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CAN PERVERSELY
DISCOURAGE DISCOUNTING

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HOW LOYALTY DISCOUNTS CAN PERVERSELY DISCOURAGE DISCOUNTING

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ABSTRACT

Loyalty discounts are agreements to sell at a lower price to buyers who buy all or most of their purchases from the seller. This article proves that (assuming no efficiency justifications) loyalty discounts can create anticompetitive effects, not only because they can impair rival efficiency, but because loyalty discounts perversely discourage discounting even when they have no effect on rival efficiency. The essential reason, missed in prior work, is that firms using loyalty discounts have less incentive to compete for free buyers, because any price reduction to win sales to free buyers will, given the loyalty discount, also lower prices to loyal buyers. This in turn reduces the incentive of rivals to cut prices, because there will exist an above-cost price that rivals can charge to free buyers without being undercut by the firm using loyalty discounts. These anticompetitive effects occur even if buyers can breach or terminate commitments, and even if the loyalty conditions require no buyer commitments and less than 100% loyalty. These anticompetitive effects also differ from those created by most-favored-nation or price matching clauses, neither of which require the seller to commit to maintain a price difference between loyal and disloyal buyers. Further, I prove that these anticompetitive effects are exacerbated if multiple sellers use loyalty discounts. None of the results depend on switching costs, market differentiation, imperfect competition, or whether the loyalty discount bundles contestable and incontestable demand. Contrary to commonly held views, I prove these anticompetitive effects exist even (1) when all prices are above seller or rival costs, (2) buyers voluntarily agree to the conditions, and (3) discount and foreclosure levels are low, although such low levels do lower the likelihood buyers would agree to anticompetitive loyalty discounts. I also derive formulas for calculating the inflated price levels in each situation. However, because loyalty discounts can have efficiencies, rule of reason analysis remains appropriate.

JEL Codes: C72, K21, L12, L40, L41, L42.

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HOW LOYALTY DISCOUNTS CAN PERVERSELY DISCOURAGE DISCOUNTING

Exclusionary agreements condition favorable terms on buyers restricting their purchases from rivals. One wing of the Chicago School has long asserted that such exclusionary agreements could never be anticompetitive because, if they were, the harm to buyers would exceed the benefits to buyers from agreeing, and thus buyers would not agree.¹ An important set of articles has proven that this Chicago School assertion is false.

One pair of seminal articles showed that, if buyers honor their exclusionary commitments, then a seller who makes discriminatory or sequential offers can get buyers to agree to anticompetitive exclusionary agreements that deprive rivals of economies of scale, even when buyers coordinate and Bertrand competition is assumed. (Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000), henceforth RRW-SW.) The essential reason is that each buyer's decision to agree does not consider the externality imposed on other buyers by the exclusionary agreement's contribution to the marketwide harm of excluding a rival that would lower prices for all buyers. RRW-SW also show that, if buyers cannot coordinate with each other, then even a seller that makes nondiscriminatory simultaneous offers can get buyers to agree to exclusionary commitments that harm them.² Buyer coordination is generally unlikely because antitrust law makes it illegal for rival buyers to agree on the terms they will accept, or even to exchange information about the terms being offered by specific sellers, with the violation subject to treble damages and possible criminal punishment.³ In any event, because discriminatory or sequential

¹ Bork, pp. 304-09 (1978); Sullivan & Harrison, p.250 (1998). Richard Posner has often been miscited for this proposition in the economic literature, but as he pointed out, he does not hold this view. See Posner, p.194 & n.2 (2001).

² Although nondiscriminatory simultaneous offers can theoretically produce a failure to agree as well, an excellent recent experimental study showed that noncoordinating buyers facing nondiscriminatory simultaneous offers agreed to anticompetitive exclusionary agreements 92% of the time. Landeo and Spier (2007). This was true even though the experiments used only two buyers, which should make the odds of rejection higher than typical because the greater the number of buyers, the less likely it is that any individual buyer's agreement will make a decisive difference to whether the marketwide foreclosure results, resulting in less buyer incentive to resist.

³ *Mandeville Island Farms v. American Crystal Sugar*, 334 U.S. 219 (1948) (illegal for rival buyers to agree on terms they will pay); *United States v. Container Corp.*, 393 U.S. 333 (1969) (illegal for rivals to exchange information on terms each is offering); *United States v. United States Gypsum*, 438 U.S. 422 (1978) (rival information exchange on terms each is offering is subject to possible criminal penalties).

offers are generally possible, it would seem a seller can usually overcome buyer coordination even if it were allowed.

However, another important recent article argues that, if buyers can breach their exclusionary commitments upon a payment of expectation damages equal to the difference between the monopoly price and the rival price, then (if we assume Bertrand competition) such a commitment cannot prevent a rival from inducing consumers to breach their exclusionary commitments. (Simpson & Wickelgren, 2007, henceforth S&W). They reason that breaching buyers will save an amount equal to expectation damages by shifting the purchases they would have made without breach to a rival that offers to sell at cost,⁴ and in addition buyers would save the deadweight loss because they would buy a greater quantity when buying at cost. However, this article also shows that, if the buyers are not consumers, but rather are intermediate buyers who compete downstream in a competitive market, then sellers can get them to accept an anticompetitive exclusionary agreement in exchange for a small sidepayment, because the intermediate buyers externalize the anticompetitive harm onto downstream consumers.⁵ They further prove that this latter point is true even if there are no relevant economies of scale.

Although these models are highly illuminating, in all of them the seller offers a form of exclusionary agreement one does not often observe in the real world. Namely, these models assume the seller offers a payment in period 1 for the buyer agreeing to buy exclusively from the seller in period 2 at whatever price the seller chooses to set in period 2. This price will be set at the monopoly level, which if a rival enters will be higher not only than the rival price, but also higher than the price the seller charges to nonexclusive buyers. That is, these models assume that exclusive dealing will lead to loyalty penalties, with sellers charging exclusive buyers a higher price than they charge to nonexclusive buyers. One does not often observe sellers in real markets punishing their most loyal buyers with higher prices.

What is commonly observed, and a very hot topic of antitrust debate recently, are loyalty discounts. With a loyalty discount, a seller agrees to charge loyal buyers a price that is lower than the price (often called the list price) that the seller charges to

⁴ The Bertrand assumption is necessary to justify this premise that a rival facing substantial foreclosure would be able to instantly produce at scale sufficient to price at cost.

⁵ This point had previously been made, without formal proof, in Elhauge (2003).

free purchasers. Sometimes, the agreements involve a return buyer commitment of exclusivity that cannot be violated without committing contractual breach. Other times, such contracts are terminable by the buyer, and sometimes the buyer makes no commitment at all, but simply buys under a contract that sets one price if it complies with the loyalty condition and a higher price if it does not. The loyalty condition may require the buyer to buy 100% from the seller to get the discount, or instead some lower threshold, such as 80% or 90%.

True loyalty discounts of the above sort should be distinguished from other arrangements that, while often called “loyalty discounts,” lack the crucial feature that the seller has committed to charge loyal buyers some discount from the price charged disloyal buyers. Prior papers addressing what they call “loyalty discounts” actually involve what are simply prices conditioned on exclusivity, without any restriction on the seller later charging lower prices to disloyal buyers.⁶ While these models of price-conditioned exclusivity have also found anticompetitive effects, they differ in category from those found here, and the models finding such effects generally invoke assumptions that competition is imperfect (e.g., demand is differentiated or switching costs exist) or that prices or incremental prices are below cost.⁷

Loyalty discounts should also be distinguished from the best price clauses considered in prior articles. Loyalty discounts differ from price matching clauses because loyalty discounts include no seller commitment to match a lower rival price.⁸ To the contrary, we shall see that the anticompetitive mechanism of loyalty discounts is precisely the

⁶ See Ordovery & Shaffer (2007); Greenlee & Reitman (2006). The latter also assumes that the discount is from a spot price that is offered to buyers ineligible for loyalty discount and constrains the price that can be charged disloyal buyers, and assumes away any possibility that rival efficiency might be impaired by assuming no buyer commitments and constant costs.

⁷ Another literature deals with *bundled* loyalty discounts. See, e.g., Greenlee, Reitman & Sibley (2008). The element of bundling a discount across multiple products makes the economics of these arrangements more akin to tying agreements. See Elhauge (2008), pp. 350-357, 408-416. This paper just addresses loyalty discounts on a single product. Different issues are also raised by *volume-based* discounts, where the seller offers a lower price to any buyers who purchase a certain quantity. See, e.g., Kolay, Shaffer & Ordovery (2004). This paper addresses only loyalty discounts, which offer a lower price to buyers who buy a minimum *percentage* of their purchases from the seller than to buyers who do not. The two raise different issues, though volume-based discounts can be quite similar to loyalty discounts if the volume threshold is varied for each buyer in proportion to their requirements in a way that effectively sets a share-based discount.

⁸ See, e.g., Edlin (1997); Schnitzer (1994).

opposite: discouraging firms from matching rivals. Loyalty discounts also differ from most-favored-nation clauses, where a seller commits that agreeing buyers will get at any lower prices it offers to other buyers. Unlike most-favored-nations clauses, loyalty discounts (1) include a seller commitment to maintain an affirmative price difference between agreeing and nonagreeing buyers; and (2) often involve buyer commitment to buy all or a high share of purchases from the rival. When neither factor is present, then loyalty discounts would be the same as a most-favored-nation clause.⁹

This article analyzes true loyalty discounts that lack any efficiency justification, and proves that they can raise prices above competitive levels for both loyal and free buyers, even if we assume Bertrand competition without economies of scale or without any impairment of rival efficiency. The essential reason, missed in prior work, is that firms using loyalty discounts have less incentive to compete for free buyers, because any price reduction to win sales to free buyers will, given the loyalty discount, also lower prices to loyal buyers. Loyalty discounts thus make it more costly to compete for free buyers. This in turn reduces the incentive of rivals to cut prices, because there will exist an above-cost price that rivals can charge to free buyers without being undercut by the firm using loyalty discounts. This is true even though I assume Bertrand competition in a homogeneous product for buyers who are ultimate consumers and thus cannot pass along any portion of the price increase. These adverse price effects are worsened if the exclusionary agreements do exclude rivals or impair rival efficiency.

I prove that these anticompetitive effects exist even if we assume buyers would breach loyalty commitments when the gains exceed expectation damages. Further, I go beyond that to prove that anticompetitive effects persist even if buyers make no commitments, and thus are free to violate the loyalty condition without paying any damages whenever they can get a better deal from the rival. Indeed, in such a case, the fact that accepting buyers never have to pay more than the rival would charge makes it even easier to show that buyers will agree to anticompetitive loyalty discounts, and thus prove that the equilibrium will produce anticompetitive results.

⁹ In this case (no commitments and $d=0$), my model finds no anticompetitive effects because it assumes single period Bertrand competition. Articles on most favored nations clauses have found anticompetitive effects because they assumed oligopolistic coordination, *see* Cooper (1986), or because they assumed a monopolist selling a durable good that might use such clauses to restrain competition by itself later in time, Butz (1990); Marx & Shaffer (2004).

I also prove these anticompetitive effects exist even when less than 100% loyalty is required to trigger the loyalty discounts.

I extend the analysis to cases where multiple firms offer loyalty discounts with commitments, and prove that this exacerbates the anticompetitive effects. The resulting cumulative foreclosure leaves fewer uncommitted buyers available, and thus creates even less incentive for either firm to undercut uncommitted prices to get them, given that doing so reduce the committed prices of each. Cumulative foreclosure also makes it even more likely that other rivals will be unable to achieve economies of scale. Finally, when both firms offer loyalty discounts, the anticompetitive equilibria are even more likely and less vulnerable to defection.

Following the convention in this area, I analyze only “naked exclusion”, that is, loyalty discounts that are naked of any efficiency justification. The reason for this convention is because the focus has been on disproving the Chicago School assertion that exclusionary agreements, such as loyalty discounts, can never have any anticompetitive effect (unless perhaps they are below cost). However, in reality loyalty discounts can have efficiencies that make rule of reason analysis (rather than rules of per se legality or illegality) appropriate when assessing them.¹⁰

I. The Model

Assume the market has N buyers, each of which have the same downward sloping demand function, $q = (1/N)(A - P)$, where q is the quantity demanded by each buyer, A is a constant, and P is the price the buyer pays. If all buyers pay the same price, the total quantity demanded $Q = qN = A - P$.¹¹ The market has two potential producers, the incumbent monopolist and a potential rival. If the rival enters, the two produce identical products and have the same average cost function. I analyze the situation under two alternative assumptions about the cost function. One is that average and marginal costs are a constant C at all output levels. The other assumes, like the prior literature, that average costs decline with output until they reach the minimum efficient scale and then are constant thereafter. See RRW-SW. To give this cost

¹⁰ See Elhauge (2008), at 414-416.

¹¹ The analysis extends to any linear demand curve $Q = A - BP$ because one could convert that into an equation that takes the form $Q = A - P$ by using a measure of units that makes $B = 1$.

assumption a specific form, I assume that average cost depends on the quantity each firm produces, Q_i , and a recurring fixed cost F , with $C(Q_i) = F/Q_i$ for $Q_i < Q^*$ and $C(Q_i) = F/Q^* = C$ for all $Q_i \geq Q^*$. The minimum efficient scale is thus Q^* , and I assume that the market is not a natural monopoly by assuming $Q^* < (1/2)(A - C)$. Given these assumptions, the competitive cost and price = $C = F/Q^*$, the competitive output = $A - C$, the monopoly output of $Q_m = (A - C)/2$, and the monopoly price of $P_m = (A + C)/2$.

In period 1, the incumbent offers a loyalty discount agreement to buyers. I will begin with the assumption that accepting the loyalty discount commits buyers to buy 100% from the incumbent in period 2 and that buyers always comply with their commitments. Later, I extend the analysis to cases where (1) buyers commit but breach when that is profitable, (2) buyers make no contractual commitment, (3) less than 100% loyalty is required, and (4) both the incumbent and rival offer loyalty discounts with commitments. In all these cases, I assume a loyalty discount agreement requires the incumbent to charge $P_f - d$ to loyal buyers, where P_f is the price the incumbent charges to buyers free of loyalty conditions and d is the loyalty discount. Thus, the loyalty discount commits the incumbent to charge loyal buyers less than it charges free buyers. I assume buyer coordination is impossible, which is realistic given the legal penalties on it and the large number of buyers in many markets. The loyalty discount agreement is signed by S buyers. I will use θ to denote S/N , the share of buyers that agreed to loyalty discounts.

Between periods 1 and 2, the rival decides whether to enter the market and make a product. I initially assume there is only one rival. This is often the case in reality if patents or FDA approval restrict a market to two firms with attractive products, or when only two firms could achieve the minimum efficient scale given the size of the market. I also assume the rival does not offer loyalty discounts itself. Later I extend the analysis to multiple rivals where multiple firms can adopt loyalty discounts.

In period 2, I adopt the assumption, like prior papers, that if the rival enters, the incumbent and rival engage in Bertrand competition. The Bertrand model is extreme because it unrealistically assumes that output is infinitely and instantly expandable, that there is no product differentiation or switching costs, and that competition is a single period game so that firms need not fear reactions in subsequent periods, all of which results in the “strained” conclusion that (without loyalty discounts) a duopoly

will produce the same prices as a perfectly competitive market.¹² Nonetheless, I here adopt Bertrand assumptions for two reasons. First, it biases the case against finding anticompetitive effects.¹³ Second, it makes it easier to compare the conclusions here with those reached in prior papers about naked exclusion because they used Bertrand models.

My initial analysis in each section assumes the rival picks price first. The intuition is that, because buyers are buying from the incumbent already, the rival must first offer buyers a better price, and that the incumbent will have a chance to respond before the buyer switches. This assumption also makes the conclusions more conservative. I also consider the possibilities that the incumbent picks price first or that they pick price simultaneously, and show that under those scenarios rival prices will be higher and closer to incumbent prices, making it even more likely that buyers would agree to loyalty commitments.

Given the assumption of Bertrand competition, if $S = 0$ (i.e., there are no loyalty discounts), then both the rival and incumbent will set prices equal to C . They will do so if either picks price first, because any price higher than C would be undercut by the other. They will also do so if they pick simultaneously under the standard Bertrand model. This is thus the but-for baseline without any loyalty discounts.

II. If Buyers Honor 100% Loyalty Commitments

I begin with the case where the loyalty discount agreement requires buyers to commit to make 100% of their purchases from the incumbent. Like the prior papers, I first analyze the period 2 outcomes if the rival does enter, then consider the effects of those possible outcomes on the likelihood of rival entry and on buyer willingness to agree in stage 1.

a. No Loss of Rival Efficiency. Take first the case where either incremental costs C are constant or the uncommitted market is large enough to allow the rival to operate at minimum efficient scale if it can win all uncommitted buyers, that is $(N-S)q(C) \geq Q^*$.

¹² Tirole (1988), at 211.

¹³ *Id.* at 212. The reasons will be explained in the Implications section below.

Rival Picks Price First. For any rival price, P_r , that the rival chooses, the incumbent has two options. First, it can deprive its rival of all sales by lowering its uncommitted price to some infinitesimal amount less than the rival price, $P_r - \varepsilon$, thus earning $P_r - \varepsilon$ to $N-S$ buyers and $P_r - \varepsilon - d$ to S buyers. Second, it can concede all uncommitted buyers, but still make all sales to the committed buyers, in which case it will maximize profits by charging $P_r - d = P_m$ to these S buyers.

The rival earns zero profits from the first option or from pricing at $P_r = C$. Thus, the rival will want to set $P_r > C$ but sufficiently low that the incumbent finds it more profitable to sell to the committed buyers at the monopoly price, rather than try to undercut the rival price for uncommitted buyers. Ignoring the ε , since it is infinitesimally small, this condition is met when:

$$(P_m - C)(S/N)(A - P_m) > (P_r - C)((N-S)/N)(A - P_r) + (P_r - d - C)(S/N)(A - P_r + d).$$

Given that $A = 2P_m - C$, and $\theta = S/N$, this can be rearranged as:

$$P_r^2 - 2(P_m + \theta d)P_r + 2CP_m - C^2 + 2\theta dP_m + \theta d^2 + \theta \cdot (P_m - C)^2 > 0.$$

The Appendix proves that this inequality will be satisfied as long as the rival charges no more than

$$P_r^* = P_m + \theta d - [(1-\theta)((P_m - C)^2 - \theta d^2)]^{1/2},$$

and that P_r^* is always above cost and more profitable for the rival than any alternative rival price as long as $P_r^* < P_m$. If $P_r^* \geq P_m$, then the rival will find it more profitable to charge P_m , which the Appendix proves will be true if $d \geq (P_m - C)/[\theta/(1-\theta)]^{1/2}$. As long as the rival charges P_r^* or less, the most profitable price for the incumbent is to charge the committed buyers P_m , thus losing all uncommitted buyers by offering them a price $(P_m + d)$ that the rival has undercut.

This is a subgame perfect Nash equilibrium. The rival will not charge $P_r > P_r^*$ because that would cause the incumbent to charge $P_f < P_r$ to uncommitted buyers, which would reduce rival sales and profits to zero. If $P_r^* > P_m$, the rival will charge P_m and would not charge any less because that would result in lower profits. If $P_r^* \leq P_m$, the rival will charge P_r^* because any lower price would bring the rival further below the profit-maximizing level and thus earn it less money. Given that the rival is charging no more than P_r^* , the incumbent will not have any incentives to charge committed buyers less than P_m because the incumbent cannot undercut the rival price to uncommitted buyers without resulting in lower overall profits.

Lemma 1a. Suppose there are no economies of scale or the rival produces enough to reach its minimum efficient scale, and the rival and incumbent engage

in Bertrand competition, but with the rival picking price first. If the loyalty discounts have commitments with which buyers comply, then the incumbent will make all sales to committed buyers at P_m . The rival makes all sales to uncommitted buyers at:

(i) $P_r^* = P_m + \theta d - [(1-\theta)((P_m-C)^2 - \theta d^2)]^{1/2}$ if $P_r^* < P_m$

(ii) P_m if $P_m \leq P_r^*$.

$P_m \leq P_r^*$ if $d \geq (P_m-C)/[\theta/(1-\theta)]^{1/2}$. All these prices will exceed the but-for competitive level, C , which would have prevailed without the loyalty discounts on the same market assumptions.

Incumbent Picks Price First. Now suppose the incumbent picks price first. It knows that no matter what above-cost price it picks for uncommitted buyers, the rival can undercut that price, and that if it prices at cost to uncommitted buyers, it is committing to price at a loss to committed buyers. Thus, as long as there is at least one committed buyer, the incumbent will charge P_m to committed buyers, making its offer to uncommitted buyers $P_m + d$. The rival will charge P_m to uncommitted buyers and will win all sales to them.

Lemma 1b. With the same assumptions as Lemma 1a, but instead assuming the incumbent picks price first, then as long as there is at least one committed buyers, the incumbent will make all sales to committed buyers at P_m and the rival will make all sales to uncommitted buyers at P_m .

In short, when either the rival or incumbent pick prices first, loyalty discounts cause an effective market division, where both the incumbent and the rival price above competitive levels without any agreement or coordination. When the incumbent picks price first, both always price at monopoly levels. When the rival picks price first, both price at monopoly levels when the foreclosure share and discount are sufficiently large. Otherwise, if the rival picks price first, the incumbent prices at monopoly levels, while the rival prices at a submonopoly level that is still well above the competitive level.

Simultaneous Pricing. Now assume the rival and incumbent pick prices simultaneously. If $P_r^* \geq P_m$, then the rival will sell to uncommitted buyers at P_m and the incumbent will sell to committed buyers at P_m . If $P_r^* < P_m$, then the Appendix proves the Nash equilibrium will be a mixed strategy equilibrium where both offer uncommitted buyers a distribution of prices between P_r^* and P_m , with the incumbent pricing d below those prices to committed buyers. The floor on prices remains P_r^* ,

and thus all prices will be greater than C .

Lemma 1c. With the same assumptions as Lemma 1a, but instead assuming simultaneous pricing, then:

(i) If $P_r^* \geq P_m$, the rival will sell to all uncommitted buyers at P_m and the incumbent will sell to all committed buyers at P_m .

(ii) If $P_r^* < P_m$, a mixed strategy equilibrium will result where both offer uncommitted buyers a distribution of prices that are between P_r^* and P_m , with the incumbent selling to all committed buyers at d less than it offers uncommitted buyers.

All prices will exceed the but-for competitive level.

b. Rival Efficiency Impaired. Now suppose there are economies of scale and $(N-S)q(C) < Q^*$. That is, the uncommitted buyers do not buy enough to allow the rival to achieve its minimum efficient scale, even if it wins all the uncommitted buyers and prices at cost.

Rival Picks Price First. The rival cannot charge any more than P_r^* without the incumbent undercutting its price to uncommitted buyers, resulting in zero profits to the rival. But it also cannot charge any less than $C_r = F/Q_r = F/[(1-\theta)(A-C_r)]$, which means

$$C_r^2 - (2P_m - C)C_r + F/(1-\theta) = 0,$$

the lowest quadratic solution to which is

$$C_r = P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta)]^{1/2}.$$

Thus, the rival will not produce if

$$P_r^* < C_r = P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta)]^{1/2}$$

If the rival does not produce, then the incumbent will maximize its total profits from committed and uncommitted buyers by maximizing the following:

$$(P_f - d - C)(S/N)(A - P_f + d) + (P_f - C)((N-S)/N)(A - P_f).$$

Taking the derivative with respect to P_f , this is maximized when

$$(A - P_f + d)\theta - (P_f - d - C)\theta + (A - P_f)(1-\theta) - (P_f - C)(1-\theta) = 0,$$

which boils down to $P_f = P_m + \theta d$. The price the incumbent charges committed buyers will then be $P_f - d = P_m - (1-\theta)d$.

If $P_r^* > C_r$, then the rival charges up to P_r^* and the incumbent charges P_m . If the profit-maximizing price the rival can charge uncommitted buyers is less than P_r^* , then the rival will price to maximize $(P_r - F/Q_r)(Q_r)$, which is the same as $-(1-\theta)P_r^2 + (1-$

$\theta)AP_r - F$. Taking the first and second derivative shows that profits are maximized at $P_r = A/2 = P_m - C/2$. Thus, the rival will charge the lesser of P_r^* or $P_m - C/2$.¹⁴ This proves Lemma 2a.

Lemma 2a. Suppose buyers comply with loyalty commitments, these commitments foreclose enough of the market to prevent the rival from reaching its minimum efficient scale, and the rival and the incumbent would, if the rival enters, engage in Bertrand competition in period 2 with the rival picking price first. Then

(i) if $P_r^* < C_r$, the rival will not produce, and the incumbent will sell to uncommitted buyers at $P_m + \theta d$ and to committed buyers at $P_m - (1-\theta)d$, for an average price of P_m to all buyers.

(ii) if $P_r^* \geq C_r$, the incumbent will sell to committed buyers at P_m , and the rival will sell to uncommitted buyers at the smaller of $P_m - C/2$ or P_r^* .

All these prices will exceed the but-for competitive level C that would have prevailed without the loyalty discounts.

In short, if the foreclosure is significant enough, the rival cannot profitably produce on the market, creating a monopoly that would not have existed in the but-for world. Even if the foreclosure is lower than that, it will result in the incumbent pricing at monopoly levels and the rival pricing at levels above the but-for competitive level, as well as creating productive inefficiency because the rival will be producing at higher costs than it would have incurred in the but-for world.

Incumbent Picks Price First. Now suppose the incumbent picks price first. It knows that if it picks any price for uncommitted buyers that is higher than $C_r = P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta)]^{1/2}$, then the rival will undercut it and win all uncommitted buyers. Thus, if it prices higher than C_r , it earns higher profits by charging P_m to committed buyers and forgoing all uncommitted buyers, as long as there is at least one committed buyers. The incumbent thus has to decide whether its profits are greater by pricing at $C_r - \varepsilon$ to the N - S uncommitted buyers, and $C_r - \varepsilon - d$ to the S committed buyers, or by pricing at P_m to the S committed buyers.

Ignoring the ε , since it is infinitesimally small, charging a monopoly price to the committed buyers will be more profitable if:

¹⁴ P_r^* will be the lower figure if $(1-\theta)(P_m - C)^2 > \theta d^2 + \theta dC + C^2/4$.

$$(P_m - C)(S/N)(A - P_m) > (C_r - C)((N - S)/N)(A - C_r) + (C_r - d - C)(S/N)(A - C_r + d).$$

This is true if

$$C_r < P_m + \theta d - [(1 - \theta)((P_m - C)^2 - \theta d^2)]^{1/2} = P_r^*.$$

If the incumbent charges a monopoly price to committed buyers, then its price to uncommitted buyers will be $P_m + d$, and thus the rival can win all uncommitted buyers by pricing at the monopoly level. When the above inequality is not satisfied, then the rival will anticipate that if it enters, the incumbent will price low enough to take all sales to uncommitted buyers. Thus, the rival will not enter the market at all, and the incumbent will maximize profits by selling to uncommitted buyers at $P_m + \theta d$ and to committed buyers at $P_m - (1 - \theta)d$, for an average price of P_m to all buyers. This proves Lemma 2b.

Lemma 2b. Assume the same conditions as in Lemma 2a, but that the incumbent picks price first. Then:

(i) if $P_r^* \geq C_r$, and there is at least one committed buyer, the incumbent will sell to all committed buyers at P_m and the rival will sell to all uncommitted buyers at P_m .

(ii) If $P_r^* < C_r$, the rival will not enter, and the incumbent will sell to uncommitted buyers at $P_m + \theta d$ and to committed buyers at $P_m - (1 - \theta)d$, for an average price of P_m to all buyers.

All these prices will exceed the but-for competitive level C that would have prevailed without the loyalty discounts.

Simultaneous Pricing. Now suppose the rival and incumbent pick prices simultaneously. Then, if $P_r^* < C_r$, the rival will not enter because it cannot make profitable sales to uncommitted buyers at even the lowest price that the incumbent might charge, and the incumbent will sell to uncommitted buyers at $P_m + \theta d$ and to committed buyers at $P_m - (1 - \theta)d$. If $P_r^* \geq C_r$, then the rival will enter, and the Appendix proves that a mixed strategy equilibrium will result, where the rival charges uncommitted buyers a distribution of prices between P_r^* and $P_m - C/2$, and the incumbent either competes for uncommitted buyers by offering them a distribution of prices between P_r^* and $P_m - C/2$ (thus charging d less than that price to committed buyers), or foregoes sales to uncommitted buyers and just charges P_m to committed buyers (with uncommitted buyers offered a price of $P_m + d$ that none accept).

Lemma 2c. Assume the same conditions as in Lemma 2a, but that the rival and incumbent price simultaneously. Then:

(i) if $P_r^* < C_r$, the rival will not enter, and the incumbent will sell to

uncommitted buyers at $P_m + \theta d$ and to committed buyers at $P_m - (1-\theta)d$.

(ii) if $P_r^* \geq C_r$, a mixed strategy equilibrium will result, where the rival charges uncommitted buyers a distribution of prices between P_r^* and $P_m - C/2$, and the incumbent offers uncommitted buyers a distribution of prices that is either between P_r^* and $P_m - C/2$ or equals $P_m + d$.

All these prices will be above the but-for price of C .

c. Will Buyers Accept Simultaneous Nondiscriminatory Offers? Assume the incumbent makes a simultaneous nondiscriminatory offer to charge $P_f - d$ to any buyer who will commit to buy exclusively from the incumbent in period 2. Assume buyers have uniform expectations about what other buyers will do. Then two equilibria are possible. Either all buyers will agree because they expect other buyers to agree, or no buyer will agree because they do not expect other buyers to agree.

Suppose each buyer expects all other buyers to agree. Then, if there are economies of scale, each buyer will expect that the rival will not enter, and thus each buyer would (as long as $d > 0$) expect to pay less in period 2 if it agrees to the loyalty commitment in period 1. See Lemmas 2a(i), 2b(ii), 2c(i). Thus, each buyer would agree. If there are no economies of scale, then each buyer would, if it rejects the offer, expect the rival to enter and charge uncommitted buyers P_m in period 2 if all other buyers agree and $d > 0$. (The rival would clearly charge P_m in period 2 if the incumbent picks price first. See Lemmas 1b, 2b(i). The rival would also charge P_m in period 2 if the rival picks price first or both pick simultaneously, as long as $d \geq (P_m - C)/[\theta/(1-\theta)]^{1/2}$. See Lemmas 1a(ii), 1c(i). If all other buyers are expected to agree, θ will approach 1, and the above inequality will be true for all $d > 0$.) Thus, each buyer would conclude that accepting will not make it any worse off in period 2, and it would thus agree to an anticompetitive loyalty commitment for any trivial sidepayment in period 1. As long as other buyers adhere to a strategy of accepting, no individual buyer has any incentive to deviate from that strategy, making this a subgame perfect Nash equilibrium.

Now suppose each buyer expects all other buyers to reject the offered loyalty commitment. Then each buyer knows that if it rejects the offer, there will be no loyalty commitments, and it will pay C , which is less than it would pay the incumbent if it accepts the offer. See Lemma 1a, 1b, 1c. This is also a subgame perfect Nash equilibrium because no individual buyer has incentives to deviate from a strategy of rejecting the offer if the other buyers adhere to it.

Each buyer might also expect that other buyers will agree to the loyalty commitment with some probability θ between 0 and 1, but such expectations are unstable. This is easiest to see when the incumbent picks price in period 2, because then if each buyer expects any $\theta > 0$, it will expect the rival to charge P_m in period 2, making it in the interests of all buyers to agree in period 1 for any trivial sidepayment. *See* 1b, 2b. Thus, any buyer with this expectation would conclude that all buyers will agree, which would change expected θ to 1. This analysis also suggests that while rejection may be a Nash equilibrium when the incumbent picks price first, it is not a trembling hand equilibrium because if buyers anticipate that any single other buyer will accept, then it will be in the interest of all buyers to accept. Acceptance, on the other hand, is a trembling hand equilibrium because it remains the profitable strategy even if one or a few buyers rejects the offer.

If the rival picks price first, or the two pick prices simultaneously, then any θ between 0 and 1 is likewise unstable. For example, suppose that expected θ were high enough that (with economies of scale) the rival would not enter because $P_r^* < Cr$. Then each buyer would expect to pay less in period 2 if it agrees to the loyalty commitment. *See* Lemmas 2a(i), 2c(i). Thus, any buyer with this expectation would conclude that all buyers will agree, which would change expected θ to 1. Likewise, if expected θ were high enough that $d \geq (P_m - C)/[\theta/(1-\theta)]^{1/2}$, so that $P_r^* \geq P_m$, then (even without any economies scale or impairment of rival efficiency) each buyer will conclude that the rival would charge the monopoly price in period 2. *See* Lemmas 1a(ii), 1c(i). This makes it profitable (for it and all other buyers) to agree for any trivial sidepayment in period 1, so that each buyer will conclude that all buyers will agree, which will change expected θ to 1. Setting a high enough loyalty discount level is costless to the incumbent because it induces all buyers to accept, and thus means the incumbent charges all buyers P_m in period 2.¹⁵

On the other hand, if expected θ were low enough that $P_r^* \geq Cr$ and $d < (P_m - C)/[\theta^*/(1-\theta^*)]^{1/2}$, then each buyer would conclude that the rival will enter in period 2 and charge potentially lower prices than the incumbent. If the rival picks price first in period 2, then each buyer would definitely expect the rival to charge prices that are lower than the prices the incumbent will charge committed buyers. *See* Lemmas 1a(i), 2a(ii). This will make it unprofitable (for it and all other buyers) to agree in period 1, and

¹⁵ The monopoly profit per unit need not exceed the loyalty discount because the latter merely reflects the difference between the prices offered loyal and disloyal buyers.

thus each buyer will conclude that all buyers will reject an offer with trivial sidepayments, changing the expected θ to 0. If the rival and incumbent pick prices simultaneously in period 2, then each buyer would expect the rival and incumbent to pursue a mixed strategy, with each charging a different distribution of prices to uncommitted buyers and the incumbent charging d less than its distribution to committed buyers. See Lemmas 1c(ii), 2c(ii). This creates two possibilities, depending on whether the expected rival price to uncommitted buyers would be lower or higher than the expected incumbent price to committed buyers. If the expected rival price would be lower, then each buyer will conclude that it (and all other buyers) would be better off rejecting the offer, thus changing the expected θ to 0. If the expected rival price would be higher, then each buyer will conclude that it (and all other buyers) would be better off accepting the offer, thus changing the expected θ to 1.

This above thus proves Proposition 1.

Proposition 1. Suppose that in period 1 the incumbent offers binding nondiscriminatory loyalty commitments simultaneously to noncoordinating buyers, and in period 2 the rival and incumbent engage in Bertrand competition. Then there are two possible subgame perfect Nash equilibria. In one equilibrium, all buyers reject the offered loyalty commitments. In the other equilibrium, all buyers accept in exchange for at most a trivial sidepayment in period 1, and the incumbent makes all sales (which are all to committed buyers) at P_m in period 2. In the latter equilibrium, prices will be above the but-for price of C that would have prevailed without the loyalty commitments.

A simultaneous offer with substantial sidepayments can push expectations toward concluding all buyers will accept, but both equilibria remain possible. To be conservative, assume the case where the rival picks price first, because that produces the lowest rival price in period 2 and thus requires the largest sidepayment. The Appendix proves that the incumbent can profitably pay a sidepayment in period 1 to induce buyer agreement if and only if buyers expect θ to be high enough that $P_r^* \geq .27P_m + 0.73C$, which I will call Lemma 3.

Lemma 3. Assuming the rival picks price first in period 2, incumbents can always profitably pay a period 1 sidepayment to induce buyers to accept a loyalty commitment for a sidepayment if and only if $P_r^* \geq .27P_m + 0.73C$.

Thus, if each buyer expects that no other buyers will agree, or begin with any probabilistic expectation of θ that is low enough to make expected $P_r^* < .27P_m + 0.73C$, then the incumbent cannot offer any sidepayment in period 1 that is large enough to make it profitable for each buyer to agree, which will make expected $\theta = 0$. On the other hand, if each buyer begins with an expected θ that is high enough that $P_r^* \geq .27P_m + 0.73C$, then the incumbent can offer a sidepayment in period 1 that is high enough that all buyers will conclude they are better off agreeing to the loyalty commitment even though doing so collectively harms them all. This will raise expected θ and lower the necessary sidepayment, until a trivial sidepayment suffices.

d. Will Buyers Accept Sequential Offers? Now consider the possibility that the incumbent can make sequential offers of a loyalty commitment. Take the case where the rival picks price first, because that creates the greatest disincentive for buyers to agree to loyalty commitments without substantial sidepayments. If a buyer expects that the likelihood that other buyers will agree is high enough that $P_r^* \geq .27P_m + 0.73C$, then the buyer would accept in exchange for a sidepayment. However, the buyer also knows that if it rejects any offer, an offer can be made sequentially to the other buyers, and that as more buyers accept, that will raise P_r^* and lower the required sidepayment in later rounds, until P_r^* rises to P_m and no sidepayment is required at all. This will often make it possible to get all buyers to accept sequential offers for a trivial sidepayment, for reasons parallel to those explained in RRW-SW for sequential offers.

III. If Buyers Breach Loyalty Commitments When Profitable

Now consider the possibility, raised by S&W, that buyers would breach their exclusionary commitments if the gains from doing so exceed their contract expectation damages. This assumption is actually quite debatable. As they acknowledge in their thoughtful article, reputational considerations and legal costs will often deter breach in such a case. Indeed, some contract scholarship indicates that reputational sanctions are often more important in securing compliance than legal penalties.¹⁶

More important, legal penalties for breach of contract are not limited to expectation

¹⁶ See Schwartz & Scott (2003), at 557.

damages. S&W assume otherwise because of the contract rule barring penalty clauses that set damages higher than expectation damages, but their assumption ignores forfeiture penalties. Under standard contract law, a buyer's intentional breach of an exclusivity commitment would allow any seller to decline to fulfil any of its own contractual commitments.¹⁷ If the seller wished to remove any doubt about the matter, it could simply make its duties explicitly conditional on the buyer honoring its exclusivity commitment. Thus, in addition to expectation damages, a buyer will suffer the harm of forfeiting the value of its other contract rights. If the relevant contract includes products other than the one in question, the penalty of forfeiting these contractual rights could be enormous. This may help explain why loyalty discounts are often bundled with loyalty discounts on other products.

Even if the contract is limited to the particular product, breach can also allow the seller to suspend duties as to past sales, such as a duty to repair or pay rebates on past sales. Those can create large penalties that exceed expectation damages. Indeed, it is relatively easy to evade the ban on penalty clauses by reframing them as conditional bonuses or rebates. For example, suppose expectation damages of X per unit would not deter breach of the exclusivity commitment, but $2X$ would. If the contract just had a clause making breach punishable by $2X$, then S&W would be right that this would violate the ban on penalty clauses. But the ban would not prevent the incumbent from charging $P_m + 2X$ with a rebate of $2X$ to buyers who comply with the exclusivity condition.¹⁸ Then buyers would comply because failure to do so would result in a loss of $2X$, and because compliant buyers would on net pay P_m they would behave just like committed buyers who comply under Proposition 1. Thus, a incumbent could always evade the obstacle observed by S&W by having a rebate conditional on compliance with the exclusivity condition, as long as the rebate exceeds the consumer welfare gain from breaching the exclusivity commitment. This may help explain why loyalty rebates are often used instead of, or in addition to, loyalty discounts.

¹⁷ If the market involves the sale of goods, this is true even for unintentional non-material breaches under the perfect tender rule. *See* Farnsworth, 551-52 (2004). For non-goods, a seller has the right to suspend performance only if the breach is material, but intentionality itself likely makes a breach material. *Id.* at 550-551. Even if the breach were unintentional, breaching a central commitment like exclusivity would likely be deemed material.

¹⁸ Because the rebate is not a “penalty” but a bonus, it does not violate the ban on penalty clauses. Nor would courts review whether the rebate exceeds the value of performance because another doctrine prohibits inquiry into the adequacy of consideration for a promise.

However, I need not rely on those additional reasons to expect compliance with commitments, because it turns out that expectation damages will alone suffice to deter breach in the cases where sellers can obtain loyalty discounts with commitments. S&W conclude otherwise for buyers who are ultimate consumers, but their conclusion depends critically on the assumption that the rival will enter at a price equal to cost. If the rival does so, then consuming buyers who switch to the rival will have to pay contract damages of $P_m - C$ on every purchase they would have made from the incumbent, which will be offset by an equivalent gain of $P_m - C$ in lower prices on every such purchase, and in addition buyers will gain the deadweight loss they otherwise would have suffered because buying at P_m would cause them to buy less than the efficient amount.

The reason S&W's analysis is inapplicable here is that the above shows that, given loyalty commitments, the rival will price at $P_r^* > C$ even under the extreme assumption of Bertrand competition.¹⁹ In the possibilities where the rival price would equal P_m , there will be no gain to buyers from breaching, and thus expectation damages will clearly deter breach. In the possibilities where the rival charges $P_r^* < P_m$, we can determine when expectation damages will exceed buyer gains from breach by using the sidepayment analysis above. Because that sidepayment analysis showed when an agreement creates incumbent gains that exceed individual buyer losses, it also shows when breach of an agreement creates expectation damages to the incumbent that exceed individual buyer gains from breach. Thus, expectation damages will make breach unprofitable whenever $P_r^* \geq .27P_m + 0.73C$. Because that is the condition to get the last buyer to agree to loyalty commitments (even if we assume the case where rivals pick price first, which is what generates the greatest difference between rival and incumbent prices), it should be met for any set of loyalty commitments that actually exist.

¹⁹ Likewise, S&W's conclusions would not hold if we more realistically assumed imperfect competition or lags in growing rival output, which would also make the rival price above C .

IV. If Loyalty Discounts Require No Buyer Commitments and Less Than 100% Loyalty

Now consider the case where the loyalty discounts reflect conditions without any buyer commitments or rebates. That is, the loyalty discount agreement binds the seller to maintain a discount between loyal and disloyal buyers, but does not bind buyers to buy exclusively from the incumbent; buyers instead agree only that exclusivity will be a condition of getting the loyalty discount. At any time, the buyers can buy from the rival without incurring contractual liability or lost rebates, as long as they are willing to forgo future loyalty discounts. Even then, it turns out the effects of the loyalty discounts are anticompetitive. Further, it is even clearer that buyers will accept them because there is no downside to doing so.

a. Unconditioned Market Suffices to Achieve Minimum Efficient Scale. Take first the case where either (1) there are no economies of scale or (2) selling to buyers who have not agreed to the condition suffices to obtain all economies of scale, that is $(N-S)q(C) \geq Q^*$.

Rival Picks Price First. I first assume the rival sets a price first in the second period and the incumbent has an opportunity to respond. For any rival price, P_r , greater than C , the incumbent has two options. First, it can deprive its rival of all sales by lowering its unconditioned price to some infinitesimal amount less than the rival price, $P_r - \varepsilon$, thus earning $P_r - \varepsilon$ to $N-S$ buyers and $P_r - \varepsilon - d$ to S buyers. Second, it can concede all unconditioned buyers, but still make all sales to the conditioned buyers, in which case it will maximize profits by charging $P_f - d = P_r - \varepsilon$ to these S buyers.

As above, the rival will want to set P_r sufficiently low to trigger the second response by the incumbent because otherwise the rival will earn zero profits. This condition is met when:

$$(P_r - C)(S/N)(A - P_r) > (P_r - C)((N-S)/N)(A - P_r) + (P_r - d - C)(S/N)(A - P_r + d),$$

which boils down to

$$(1-\theta)P_r^2 - 2[(1-\theta)P_m + \theta d]P_r + (1-\theta)2CP_m - (1-\theta)C^2 + 2\theta dP_m + \theta d^2 > 0$$

The Appendix proves this will be true as long as the rival charges no more than

$$P_r^{**} = P_m + d\theta/(1-\theta) - [(P_m - C)^2 + \theta d^2(2\theta - 1)/(1-\theta)^2]^{1/2}.$$

and that P_r^{**} is always above cost and more profitable for the rival than any alternative rival price as long as $P_r^{**} < P_m$. If $P_r^{**} \geq P_m$, then the rival will find it more profitable to charge P_m , which the Appendix proves will be true if $d \geq (P_m - C)/[\theta/(1-\theta)]^{1/2}$. Thus, the same formula that determines when P_r^* exceeds P_m also

determines whether P_r^{**} exceeds P_m . This thus proves Lemma 4a.

Lemma 4a. Suppose there are no economies of scale or the unconditioned market is large enough for the rival to reach its minimum efficient scale, the incumbent uses loyalty discounts that require no buyer commitment, and the rival and incumbent engage in Bertrand competition in period 2, with the rival picking price first. Then:

a. If $P_m > P_r^{**} = P_m + d\theta/(1-\theta) - [(P_m - C)^2 + \theta d^2(2\theta - 1)/(1-\theta)^2]^{1/2}$, the rival will sell to unconditioned buyers at P_r^{**} and the incumbent will sell to conditioned buyers at $P_r^{**} - \varepsilon$.

b. If $P_m \leq P_r^{**}$, then the rival will sell to unconditioned buyers at P_m and the incumbent will sell to conditioned buyers at $P_m - \varepsilon$.

$P_m \leq P_r^{**}$ if $d \geq (P_m - C)/[\theta/(1-\theta)]^{1/2}$. As long as $d > 0$, all these prices will exceed the but-for competitive level, C , which would have prevailed without the loyalty discounts on the same market assumptions.

Given the assumptions, this is a subgame perfect Nash equilibrium. The rival will not charge $P_r > P_r^{**}$ because that would cause the incumbent to charge $P_f < P_r$ to unconditioned buyers and lower rival sales and profits to zero. If $P_r^{**} > P_m$, the rival and incumbent will respectively charge P_m to unconditioned buyers and $P_m - \varepsilon$ to conditioned buyers, and would not charge any less because that would result in lower profits. If $P_r^{**} \leq P_m$, the rival will charge P_r^{**} because any lower price will bring it further below the profit-maximizing level and thus earn it less money. Given that the rival is charging no more than P_r^{**} , the incumbent will not have any incentives to charge conditioned buyers less than $P_r^{**} - \varepsilon$ because it cannot undercut the rival price to unconditioned buyers without resulting in lower overall profits.

Incumbent Picks Price First. If we assumed the incumbent picked price first, then the rival could undercut a incumbent price of $P_r^{**} - \varepsilon$ to the conditioned buyers. To avoid this, the incumbent would thus want to pick the highest price, P_x , for conditioned buyers that is low enough that the rival finds it less profitable to sell to all buyers at $P_x - \varepsilon$ than to sell at $P_x + d$ or P_m (whichever is lower) to just the unconditioned buyers.

If $P_x + d \geq P_m$, then the incumbent must pick the highest price that satisfies

$$(1-\theta)(P_m - C)(A - P_m) > (P_x - C)(A - P_x),$$

which is true when

$$P_x^2 - 2P_m P_x + (1-\theta)P_m^2 + 2\theta C P_m - \theta C^2 \geq 0.$$

This has a minimum at $P_x = P_m$ where it is negative. We can ignore solutions above

P_m because the parallel solutions below P_m that will be more profitable. The lefthand formula equals zero when

$$P_x = P_m - (P_m - C)[\theta]^{1/2}.$$

Thus, the incumbent will charge a P_x of $P_m - (P_m - C)[\theta]^{1/2}$ if $P_x + d \geq P_m$, which is true when $P_m - (P_m - C)[\theta]^{1/2} + d \geq P_m$, which is when $d \geq (P_m - C)/\theta^5$.

If $P_x + d < P_m$, then the incumbent must pick the highest price P_x that satisfies:

$$(1-\theta)(P_x + d - C)(A - P_x - d) > (P_x - C)(A - P_x),$$

which is true when

$$\theta P_x^2 + 2(\theta d - \theta P_m - d)P_x + 2(d - d\theta + \theta C)P_m - \theta C^2 - (1-\theta)d^2 > 0.$$

This has a minimum at

$$P_x = P_m + d(1-\theta)/\theta.$$

It crosses zero at

$$P_x = P_m - d + d/\theta - [(P_m - C)^2 + (1-\theta)d^2/\theta^2]^{1/2},$$

which one can show is always greater than C .

This thus proves Lemma 4b.

Lemma 4b. With the same assumptions as Lemma 4a, but instead assuming the incumbent picks price first in period 2, then

(1) the incumbent will sell to all conditioned buyers at $P_x = P_m - d + d/\theta - [(P_m - C)^2 + (1-\theta)d^2/\theta^2]^{1/2}$ if $P_x + d < P_m$, and the rival will sell to all unconditioned buyers at $P_x + d$.

(2) the incumbent will sell to all conditioned buyers at $P_x = P_m - (P_m - C)[\theta]^{1/2}$ if $P_x + d \geq P_m$, and the rival will sell to all unconditioned buyers at P_m .

All prices will exceed the but-for competitive level.

Simultaneous Choice. If one instead assumes simultaneous choice, the situation gets complicated. The rival knows that if the incumbent picks a price to conditioned buyers = P_x , then its best price to unconditioned buyers would be $P_x + d$ or P_m (whichever is lower). But if the rival picks that price, then the best price for the incumbent to charge conditioned buyers is epsilon below the rival price rather than charging just P_x . And if the rival expects the incumbent to charge that high a price, it has an incentive to undercut the incumbent price to win sales to conditioned buyers. The Appendix proves that a Nash equilibrium thus requires the mixed strategy set forth in Lemma 4c. All the prices in this range will exceed but-for competitive levels.

Lemma 4c. With the same assumptions as Lemma 4a, but instead assuming simultaneous pricing, then a mixed strategy equilibrium will result where both offer committed buyers a distribution of prices that are between P_x and the minimum of $P_x + d$ and P_m , with the rival selling to all uncommitted buyers. All prices will exceed the but-for competitive level.

b. Unconditioned Market Does Not Suffice to Achieve Minimum Efficient Scale.

Now suppose the unconditioned buyers do not buy enough to allow the rival to achieve its minimum efficient scale. Assume the rival picks its price first. (If one assumes the incumbent picks first, one can substitute P_x for P_r^{**} below.) Given the analysis above, the rival can profitably restrict itself to the unconditioned buyers if $P_r^{**} \geq C_r = P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta)]^{1/2}$. If so, the rival will sell to unconditioned buyers for the lesser of P_r^{**} or $P_m - C/2$, and the incumbent will sell to the conditioned buyers for the lesser of $P_r^{**} - \varepsilon$ or $P_m - C/2 - \varepsilon$.

If $P_r^{**} < C_r$, then the rival cannot profitably restrict itself to selling to the unconditioned buyers. The rival will thus have to set a price low enough that the incumbent would not have incentives to undercut it even as to conditioned buyers. The only price that satisfies this condition is C . Thus, under these assumptions, the rival will price at C and make all sales to unconditioned buyers. Assuming they split sales to conditioned prices at the same price, each will make half the sales to conditioned buyers. This proves Lemma 5.

Lemma 5. Suppose the incumbent uses loyalty discounts that require no buyer commitment, the unconditioned market is not large enough for the rival to reach its minimum efficient scale, and the rival and incumbent engage in Bertrand competition, but with the rival picking price first. Then

a. if $P_r^{**} \geq C_r$, the rival will sell to unconditioned buyers for the lesser of P_r^{**} or $P_m - C/2$, and the incumbent will sell to the conditioned buyers for the lesser of $P_r^{**} - \varepsilon$ or $P_m - C/2 - \varepsilon$. All these prices will exceed but-for competitive level C that would have prevailed without the loyalty discounts.

b. if $P_r^{**} < C_r$, then the incumbent will sell at C to half the conditioned buyers, and the rival will sell at C to the other half of the conditioned buyers and to all the unconditioned buyers.

c. Will Buyers Accept? When loyalty discounts do not require commitments, each buyer will always accept because it is never individually worse off doing so. This is

because under every scenario, each buyer is always better off having agreed to the loyalty discount, because the incumbent price to conditioned buyers is always below or equal to the rival price to unconditioned buyers. Thus, all buyers have incentives to accept without need of any sidepayments.

If there are no economies of scale, then the incumbent will not want to make a simultaneous nondiscriminatory offer, because then all buyers would accept and be conditioned, which means that the rival could win sales only by offering a price lower than the incumbent price to conditioned buyers. Bertrand competition would then drive prices down to costs. Instead, the incumbent would want to make sequential offers until all but one buyer accepts, because then the incumbent gets to price at $P_m - \varepsilon$ to the maximum number of buyers. It will set $d \geq (P_m - C)/[\theta/(1-\theta)]^{1/2}$, which here will be $(P_m - C)/[N-1]^{1/2}$, to assure the rival maximizes profits by selling at P_m . We now have proven Proposition 2.

Proposition 2. Suppose there are no economies of scale or the unconditioned market is large enough for the rival to reach its minimum efficient scale, the incumbent sequentially offers loyalty discounts that require no buyer commitment, and the rival and incumbent engage in Bertrand competition in period 2, with the rival picking price first. Then an equilibrium will result where all but one buyer accepts the loyalty discount, the incumbent charges $P_m - \varepsilon$ to the conditioned buyers and the rival charges P_m to the unconditioned buyer. All these prices will exceed the but-for competitive level. The loyalty discount will be set so $d \geq (P_m - C)/[\theta/(1-\theta)]^{1/2}$.

If economies of scale do exist, then the incumbent will want to stop offering loyalty discounts before θ rises to a level that drives P_r^{**} below C_r because that would trigger a price war that drives prices for both firms down to costs. However, the incumbent will want to keep offering loyalty discounts until $P_r^{**} \geq P_m - C/2$ because that maximizes the price it can charge conditioned buyers. It will also want to maximize the number of conditioned buyers that purchase at this price. Thus, the incumbent will keep offering sequential loyalty discounts until it reaches the foreclosure share where $P_r^{**} = C_r$, which is the same as

$$P_m + d\theta/(1-\theta) - [(P_m - C)^2 + \theta d^2(2\theta - 1)/(1-\theta)^2]^{1/2} = P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta)]^{1/2}.$$

Call the solution to this equation, which unfortunately does not simplify nicely, θ^* . Call d^* the discount level that maximizes the size of θ^* . Call C_r at this foreclosure level $C_r^* = P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta^*)]^{1/2}$. We now have proposition 4.

Proposition 3. Suppose the unconditioned market is not large enough for the rival to reach its minimum efficient scale, the incumbent sequentially offers loyalty discounts that require no buyer commitment, and the rival and incumbent engage in Bertrand competition in period 2, with the rival picking price first. Then an equilibrium will result where the loyalty discount level is d^* , θ^* of buyers accept, the incumbent sells to conditioned buyers for $P_m - C/2 - \varepsilon$, and the rival sells to all unconditioned buyers for Cr^* . All these prices will exceed the but-for competitive level.

In short, when loyalty discounts do not require commitments, buyers will always accept them when offered, and an incumbent making sequential offers should always be able to offer enough loyalty discount agreements to make the prices of both the incumbent and rival greater than their but-for levels.

d. Thresholds Less Than 100%. Now consider the possibility that the loyalty condition does not require 100% exclusivity, but rather requires some threshold percentage $T < 1$ of purchases from the incumbent. This does not change any of the analysis in the case of conditions without commitments. The reason is that the buyers who meet this threshold T will pay $P_f - d$ on all their purchases from the incumbent, and because that is always less than P_r , the compliant buyers will make all their purchases from the incumbent.

In the case of loyalty commitments with sub-100% thresholds, the analysis is more complicated because now the incumbent has three options. First, the incumbent can deprive its rival of all sales by lowering its uncommitted price to $P_r - \varepsilon$, thus earning $P_r - \varepsilon$ to $N-S$ buyers and $P_r - \varepsilon - d$ to S buyers. Second, it can concede all uncommitted buyers ($N-S$), but still make all sales to the committed buyers, by lowering its committed price to $P_f - d = P_r - \varepsilon$, thus earning $P_r - \varepsilon$ to S buyers. Third, it can concede all uncommitted purchases ($N-ST$), and just sell T times the quantity purchased by committed buyers, by keeping $P_f - d = P_m$. The first two are the same as the two options with 100% loyalty conditions without commitments. Thus, the rival can always price at least at the levels indicated in Lemmas 4 and 5 without triggering the first option. However, sometimes the incumbent will find the third option more profitable than the second at those prices, in which case the rival faces less of a

constraint and can price somewhat higher than in Lemmas 4 and 5.²⁰

V. When Multiple Firms Use Loyalty Discounts With Commitments

Now suppose a case where loyalty discounts are used by multiple firms. Take the case where there are no economies of scale or both firms achieve them, and the loyalty discounts require commitment with which buyers comply. For simplicity, assume firms 1 and 2 offer the same loyalty discount d , and have respectively signed up a θ_1 and θ_2 share of buyers, where $0 < \theta_1 + \theta_2 < 1$. Call θ_1 whichever is larger, so that $\theta_1 > \theta_2$.

Assume firm 2 picks price first. For any uncommitted price, P_2 , that firm 2 chooses, firm 1 has two options. First, it can deprive firm 2 of all uncommitted sales by lowering firm 1's uncommitted price to $P_2 - \varepsilon$, thus selling at $P_2 - \varepsilon$ to a $1 - \theta_1 - \theta_2$ share of buyers, and at $P_2 - \varepsilon - d$ to a θ_1 share of buyers. Second, it can concede all uncommitted buyers, but still make all sales to the committed buyers, in which case it will maximize profits by charging $P_1 - d = P_m$ to a θ_1 share of buyers.

The second option will be more profitable to firm 1 if:

$$\theta_1 \cdot (P_m - C)(A - P_m) > (1 - \theta_1 - \theta_2)(P_2 - C)(A - P_2) + \theta_1 \cdot (P_2 - d - C)(A - P_2 + d).$$

This can be rearranged as:

$$(1 - \theta_2)P_2^2 - 2[(1 - \theta_2)P_m + \theta_1 d]P_2 + (1 - \theta_2)C(2P_m - C) + [2dP_m + d^2 + (P_m - C)^2]\theta_1 > 0.$$

The Appendix proves this will be true as long as firm 2 charges no more than

$$P_2^* = P_m + \theta_1 d / (1 - \theta_2) - [(1 - \theta_1 - \theta_2)((P_m - C)^2 / (1 - \theta_2) - \theta_1 d^2 / (1 - \theta_2)^2)]^{1/2}$$

and that P_2^* is always above P_r^* and more profitable for firm 2 than any alternative price as long as $P_2^* < P_m + d\theta_2 / (1 - \theta_1)$. If $P_2^* \geq P_m + d\theta_2 / (1 - \theta_1)$, the Appendix proves that firm 2 will find it more profitable to charge uncommitted buyers $P_m + d\theta_2 / (1 - \theta_1)$, and to charge its committed buyers $P_m - d(1 - \theta_1 - \theta_2) / (1 - \theta_1)$, for an average price of P_m .

This proves Lemma 6.

Lemma 6. Suppose there are no economies of scale or both firms achieve them, and two firms engage in Bertrand competition and offer discounts for loyalty commitments, with firm 1 getting the larger foreclosure share, $\theta_1 > \theta_2$ and firm

²⁰ I omit the math to determine the precise price under this scenario because it takes up too much space given the complexity of the resulting formulas.

2 picking price first. Define $P_2^* = P_m + \theta_1 d / (1 - \theta_2) - [(1 - \theta_1 - \theta_2) ((P_m - C)^2 / (1 - \theta_2) - \theta_1 d^2 / (1 - \theta_2)^2)]^{1/2}$.

a. If $P_2^* \geq P_m + d\theta_2 / (1 - \theta_1)$, then firm 2 will sell to all uncommitted buyers at $P_m + d\theta_2 / (1 - \theta_1)$ and sell to all its committed buyers at $P_m - d(1 - \theta_1 - \theta_2) / (1 - \theta_1)$, for an average price of P_m . Firm 1 will sell to all its committed buyers at P_m .

b. If $P_2^* < P_m + d\theta_2 / (1 - \theta_1)$, then firm 2 will sell to all uncommitted buyers at P_2^* , and sell to all its committed buyers at $P_2^* - d$. Firm 1 will sell to all its committed buyers at P_m .

The prices firm 2 charges to uncommitted buyers will always exceed P_r^* , the price it would have charged uncommitted buyers if only firm 1 had loyalty commitments. All the prices will exceed the but-for competitive level, C , which would have prevailed if neither offered loyalty discounts.

Because firms 1 and 2 are both offering loyalty commitments, buyers are better off picking a loyalty commitment from either firm 1 or firm 2 than remaining uncommitted. Thus, one would expect all buyers to accept a commitment from one of the firms, until there are no uncommitted buyers. When this equilibrium is reached, both firm 1 and firm 2 will charge their committed buyers P_m with a nominal list price of $P_m + d$ that no buyer pays.

Now suppose there is a third firm, firm 3, deciding whether to enter. Firm 3 faces precisely the same situation as the rival faced in Section II, only with a cumulative foreclosure share that exceeds the single firm foreclosure share because $\theta_1 + \theta_2 > \theta_1$. This higher foreclosure share makes it more likely that the rival cannot achieve its minimum efficient scale. Further, if the rival does enter and achieve minimum efficient scale, the higher foreclosure share raises the rival's prices, because all the price formulas increase with increasing total θ .

Take the case where firm 3 does enter, but assume now firm 4 is considering entering and that the unforeclosed market is large enough for both of them to achieve their economies of scale. Then firm 3 will have incentives to adopt loyalty discounts as well, because otherwise firm 3 and 4 will compete prices down to cost in the unforeclosed market. The cumulative foreclosure will be even higher, and firm 4 will thus either be deterred from entering or enter at higher prices.

VI. Implications

The analysis here disproves many commonly held beliefs about loyalty discounts. Most basically, many hold the misconception that loyalty discounts presumptively lower prices.²¹ The above proves this is untrue – in every situation analyzed above, loyalty discounts raised prices above but-for levels. The word “discounts” deceptively suggests otherwise, but the nominal “discount” is just the difference between the compliant and noncompliant prices that a firm chooses, and does not indicate prices lower than the levels that would have resulted without loyalty discounts. There is no sound economic reason to conflate real discounts from but-for levels with price differences conditioned on compliance with exclusionary terms. To the contrary, loyalty discounts perversely discourage discounting.

Several courts and scholars have claimed that loyalty discounts should be deemed presumptively or conclusively procompetitive if the discounted price is above cost,²² or if the rival is pricing above its own costs.²³ The above disproves both these claims. In every situation, the discounted and rival prices are above cost, but the loyalty discount results in anticompetitive effects. Both claims miss the point that loyalty discounts discourage price-cutting by both the firms that use them and their rivals, and cause prices to be above cost. The first claim also missed the point that loyalty commitments can raise rivals’ costs above but-for levels.

More generally, many have argued that exclusionary conduct should not be condemned unless it involves a short-term profit sacrifice,²⁴ would not be profitable if it did not eliminate or impair rivals,²⁵ or does or could exclude an equally efficient rival.²⁶ The above undercuts these claims. In all the above situations, the loyalty discounts have anticompetitive effects even though the conduct is always profitable,

²¹ See, e.g., Hovenkamp (2006).

²² Hovenkamp (2006); Hovenkamp (2005), at 129, 132; Lambert (2005); *NicSand v. 3M*, 507 F.3d 442 (6th Cir. 2007); *Concord Boat v. Brunswick Corp.*, 207 F.3d 1039 (8th Cir. 2000). Others have rejected this claim, but without rigorous economic proof. *FTC v. Brown Shoe*, 384 U.S. 316 (1966); *LePage’s v. 3M*, 324 F.3d 141 (3^d Cir. 2003) (en banc).

²³ Lambert (2005); *NicSand*.

²⁴ Bork, p.144 (1978); Ordovery & Willig (1981); Patterson (2003).

²⁵ Melamed (2006); Werden (2006).

²⁶ Posner, pp.194-96 (2001); Hovenkamp (2005), at 129, 132; Lambert (2005); Lave (2005); DG Competition (2005); *Cascade Health Solutions v. Peacehealth*, 502 F.3d 895 (9th Cir. 2007).

would remain profitable even without eliminating or impairing rivals, and whether or not the rival is equally efficient and stays in the market. Further, the equally efficient rival test misses the point that sometimes the loyalty discounts will create anticompetitive effects by making the rival less efficient.

Another common general claim is that exclusionary agreements cannot be anticompetitive if buyers voluntarily agree to them.²⁷ Again, the above analysis disproves this claim. In all the situations, the buyers voluntarily agree to the loyalty discounts because doing so makes each individually better off, even though collectively they would be better off if none of them accepted. Relatedly, some have argued that loyalty discounts are anticompetitive only when they create a form of intraproduct bundling, by bundling each buyer's contestable demand for a product with its incontestable demand, such as when the buyer is a dealer with two sets of downstream buyers.²⁸ The above again proves this is untrue because none of the models assumed buyers had such divergent demands for the product of the firm using loyalty discounts.

Others more modestly assume that loyalty discounts cannot be anticompetitive unless they create a large enough foreclosure to impair rival efficiency.²⁹ The above proves that even this claim is untrue, because it turns out to miss the fact that loyalty discounts discourage discounting even if they do not affect rival efficiency at all. Relatedly, courts or scholars often say that exclusionary agreements should not be deemed anticompetitive unless they foreclose a substantial share of the market, with 20-40% often stated to be the level necessary to be "substantial" under U.S. antitrust law.³⁰ However, because the anticompetitive effects of loyalty discounts do not depend on the rival losing economies of scale, they persist even at low foreclosure levels. For example, suppose the foreclosure share were only 10%, with $P_m = 100$, $C = 20$, and $d = 20$. Then, even if rivals pick price first (the assumption that leads to

²⁷ Bork, pp.304-09 (1978); Sullivan & Harrison, p.250 (1998); *Concord Boat*. Others have rejected this claim. *Standard Fashion*; *Brown Shoe*; Posner, p.194 & n.2 (2001).

²⁸ DG Competition (2005); *Canada v. Canada Pipe*, 2005 Canada Comp. Trib. 3; Hovenkamp (2005), at 129-130.

²⁹ Jacobson (2002); Brodley & Ma (1993).

³⁰ Wright (2006); *Twin City Sportservice v. Charles O. Finley & Co.*, 676 F.2d 1291 (9th Cir. 1982) (24% sufficed); *Stop & Shop Supermarket v. Blue Cross*, 373 F.3d 57 (1st Cir. 2004) (30–40%); *Concord Boat* (must be substantial); Areeda (1991) (20% presumptively unreasonable); Hovenkamp (1998) (20% with HHI over 1800 presumptively unreasonable).

the lowest rival prices), Lemma 1a shows that the loyalty commitments would still cause incumbent prices that are 400% over but-for levels and a rival price of $P_r = 26.3$ that is 31.5% above the but-for level. Increasing the foreclosure level does increase the anticompetitive effect, but even a relatively low foreclosure share can elevate prices substantially above but-for levels. This seems to support the position in EC competition caselaw, as well some U.S. cases, which have found loyalty discounts by firms with market power illegal without proof of a substantial foreclosure share, in cases where the loyalty conditions lacked any efficiency justification.³¹

At least one court has suggested that loyalty discounts cannot be anticompetitive if the discount levels are low, such as 1-3%.³² However, while the above proves that increasing the discount level does increase rival prices further above but-for levels, it also proves that even small discount levels can elevate prices substantially. For example, suppose the foreclosure share was 50%, $P_m = 100$, $C = 20$, and the discount level was 1 or 3%. Then, even if rivals pick price first, Lemma 1a shows that the loyalty commitment would still cause the incumbent to price 400% above the competitive level, and cause the rival to charge 43.93 (if the discount is 1%) or 44.95 (if the discount is 3%), which are 120-125% above but-for levels.³³ Even without commitment, Lemma 4a shows that a loyalty discount of 1-3% would raise rival and incumbent prices to 21-23, which is 5-15% above but-for levels, more than significant given the 5% standard of significance used in the U.S. merger guidelines.³⁴

Another issue of lively debate is whether exclusionary agreements should be deemed presumptively or conclusively procompetitive if they are terminable or require no commitment, with many courts and scholars asserting the answer is yes.³⁵ The above

³¹ Case 85-76, *Hoffmann-La Roche v. Commission*, 1979 E.C.R. 461; *Brown Shoe; Le Page's*; *Microsoft v. U.S.*, 253 F.3d 34, 68 (D.C. Cir. 2001) (en banc).

³² *Concord Boat*.

³³ Although the formulas also indicate anticompetitive price effects from loyalty commitments even if $d = 0$, the analysis also shows that buyers would not agree to loyalty commitments unless $d > 0$.

³⁴ U.S. Department of Justice & Federal Trade Commission, *Horizontal Merger Guidelines*, §1.11 (1992, revised 1997).

³⁵ Hovenkamp (1998); Hovenkamp (2005), at 129; Wright (2006); *Concord Boat*; *CDC Technologies, Inc. v. IDEXX Labs, Inc.*, 186 F.3d 74 (2d Cir. 1999); *Omega Envtl. v. Gilbarco, Inc.*, 127 F.3d 1157 (9th Cir. 1997); *Thompson Everett, Inc. v. Nat'l Cable Adver.*, 57 F.3d 1317 (4th Cir. 1995); *U.S. Healthcare v. Healthsource*, 986 F.2d 589, 596 (1st Cir. 1993); *Roland Mach. v.*

disproves that claim. Indeed, Lemmas 4a-4c, 5, and Propositions 2-3 prove that, even when loyalty discount agreements require no commitment at all, they can raise prices greatly above but-for levels. For example, if $P_m = 100$, $\theta = .5$, $C = 20$, and $d = 20$, and rivals pick price first, then Lemma 4a shows that a loyalty discount without any commitment would cause the incumbent and rival to both price at 40, which is 100% above the but-for level. For any given discount and foreclosure level, loyalty discounts without commitments result in somewhat lower prices than those with commitments. However, the resulting prices are still above cost, and the anticompetitive result is more stable because buyers who agree to the loyalty discounts never do any worse than those who do not.

Others have stated that loyalty discounts cannot be anticompetitive if they require significantly less than 100% exclusivity.³⁶ The above shows this is false. A threshold lower than 100% does not at all alter the anticompetitive effects of loyalty discounts without commitments. While a lower threshold reduces the anticompetitive effects for loyalty discounts with commitments, they remain significant and at least as high as the anticompetitive effects of loyalty discounts without commitments. Relatedly, some have suggested that if buyers buy more from the incumbent than the sub-100% threshold required by their loyalty discount, then it is unlikely to be anticompetitive.³⁷ The above again disproves this. Indeed, for loyalty discounts without commitments, anticompetitive effects result though buyers always buy more than the threshold from the incumbent. For loyalty discounts with commitments, they result even though buyers often make above-threshold purchases from the incumbent.

Some have argued that loyalty discounts cannot create any anticompetitive effects if other firms can also use them.³⁸ Lemma 6 proves, to the contrary, that the anticompetitive effects are exacerbated if multiple firms use loyalty discounts. Lemma 6 also bears on the appropriateness of using a cumulative foreclosure

Dresser Indus., 749 F.2d 380 (7th Cir. 1984). Others have rejected this claim. *Brown Shoe* (condemning agreement terminable at will); *Standard Oil v. United States*, 337 U.S. 293 (1949) (condemning agreement terminable upon thirty days notice); *Standard Fashion v. Magrane-Houston Co.*, 258 U.S. 346 (1922) (condemning agreement terminable upon three months notice); *United States v. Dentsply, Intl.*, 399 F.3d 181 (3d Cir. 2005); *LePage's*.

³⁶ *Concord Boat*. Others have rejected this claim. *Brown Shoe* (75% threshold sufficed); *Microsoft* (D.C. Cir. 2001) (en banc)(75% threshold sufficed).

³⁷ *Concord Boat*.

³⁸ *NicSand*.

approach that aggregates the foreclosure shares produced by multiple sellers. Although U.S. Supreme Court cases and EC guidelines have long used a cumulative foreclosure approach,³⁹ some have argued cumulative foreclosure has no economic basis.⁴⁰ The above disproves this argument. Although anticompetitive effects persist at low foreclosure levels, the cumulative effect of foreclosure by two firms is to raise prices above the levels that would have been created by the foreclosure of only one of the firms. Further, the effect is to make loyalty discounts more stable by driving buyers into commitments with one of the firms offering them and deterring production by other firms. Thus, if foreclosure levels are used to screen out cases based on the likely size of anticompetitive effects, then it makes more sense to look at cumulative foreclosure than single firm foreclosure.

Any of the above anticompetitive effects might be offset by efficiencies. Such efficiencies were excluded from my model, and the models used in prior articles, because the models all assume cost and demand curves that are not altered by the existence of loyalty discounts. This precludes the possibilities that loyalty discounts might lower production costs or increase product value. In other words, it assumes exclusion “naked” of any efficiency justifications. If loyalty discounts can be demonstrated to have efficiencies that cannot be advanced by less restrictive alternatives, such as volume-based discounts, then the net effects of loyalty discounts might increase net efficiency, lower prices, or otherwise benefit consumers despite some anticompetitive effects. This article proves the effects of loyalty discounts only on the assumption that they are not necessary to achieve efficiencies. However, lower prices are not themselves an efficiency justification for loyalty discounts, as some have thought,⁴¹ both because firms can lower prices without conditioning those prices on loyalty, and because this article proves that, absent some productive efficiency, conditioning price reductions on such loyalty conditions tends to raise, not lower, prices.

On the other hand, the anticompetitive effects predicted above are understated because of the extreme assumption of Bertrand competition, especially as to anticompetitive effects on rivals. If we made more realistic assumptions of imperfect competition,

³⁹ *FTC v. Motion Picture Advertising Service*, 344 U.S. 392 (1953); *Standard Oil*; EC Guidelines on Vertical Restraints §2.1, O.J. 2000, C291/1.

⁴⁰ Hovenkamp (2004); *Paddock Publ'ns. v. Chicago Tribune*, 103 F.3d 42 (7th Cir. 1996).

⁴¹ *Concord Boat*.

loyalty discounts would be more likely to both create adverse effects on rival competitiveness and lead to anticompetitive equilibria. The former would be true if, for example, one more realistically assumed that switching costs exist, that supply elasticity is limited so that output cannot instantly be expanded, or that differentiated demand meant that loyalty discounts bundled contestable with incontestable demand. The latter would be true if imperfect competition meant that even two firms operating at efficient scale would produce above-cost prices for free buyers from which loyalty discounts could be offered, making it even easier to arrive at equilibria in which buyers agree to anticompetitive loyalty commitments and do not breach or terminate them. Such above-cost pricing would result if we assumed that firms either operate on a differentiated market, view competition as a multi-period game with no fixed endpoint (and thus coordinate on uncommitted prices), or that expanding output requires advance planning so that firms pick outputs rather than price (and thus engage in Cournot or Stackelberg competition).

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APPENDIX

Proof of Lemma 1a. As the paper showed, the incumbent will not undercut the rival price to uncommitted buyers as long as

$$P_r^2 - 2(P_m + \theta d)P_r + 2CP_m - C^2 + 2\theta dP_m + \theta d^2 + \theta \cdot (P_m - C)^2 > 0.$$

Because this form repeats throughout this article, it is worth pointing out that for any inequality $ax^2 - 2bx + c > 0$, where a is positive, taking the first and second derivative will show there is a minimum at $x = b/a$. The value of the left hand formula at the minimum will be $-b^2/a + c$. Thus, if $c > b^2/a$, the inequality will always be satisfied. If $c < b^2/a$, the formula is negative at its minimum and will become positive (and thus satisfy the inequality) only if x is below the lower quadratic root or above the higher root, which are $b/a \pm [(b/a)^2 - c/a]^{1/2}$.

Thus, the above inequality is always satisfied if

$$2CP_m - C^2 + 2\theta dP_m + \theta d^2 + \theta \cdot (P_m - C)^2 > (P_m + \theta d)^2$$

which can be rearranged as $d\theta^{1/2} > P_m - C$. When that is the case, the rival can charge any price to uncommitted buyers without causing the incumbent to try to undercut it, and thus the rival will pick the profit-maximizing price of P_m

If $d\theta^{1/2} < P_m - C$, then using the above and simplifying, the inequality will be satisfied only if P_r is above or below the respective quadratic roots, which we can simplify as:

$$P_m + \theta d \pm [(1-\theta)((P_m - C)^2 - \theta d^2)]^{1/2}$$

Because the midpoint of the two roots is higher than P_m , then at the higher root P_r must be further away from P_m , which we can show is always less profitable. To see why assume any set of possible prices $P_m + X \pm Y$, where X and Y are both positive. Then the lower solution will earn more than the higher solution if

$$(P_m + X - Y - C)(A - P_m - X + Y) > (P_m + X + Y - C)(A - P_m - X - Y)$$

which because $A = 2P_m - C$, can be simplified to being true whenever

$$XY > 0,$$

which is always true because X and Y are both positive. Thus, the rival will always choose the lower solution over any price at or above the higher solution.

The rival will thus charge a price up to

$$P_r^* = P_m + \theta d - [(1-\theta)((P_m - C)^2 - \theta d^2)]^{1/2}$$

and be able to sell to all the uncommitted buyers without inducing the incumbent to undercut its price. If $P_r^* \geq P_m$, then the rival will charge P_m since that price will earn more profits from uncommitted buyers than a higher price. P_r^* will be $\geq P_m$ only if

$$\theta d \geq [(1-\theta)((P_m - C)^2 - \theta d^2)]^{1/2},$$

which simplifies to being true only if

$$d \geq (P_m - C) / [\theta / (1 - \theta)]^{1/2}.$$

If $P_r^* < P_m$, then the rival will charge P_r^* because any lower price earns less profit.

$P_r^* > C$ if $P_m + \theta d - [(1 - \theta)((P_m - C)^2 - \theta d^2)]^{1/2} > C$, which simplifies to,

$$\theta(P_m - C)^2 + 2(P_m - C)\theta d + \theta d^2 > 0.$$

Since $P_m > C$, this is true whenever $\theta > 0$, that is whenever any buyer accepts the loyalty discount. Note that d need not be > 0 .

Thus, as long as any buyer accepts the loyalty discount, there is always a rival price $P_r^* > C$ that the rival can charge that will cause the incumbent to keep the price to committed buyers equal to P_m , with the incumbent thus offering uncommitted buyers $P_m + d$ but being undercut by P_r^* , so that the rival makes all sales to uncommitted buyers at P_r^* and the incumbent makes all sales to committed buyers at P_m .

Proof of Lemma 1c. Because the rival will simply charge uncommitted buyers P_m if $P_r^* \geq P_m$, the interesting cases to address here are when $P_r^* < P_m$. In a simultaneous game the rival will not charge a price below P_r^* because the incumbent prefers to forfeit sales to uncommitted buyers for any price below P_r^* . Nor will the rival charge a price above P_m because it would make more profit at the monopoly price. In an equilibrium, the rival cannot play a pure strategy $P_r \in (P_r^*, P_m]$ because it will be undercut by the incumbent. The best response by the incumbent to a rival strategy of always charging P_r^* would be to charge the monopoly price to the committed buyers and ignore the uncommitted buyers, but if the incumbent adopted that strategy it would not be an equilibrium strategy for the rival to charge P_r^* because the rival could increase its profits by raising its price to P_m . Thus, there is no Nash equilibrium in this game in pure strategies, and we need to find a mixed strategy equilibrium.

In a mixed strategy equilibrium, the incumbent charges uncommitted buyers price P_i between P_r^* and P_m when it competes for them (the committed buyers are charged $P_i - d$), and it charges price $P_m + d$ to the uncommitted buyers, and therefore price P_m to the committed buyers, when it withdraws from competition for the uncommitted buyers. Denote the cumulative distribution function according to which the incumbent sets its price to uncommitted buyers by $G(P)$, which indicates the probability for each price P that the firm will charge a price that is less than P . The incumbent will not charge uncommitted buyers any price $P_i \in [P_m, P_m + d)$ because it will be undercut in this price range among the uncommitted buyers by the rival. Therefore, $G(P) = 1 - \text{Prob}(P_i = P_m + d)$ for all $P \in [P_m, P_m + d)$.

The rival can always earn $(1-\theta)(P_r^* - C)(A - P_r^*)$ by charging P_r^* , thus in equilibrium its expected profit by charging $P \in (P_r^*, P_m]$ must equal $(1-\theta)(P_r^* - C)(A - P_r^*)$ as well, otherwise the rival would simply charge the price that earned a higher profit. When the rival charges $P \in (P_r^*, P_m]$, then with probability $G(P)$ it is undercut by the incumbent and receives zero profits and with probability $1 - G(P)$ it wins over the uncommitted buyers and receives profits $(1-\theta)(P - C)(A - P)$. Therefore, the distribution function $G(P)$ satisfies the following equation:

$$(1 - G(P))(1 - \theta)(P - C)(A - P) = (1 - \theta)(P_r^* - C)(A - P_r^*),$$

which is equivalent to

$$G(P) = 1 - [(P_r^* - C)(A - P_r^*) / (P - C)(A - P)]$$

for $P \in (P_r^*, P_m]$. This means that

$$1 - G(P_m) = (P_r^* - C)(A - P_r^*) / (P_m - C)(A - P_m) < 1$$

equals the probability that the incumbent ignores the uncommitted buyers and charges the committed buyers P_m .

The rival chooses a price P_r between P_r^* and P_m according to a cumulative distribution function denoted as $H(P)$. The incumbent can always earn $\theta(P_m - C)(A - P_m)$ by selling to the committed buyers only, thus its expected profits when it competes for the uncommitted buyers should be equal to $\theta(P_m - C)(A - P_m)$ as well. When the incumbent charges uncommitted buyers $P \in (P_r^*, P_m)$, then with probability $H(P)$ it is undercut by the rival and profits only from its sales to the committed buyers and with probability $1 - H(P)$ it captures both groups of buyers. Therefore, the equation on the incumbent's expected profits determines the distribution function $H(P)$:

$$\theta(P - d - C)(A - P + d) + (1 - H(P))(1 - \theta)(P - C)(A - P) = \theta(P_m - C)(A - P_m),$$

which is equivalent to

$$H(P) = 1 - [\theta(P_m - C)(A - P_m) - \theta(P - d - C)(A - P + d)] / [(1 - \theta)(P - C)(A - P)].$$

Because $(P_m - C)(A - P_m) > (P_m - C - d)(A - P_m + d)$, then $H(P_m) < 1$, and the rival charges P_m with probability

$$1 - H(P_m) = [\theta(P_m - C)(A - P_m) - \theta(P - d - C)(A - P + d)] / [(1 - \theta)(P - C)(A - P)].$$

The strategies described above describe a Nash equilibrium because neither the incumbent nor the rival can increase its profits by deviating from its strategy given that the other party follows its strategy.

Proof of Lemma 2c. Because the rival does not enter if $P_r^* < C_r$, the interesting cases to address here are where $P_r^* \geq C_r$. In such a case, there is a mixed strategies Nash equilibrium. The rival will not charge below P_r^* because the incumbent prefers to

forfeit competition for the uncommitted buyers for any price up to P_r^* . The rival will not charge above $P_m - C/2$ because, given the cost curve and the buyers available to it, the rival maximizes its profits at that price even if not undercut by the incumbent for uncommitted buyers.

The incumbent charges uncommitted buyers price P_i between P_r^* and $P_m - C/2$ when it competes for them (the committed buyers are charged $P_i - d$) and it charges price $P_m + d$ to the uncommitted buyers, and therefore price P_m to the committed buyers, when it does not compete for the uncommitted buyers. Denote the cumulative distribution function according to which the incumbent randomizes its price by $G(P)$. The incumbent will not charge any price $P_i \in [P_m - C/2, P_m + d)$ because it will be undercut in this price range among the uncommitted buyers by the rival. Therefore, $G(P) = 1 - \text{Prob}(P_i = P_m + d)$ for all $P_i \in [P_m - C/2, P_m + d)$.

Given the cost curve over the relevant output range, the rival can always earn $(1 - \theta)P_r^*(A - P_r^*) - F$ by charging P_r^* , thus its expected profit by charging $P \in (P_r^*, P_m - C/2]$ should be equal to $(1 - \theta)P_r^*(A - P_r^*) - F$ as well. When the rival charges $P \in (P_r^*, P_m - C/2]$, then with probability $G(P)$ it is undercut by the incumbent and receives zero profits and with probability $1 - G(P)$ it wins over the uncommitted buyers and receives profits $(1 - \theta)P(A - P) - F$. Therefore, the distribution function $G(P)$ satisfies the following equation:

$$(1 - G(P))[(1 - \theta)P(A - P) - F] = (1 - \theta)P_r^*(A - P_r^*) - F,$$

which is equivalent to

$$G(P) = 1 - [(1 - \theta)P_r^*(A - P_r^*) - F] / [(1 - \theta)P(A - P) - F]$$

for $P \in [P_r^*, P_m - C/2]$. Further,

$$1 - G(P_m - C/2) = [(1 - \theta)P_r^*(A - P_r^*) - F] / [(1 - \theta)(P_m - C/2)(A - P_m + C/2) - F] < 1$$

equals the probability that the incumbent foregoes sales to the uncommitted buyers and charges the committed buyers P_m .

The rival chooses price P_r between P_r^* and $P_m - C/2$ according to a cumulative distribution function denoted as $H(P)$. The incumbent can always earn $\theta(P_m - C)(A - P_m)$ by selling to the committed buyers only, thus its expected profits when it competes for the uncommitted buyers should be equal to $\theta(P_m - C)(A - P_m)$ as well. When the incumbent charges $P \in (P_r^*, P_m - C/2)$ then with probability $H(P)$ it is undercut by the rival and profits only from its sales to the committed buyers and with probability $1 - H(P)$ it captures both groups of buyers. Therefore, the equation on the incumbent's expected profits determines the distribution function $H(P)$:

$$\theta(P - d - C)(A - P + d) + (1 - H(P))(1 - \theta)(P - C)(A - P) = \theta(P_m - C)(A - P_m),$$

which can be rearranged as:

$$H(P) = 1 - [\theta(P_m - C)(A - P_m) - \theta(P - d - C)(A - P + d)] / [(1 - \theta)(P - C)(A - P)].$$

Because $(P_m - C)(A - P_m) > (P_m - 3C/2 - d)(A - P_m + C/2 + d)$, then $H(P_m - C/2) < 1$, and the rival charges $P_m - C/2$ with probability $1 - H(P_m - C/2)$, which is equal to:

$$[\theta(P_m - C)(A - P_m) - \theta(P_m - 3C/2 - d)(A - P_m + C/2 + d)] / [(1 - \theta)(P_m - 3C/2)(A - P_m + C/2)].$$

The strategies described above describe a Nash equilibrium because neither the incumbent nor the rival can increase its profits by deviating from its strategy given that the other party follows its strategy.

Proof of Lemma 3. The gain to the incumbent from the agreement of each buyer will be $(P_m - C)(Q_m/N)$, which since $Q_m = P_m - C$ is the same as $(1/N)(P_m - C)^2$. The loss to each buyer from agreeing will be $(1/2N)(2P_m - C - P_r^*)^2 - (1/2N)(P_m - C)^2$. The incumbent can thus offer a sidepayment that induces buyers with this expectation to agree as long as

$$(1/N)(P_m - C)^2 > (1/2N)(2P_m - C - P_r^*)^2 - (1/2N)(P_m - C)^2,$$

which can be expressed as

$$P_r^{*2} - 2(2P_m - C)P_r^* - 2C^2 + 2P_m C + P_m^2 < 0.$$

The left hand formula has a minimum at $P_r^* = 2P_m - C$, at which it takes the value $-3(P_m - C)^2$, which is always negative. It stays negative (and thus satisfies the inequality) as long as P_r^* is between the roots $2P_m - C \pm (3)^{.5}(P_m - C)$. We can ignore the upper bound because no rival would not offer such a price, given that doing it is greater than P_m . Thus, a profitable sidepayment can be made as long as buyers expect $P_r^* \geq 2P_m - C - (3)^{.5}(P_m - C)$, which with rounding can be simplified as $P_r^* \geq .27P_m + 0.73C$.

Proof of Lemma 4a. As the paper showed, the incumbent will not undercut the rival price to unconditioned buyers as long as

$$(1 - \theta)P_r^2 - 2[(1 - \theta)P_m + \theta d]P_r + (1 - \theta)2CP_m - (1 - \theta)C^2 + 2\theta dP_m + \theta d^2 > 0$$

This inequality is always satisfied if $P_m - C < [\theta - 2\theta^2]^{1/2}d/(1 - \theta)$, in which case the rival can pick any price without being undercut on unconditioned buyers, so it will price at the profit-maximizing price, P_m .

If $P_m - C > [\theta - 2\theta^2]^{1/2}d/(1 - \theta)$, then the inequality will be satisfied only if P_r is above or below the respective quadratic roots, which we can simplify as:

$$P_r = P_m + d\theta/(1 - \theta) \pm [(P_m - C)^2 + \theta d^2(2\theta - 1)/(1 - \theta)^2]^{1/2}.$$

The higher of the two solutions is above P_m and thus we know that any price above it will be even further away from the profit-maximizing price and thus earn less

profits. Further, because the higher solution is further away from P_m , we know it is less profitable than the lower solution, given the proof above. Thus, the rival will always choose the lower solution over any price at or above the higher solution.

The rival will thus charge a price up to

$$P_r^{**} = P_m + d\theta/(1-\theta) - [(P_m - C)^2 + \theta d^2(2\theta - 1)/(1-\theta)^2]^{1/2}.$$

and be able to sell to all the unconditioned buyers without triggering a price cut that undercuts its price. If $P_r^{**} \geq P_m$, then the rival will charge P_m since that will earn more profits from unconditioned buyers than a higher price. P_r^{**} will be $\geq P_m$ only if

$$d\theta/(1-\theta) \geq [(P_m - C)^2 + \theta d^2(2\theta - 1)/(1-\theta)^2]^{1/2}.$$

which simplifies to being true only if

$$d \geq (P_m - C)/[\theta/(1-\theta)]^{1/2}.$$

If $P_r^{**} < P_m$, then the rival will charge P_r^{**} as long as $P_r^{**} > C$. This condition will be met whenever

$$P_m + d\theta/(1-\theta) - [(P_m - C)^2 + \theta d^2(2\theta - 1)/(1-\theta)^2]^{1/2} > C,$$

which can be rearranged as when

$$2(P_m - C)d > d^2(\theta - 1)/(1-\theta)$$

Because $\theta \leq 1$, this is always true as long as $d > 0$. Thus, there always exists a $P_r^{**} > C$ that the rival can charge that will cause the incumbent to sell to conditioned buyers at $P_r^{**} - \varepsilon$, while the rival sells to unconditioned buyers at P_r^{**} .

Proof of Lemma 4c. Start with the case where $P_x + d < P_m$. In a simultaneous game, there is an equilibrium in which the rival charges between P_x and $P_x + d$ and the incumbent concedes unconditioned buyers to the rival and competes only for the conditioned buyers by offering the price between P_x and $P_x + d$.

The incumbent charges conditioned buyers price P_i and it charges price $P_i + d$ to the unconditioned buyers. Denote the cumulative distribution function according to which the incumbent randomizes its price by $G(P)$. Because the loyalty discount prevents the incumbent from charging unconditioned buyers less than $P_x + d$, the rival can always earn $(1-\theta)(P_x + d - C)(A - P_x - d)$ by charging $P_x + d$. Thus, the rival's expected profit by charging $P \in (P_x, P_x + d]$ should equal $(1-\theta)(P_x + d - C)(A - P_x - d)$ as well. Therefore, the distribution function $G(P)$ satisfies the following equation:

$$(1-\theta)(P - C)(A - P) + (1 - G(P))\theta(P - C)(A - P) = (1-\theta)(P_x + d - C)(A - P_x - d),$$

which is equivalent to

$$G(P) = [(P - C)(A - P) - (1-\theta)(P_x + d - C)(A - P_x - d)] / [\theta(P - C)(A - P)]$$

for $P \in [P_x, P_x + d]$. Note that $G(P_x) = 0$ and $G(P_x + d) = 1$.

Denote the cumulative distribution function according to which the rival randomizes its price by $H(P)$. The incumbent can always earn $\theta(P_x - C)(A - P_x)$ by selling to the conditioned buyers only, thus its expected profits when it competes for the unconditioned buyers should be equal to $\theta(P_x - C)(A - P_x)$ as well. Therefore, the equation on the incumbent's expected profits determines $H(P)$:

$$(1 - H(P))\theta(P - C)(A - P) = \theta(P_x - C)(A - P_x),$$

which is equivalent to

$$H(P) = 1 - [(P_x - C)(A - P_x)] / [(P - C)(A - P)].$$

Because $H(P_x + d) < 1$, the rival charges $P_x + d$ with probability $[(P_x - C)(A - P_x)] / [(P_x + d - C)(A - P_x - d)] < 1$.

If $P_x + d > P_m$, then the rival can charge the monopoly price to the unconditioned buyers and it will not be undercut by the incumbent. For $P \in (P_x, P_m)$, the rival will select price P_r according to the distribution function

$$H(P) = 1 - [(P_x - C)(A - P_x)] / [(P - C)(A - P)].$$

The rival will charge P_m with probability

$$[(P_x - C)(A - P_x)] / [(P_m - C)(A - P_m)] < 1.$$

The equation for the distribution function $G(P)$ becomes

$$(1 - \theta)(P - C)(A - P) + (1 - G(P))\theta(P - C)(A - P) = (1 - \theta)(P_m - C)(A - P_m),$$

which is equivalent to

$$G(P) = [(P - C)(A - P) - (1 - \theta)(P_m - C)(A - P_m)] / [\theta(P - C)(A - P)]$$

for $P \in [P_x, P_m]$. Note that $G(P_x) = 0$ and $G(P_m) = 1$.

The strategies described above describe a Nash equilibrium because neither the incumbent nor the rival can increase its profits by deviating from its strategy given that the other party follows its strategy.

Proof of Lemma 6. As the paper showed, firm 1 will not undercut firm 2's price to uncommitted buyers as long as

$$(1 - \theta_2)P_2^2 - 2[(1 - \theta_2)P_m + \theta_1 d]P_2 + (1 - \theta_2)C(2P_m - C) + [2dP_m + d^2 + (P_m - C)^2]\theta_1 > 0$$

This inequality is always satisfied if $d[\theta_1 / (1 - \theta_2)]^{1/2} > P_m - C$. When that is the case, firm 2 can charge any price to uncommitted buyers without causing firm 1 to try to undercut it. Firm 2 will thus pick the price that maximizes its profits for both the combination of its committed buyers and these uncommitted buyers. The tradeoff is the same as that faced by an incumbent without any rival if we adjust for the different ratios of uncommitted to committed buyers. Thus, given Proposition 2a, firm 2 will

charge uncommitted buyers $P_m + d\theta_2/(1-\theta_1)$ and charge committed buyers $P_m - d \cdot (1-\theta_1 - \theta_2)/(1-\theta_1)$, for an average price of P_m .

If $d[\theta_1/(1-\theta_2)]^{1/2} < P_m - C$, then the inequality will be satisfied only if P_r is above or below the respective quadratic roots, which we can simplify as:

$$P_m + \theta_1 d/(1-\theta_2) \pm [(1-\theta_1-\theta_2)((P_m-C)^2/(1-\theta_2) - \theta_1 d^2/(1-\theta_2)^2)]^{1/2}$$

The solutions above the upper root can be rejected for reasons noted in prior proofs. Thus, firm 2 can charge up to

$$P_2^* = P_m + \theta_1 d/(1-\theta_2) - [(1-\theta_1-\theta_2)((P_m-C)^2/(1-\theta_2) - \theta_1 d^2/(1-\theta_2)^2)]^{1/2}$$

and be able to sell to all the uncommitted buyers without firm 1 undercutting its price to uncommitted buyer. Because $\theta_1 > \theta_2$, and this price is the price at which firm 1 just breaks even between selling to the uncommitted buyers and selling to θ_1 buyers at the monopoly price, then it must be the case that this price is more profitable to firm 2 than forgoing the uncommitted buyers and selling to θ_2 buyers at P_m .

If $P_2^* > P_m + d\theta_2/(1-\theta_1)$, then firm 2 will charge $P_m + d\theta_2/(1-\theta_1)$ to uncommitted buyers since that price will earn firm 2 more profits than a higher price. This will be the case only if $P_2^* > P_m + d\theta_2/(1-\theta_1)$, which unfortunately does not simplify nicely. Otherwise, firm 2 will charge P_2^* to uncommitted buyers. Because every term in P_2^* makes it larger than P_r^* , it must be true that $P_2^* > P_r^*$.