The comparisons are summarized in the following proposition.

**Proposition 1 (No Misperception; NPD v. PD):**
**(a) When $v < 1$, price discrimination increases efficiency and reduces consumer surplus.**
**(b) When $v > 1$, price discrimination does not affect efficiency and increases consumer surplus.**

When $v$ is small ($v < 1$), price discrimination increases social welfare and reduces consumer surplus. Starting with social welfare: With NPD, the firms set a single, high price and fewer consumers are served. With PD, firms set lower prices for marginal consumers and so more consumers are served. Therefore, PD increases social welfare.[4] In terms of consumer surplus: With

[4] If we decide to develop a $c > 0$ extension, we need to check if this result still holds.

PD, while more consumers are served, the firms extract the entire surplus and consumers are left with nothing. With NPD, the fewer consumers who are served enjoy a positive surplus. Therefore, PD reduces consumer surplus.

When $v$ is large ($v > 1$), price discrimination does not affect efficiency and increases consumer surplus. Since the market is covered with and without price discrimination, PD does not affect social welfare. In terms of consumer surplus, PD helps consumers located towards the middle of the Hotelling line and hurts consumers located towards the ends of the Hotelling line. Overall, PD increases the consumer surplus. The reason is that the single-price constraint under NPD softens competition and thus harms consumers. Specifically, to win over the marginal consumers, near the middle of the Hotelling line, the firm has to reduce prices for all consumers, and is reluctant to do so.

## 4. Common Misperception

The No Misperception benchmark, in Section 3, serves as a reference point for the analysis of consumer misperception and its effects on market outcomes, on social welfare and on consumer surplus. Section 4.1 studies the NPD case and Section 4.2 studies the PD case. Section 4.3 compares NPD and PD in the presence of consumer misperception.

### 4.1 No Price Discrimination

Consumer misperceptions affect the firms' optimization problems. Specifically, in the constraints – (1a) and (1b) for F1 and (2a) and (2b) for F2 – the actual value, $v$, is replaced by the perceived value, $\hat{v} = v + \delta$. The market equilibrium is characterized in the following lemma.

*Lemma 4 (Common Misperception; No Price Discrimination): With misperception and no price discrimination –*

*(a) For* $\hat{v} \leq 1$,

*(a.1) Prices are:* $p_1 = p_2 = \frac{1}{2}\hat{v}$.

*(a.2) The market is not covered. Consumers with* $x \in \left(\frac{1}{2}\hat{v}, 1 - \frac{1}{2}\hat{v}\right)$ *do not buy the good. Social welfare is:* $\hat{v}\left(v - \frac{1}{4}\hat{v}\right)$.

*(a.3) Consumer* $x$ *enjoys a payoff of:* $v - \frac{1}{2}\hat{v} - x$ *for* $x \leq \frac{1}{2}\hat{v}$, $v - \frac{1}{2}\hat{v} - (1 - x)$ *for* $x \geq 1 - \frac{1}{2}\hat{v}$, *and zero for* $x \in \left(\frac{1}{2}\hat{v}, 1 - \frac{1}{2}\hat{v}\right)$. *Consumer surplus is:* $\left(v - \frac{1}{2}\hat{v}\right)^2 - (\hat{v} - v)^2$.

*(b) For* $\hat{v} \in \left(1, \frac{3}{2}\right)$,

*(b.1) Prices are:* $p_1 = p_2 = \hat{v} - \frac{1}{2}$.

*(b.2) The market is covered. Social welfare is:* $v - \frac{1}{4}$.

*(b.3) Consumer* $x$ *enjoys a payoff of:* $\frac{1}{2} - (\hat{v} - v) - x$ *for* $x \leq \frac{1}{2}$, *and* $\frac{1}{2} - (\hat{v} - v) - (1 - x)$ *for* $x > \frac{1}{2}$. *Consumer surplus is:* $\left(\frac{1}{2} - (\hat{v} - v)\right)^2 - (\hat{v} - v)^2$.

*(c) For $\hat{v} \geq \frac{3}{2}$,*

*(c.1) Prices are: $p_1 = p_2 = 1$.*

*(c.2) The market is covered. Social welfare is: $v - \frac{1}{4}$.*

*(c.3) Consumer $x$ enjoys a payoff of: $v - 1 - x$ for $x \leq \frac{1}{2}$, and $v - 1 - (1 - x)$ for $x > \frac{1}{2}$. Consumer surplus: $v - \frac{5}{4}$.*

The effects of common misperception, derived from a comparison between Lemma 4 and Lemma 2, are summarized in the following proposition.

**Proposition 2 (No Price Discrimination; the Effects of Common Misperception):**

**(a) When $\hat{v} \leq 1$, higher levels of misperception increase social welfare when $\delta < v$ and reduce social welfare when $\delta > v$, and when $\delta > (1 + \sqrt{7})v$ they reduce social welfare below the no-misperception benchmark; higher levels of misperception always harm consumers.**

**(b) When $\hat{v} \in \left(1, \frac{3}{2}\right)$, higher levels of misperception have no effect on social welfare and harm consumers.**

**(c) When $\hat{v} \geq \frac{3}{2}$, higher levels of misperception have no effect – neither on social welfare nor on consumer surplus.**

When $\hat{v}$ is small ($\hat{v} \leq 1$), misperception can correct for the inadequate quantity problem and thus increase social welfare. But only as long as the level of misperception is not too large. When the level of misperception is larger ($\delta > v$), the misperception results in some inefficient purchases that reduce social welfare (as compared to $\delta = v$). And when the level of misperception is sufficiently large ($\delta > (1 + \sqrt{7})v$), social welfare is below the no-misperception ($\delta = 0$) benchmark. In terms of consumer surplus, misperception allows the firms to charge a higher price and this reduces consumer surplus. (And some consumers – those near the middle of the Hotelling line – incur an actual loss.)

For intermediate levels of $\hat{v}$ ($\hat{v} \in \left(1, \frac{3}{2}\right)$), higher levels of misperception have no effect on social welfare. In terms of consumer surplus, higher levels of misperception allow the firms to charge a higher price and this reduces consumer surplus.

When $\hat{v} \geq \frac{3}{2}$, higher levels of misperception have no effect. The market is covered regardless of any additional increase in the level of misperception. Turning to consumer surplus, the large perceived surplus triggers competition that prevents firms from raising prices. Thus, any additional increase in the level of misperception does not affect consumer surplus.

4.2 Price Discrimination

Here too consumer misperceptions affect the firms' optimization problems. Specifically, in the constraints – (3a) and (3b) for F1 and (4a) and (4b) for F2 – the actual value, $v$, is replaced by the perceived value, $\hat{v} = v + \delta$. The market equilibrium is characterized in the following lemma.

*Lemma 5 (Common Misperception; Price Discrimination): With misperception and price discrimination –*

*(a) For $\hat{v} \leq \frac{1}{2}$,*

*(a.1) Prices are: $p_1(x) = \hat{v} - x$ and $p_2(x) = 0$ for $x \leq \hat{v}$; and $p_2(x) = \hat{v} - (1-x)$ and $p_1(x) = 0$ for $x \geq 1 - \hat{v}$.*

*(a.2) The market is not covered. Consumers with $x \in (\hat{v}, 1-\hat{v})$ do not buy the good. Social welfare is: $v^2 - (\hat{v} - v)^2$.*

*(a.3) Consumer $x$ enjoys a payoff of $v - \hat{v}$ for $x \leq \hat{v}$ and $x \geq 1 - \hat{v}$; and zero for $x \in (\hat{v}, 1-\hat{v})$. Consumer surplus is $2\hat{v}(v - \hat{v})$.*

*(b) For $\hat{v} \in \left(\frac{1}{2}, 1\right)$,*

*(b.1) Prices are: For $x \leq \frac{1}{2}$, $p_1(x) = \hat{v} - x$ for $x \leq 1 - \hat{v}$ and $p_1(x) = 1 - 2x$ for $x > 1 - \hat{v}$, and $p_2(x) = 0$; and for $x > \frac{1}{2}$, $p_2(x) = \hat{v} - (1-x)$ for $x \geq \hat{v}$ and $p_2(x) = -(1-2x)$ for $x < \hat{v}$, and $p_1(x) = 0$.*

*(b.2) The market is covered. Social welfare is: $v - \frac{1}{4}$.*

*(b.3) Consumer $x$ enjoys a payoff of $v - (1-x) \geq 0$ for $x \in \left(1 - \hat{v}, \frac{1}{2}\right]$; $v - x \geq 0$ for $x \in \left(\frac{1}{2}, \hat{v}\right)$; and $v - \hat{v}$ for $x \leq 1 - \hat{v}$ and $x \geq \hat{v}$. Consumer surplus is: $\left(\hat{v} - \frac{1}{2}\right)^2 + (v - \hat{v})$.*

*(c) For $\hat{v} \geq 1$,*

*(c.1) Prices are: $p_1(x) = 1 - 2x$ and $p_2(x) = 0$ for $x \leq \frac{1}{2}$; and $p_2(x) = -(1-2x)$ and $p_1(x) = 0$ for $x > \frac{1}{2}$.*

*(c.2) The market is covered. Social welfare is: $v - \frac{1}{4}$.*

*(c.3) Consumer $x$ enjoys a payoff of $v - (1-x) \geq 0$ for $x \leq \frac{1}{2}$; and $v - x \geq 0$ for $x > \frac{1}{2}$. Consumer surplus is: $v - \frac{3}{4}$.*

The effects of common misperception, derived from a comparison between Lemma 5 and Lemma 3, are summarized in the following proposition.

**Proposition 3 (Price Discrimination; the Effects of Common Misperception):**

**(a) When $\hat{v} \leq \frac{1}{2}$, higher levels of misperception reduce social welfare and harm consumers.**

**(b) When $\hat{v} \in \left(\frac{1}{2}, 1\right)$, higher levels of misperception have no effect on social welfare and harm consumers.**

**(c) When $\hat{v} \geq 1$, higher levels of misperception have no effect – neither on social welfare nor on consumer surplus.**

When $\hat{v}$ is small ($\hat{v} < \frac{1}{2}$), the misperception reduces social welfare and harms consumers. Social welfare is reduced, because consumers who should not be served are served because of the misperception. Consumer surplus is also reduced: The consumers who are served with and without the misperception are now incurring an actual loss (since the misperception increases prices). And the consumers who should not be served, but are served because of the misperception, i.e., consumers with $x \in (v, \hat{v})$ and with $x \in (1-\hat{v}, 1-v)$, also incur a loss.

For intermediate levels of $\hat{v}$ ($\hat{v} \in \left(\frac{1}{2}, 1\right)$), higher levels of misperception have no effect on social welfare. In terms of consumer surplus, higher levels of misperception allow the firms to charge a higher price to consumers closer to the ends of the Hotelling line (specifically, consumers with $x < 1-\hat{v}$ and $x > \hat{v}$) and this reduces consumer surplus. Higher levels of misperception do not affect the price paid by consumers closer to the middle of the Hotelling line (specifically, consumers with $x \in (1-\hat{v}, \hat{v})$, but a higher level of misperception increases the number of consumers whose price is affected by the misperception.

When $\hat{v}$ is large ($\hat{v} \geq 1$), higher levels of misperception have no effect on prices or payoffs and thus do not affect social welfare or consumer surplus.

### 4.3 Comparison

Collecting the results from the preceding analysis, we compare NPD and PD in terms of social welfare (Table 3 and Figure 2) and consumer surplus (Table 4 and Figure 3).

| | No Price Discrimination (NPD) | Price Discrimination (PD) | Comparison |
|---|---|---|---|
| $v + \delta \leq \frac{1}{2}$ | $\hat{v}\left(v - \frac{1}{4}\hat{v}\right)$ | $v^2 - (\hat{v} - v)^2$ | PD is better if $\delta < \frac{1}{3}v$<br>NPD is better if $\delta > \frac{1}{3}v$ |
| $v + \delta \in \left(\frac{1}{2}, 1\right)$ | $\hat{v}\left(v - \frac{1}{4}\hat{v}\right)$ | $v - \frac{1}{4}$ | PD is better if $\delta < 3v - 1$<br>NPD is better if $\delta > 3v - 1$ |
| $v + \delta \geq 1$ | $v - \frac{1}{4}$ | $v - \frac{1}{4}$ | Same |

<u>Table 3: Comparing Social Welfare with NPD and PD, with Common Misperception</u>

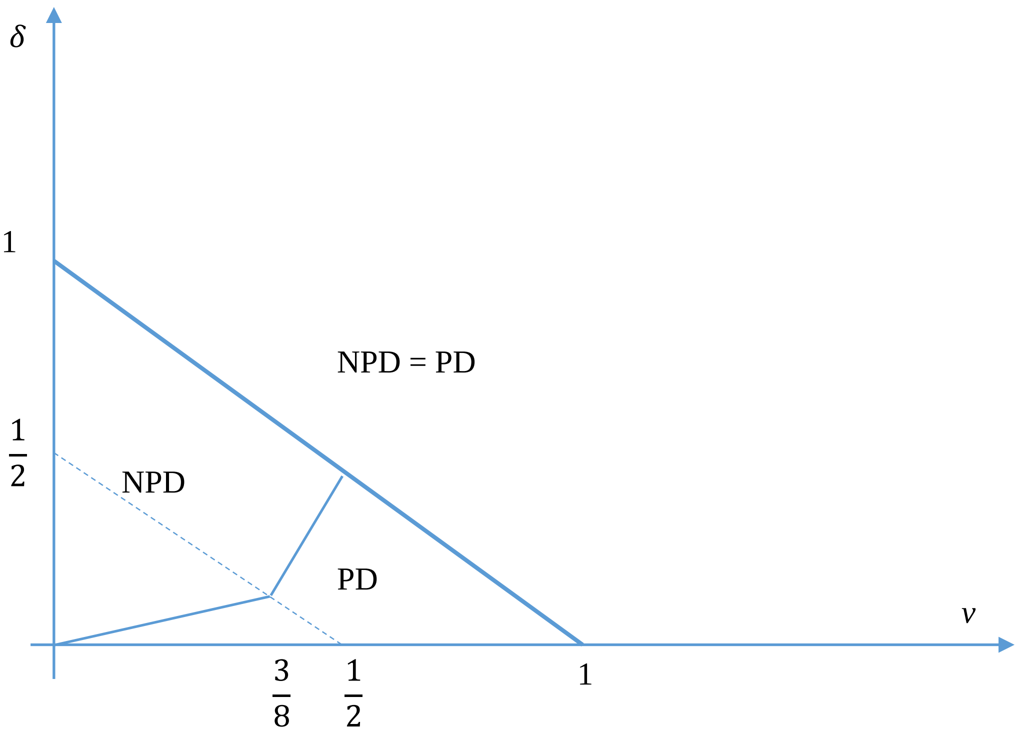


Figure 2: Comparing Social Welfare with NPD and PD, with Common Misperception

When $\hat{v} \leq \frac{1}{2}$, PD increases social welfare by: $\hat{v}\left(v - \frac{3}{4}\hat{v}\right)$. Without misperception, the difference is $\frac{1}{4}v^2$. Misperception reduces the magnitude of PD's advantage. And when $\delta > \frac{1}{3}v$, NPD induces greater social welfare. When $\hat{v} \in \left(\frac{1}{2}, 1\right)$, PD increases social welfare by: $\frac{1}{4}\hat{v}^2 - v\hat{v} + v - \frac{1}{4}$. Without misperception, the difference is $-\frac{3}{4}v^2 + v - \frac{1}{4}$. Misperception reduces the magnitude of PD's advantage. And when $\delta > 3v - 1$, NPD induces greater social welfare. When $\hat{v} \geq 1$, NPD and PD generate the same social welfare.[5]

[5] The comparison to the no-misperception benchmark is only suggestive, since the three ranges are defined differently – as a function of $v$ in the no-misperception benchmark and as a function of $\hat{v}$ with common misperception.

| | No Price Discrimination (NPD) | Price Discrimination (PD) | Comparison |
|---|---|---|---|
| $v+\delta \le \frac{1}{2}$ | $\left(v-\frac{1}{2}\hat{v}\right)^2-(\hat{v}-v)^2$ | $-2\hat{v}(\hat{v}-v)$ | NPD is better |
| $v+\delta \in \left(\frac{1}{2},1\right)$ | $\left(v-\frac{1}{2}\hat{v}\right)^2-(\hat{v}-v)^2$ | $\left(\hat{v}-\frac{1}{2}\right)^2-(\hat{v}-v)$ | NPD is better |
| $v+\delta \in \left(1,\frac{3}{2}\right)$ | $\left(\frac{1}{2}-(\hat{v}-v)\right)^2-(\hat{v}-v)^2$ $=\frac{1}{4}-(\hat{v}-v)$ | $v-\frac{3}{4}$ | PD is better |
| $v+\delta \ge \frac{3}{2}$ | $v-\frac{5}{4}$ | $v-\frac{3}{4}$ | PD is better |

Table 4: Comparing Consumer Surplus with NPD and PD, with Common Misperception

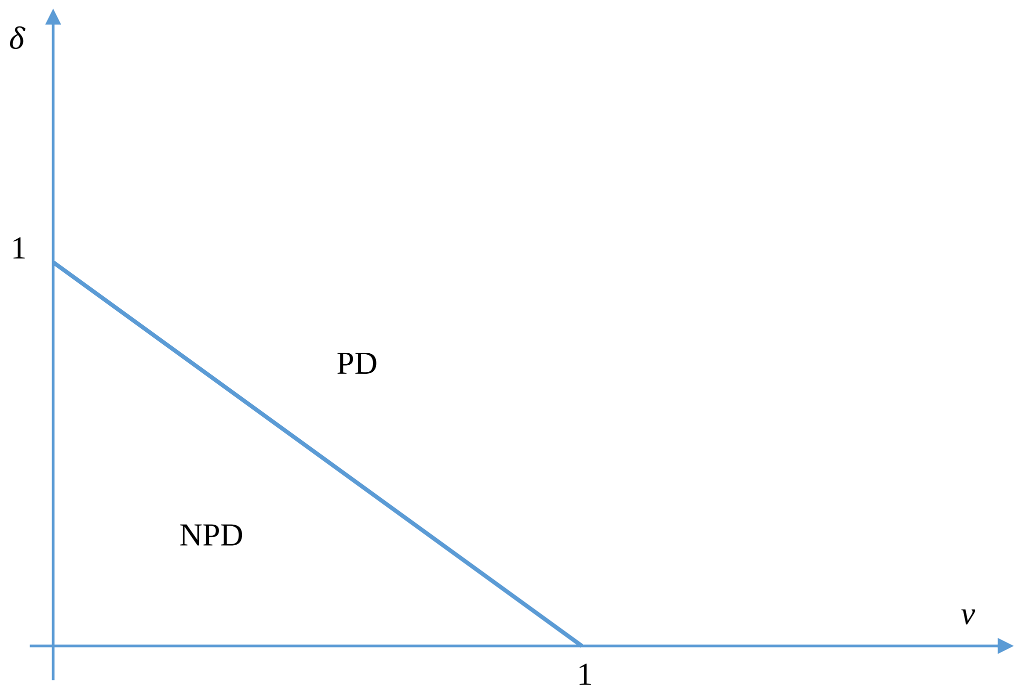


Figure 3: Comparing Consumer Surplus with NPD and PD, with Common Misperception

When $\hat{v} \le \frac{1}{2}$, PD decreases the consumer surplus by: $\hat{v}\left(\frac{5}{4}\hat{v}-v\right)$. Without misperception, the difference is $\frac{1}{4}v^2$. Misperception increases the magnitude of NPD's advantage. When $\hat{v} \in \left(\frac{1}{2},1\right)$, PD decreases the consumer surplus by: $-\frac{7}{4}\hat{v}^2+(v+2)\hat{v}-\left(v+\frac{1}{4}\right)$. Without misperception, the difference is $-\frac{3}{4}v^2+v-\frac{1}{4}$. For $v<\frac{4}{5}$, misperception initially increases the magnitude of NPD's advantage until $\delta = \frac{5}{7}\left(\frac{4}{5} - v\right)$; a larger bias then begins to reduce NPD's advantage, and with $\delta > \frac{10}{7}\left(\frac{4}{5} - v\right)$, NPD's advantage with misperception is smaller than NPD's advantage without misperception. For $v < \frac{4}{5}$, misperception monotonically increases the magnitude of NPD's advantage. When $\hat{v} \in \left(1, \frac{3}{2}\right)$, PD increases the consumer surplus by: $\hat{v} - 1$. Without misperception, the difference is $v - 1$. Misperception increases the magnitude of PD's advantage. When $\hat{v} \geq \frac{3}{2}$, PD increases the consumer surplus by: $\frac{1}{2}$. The magnitude of the misperception has no effect on PD's advantage.

The comparisons are summarized in the following proposition.

**Proposition 4 (NPD v. PD, the Effects of Common Misperception)**
**(a) When $\hat{v} \leq \frac{1}{2}$, NPD induces greater social welfare than PD iff $\delta > \frac{1}{3}v$, and NPD always induces greater consumer surplus than PD.**
**(b) When $\hat{v} \in \left(\frac{1}{2}, 1\right)$, NPD induces greater social welfare than PD iff $\delta > 3v - 1$, and NPD always induces greater consumer surplus than PD.**
**(c) When $\hat{v} \geq 1$, NPD and PD induce the same level of social welfare, and PD induces greater consumer surplus than NPD.**

The following corollary states related results, focusing on the actual value, *v*.

Corollary 1:
(a) When $v < 1$, the market is not covered with NPD. Overestimation reduces this problem and with it the advantage of PD over NPD. Consumer surplus is higher with NPD, since PD allows firms to extract more surplus. Misperception allows firms to price higher with both NPD and PD. This higher-price problem initially increases NPD's advantage, but then reduces it. Eventually, PD is better for consumers.
(b) When $v \geq 1$, the market is covered with both NPD and PD, so both induce the same level of social welfare. Misperception has no effect on social welfare. PD induces a larger consumer surplus, through more intense competition and lower prices. NPD mutes competitive forces; firms want to exploit their market power vis-à-vis consumers at the ends of the Hotelling line and thus set a high price for all consumers. Misperception has no effect on consumer surplus.

Note that for consumers who are close to the end (either end) of the Hotelling line, the firm close to them enjoys market power. As in the monopoly case, with common misperception, PD harms consumers and may even reduce social welfare. Compare Bar-Gill (2019).

## 5. Relative Misperception

Next, consider the effects of relative misperception. As explained in Section 2, relative misperception affects the relative attraction of the two firms, and of their respective products, as perceived by the consumer. Formally, this type of misperception is captured by a perceived location (on the Hoteling line), $\hat{x}$, which is different from the actual location, *x*. Section 5.1 studies the NPD case and Section 5.2 studies the PD case. Section 5.3 compares NPD and PD in the presence of relative misperception.

5.1 No Price Discrimination

Consumer misperceptions affect the firms' optimization problems. Specifically, in the constraints – (1a) and (1b) for F1 and (2a) and (2b) for F2 – the actual location, $x_1$ (or $x_2$), is replaced by the perceived location, $\hat{x}_1 = x_1 + \delta$ (or $\hat{x}_2 = x_2 + \delta$).[6] (To simplify the analysis, I assume that $\delta < v$. See Proof of Lemma 6 in the Appendix.) The market equilibrium is characterized in the following lemma.

*Lemma 6 (Relative Misperception; No Price Discrimination): With misperception and no price discrimination –*

*(a) For $v \leq 1$,*

*(a.1) Prices are: $p_1 = \frac{1}{2}(v-\delta)$ and $p_2 = \frac{1}{2}(v+\delta)$*

*(a.2) The market is not covered. Consumers with $x \in \left(\frac{1}{2}(v-\delta), 1-\frac{1}{2}(v+\delta)\right)$ do not buy the good. Social welfare is: $\int_0^{\frac{1}{2}(v-\delta)}(v-x)dx + \int_{1-\frac{1}{2}(v+\delta)}^{1}\big(v-(1-x)\big)dx$.*

*(a.3) Consumer x enjoys a payoff of: $v-\frac{1}{2}(v-\delta)-x$ for $x \leq \frac{1}{2}(v-\delta)$,$v-\frac{1}{2}(v+\delta)-(1-x)$ for $x \geq 1-\frac{1}{2}(v+\delta)$, and zero for $x \in \left(\frac{1}{2}(v-\delta), 1-\frac{1}{2}(v+\delta)\right)$. Consumer surplus is: $\int_0^{\frac{1}{2}(v-\delta)}\left(v-\frac{1}{2}(v-\delta)-x\right)dx + \int_{1-\frac{1}{2}(v+\delta)}^{1}\left(v-\frac{1}{2}(v+\delta)-(1-x)\right)dx$.*

*(b) For $v \in \left(1,\frac{3}{2}\right)$, let $\phi_p = 1+\frac{2}{3}(v-1) \in \left[1,\frac{4}{3}\right]$ and $\phi_x = \frac{1}{2}\left[1-\frac{2}{3}(v-1)\right] \in \left[\frac{1}{3},\frac{1}{2}\right]$,*

*(b.1) Prices are: $p_1 = v-\frac{1}{2}-\frac{1}{2}\phi_p\delta$* and *$p_2 = v-\frac{1}{2}+\frac{1}{2}\phi_p\delta$.*

*(b.2) The market is covered, such that consumers with $x \leq \frac{1}{2}-\phi_x\delta$ buy from F1 and consumers with $x > \frac{1}{2}-\phi_x\delta$ buy from F2. Social welfare is: $\int_0^{\frac{1}{2}-\phi_x\delta}(v-x)dx + \int_{\frac{1}{2}-\phi_x\delta}^{1}\big(v-(1-x)\big)dx = W(\delta=0) - \int_{\frac{1}{2}-\phi_x\delta}^{\frac{1}{2}}(1-2x)dx = v-\frac{1}{4}-(\phi_x\delta)^2$.*

*(b.3) Consumer x enjoys a payoff of: $\frac{1}{2}+\frac{1}{2}\phi_p\delta - x$ for $x \leq \frac{1}{2}-\phi_x\delta$, and $\frac{1}{2}-\frac{1}{2}\phi_p\delta-(1-x)$ for $x > \frac{1}{2}-\phi_x\delta$. Consumer surplus is: $\int_0^{\frac{1}{2}-\phi_x\delta}\left(\frac{1}{2}+\frac{1}{2}\phi_p\delta-x\right)dx + \int_{\frac{1}{2}-\phi_x\delta}^{1}\left(\frac{1}{2}-\frac{1}{2}\phi_p\delta-(1-x)\right)dx$.*

*(c) For $v \geq \frac{3}{2}$,*

*(c.1) Prices are: $p_1 = 1-\frac{2}{3}\delta$ and $p_2 = 1+\frac{2}{3}\delta$.*

[6] In this framework, it is possible to get $\hat{x}_1 > 1$ (and $\hat{x}_2 > 1$). We could interpret this as a special advantage that this consumer gets by purchasing from F2. Alternatively, we could impose a constraint that $\hat{x}_1$ (and $\hat{x}_2$) cannot exceed 1.

*(c.2)* *The market is covered. Social welfare is:* $\int_0^{\frac{1}{2}-\frac{1}{3}\delta}(v-x)dx+\int_{\frac{1}{2}-\frac{1}{3}\delta}^{1}\big(v-(1-x)\big)dx=W(\delta=0)-\int_{\frac{1}{2}-\frac{1}{3}\delta}^{\frac{1}{2}}(1-2x)dx=v-\frac{1}{4}-\left(\frac{1}{3}\delta\right)^2$.

*(c.3)* *Consumer x enjoys a payoff of:* $v-\left(1-\frac{2}{3}\delta\right)-x$ *for* $x\le\frac{1}{2}-\frac{1}{3}\delta$, *and* $v-\left(1+\frac{2}{3}\delta\right)-(1-x)$ *for* $x>\frac{1}{2}-\frac{1}{3}\delta$. *Consumer surplus:* $\int_0^{\frac{1}{2}-\frac{1}{3}\delta}\left(v-\left(1-\frac{2}{3}\delta\right)-x\right)dx+\int_{\frac{1}{2}-\frac{1}{3}\delta}^{1}\left(v-\left(1+\frac{2}{3}\delta\right)-(1-x)\right)dx$.

The effects of relative misperception, derived from a comparison between Lemma 6 and Lemma 2, are summarized in the following proposition.

**Proposition 5 (No Price Discrimination; the Effects of Relative Misperception): For all *v*, higher levels of misperception reduce social welfare and harm consumers.**

When $v$ is small, the misperception leads to insufficient purchases from F1 and to excessive purchases from F2. When $v$ is large, the misperception inefficiently shifts purchases from F1 to F2.

5.2 Price Discrimination

Here too consumer misperceptions affect the firms' optimization problems. Specifically, in the constraints – (3a) and (3b) for F1 and (4a) and (4b) for F2 – the actual location, *x*, is replaced by the perceived location, $\hat{x}=x+\delta$. The market equilibrium is characterized in the following lemma.

*Lemma 7 (Relative Misperception; Price Discrimination): With relative misperception and price discrimination –*

*(a) For* $v\le\frac{1}{2}$,

*(a.1)* *Prices are:* $p_1(x)=v-x-\delta$ *and* $p_2(x)=0$ *for* $x\le v-\delta$; *and* $p_2(x)=v-(1-x-\delta)$ *and* $p_1(x)=0$ *for* $x\ge 1-v-\delta$.

*(a.2)* *The market is not covered. Consumers with* $x\in(v-\delta,1-v-\delta)$ *do not buy the good. Social welfare is:* $\int_0^{v-\delta}(v-x)dx+\int_{1-v-\delta}^{1}(v-(1-x))dx$.

*(a.3)* *Consumer x enjoys a payoff of* $\delta$ *for* $x\le v-\delta$ *and a payoff of* $-\delta$ *for* $x\ge 1-v-\delta$. *Consumer surplus is:* $\int_0^{v-\delta}\delta dx+\int_{1-v-\delta}^{1}(-\delta)dx$.

*(b) For* $v\in\left(\frac{1}{2},1\right)$,

*(b.1)* *Prices are:* $p_1(x)=v-x-\delta$ *and* $p_2(x)=0$ *for* $x<1-v-\delta$; $p_1(x)=1-2(x+\delta)$ *and* $p_2(x)=0$ *for* $x\in\left[1-v-\delta,\frac{1}{2}-\delta\right]$; $p_1(x)=0$ *and* $p_2=-\big(1-2(x+\delta)\big)$ *for* $x\in(\frac{1}{2}-\delta,v-\delta]$; *and* $p_1(x)=0$ *and* $p_2(x)=v-(1-x-\delta)$ *for* $x>v-\delta$.

*(b.2) The market is covered. Social welfare is:* $\int_0^{\frac{1}{2}-\delta}(v-x)dx+\int_{\frac{1}{2}-\delta}^{1}(v-(1-x))dx$.

*(b.3) Consumer* $x$ *enjoys a payoff of* $\delta$ *for* $x<1-v-\delta$; $v-1+x+2\delta$ *for* $x\in\left[1-v-\delta,\frac{1}{2}-\delta\right]$; $v-x-2\delta$ *for* $x\in(\frac{1}{2}-\delta,v-\delta]$; *and* $-\delta$ *for* $x>v-\delta$. *Consumer surplus is:* $\int_0^{1-v-\delta}\delta dx+\int_{1-v-\delta}^{\frac{1}{2}-\delta}(v-1+x+2\delta)dx+\int_{\frac{1}{2}-\delta}^{v-\delta}(v-x-2\delta)dx+\int_{v-\delta}^{1}(-\delta)dx$.

*(c) For* $v\geq 1$,

*(c.1) Prices are:* $p_1(x)=1-2(x+\delta)$ *and* $p_2(x)=0$ *for* $x\leq\frac{1}{2}-\delta$; *and* $p_2=-\left(1-2(x+\delta)\right)$ *and* $p_1(x)=0$ *for* $x>\frac{1}{2}-\delta$.

*(c.2) The market is covered. Social welfare is:* $\int_0^{\frac{1}{2}-\delta}(v-x)dx+\int_{\frac{1}{2}-\delta}^{1}(v-(1-x))dx$.

*(c.3) Consumer* $x$ *enjoys a payoff of* $v-\left(1-2(x+\delta)\right)-x$ *for* $x\leq\frac{1}{2}-\delta$; *and* $v+\left(1-2(x+\delta)\right)-(1-x)$ *for* $x>\frac{1}{2}-\delta$. *Consumer surplus is:* $\int_0^{\frac{1}{2}-\delta}\left(v-\left(1-2(x+\delta)\right)-x\right)dx+\int_{\frac{1}{2}-\delta}^{1}\left(v+\left(1-2(x+\delta)\right)-(1-x)\right)dx$.

The effects of misperception, derived from a comparison between Lemma 7 and Lemma 3, are summarized in the following proposition.

**Proposition 6 (Price Discrimination; the Effects of Relative Misperception): For all *v*, higher levels of misperception reduce social welfare and harm consumers.**

As before, relative misperception distorts purchasing decisions and thus reduces social welfare and harms consumers.

5.3 Comparison

Collecting the results from the preceding analysis, we compare NPD and PD in terms of social welfare and consumer surplus. For $v\leq\frac{1}{2}$, we have: $W^{NPD}-W^{PD}=\frac{1}{4}(-v^2+3\delta^2)$. Therefore, NPD generates higher social welfare *iff* $\delta>\frac{\sqrt{3}}{3}v$. PD increases the number of purchases. For consumers buying from F1, this is clearly welfare enhancing. But for consumers buying from F2, at least some of these added purchases are welfare reducing. In terms of consumer surplus, we have: $CS^{NPD}-CS^{PD}=\frac{1}{4}v^2+\frac{5}{4}\delta^2$. Therefore, NPD generates more consumer surplus. For consumers, any gain from the larger number of purchases is stripped away through higher prices.

For $v \in \left(\frac{1}{2}, 1\right)$, we have: $W^{NPD} - W^{PD} = \frac{1}{4}(3v^2 - 4v + 1 + 3\delta^2)$. Therefore, NPD generates higher social welfare *iff* $\delta > \frac{\sqrt{3}}{3}\sqrt{(1-v)(3v-1)}$.[7] PD results in more purchases from F2. Consumers who did not buy with NPD, now buy from F2. Some of these additional purchases are welfare enhancing (as long as $v > \delta$), but others are welfare reducing. And when $v$ is higher (i.e., closer to 1), some of the additional F2 purchases replace more efficient F1 purchases. In terms of consumer surplus, we have: $CS^{NPD} - CS^{PD} = -\frac{3}{4}v^2 + v - \frac{1}{4} + \frac{5}{4}\delta^2 = \frac{5}{4}\delta^2 + \frac{1}{4}(1-v)(3v-1)$, which is always positive for $v \in \left(\frac{1}{2}, 1\right)$. Therefore, NPD generates more consumer surplus.

For $v \in \left(1, \frac{3}{2}\right)$, we have: $W^{NPD} - W^{PD} = (1 - \phi_x^2)\delta^2$. Therefore, NPD generates higher social welfare. The market is covered with both NPD and PD. PD inefficiently shifts purchases from F1 to F2. In terms of consumer surplus, we have: $CS^{NPD} - CS^{PD} = 1 - v + \left(3 - \phi_x^2 - \phi_x\phi_p\right)\delta^2$. NPD generates more consumer surplus *iff* $\delta > \sqrt{\frac{v-1}{3-\phi_x^2-\phi_x\phi_p}}$.

For $v \geq \frac{3}{2}$, we have: $W^{NPD} - W^{PD} = \frac{8}{9}\delta^2$. Therefore, NPD generates higher social welfare. The market is covered with both NPD and PD. PD inefficiently shifts purchases from F1 to F2. In terms of consumer surplus, we have: $CS^{NPD} - CS^{PD} = \frac{22}{9}\delta^2 - \frac{1}{2}$. Therefore, NPD generates more consumer surplus *iff* $\delta > \frac{3\sqrt{11}}{22}$. We saw in Section 3 that, without misperception, PD generates more consumer surplus. For sufficiently strong misperception, this benefit of PD is outweighed by the shift to F2 who can charge higher prices because of the misperception.

These comparisons are depicted graphically in Figure 4 (social welfare) and Figure 5 (consumer surplus).

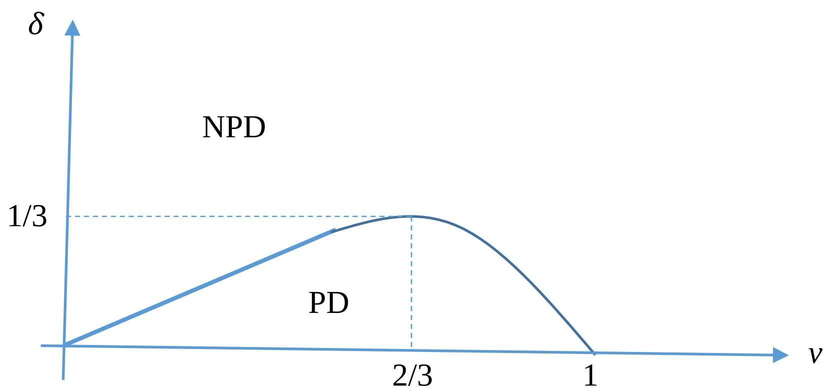


Figure 5: Comparing Social Welfare with NPD and PD, with Relative Misperception

[7] Let $f(v) = \frac{\sqrt{3}}{3}\sqrt{(1-v)(3v-1)}$, and note that $f\left(v = \frac{1}{2}\right) = \frac{\sqrt{3}}{6}$, $f(v = 1) = 0$, and that $argmax\langle f(v)\rangle = \frac{2}{3}$ and $max\langle f(v)\rangle = \frac{1}{3}$.

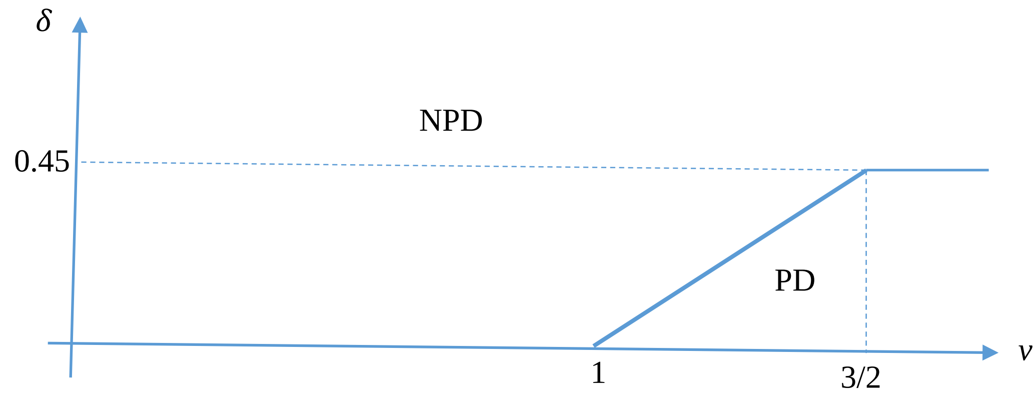


Figure 6: Comparing Consumer Surplus with NPD and PD, with Relative Misperception

The comparisons are summarized in the following proposition.

**Proposition 7 (NPD v. PD, the Effects of Relative Misperception)**
**(a) When $\boldsymbol{v \le \frac{1}{2}}$, NPD induces greater social welfare than PD iff $\boldsymbol{\delta > \frac{\sqrt{3}}{3} v}$, and NPD always induces greater consumer surplus than PD.**
**(b) When $\boldsymbol{v \in \left(\frac{1}{2}, 1\right)}$, NPD induces greater social welfare than PD iff $\boldsymbol{\delta > \frac{\sqrt{3}}{3}\sqrt{(1-v)(3v-1)}}$, and NPD always induces greater consumer surplus than PD.**
**(c) When $\boldsymbol{v \in \left(1, \frac{3}{2}\right)}$, NPD always induces greater social welfare than PD, and NPD induces greater consumer surplus than PD iff $\boldsymbol{\delta > \sqrt{\frac{v-1}{3-\phi_x^2-\phi_x\phi_p}}}$.**
**(d) When $\boldsymbol{v \ge \frac{3}{2}}$, NPD always induces greater social welfare than PD, and NPD induces greater consumer surplus than PD iff $\boldsymbol{\delta > \frac{3\sqrt{11}}{22}}$.**

Starting with social welfare, for lower *v* ($v < 1$), PD induces greater social welfare, but only for low levels of misperception ($\delta < \frac{\sqrt{3}}{3} v$ for $v < \frac{1}{2}$ and $\delta < \frac{\sqrt{3}}{3}\sqrt{(1-v)(3v-1)}$ for $v \in \left[\frac{1}{2}, 1\right)$). For higher levels of misperception, NPD induces greater social welfare. For higher *v* ($v > 1$), NPD induces greater social welfare for any level of misperception. Intuitively, PD increases efficiency when *v* is low, since it allows for more transactions. When *v* is higher, the problem of insufficient transacting is smaller (or non-existent) and the misperception generates more distortions with PD. Next consider consumer surplus: For lower *v* ($v < 1$), NPD induces greater consumer surplus. For higher *v* ($v \ge 1$), NPD induces greater social surplus, when the level of misperception is high; and PD induces greater social surplus, when the level of misperception is low.

## 6. Concluding Remarks

In these brief concluding remarks, I summarize the main findings and their normative implications (section 6.1), discuss policy implications (section 6.2) and list some possible extensions (section 6.3).

6.1 Summary and Normative Implications

This paper studied the implications of misperceptions and of price discrimination. Starting with misperceptions, the analysis shows that common misperception can increase efficiency when *v* is small and the firms cannot price discriminate. And, even then, this efficiency benefit must be balanced against the harm to consumers that the misperception causes. Relative misperception always reduces efficiency and harms consumers.

The effects of price discrimination depend on the magnitude of *v*. For small *v* and common misperception, the normative assessment of price discrimination is summarized in Table 5:

| Misperception Level | Efficiency | Consumer Surplus | Overall |
|---|---|---|---|
| Zero – Low | PD is Good | PD is Bad | ? |
| Intermediate | PD is Bad | PD is Bad | PD is Bad |
| High | PD is Neutral | PD is Good | PD is Good |

Table 5: The Effects of Price Discrimination with Small *v* and Common Misperception

For small *v* and relative misperception, the normative assessment of price discrimination is summarized in Table 6:

| Misperception Level | Efficiency | Consumer Surplus | Overall |
|---|---|---|---|
| Zero – Low | PD is Good | PD is Bad | ? |
| Intermediate – High | PD is Bad | PD is Bad | PD is Bad |

Table 6: The Effects of Price Discrimination with Small *v* and Relative Misperception

For large *v* and common misperception, price discrimination is good: It does not affect efficiency and it helps consumers. For large *v* and relative misperception, the normative assessment of price discrimination is summarized in Table 7:

| **Misperception Level** | **Efficiency** | **Consumer Surplus** | **Overall** |
|---|---|---|---|
| Zero – Low | PD is Bad | PD is Good | ? |
| High | PD is Bad | PD is Bad | PD is Bad |

Table 7: The Effects of Price Discrimination with High *v* and Relative Misperception

Notice how the introduction of misperception can eliminate the ambiguity about the normative implications of price discrimination. In many cases, without misperception price discrimination improves efficiency but harms consumers, or vice versa. With misperception both normative criteria often point in the same direction.

### 6.2 Policy Implications

The normative implications summarize in Section 6.1, provide a starting point for policymakers who are considering possible interventions in oligopolistic markets with horizontally differentiated goods and services. First, since misperception is generally harmful, policies that reduce misperception – such as restrictions on deceptive advertising[8] or government-sponsored information campaigns – should be considered. Policymakers can also provide incentives for firms to reduce misperception (or refrain from creating misperception) by imposing statutory warranties (or products liability) that require firms to rebate the difference between the perceived and actual value that the consumer gets from the product or service.[9] Second, to the extent that policymakers (e.g., competition authorities) can restrict price discrimination, such efforts should be focused on (i) markets with small $v$ and an intermediate level of common misperception or an intermediate-to-high level of relative misperception, and (ii) markets with large $v$ and high levels of relative misperception.

### 6.3 Extensions

The simple model studied in this paper can be extended in several ways. First, underestimation ($\delta < 0$) can be added to the overestimation studied in this paper, for common misperception. In most markets, firms' incentives suggest that overestimation is more likely than underestimation, but there are products and services (e.g., in the healthcare market) where underestimation of benefits can be a concern. Second, the Hotelling transportation costs can be generalized, from $x$ in the simple model to a possibly non-linear $t(x)$. Related: the distribution of consumers on the Hotelling line can be generalized beyond the Uniform distribution studied in this paper. Third, more general formulations of the consumers' utility and perceived utility functions can be considered. Finally, while this paper fixes firm locations at the ends of the Hotelling line, it may

[8] See, e.g., Federal Trade Commission Act, Sec. 5; Lanham Act, Sec. 43.
[9] See Nicola Persico's contribution to this Symposium.

be interesting to endogenize location decisions in a setting with misperception. It may also be interesting to study the effects of price discrimination on location decisions.[10]

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[10] I thank Olga Gorelkina for suggesting many of these extensions. See also Gorelkina's contribution to this Symposium.

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## Appendix

Proof of Lemma 2 (No Misperception; No Price Discrimination):

F1's maximization problem is equivalent to: $max\langle p_1 \cdot x_1(p_1; p_2)\rangle$, where

$$x_1(p_1; p_2) = \begin{cases} \frac{1}{2}\big(1 - (p_1 - p_2)\big) & , \ p_1 \leq 2v - 1 - p_2 \\ v - p_1 & , \ p_1 > 2v - 1 - p_2 \end{cases}$$

F1's revenue function is:

$$R_1(p_1; p_2) = p_1 \cdot x_1(p_1; p_2) = \begin{cases} p_1 \frac{1}{2}\big(1 - (p_1 - p_2)\big) & , \ p_1 \leq 2v - 1 - p_2 \\ p_1(v - p_1) & , \ p_1 > 2v - 1 - p_2 \end{cases}$$

When IC is binding, $p_1(p_2) = \frac{1}{2}(1+p_2)$ maximizes $R_1(p_1; p_2) = p_1 \frac{1}{2}\big(1 - (p_1 - p_2)\big)$; and $p_1(p_2) = \frac{1}{2}(1+p_2)$ satisfies the boundary condition $p_1 \leq 2v - 1 - p_2$, when $p_2 < \frac{4}{3}v - 1$. When IR is binding, $p_1(p_2) = \frac{1}{2}v$ maximizes $R_1(p_1; p_2) = p_1(v - p_1)$; and $p_1(p_2) = \frac{1}{2}v$ satisfies the boundary condition, when $p_2 > \frac{3}{2}v - 1$.

What happens when $p_2 \in \left(\frac{4}{3}v - 1, \frac{3}{2}v - 1\right)$?

$R_1(p_1; p_2) = p_1 \frac{1}{2}\big(1 - (p_1 - p_2)\big)$ is maximized at $p_1(p_2) = \frac{1}{2}(1+p_2)$. When $p_2 > \frac{4}{3}v - 1$, $p_1(p_2) = \frac{1}{2}(1+p_2)$ violates the boundary condition $p_1 \leq 2v - 1 - p_2$, and so the maximal value of $p_1 \frac{1}{2}\big(1 - (p_1 - p_2)\big)$ is $(2v - 1 - p_2)\frac{1}{2}\big(1 - (2v - 1 - p_2 - p_2)\big) = (2v - 1 - p_2)(1 - v + p_2)$.

$R_1(p_1; p_2) = p_1(v - p_1)$ is maximized at $p_1(p_2) = \frac{1}{2}v$. When $p_2 < \frac{3}{2}v - 1$, $p_1(p_2) = \frac{1}{2}v$ violates the boundary condition $p_1 > 2v - 1 - p_2$, and so the maximal value of $p_1(v - p_1)$ is $(2v - 1 - p_2)\big(v - (2v - 1 - p_2)\big) = (2v - 1 - p_2)(1 - v + p_2)$.

By definition, the two expressions are equivalent, since at $p_1 = 2v - 1 - p_2$ IC and IR converge. We thus have:

$$p_1(p_2) = \begin{cases} \frac{1}{2}(1+p_2) & , \quad p_2 < \frac{4}{3}v - 1 \\ 2v - 1 - p_2 & , \ p_2 \in \left(\frac{4}{3}v - 1, \frac{3}{2}v - 1\right) \\ \frac{1}{2}v & , \quad p_2 > \frac{3}{2}v - 1 \end{cases}$$

Combining F1's reaction function, $p_1(p_2)$, with F2's symmetric reaction function, $p_2(p_1)$, we obtain the equilibrium described in Lemma 2.

QED

Proof of Lemma 3 (No Misperception; Price Discrimination):

Competition implies that $min\langle p_1, p_2\rangle = 0$.

For $x \leq \frac{1}{2}$: If Consumer x buys the product, it will be from F1. F2 sets $p_2 = 0$ and F1 sets $p_1$ depending on which constraint is binding. If IC is binding, then F1 sets $p_1 = 1 - 2x$; if IR is binding, then F1 sets $p_1 = v - x$. Therefore, $p_1 = min(1 - 2x, v - x)$. And, of course, F1 will not sell if $p_1 < 0$. This means that if $min(1 - 2x, v - x) = v - x$ and $v - x < 0$, Consumer x will not be served.

For $x > \frac{1}{2}$: If Consumer x buys the product, it will be from F2. F1 sets $p_1 = 0$ and F2 sets $p_2$ depending on which constraint is binding. If IC is binding, then F2 sets $p_2 = -(1 - 2x)$; if IR is binding, then F2 sets $p_2 = v - (1 - x)$. Therefore, $p_2 = min\big(-(1 - 2x), v - (1 - x)\big)$. And, of course, F2 will not sell if $p_2 < 0$. This means that if $min\big(-(1 - 2x), v - (1 - x)\big) = v - (1 - x)$ and $v - (1 - x) < 0$, Consumer x will not be served.

Which constraint – IC or IR – is binding plays a central role in the analysis. For $x \leq \frac{1}{2}$, IC is binding, when $1 - 2x \leq v - x$ or $x \geq 1 - v$. For $x > \frac{1}{2}$, IC is binding, when $-(1 - 2x) \leq v - (1 - x)$ or $x \leq v$. Note that, for both $x \leq \frac{1}{2}$ and $x > \frac{1}{2}$, when $v \geq 1$, IC is always binding, and when $v \leq \frac{1}{2}$, IR is always binding. When $v \in \left(\frac{1}{2}, 1\right)$, the IC constraint is binding for $x \in [1 - v, v]$ and the IR constraint is binding for $x < 1 - v$ and $x > v$.

Case 1: $v \geq 1$

- For $x \leq \frac{1}{2}$: F1 sets $p_1 = 1 - 2x$. Consumer x enjoys a payoff of $v - x - (1 - 2x) = v - (1 - x) \geq 0$. And F1 enjoys a payoff of $p_1 = 1 - 2x \geq 0$.
- For $x > \frac{1}{2}$: F2 sets $p_2 = -(1 - 2x)$. Consumer x enjoys a payoff of $v - (1 - x) + (1 - 2x) = v - x \geq 0$. And F2 enjoys a payoff of $p_2 = -(1 - 2x) = 2x - 1 > 0$.
- Consumer surplus: $\frac{1}{4} + (v - 1) = v - \frac{3}{4}$.

Case 2: $v \leq \frac{1}{2}$

- For $x \leq \frac{1}{2}$: When $x \leq v$, F1 sets $p_1 = v - x$, Consumer x buys the product, but enjoys a surplus of zero. And when $x > v$, Consumer x will not be served.
- For $x > \frac{1}{2}$: When $x \geq 1 - v$, F2 sets $p_2 = v - (1 - x)$, Consumer x buys the product, but enjoys a surplus of zero. And when $x < 1 - v$, Consumer x will not be served.
- Combining the two sets of results, we have:
  - For $x \leq v$, F1 sets $p_1 = v - x$ and F2 sets $p_2 = 0$, Consumer x buys the product from F1, but enjoys a surplus of zero.
  - For $x \in (v, 1 - v)$, Consumer x will not be served.
  - For $x \geq 1 - v$, F2 sets $p_2 = v - (1 - x)$ and F1 sets $p_1 = 0$, Consumer x buys the product from F2, but enjoys a surplus of zero.

- Consumer surplus is zero.

Case 3: $v \in \left(\frac{1}{2}, 1\right)$

- The analysis of Case 3 is a combination of the analysis of Case 1 and Case2.
- Consumer surplus: $\left(v - \frac{1}{2}\right)^2$

QED

Proof of Lemma 4 (Common Misperception; No Price Discrimination):

The proof of Lemma 4 is similar to the proof of Lemma 2 (after replacing $v$ with $\hat{v}$) and is, therefore omitted.

Proof of Lemma 5 (Common Misperception; Price Discrimination):

Competition implies that $min\langle p_1, p_2\rangle = 0$.

For $x \le \frac{1}{2}$: If Consumer x buys the product, it will be from F1. F2 sets $p_2 = 0$ and F1 sets $p_1$ depending on which constraint is binding. If IC is binding, then F1 sets $p_1 = 1 - 2x$; if IR is binding, then F1 sets $p_1 = \hat{v} - x$. Therefore, $p_1 = min(1 - 2x, \hat{v} - x)$. And, of course, F1 will not sell if $p_1 < 0$. This means that if $min(1 - 2x, \hat{v} - x) = \hat{v} - x$ and $\hat{v} - x < 0$, Consumer x will not be served.

For $x > \frac{1}{2}$: If Consumer x buys the product, it will be from F2. F1 sets $p_1 = 0$ and F2 sets $p_2$ depending on which constraint is binding. If IC is binding, then F2 sets $p_2 = -(1 - 2x)$; if IR is binding, then F2 sets $p_2 = \hat{v} - (1 - x)$. Therefore, $p_2 = min\big(-(1 - 2x), \hat{v} - (1 - x)\big)$. And, of course, F2 will not sell if $p_2 < 0$. This means that if $min\big(-(1 - 2x), \hat{v} - (1 - x)\big) = \hat{v} - (1 - x)$ and $\hat{v} - (1 - x) < 0$, Consumer x will not be served.
At $x = \frac{1}{2}$, IC is binding iff $min\left(\left(1 - 2 \cdot \frac{1}{2}\right), \hat{v} - \frac{1}{2}\right) = \left(1 - 2 \cdot \frac{1}{2}\right)$, or $\hat{v} \ge \frac{1}{2}$.

Which constraint – IC or IR – is binding plays a central role in the analysis. For $x \le \frac{1}{2}$, IC is binding, when $1 - 2x \le \hat{v} - x$ or $x \ge 1 - \hat{v}$. For $x > \frac{1}{2}$, IC is binding, when $-(1 - 2x) \le \hat{v} - (1 - x)$ or $x \le \hat{v}$. Note that, for both $x \le \frac{1}{2}$ and $x > \frac{1}{2}$, when $\hat{v} \ge 1$, IC is always binding, and when $\hat{v} \le \frac{1}{2}$, IR is always binding. When $\hat{v} \in \left(\frac{1}{2}, 1\right)$, the IC constraint is binding for $x \in [1 - \hat{v}, \hat{v}]$ and the IR constraint is binding for $x < 1 - \hat{v}$ and $x > \hat{v}$.

We divide the rest of the analysis into three cases.

1.1 Case 1: The IC Constraint Is Always Binding and the Market is Covered ($\hat{v} \geq 1$)

Social welfare is: $v - \frac{1}{4}$

Overall consumer surplus is: $\frac{1}{4} + (v - 1) = v - \frac{3}{4}$

1.2 Case 2: The IR Constraint Is Always Binding ($\hat{v} \leq \frac{1}{2}$)

For $x \leq \frac{1}{2}$: When $x \leq \hat{v}$, F1 sets $p_1 = \hat{v} - x$, Consumer x buys the product, and loses $\hat{v} - v$. And when $x > \hat{v}$, Consumer x will not be served.

For $x > \frac{1}{2}$: When $x \geq 1 - \hat{v}$, F2 sets $p_2 = \hat{v} - (1 - x)$, Consumer x buys the product, and loses $\hat{v} - v$. And when $x < 1 - \hat{v}$, Consumer x will not be served.

Combining the two sets of results, we have:

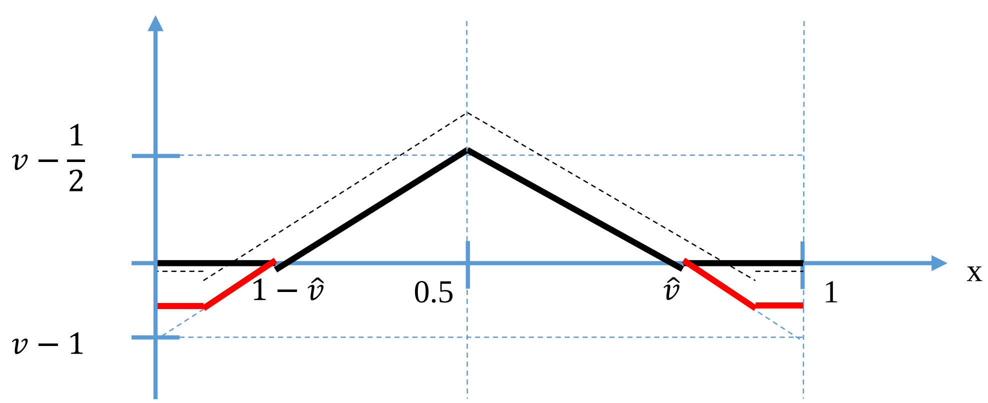
- For $x \leq \hat{v}$, F1 sets $p_1 = \hat{v} - x$ and F2 sets $p_2 = 0$, Consumer x buys the product from F1, and loses $\hat{v} - v$.
- For $x \in (\hat{v}, 1 - \hat{v})$, Consumer x will not be served.
- For $x \geq 1 - \hat{v}$, F2 sets $p_2 = \hat{v} - (1 - x)$ and F1 sets $p_1 = 0$, Consumer x buys the product from F2, and loses $\hat{v} - v$.

Overall consumer surplus is: $2\hat{v}(v - \hat{v})$.

1.3 Case 3: Either the IC Constraint or the IR Constraint Is Binding ($\hat{v} \in \left(\frac{1}{2}, 1\right)$)

Case 3 is a combination of Case 1 and Case2.

Consumer surplus as a function of x:




Overall consumer surplus: $\left(v - \frac{1}{2}\right)^2 - (\hat{v} - v)^2 - 2(\hat{v} - v)(1 - \hat{v}) = \left(\hat{v} - \frac{1}{2}\right)^2 + (v - \hat{v})$.

QED

Proof of Proposition 4 (Common Misperception; NPD v. PD)

Social Welfare: The social welfare levels, with NPD and PD, for different parameter values are summarized in Table 3.

- For $\hat{v} \leq \frac{1}{2}$, social welfare is $\hat{v}\left(v - \frac{1}{4}\hat{v}\right)$ with NPD and $v^2 - (\hat{v} - v)^2$ with PD. Therefore, the difference in welfare is $\Delta(\hat{v}; v) = \hat{v}\left(v - \frac{1}{4}\hat{v}\right) - [v^2 - (\hat{v} - v)^2]$. When $\hat{v} \leq \frac{1}{2}$, the difference function, $\Delta(\hat{v}; v)$, is positive if $\hat{v} > \frac{4}{3}v$ and negative if $\hat{v} < \frac{4}{3}v$.
- For $\hat{v} \in \left(\frac{1}{2}, 1\right)$, social welfare is $\hat{v}\left(v - \frac{1}{4}\hat{v}\right)$ with NPD and $v - \frac{1}{4}$ with PD. Therefore, the difference in welfare is $\Delta(\hat{v}; v) = \hat{v}\left(v - \frac{1}{4}\hat{v}\right) - \left[v - \frac{1}{4}\right]$. When $\hat{v} \in \left(\frac{1}{2}, 1\right)$ and $v < \frac{1}{2}$, the difference function, $\Delta(\hat{v}; v)$, is positive if $\hat{v} > 4v - 1$ and negative if $\hat{v} < 4v - 1$. When $\hat{v} \in \left(\frac{1}{2}, 1\right)$ and $v \geq \frac{1}{2}$, the difference function, $\Delta(\hat{v}; v)$, is positive throughout the $\hat{v} \in \left(\frac{1}{2}, 1\right)$ range.
- For $\hat{v} \geq 1$, social welfare is $v - \frac{1}{4}$ with both NPD and PD.

Consumer Surplus: The consumer surplus levels, with NPD and PD, for different parameter values are summarized in Table 4.

- For $\hat{v} \leq \frac{1}{2}$, consumer surplus is $\left(v - \frac{1}{2}\hat{v}\right)^2 - (\hat{v} - v)^2$ with NPD and $-2\hat{v}(\hat{v} - v)$ with PD. Therefore, the difference in welfare is $\Delta(\hat{v}; v) = \left(v - \frac{1}{2}\hat{v}\right)^2 - (\hat{v} - v)^2 - [-2\hat{v}(\hat{v} - v)]$. When $\hat{v} \leq \frac{1}{2}$, the difference function, $\Delta(\hat{v}; v)$, is always positive.
- For $\hat{v} \in \left(\frac{1}{2}, 1\right)$, consumer surplus is $\left(v - \frac{1}{2}\hat{v}\right)^2 - (\hat{v} - v)^2$ with NPD and $\left(\hat{v} - \frac{1}{2}\right)^2 - (\hat{v} - v)$ with PD. Therefore, the difference in welfare is $\Delta(\hat{v}; v) = \left(v - \frac{1}{2}\hat{v}\right)^2 - (\hat{v} - v)^2 - \left[\left(\hat{v} - \frac{1}{2}\right)^2 - (\hat{v} - v)\right]$. When $\hat{v} \in \left(\frac{1}{2}, 1\right)$, the difference function, $\Delta(\hat{v}; v)$, is always positive.
- For $\hat{v} \in \left(1, \frac{3}{2}\right)$, consumer surplus is $\frac{1}{4} - (\hat{v} - v)$ with NPD and $v - \frac{3}{4}$ with PD. Therefore, the difference in welfare is $\Delta(\hat{v}; v) = \frac{1}{4} - (\hat{v} - v) - \left[v - \frac{3}{4}\right]$. When $\hat{v} \in \left(1, \frac{3}{2}\right)$, the difference function, $\Delta(\hat{v}; v)$, is always negative.
- For $\hat{v} \geq 1$, consumer surplus is $v - \frac{5}{4}$ with NPD and $v - \frac{3}{4}$ with PD. Therefore, the difference in welfare is $\Delta(\hat{v}; v) = v - \frac{5}{4} - \left[v - \frac{3}{4}\right]$. When $\hat{v} \geq 1$, the difference function, $\Delta(\hat{v}; v)$, is always negative.

QED

Proof of Lemma 6 (Relative Misperception; No Price Discrimination):

F1 solves:

(1) $max\langle p_1 \cdot x_1(p_1; p_2)\rangle$
s.t.
(1a) IC: $v - \hat{x}_1 - p_1 \geq v - (1 - \hat{x}_1) - p_2$ or $p_1 \leq p_2 + (1 - 2\hat{x}_1)$ or $\hat{x}_1 \leq \frac{1}{2}\left(1 - (p_1 - p_2)\right)$
(1b) IR: $v - \hat{x}_1 - p_1 \geq 0$ or $p_1 \leq v - \hat{x}_1$ or $\hat{x}_1 \leq v - p_1$
(1c) $p_1 > 0$

F1's maximization problem is equivalent to: $max\langle p_1 \cdot (\hat{x}_1(p_1; p_2) - \delta)\rangle$, where

$$\hat{x}_1(p_1; p_2) = \begin{cases} \frac{1}{2}\left(1 - (p_1 - p_2)\right) & , \ p_1 \leq 2v - 1 - p_2 \\ v - p_1 & , \ p_1 \geq 2v - 1 - p_2 \end{cases}$$

When IC is binding, $p_1(p_2) = \frac{1}{2}(1+p_2) - \delta$ maximizes $R_1(p_1; p_2) = p_1\left(\frac{1}{2}\left(1 - (p_1 - p_2)\right) - \delta\right)$; and $p_1(p_2) = \frac{1}{2}(1+p_2) - \delta$ satisfies the boundary condition $p_1 \leq 2v - 1 - p_2$, when $p_2 \leq \frac{4}{3}v - 1 + \frac{2}{3}\delta$. When IR is binding, $p_1(p_2) = \frac{1}{2}(v - \delta)$ maximizes $R_1(p_1; p_2) = p_1(v - p_1 - \delta)$; and $p_1(p_2) = \frac{1}{2}(v - \delta)$ satisfies the boundary condition, when $p_2 \geq \frac{3}{2}v - 1 + \frac{1}{2}\delta$. A larger misperception relaxes the "IC-binding condition" and makes it more difficult to satisfy the "IR-binding condition." Namely, when the misperception is larger, it is more likely that IC will determine F1's price; and when the misperception is smaller, it is more likely that IR will determine F1's price.
What happens when $p_2 \in \left(\frac{4}{3}v - 1 + \frac{2}{3}\delta, \frac{3}{2}v - 1 + \frac{1}{2}\delta\right)$?

$$p_1(p_2) = \begin{cases} \frac{1}{2}(1+p_2) - \delta & , \quad p_2 \leq \frac{4}{3}v - 1 + \frac{2}{3}\delta \\ 2v - 1 - p_2 & , \ p_2 \in \left(\frac{4}{3}v - 1 + \frac{2}{3}\delta, \frac{3}{2}v - 1 + \frac{1}{2}\delta\right) \\ \frac{1}{2}(v - \delta) & , \quad p_2 > \frac{3}{2}v - 1 + \frac{1}{2}\delta \end{cases}$$

[I assume that $\delta < v$. Otherwise, $\frac{4}{3}v - 1 + \frac{2}{3}\delta \geq \frac{3}{2}v - 1 + \frac{1}{2}\delta$. Then the boundary conditions for both IC and IR are satisfied simultaneously. This means that neither constraint is binding. But it also means that we start bumping into the $p_1 \geq 0$ constraint.]

F2:
When IC is binding, $p_2(p_1) = \frac{1}{2}(1+p_1) + \delta$ maximizes $R_2(p_2; p_1) = p_2\left(1 - \frac{1}{2}\left(1 - (p_1 - p_2)\right) + \delta\right)$; and $p_2(p_1) = \frac{1}{2}(1+p_1) + \delta$ satisfies the boundary condition $p_1 \leq 2v - 1 - p_2$, when

$p_1 \le \frac{4}{3}v - 1 - \frac{2}{3}\delta$. When IR is binding, $p_2(p_1) = \frac{1}{2}(v+\delta)$ maximizes $R_2(p_2; p_1) = p_2(v - p_2 + \delta)$; and $p_2(p_1) = \frac{1}{2}(v+\delta)$ satisfies the boundary condition, when $p_1 > \frac{3}{2}v - 1 - \frac{1}{2}\delta$. A larger misperception makes it more difficult to satisfy the "IC-binding condition" and relaxes the "IR-binding condition." Namely, when the misperception is larger, it is more likely that IR will determine F2's price; and when the misperception is smaller, it is more likely that IC will determine F2's price.

$$p_2(p_1) = \begin{cases} \frac{1}{2}(1+p_1) + \delta & , \quad p_1 \le \frac{4}{3}v - 1 - \frac{2}{3}\delta \\ 2v - 1 - p_1 & , \quad p_2 \in \left(\frac{4}{3}v - 1 - \frac{2}{3}\delta, \frac{3}{2}v - 1 - \frac{1}{2}\delta\right) \\ \frac{1}{2}(v+\delta) & , \quad p_2 > \frac{3}{2}v - 1 - \frac{1}{2}\delta \end{cases}$$

For $v \le 1$, the prices $p_1 = \frac{1}{2}(v-\delta)$ and $p_2 = \frac{1}{2}(v+\delta)$ maximize the relevant revenue functions and satisfy the corresponding boundary constraints.
For $v \ge \frac{3}{2}$, the prices $p_1 = 1 - \frac{2}{3}\delta$ and $p_2 = 1 + \frac{2}{3}\delta$ maximize the relevant revenue functions and satisfy the corresponding boundary constraints. (We assume that $1 - \frac{2}{3}\delta > 0.$)
For $v \in \left(1, \frac{3}{2}\right)$, the boundary constraints are binding. Therefore, we know that $p_1 = 2v - 1 - p_2$ or $p_1 + p_2 = 2v - 1$. In the no misperception case (and in the common misperception case), I used symmetry ($p_1 = p_2$) to derive a unique set of prices $p_1 = p_2 = v - \frac{1}{2}$. There is no symmetry in the relative misperception case. When $v \le 1$, we have $p_1 = \frac{1}{2}(v-\delta)$ and $p_2 = \frac{1}{2}(v+\delta)$; and when $v \ge \frac{3}{2}$, we have $p_1 = 1 - \frac{2}{3}\delta$ and $p_2 = 1 + \frac{2}{3}\delta$. Note that for $v \le 1$, $p_2 = p_1 + \delta$; and for $v \ge \frac{3}{2}$, $p_2 = p_1 + \frac{4}{3}\delta$. If we assume that $p_2 = p_1 + \phi_p\delta$ with $\phi_p \in \left[1, \frac{4}{3}\right]$ and plug this into the constraint $p_1 + p_2 = 2v - 1$, we get: $p_1 = v - \frac{1}{2} - \frac{1}{2}\phi_p\delta$ and $p_2 = v - \frac{1}{2} + \frac{1}{2}\phi_p\delta$. For $v \to 1$, the market is covered, such that consumers with $x \le \frac{1}{2} - \frac{1}{2}\delta$ buy from F1 and consumers with $x > \frac{1}{2} - \frac{1}{2}\delta$ buy from F2; and, for $v \to \frac{3}{2}$, the market is covered, such that consumers with $x \le \frac{1}{2} - \frac{1}{3}\delta$ buy from F1 and consumers with $x > \frac{1}{2} - \frac{1}{3}\delta$ buy from F2. We assume that for $v \in \left(1, \frac{3}{2}\right)$ the market is covered, such that consumers with $x \le \frac{1}{2} - \phi_x\delta$ buy from F1 and consumers with $x > \frac{1}{2} - \phi_x\delta$ buy from F2, with $\phi_x \in \left[\frac{1}{3}, \frac{1}{2}\right]$. Importantly, $\phi_p$ and $\phi_x$ are correlated, such that a low $\phi_p$ corresponds to a high $\phi_x$. We assume $\phi_p = 1 + \frac{2}{3}(v-1)$ and $\phi_x = \frac{1}{2}\left[1 - \frac{2}{3}(v-1)\right]$.
QED

Proof of Proposition 5 (No Price Discrimination; The Effects of Relative Misperception):

Observe that: for $v < 1$, $\frac{dW}{d\delta} = -\frac{1}{2}\delta$ and $\frac{dCS}{d\delta} = -\frac{3}{2}\delta$; for $v \in \left(1, \frac{3}{2}\right)$, $\frac{dW}{d\delta} = -2\phi_x^2\delta$ and $\frac{dCS}{d\delta} = -2\phi_x\left(\phi_x + \phi_p\right)\delta$; and for $v \geq \frac{3}{2}$, $\frac{dW}{d\delta} = -\frac{2}{9}\delta$. $\frac{dCS}{d\delta} = -\frac{10}{9}\delta$.

QED

Proof of Lemma 7 (Relative Misperception; Price Discrimination):

For each consumer, i.e., for each *x*, F1 solves:

(3) $\max_{p_1(x)}\langle p_1(x)\rangle$
s.t.
(3a) IC: $v - \hat{x} - p_1(x) \geq v - (1 - \hat{x}) - p_2(x)$ or $p_1(x) \leq p_2(x) + (1 - 2\hat{x})$
(3b) IR: $v - \hat{x} - p_1(x) \geq 0$ or $p_1(x) \leq v - \hat{x}$
(3c) $p_1(x) > 0$

And, for each consumer, i.e., for each *x*, F2 solves:

(4) $\max_{p_2(x)}\langle p_2(x)\rangle$
s.t.
(4a) IC: $v - \hat{x} - p_1(x) < v - (1 - \hat{x}) - p_2(x)$ or $p_1(x) > p_2(x) + (1 - 2\hat{x})$
(4b) IR: $v - (1 - \hat{x}) - p_2(x) \geq 0$ or $p_2(x) \leq v - (1 - \hat{x})$
(4c) $p_2(x) > 0$

Competition implies that $min\langle p_1, p_2\rangle = 0$.

For $\hat{x} \leq \frac{1}{2}$: If Consumer x buys the product, it will be from F1. F2 sets $p_2 = 0$ and F1 sets $p_1$ depending on which constraint is binding. If IC is binding, then F1 sets $p_1 = 1 - 2\hat{x}$; if IR is binding, then F1 sets $p_1 = v - \hat{x}$. Therefore, $p_1 = min(1 - 2\hat{x}, v - \hat{x})$. And, of course, F1 will not sell if $p_1 < 0$. This means that if $min(1 - 2\hat{x}, v - \hat{x}) = v - \hat{x}$ and $v - \hat{x} < 0$, Consumer x will not be served.

For $\hat{x} > \frac{1}{2}$: If Consumer x buys the product, it will be from F2. F1 sets $p_1 = 0$ and F2 sets $p_2$ depending on which constraint is binding. If IC is binding, then F2 sets $p_2 = -(1 - 2\hat{x})$; if IR is binding, then F2 sets $p_2 = v - (1 - \hat{x})$. Therefore, $p_2 = min\left(-(1 - 2\hat{x}), v - (1 - \hat{x})\right)$. And, of course, F2 will not sell if $p_2 < 0$. This means that if $min\left(-(1 - 2\hat{x}), v - (1 - \hat{x})\right) = v - (1 - \hat{x})$ and $v - (1 - \hat{x}) < 0$, Consumer x will not be served.

Which constraint – IC or IR – is binding plays a central role in the analysis. For $\hat{x} \leq \frac{1}{2}$, IC is binding, when $1 - 2\hat{x} \leq v - \hat{x}$ or $\hat{x} \geq 1 - v$. For $\hat{x} > \frac{1}{2}$, IC is binding, when $-(1 - 2\hat{x}) \leq v -$

$(1-\hat{x})$ or $\hat{x} \leq v$. Note that, for both $\hat{x} \leq \frac{1}{2}$ and $\hat{x} > \frac{1}{2}$, when $v \geq 1$, IC is always binding, and when $v < \frac{1}{2}$, IR is always binding. When $v \in [\frac{1}{2}, 1)$, the IC constraint is binding for $\hat{x} \in [1-v, v]$ and the IR constraint is binding for $\hat{x} < 1-v$ and $\hat{x} > v$.

Case 1: $v \geq 1$

- For $\hat{x} \leq \frac{1}{2}$ or $x \leq \frac{1}{2} - \delta$: F1 sets $p_1 = 1 - 2\hat{x} = 1 - 2(x+\delta)$. Consumer x enjoys a payoff of $v - x - p_1$. And F1 enjoys a payoff of $p_1$.
- For $\hat{x} > \frac{1}{2}$ or or $x > \frac{1}{2} - \delta$: F2 sets $p_2 = -(1-2\hat{x}) = -\big(1-2(x+\delta)\big)$. Consumer x enjoys a payoff of $v - (1-x) - p_2$. And F2 enjoys a payoff of $p_2$.
- Consumer surplus: $\int_0^{\frac{1}{2}-\delta}\big(v - \big(1-2(x+\delta)\big) - x\big)dx + \int_{\frac{1}{2}-\delta}^{1}\Big(v + \big(1-2(x+\delta)\big) - (1-x)\Big)dx.$
- Social welfare: $\int_0^{\frac{1}{2}-\delta}(v-x)dx + \int_{\frac{1}{2}-\delta}^{1}(v-(1-x))dx.$
- $\frac{dW}{d\delta} = -2\delta. \frac{dCS}{d\delta} = -6\delta.$

Case 2: $v \leq \frac{1}{2}$

- For $\hat{x} \leq \frac{1}{2}$ or $x \leq \frac{1}{2} - \delta$: When $\hat{x} \leq v$, F1 sets $p_1 = v - \hat{x}$. Consumer x buys the product, and enjoys a payoff of $v - (v - \hat{x}) - x = \delta$. And when $\hat{x} > v$, Consumer x will not be served.
- For $\hat{x} > \frac{1}{2}$ or $x > \frac{1}{2} - \delta$: When $\hat{x} \geq 1 - v$, F2 sets $p_2 = v - (1-\hat{x})$. Consumer x buys the product, and enjoys a payoff of $v - p_2 - (1-x) = -\delta$. And when $\hat{x} < 1 - v$, Consumer x will not be served.
- Combining the two sets of results, we have:
  - For $\hat{x} \leq v$ or $x \leq v - \delta$, F1 sets $p_1 = v - \hat{x}$ and F2 sets $p_2 = 0$, Consumer x buys the product from F1, and enjoys a payoff of $\delta$.
  - For $\hat{x} \in (v, 1-v)$ or $x \in (v-\delta, 1-v-\delta)$, Consumer x will not be served.
  - For $\hat{x} \geq 1 - v$ or $x \geq 1 - v - \delta$, F2 sets $p_2 = v - (1-\hat{x})$ and F1 sets $p_1 = 0$, Consumer x buys the product from F2, and enjoys a payoff of $-\delta$.
- Consumer surplus is: $\int_0^{v-\delta}\delta dx + \int_{1-v-\delta}^{1}(-\delta)dx$
- Social welfare is: $\int_0^{v-\delta}(v-x)dx + \int_{1-v-\delta}^{1}(v-(1-x))dx$
- $\frac{dW}{d\delta} = -2\delta. \frac{dCS}{d\delta} = -4\delta$

Case 3: $v \in \left(\frac{1}{2}, 1\right)$

- For $\hat{x} < 1 - v$ or $x < 1 - v - \delta$: IR is binding, F1 sets $p_1 = v - \hat{x}$, Consumer x buys the product, and enjoys a payoff of $v - (v - \hat{x}) - x = \delta$.
- For $\hat{x} \in [1-v, v]$ or $x \in [1-v-\delta, v-\delta]$: IC is binding. For $x \in \left[1-v-\delta, \frac{1}{2}-\delta\right]$ F1 sets $p_1 = 1 - 2\hat{x} = 1 - 2(x+\delta)$, F2 sets $p_2 = 0$, consumer x buys from F1 and enjoys a payoff of $v - x - (1-2\hat{x}) = v - 1 + x + 2\delta \geq 0$. For $x \in (\frac{1}{2} - \delta, v - \delta]$, F2 sets $p_2 =$

$-(1-2\hat{x}) = -\big(1-2(x+\delta)\big)$, F1 sets $p_1 = 0$, consumer x buys from F2 and enjoys a payoff of $v-(1-x)+(1-2\hat{x}) = v-x-2\delta$.

- For $\hat{x} > v$ or $x > v-\delta$: IR is binding, F2 sets $p_2 = v-(1-\hat{x})$, Consumer x buys the product, and enjoys a payoff of $v-p_2-(1-x) = -\delta$.
- Consumer surplus: $\int_0^{1-v-\delta} \delta dx + \int_{1-v-\delta}^{\frac{1}{2}-\delta}(v-1+x+2\delta)dx + \int_{\frac{1}{2}-\delta}^{v-\delta}(v-x-2\delta)dx + \int_{v-\delta}^{1}(-\delta)dx$.
- Social welfare is: $\int_0^{\frac{1}{2}-\delta}(v-x)dx + \int_{\frac{1}{2}-\delta}^{1}(v-(1-x))dx$
- $\frac{dW}{d\delta} = -2\delta$. $\frac{dCS}{d\delta} = -3\delta$.

QED

Proof of Proposition 6 (Price Discrimination; The Effects of Relative Misperception):

Observe that: for $v \le \frac{1}{2}$, $\frac{dW}{d\delta} = -2\delta$ and $\frac{dCS}{d\delta} = -4\delta$; for $v \in \left(\frac{1}{2}, 1\right)$, $\frac{dW}{d\delta} = -2\delta$ and $\frac{dCS}{d\delta} = -4\delta$; and for $v \ge 1$, $\frac{dW}{d\delta} = -2\delta$. $\frac{dCS}{d\delta} = -6\delta$.

QED