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Derivatives Trading and Negative Voting

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Abstract

This paper exposits a model of parallel trading of corporate securities (shares, bonds) and derivatives (TRS, CDS) in which a large trader can sometimes profitably acquire securities with their corporate control rights for the sole purpose of reducing the corporation's value and gaining on a net short position created through off-setting derivatives. At other times, the large trader profitably takes a net long position. The large trader requires no private information beyond its own trades. The problem is most likely to manifest when derivatives trade on an exchange and transactions give blocking powers to small minorities, particularly out-of-bankruptcy restructurings and freezeouts.

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1 Introduction

Securities regulators, practitioners, and legal commentators worry that derivatives may provide shareholders and creditors incentives to destroy value in their corporation.[1] The basic concern is that if shareholders or creditors own a sufficient amount of off-setting derivatives such as put options, total return swaps (TRS), or credit default swaps (CDS), any losses on their shares or debt will be more than off-set by the corresponding gains on their derivatives ("over-hedging"). In this case, shareholders and creditors benefit by using the control rights inherent in their shares or debt to reduce the corporation's value ("negative voting"). An important question that is generally not considered, however, is whether it would ever be profitable for shareholders or creditors to acquire so many derivatives in the first place. After all, any gains to shareholders and creditors come at the expense of their counterparties on their derivative contracts. These counterparties would therefore prefer not to sell the derivatives, or only at a price that compensates them for the future payouts, thus depriving shareholders and creditors of any profit in the overall scheme.[2]

This paper shows that over-hedging and negative voting can indeed be profitable with a minimal and realistic degree of investor heterogeneity and asymmetric information. The paper presents a model of parallel markets for corporate securities (shares, bonds) and deriva-

[1] Regulators: See, e.g., Securities and Exchange Commission, Concept Release on the U.S. Proxy System, 75 Fed. Reg. 42,982, 43,017-20 (July 22, 2010); Committe of European Securities Regulators, Public Statement of the Market Participants Consultative Panel CESR/10/567 (July 5, 2010), at 3-4. Practitioners: See, e.g., Soros (2010) and Sender (2009) (quoting from David Einhorn's letter to investors). Commentators: See in particular Martin and Partnoy (2005) and Hu and Black (2006, 2007, 2008).

[2] Here and throughout, the article refers to the large trader as "buying" a derivative and its counterparties as "selling" it. This terminology only serves to clarify the exposition. It will not always match the specific terminology employed by the real-world markets trading such contracts.

tives in which a large, strategic trader interacts with liquidity traders and competitive market makers. As in the standard model of Kyle (1985), market makers cannot observe the large trader's orders directly, and cannot always infer them from aggregate order flow because of fluctuating liquidity trades. Market makers therefore cannot always predict how control rights will be exercised if the large trader only over-hedges some of the time. Prices reflect some probability of negative voting, allowing the large trader to benefit from its private information about its own trades and expected vote. The large trader benefits at the expense of liquidity traders, whose trades provide camouflage to the large trader. Other stakeholders of the corporation suffer collateral damage. Institutional and legal conditions attenuate the problem in many settings, but not in out-of-bankruptcy restructurings and freezeouts.

The large trader exploits private information about the payoff uncertainty that the large trader itself creates. The large trader can on average buy low and sell high relative to expected payoffs because the large trader itself will tilt the payoffs in the direction favorable to it. Prices cannot always anticipate this because market makers do not know in which direction the large trader will tilt. The large trader does not need, and in the model does not have, any superior information about fundamentals or liquidity trades. In particular, other security holders, or at least a majority of them, know just as well how to maximize firm value using their control rights. This has two important implications. First, the large trader's intervention unambiguously reduces welfare. Second, the derivatives market is very vulnerable to such parasitism because it does not require any special expertise beyond the ability to execute large trades.

The key assumption permitting such parasitism is that counterparties (market makers) cannot perfectly observe the large trader's positions and its concomitant incentives for exercising its control rights. This assumption is congenial to anonymous exchange trading and central clearing, which has long been the standard for equity options and is now generally mandated by the Dodd-Frank Act. But even when trading occurs over-the-counter (OTC), traders' positions and strategies are confidential and remain largely hidden from their counterparties (e.g., Avellaneda and Cont 2010 for the CDS market). To be sure, any participant in an OTC market knows the identity of its direct counterparty, at least post-trade. But since dealers routinely enter into chains of hedging transactions (e.g., Stulz 2010), the ultimate buyer of protection will usually be unaware of the identity of the ultimate seller, and vice versa.[3] In addition, investors can conceal their overall position even from their direct counterparties by splitting trades among many of them. The assumption that positions are at least partially unobservable to derivative counterparties thus seems realistic in many if not most settings.

To focus on the main point, the present paper does not explicitly model the benefits that liquidity traders derive from the derivative market, notably liquidity, diversification, or simple hedging (for such a model, see, e.g., Oehmke and Zawadowski 2014). The model presumes such benefits, for otherwise there would be no point for liquidity traders to trade derivatives in the first place. Issuers indirectly gain from such benefits as well, as liquidity traders' benefits reduce issuers' cost of external finance. There is no guarantee that these benefits will outweigh the harm from negative voting, however, as dispersed liquidity traders do not internalize effects on one another and on entrepreneurs (firms). Empirically, the effect of derivatives on firm financing remains mostly an open question. For CDS, Ashcraft and Santos (2009) find that CDS increase credit spreads, but Saretto and Tookes (2013) find that CDS increase leverage and maturity. The only point of the present paper is that over-

[3] Chen et al. (2011) report that dealers often do not hedge large trades right away but only in the course of several days. Unless default of the reference entity is imminent, however, this does not change the basic point here.

hedging and negative voting will make the derivatives market less beneficial than it could be. For example, the present paper predicts that firms with traded CDS will experience more (inefficient) bankruptcies, as documented in Peristiani and Savino (2011) and Subrahmanyam et al. (2014).

2 Relationship to the Prior Theoretical Literature

The distinguishing feature of the present model is that the large trader has private information about its own trades, but nothing else. The present paper shows that this is all it takes to generate a trading profit if cash flow rights and control rights can be decoupled using derivatives. Section 5.1 explains why such decoupling is easier in practice with derivatives than with other devices.

The leading paper on the interaction of derivatives and control rights (Bolton and Oehmke 2011) assumes that the sellers of derivatives (CDS) can observe buyers' full positions. This forces the buyers to internalize most of the harm from negative voting; they do not profit from trading as such. Creditors nevertheless acquire CDS to increase their bargaining power in renegotiation with their debtor. Like other commitment devices, this reduces the incidence of strategic default and thereby increases the debt capacity of the firm. In order to extract rents from the debtor, however, buyer-creditors may overinsure relative to the first best. In an extension of the model, Campello and Matta (2012) show that debtors may counter creditors' incentives to overinsure by choosing inefficiently riskier projects. Bolton and Oehmke and Campello and Matta thus also predict that introduction of CDS leads to a higher frequency of (inefficient) bankruptcy. Unlike in the present paper, however, this cost is fully or partially offset by the positive effect of commitment on debt capacity. All models agree that over-hedging is harmful. Empirically, only the present model predicts that the detrimental effect will increase with the liquidity of the CDS market.[4]

Brav and Mathews (2011) analyze a model with very similar assumptions and conclusions as the present paper. They frame their model as a model of *equity* trading and voting, including short sales and stock borrowing, by an *informed* large trader.[5] But their model has a more general interpretation as trading in securities and offsetting derivatives. In particular, Brav and Mathews note that borrowing stock in their model may be shorthand for the simultaneous acquisition of securities and offsetting derivatives, as in the present paper's model. If the large trader can borrow enough shares, then the large trader's equilibrium strategy in Brav and Mathews (proposition 6) is analogous to the one in the present paper, and always reduces welfare unless other shareholders are completely uninformed about the "right" vote. Besides clarifying the connection to derivatives trading, the present paper generalizes the notion of "enough" borrowing or hedging through a flexible voting threshold and trading cost.[6] Most importantly, the present paper clarifies that the large trader does not require *any* superior information relative to other security holders.

This deliberate focus on an uninformed large trader also distinguishes the present paper from other models of market manipulation. In most models of manipulation, a large trader can earn a profit only because the trader sometimes does have superior information about, or abilities to increase, firm value, such that its involvement is beneficial *ex ante* (e.g., Goldstein

[4]Subrahmanyam et al. (2014, section 3.4.1) find that the bankruptcy-increasing effect of CDS increases with the amount of CDS contracts outstanding. This might also be predicted by Bolton and Oehmke (2011), however, to the extent that the contracts are held as insurance by lenders. Subrahmanyam et al. do not reveal magnitudes of CDS outstanding by firm.

[5]This may explain why the derivatives literature has not taken note of Brav and Mathews (2011). For example, neither the survey of Bolton and Oehmke (2013) nor the empirical work of Subrahmanyam et al. (2014) cite Brav and Mathews (2011) on negative voting.

[6]In the most general formulation of their model, Brav and Mathews (2011) flexibly allow for a cost of borrowing shares. They then analyze a special case, however, in which borrowing shares is costless up to some number of shares, and prohibitively expensive beyond that.

and Guembel 2008; Collin-Dufresne and Vos 2013). In the classic models of Kyle (1985) and Kyle and Vila (1991), the large trader also observes others' liquidity trades before placing its own order; this is the large trader's only advantage in Kyle (1984). In the present paper, the large trader has neither type of superior information or ability.

The trading environment in the present model closely follows Kyle and Vila (1991), who in turn build on the model of a futures squeeze in Kyle (1984). (Following Brav and Mathews (2011), however, the present paper drops the assumption that the large trader observes the liquidity trades before placing its market orders.) Camouflaged by noise traders, a strategic trader can earn a trading profit if that trader has the power to influence the corporation's value upwards (downwards) when going long (short). As Kyle and Vila note, it is not crucial that the large trader's influence (in Kyle and Vila, a takeover) improves the value of the corporation relative to the baseline of no large trader. Any power to influence the value of the corporation produces opportunities for trading profits. Of course, the coupling of control rights to cash flow rights usually ensures that such power only inheres in those who have incentives to increase firm value. This safeguard fails when share borrowing (Brav and Mathews 2011) or hedging with derivatives (this paper) uncouple control from cash flow (Hu and Black 2006, 2007, 2008).

3 Model

The model features two types of traded assets (securities and derivatives) and three types of market participants (hedge fund, liquidity traders, and market makers). Their interaction unfolds as follows:

1. The hedge fund and liquidity traders submit their market orders.

2. The market makers observe only *net* market demand (order flow), which combines the hedge fund's and the liquidity traders' orders. Based on this observation, market makers update their beliefs about the expected value of the securities and derivatives. They fill all orders at prices equal to expected value.

3. Security holders choose between two actions by some voting mechanism, and payoffs are realized.

3.1 Traded assets

The two traded[7] assets are securities and derivatives. Holdings etc. of securities will be denoted by X, while holdings etc. of derivatives will be denoted by Y.

Securities and derivatives have perfectly negatively correlated payoffs: if the security pays v, the derivative pays $1 - v$. Consequently, the derivative can be interpreted as an insurance claim on the security. In particular, if the security were a bond, the derivative could be a credit default swap; if the security were a share, the derivative could be a total equity return swap.

The number of securities is normalized to one (of which infinitesimal divisions are traded). The derivative is a synthetic asset; hence its net supply is zero but unlimited amounts can be sold and bought. Short-selling is allowed for both derivatives and securities.

The security payoff v depends on a binary choice between two actions, which is determined by a vote of the security holders.[8] For example, if the security is a bond, the choice could

[7] "Trading" does not need to be understood literally in this model. In particular, it is possible that the derivative is a contract that is sold over the counter. What matters is that there be an active market for the contract in which various parties can act as sellers or buyers, which is true for many derivative markets.

[8] To keep things simple, the paper does not explictly model reasons for providing voting rights. Introducing such reasons would be easy and would mesh well with the mechanism of the present model. For example, voting rights would be useful if opportunistic behavior by managers or shareholders, as the case may be, were invisible to courts (perhaps because of institutional constraints) but not to investors (Tirole 2006, ch.

be whether or not to agree to a proposed restructuring; if the security is a share, it could be whether or not to agree to a merger. The payoffs are normalized to $v = 1$ when the "right" decision is taken, and $v = 0$ when the "wrong" decision is taken.

Each security provides one vote; derivatives do not provide any votes.

3.2 Voting

The model assumes that other security holders, or at least a requisite majority of them, always vote for the "right" decision. This is of course the only rational choice for any informed and *unhedged* security holder. Only the hedge fund will consider voting for the "wrong" decision (which it will rationally do if and only if it owns more derivatives than securities).

The hedge fund's ability to block the "right" decision depends on whether the hedge fund's security-holding x is above some voting threshold. The voting threshold is assumed to be a random variable distributed on $[0, 1]$ according to the continuous cdf $F(\cdot)$ with $F(0) = 0$ and $F(1) = 1$. Let $\underline{x} \equiv \max\{x|F(x) = 0\} \geq 0$. The voting threshold is assumed to be independent of other exogenous variables in the model, and it will be independent of any trading activity since it will be only revealed after all trading occurs.

The randomness captures variation in voter participation, uncertainty arising from legal concerns, different formal thresholds for different types of decisions, etc. In particular, the voting threshold may also capture the fact that some minority of security holders is either conflicted or ignorant about the "right" decision, such that the hedge fund can win with less than the nominally required percentage of the voting rights (e.g., 50%).

10). The "bad" decision could then be interpreted as support for the opportunistic behavior.

Similarly, adding payoff uncertainty beyond the outcome of the vote would provide motivation for the existence of liquidity trades (hedging), and would not change the model except for the addition of expectations operators.

3.3 Market Participants (and Prices)

There are three types of risk-neutral market participants: liquidity traders, one hedge fund, and competitive market makers.

3.3.1 Liquidity traders

The liquidity traders do not act strategically. They exogenously trade quantities $\tilde{x}$ and $\tilde{y}$ of securities and derivatives, respectively, where positive numbers indicate that the liquidity traders are buying, and negative numbers indicate that they are selling. These trades are not sensitive to price, and the source of these trades is not modelled. To motivate these trades and their price insensitivity, one may think of large institutional investors and their regulatory constraints. For example, certain pension funds might be forced to sell bonds following a credit downgrade of the borrower. Similarly, financial institutions might be forced to purchase credit default swaps on certain bonds they hold. Or one may think of mutual funds having to liquidate part of their portfolio to meet redemption requests.

Liquidity traders' demand for derivatives, $\tilde{y}$, is stochastic (keeping in mind that the "demand" can be negative). With probability $(1 - \lambda)$, the demand is low ($\tilde{y} = \underline{y}$), while with probability λ, demand is high ($\tilde{y} = \bar{y}$). Define the difference between these demand realizations as $\delta \equiv \bar{y} - \underline{y} > 0$.

For simplicity, liquidity traders' demand for securities, $\tilde{x}$, is assumed to be constant.

3.3.2 Hedge fund

The hedge fund does act strategically. Initially, the hedge fund does not hold any securities or derivatives. It purchases quantities x and y of securities and derivatives, respectively, taking into account the effect of its trades on the price (as explained below), its own voting power, and its own voting incentives.

The hedge fund has no superior information about liquidity traders' demand (unlike in Kyle 1984 and Kyle and Vila 1991), the value-maximizing vote (unlike in Brav and Mathews 2011), or exogenous security/derivative payoffs. The only private information that the hedge fund has is its trades. As explained in the previous subsection, however, holding $x > \underline{x} > 0$ securities gives the hedge fund the voting power to implement the "wrong" decision with probability $F(x) > 0$. Of course, the hedge fund will rationally always (never) do so if $y > x$ ($y < x$) ($x = y \neq 0$ will never occur because this would be unprofitable for the hedge fund, see proof of Lemma 1).

The hedge fund incurs a trading cost $C(x, y)$ with $C(0, 0) = \min_{x,y} C(x, y) = 0$, $sign[\partial C(x, y)/\partial x] = sign(x)$ and $sign[\partial C(x, y)/\partial y] = sign(y)$. It subsumes several costs that the hedge fund faces in the real world, in particular brokerage and financing fees. The latter arise in securities trades because hedge funds need to maintain margin and pay margin fees. All costs are relative to the value impact of the vote, which has been normalized to 1.

3.3.3 Market makers (and prices)

Market makers observe and absorb any excess demand $(\hat{x}, \hat{y}) \equiv (\tilde{x} + x, \tilde{y} + y)$. Market makers are assumed to be risk-neutral, infinitesimally small, and in perfect competition with one another. As a result, in equilibrium they purchase or sell $(\hat{x}, \hat{y})$ at prices that equal expected value (Kyle 1984, 1985; Kyle and Vila 1991). That is, market makers' equilibrium behavior is fully characterized by the prices of securities P_x and derivatives P_y, which are in turn pinned down by the expected derivative payoff θ:

$$P_y(\hat{x}, \hat{y}) = 1 - P_x(\hat{x}, \hat{y}) = \theta(\hat{x}, \hat{y}) \equiv \Pr(v = 0|\hat{x}, \hat{y}) \in [0, 1]. \tag{1}$$

The first two equalities in equation 1 follow simply from the absence of arbitrage. The restriction imposed by market maker rationality and competition is that $\theta(\hat{x}, \hat{y})$ is a probability belief compliant with Bayes' rule that the security will pay zero, conditional on observed net demand of securities and derivatives, $(\hat{x}, \hat{y})$.

Given the assumptions about voting and payoffs, the probability that the security will pay zero is equal to the probability that the hedge fund will vote for the "bad" decision, multiplied by the probability that the hedge fund will be able to win the vote, $F(x)$. The conditioning is on net demand (or "order flow") $(\hat{x}, \hat{y})$, however, rather than hedge fund holdings (x, y). This embodies the key assumption that the market makers only observe the former, not the latter. Market makers merely form beliefs about (x, y) upon observing the net demand of securities $(\hat{x}, \hat{y})$. One final restriction that will be imposed by the Perfect Bayesian Equilibrium concept is that market makers expect the hedge fund to vote rationally, given its holdings (x, y).

3.4 Remarks on the model setup

The model assumes that the hedge fund is able to acquire any amount x of securities that it desires. In particular, this ability does not depend on the amount $\tilde{x}$ supplied by liquidity traders. In reality, it may often be difficult or impossible to acquire large blocks of shares or bonds. There are, however, many situations in which exogenous sales of securities $\tilde{x}$ are large, and the reader may restrict the applicability of the model to such situations. For example, many institutional investors sell all their holdings of a bond if the bond's credit rating drops below investment grade (Da and Gao 2009). Moreover, in the model, an upper bound on the amount x of securities that the hedge fund can acquire would not change the hedge fund's strategy, and the only change from the results presented below would be that the hedge fund might have to settle for the upper bound rather than its preferred, higher position (i.e., one would observe corner solutions).

Relatedly, the assumption that large purchases have no price impact beyond the probability update by the market makers is not literally true. To go back to the acquisition of securities, it would presumably become harder and harder to find additional securities as the hedge fund's position grows, and this would be reflected in higher trading costs for larger positions. Mathematically, however, the assumption of a trading cost for the hedge fund has the same effect as assuming an additional price impact for larger blocks, so that nothing substantive hinges on the assumption of constant prices conditional on the updated probability.

Finally, it is a strong assumption that only the one hedge fund is ready to buy large stakes, and to consider over-hedging its securities position and to vote the securities for the "wrong" decision. This excludes, first, that any of the other market participants in the model, namely individual market makers and liquidity traders, who must hold the remaining supply of securities, would ever hold more derivatives than securities, or if they did, that they nevertheless voted for the "right" decision. One justification for this could be institutional, namely that reputational concerns or sheer apathy prevent market makers and liquidity traders to vote for the "wrong" decision, or to over-hedge their securities position in the first place. One can also view the model as an illustration of how "negative voting" can interfere with the smooth operation of a liquid, perfect market for securities; in this view, the true equilibrium would be more complicated, and the model merely illustrates why the market cannot be perfect.

Second, the above assumptions rule out strategic competition with a second large player. For example, one can imagine a second hedge fund trying to share the spoils, or to buy up enough of the security at a low price to prevent the first hedge fund from ever winning a vote for the "wrong" decision. From a practical point of view, however, adding another strategic player would complicate the model but not eliminate the underlying economic problem. For example, even if the security were trading at deep discount because of the hedge fund's presence, another large player could not necessarily profitably intervene by buying up the entire supply of securities if and because that second large player incurs similar financing cost as the first hedge fund. Moreover, even if the second large player could profitably do this, then in expectation the price of the security would re-adjust to 1, so that the strategy would end up being not profitable after all. Corollary 2 below states this argument formally.

4 Equilibrium

4.1 Equilibrium in the General Case

To characterize the model's Perfect Bayesian Equilibria (PBE), it remains to determine the market makers' equilibrium inference function $\theta(\hat{x}, \hat{y})$ and the hedge fund's trading strategy, i.e., the probability distribution $\sigma(\cdot, \cdot)$ over hedge fund trades $(x, y) \in \mathbb{R}^2$. All other actions are straightforward and have already been determined above: liquidity traders' trades are exogenous (see section 3.3.1), competitive market makers absorb all market demand at prices fully determined by $\theta(\hat{x}, \hat{y})$ (see section 3.3.3), and hedge fund voting is trivially determined by its holdings (x, y) (see section 3.3.2).

The inference function is critical. It describes how market makers will react to the uncertainty about payoffs generated by the hedge fund's voting, which in turn cannot always be predicted from the only information available, namely net market demand.

Lemma 1 *One inference function sustaining all possible equilibria is*

$$\theta_{eq}(\hat{x},\hat{y}) \equiv \begin{cases} 0 & \textit{if } \hat{x}-\tilde{x} \leq \underline{x} \textit{ or } \hat{y}-\underline{y} < \hat{x}-\tilde{x} \\ F(\hat{x}-\tilde{x}) & \textit{if } \underline{x} < \hat{x}-\tilde{x} < \hat{y}-\bar{y} \\ \max\{0, \min\{F(\hat{x}-\tilde{x}), \theta^*(\hat{x},\hat{y})\}\} & \textit{otherwise} \end{cases}, \qquad (2)$$

where $\theta^*(\hat{x},\hat{y}) \equiv \frac{F(\hat{x}-\tilde{x})(1-\lambda)\left[\left(\hat{y}-\underline{y}\right)-(\hat{x}-\tilde{x})\right]+C(\hat{x}-\tilde{x},\hat{y}-\bar{y})-C\left(\hat{x}-\tilde{x},\hat{y}-\underline{y}\right)}{\lambda[(\hat{x}-\tilde{x})-(\hat{y}-\bar{y})]+(1-\lambda)\left[\left(\hat{y}-\underline{y}\right)-(\hat{x}-\tilde{x})\right]}$. *The inference is unique for any* $(\hat{x},\hat{y})$ *actually observed in equilibrium.*

Proof. Market demand always fully reveals the hedge fund's voting power because liquidity traders' demand for securities $\tilde{x}$ is non-stochastic. If the hedge fund is powerless ($x = \hat{x}-\tilde{x} \leq \underline{x}$), the "right" decision will be adopted with certainty, and θ must be equal to zero. Moreover, some market demand realizations do fully reveal the hedge fund's incentives, i.e., that the hedge fund is long ($\hat{y}-\underline{y} < \hat{x}-\tilde{x}$) or short ($\hat{x}-\tilde{x} < \hat{y}-\bar{y}$), as the case may be. In those cases, the PBE assumption of sequentially rational behavior on- and off-equilibrium implies that the hedge fund votes for the "right" or the "wrong" decision, respectively, and θ must thus be equal to 0 or $F(\hat{x}-\tilde{x})$, respectively. The first two lines of the definition of θ_{eq} state this formally.

The third line deals with the interesting case in which market demand does not reveal if the hedge fund is short ($x \leq y$) or long ($x \geq y$). As a result, θ could lie anywhere between 0 and $F(\hat{x}-\tilde{x})$, depending on market makers' priors. In equilibrium, however, market makers' priors must match the hedge fund's actual strategy $\sigma(\cdot,\cdot)$. This rules out any equilibrium inference except θ^*. For example, if the market makers thought that the hedge fund always goes long, they would trivially infer $\theta = 0$. But then the hedge fund could do much better going short, snapping up derivatives for free (if the noise realization is favorable to the hedge fund, which happens with probability $1-\lambda$). The inverse would happen if market makers thought the hedge fund always goes short. For intermediate priors, θ will be in between these extremes. As long as either the long or the short trade is more profitable than the other for such θ, however, the hedge fund will exclusively place the more profitable trade, such that the intermediate prior turns out to be wrong. The intermediate prior can be correct, and equilibrium reached, only when the expected profits from short and long trades are equal. θ^* achieves just that. That is, θ^* solves[9]

$$\lambda\theta\left[(\hat{x}-\tilde{x})-(\hat{y}-\bar{y})\right]-C\left(\hat{x}-\tilde{x},\hat{y}-\bar{y}\right)=(1-\lambda)\left[F\left(\hat{x}-\tilde{x}\right)-\theta\right]\left[\left(\hat{y}-\underline{y}\right)-(\hat{x}-\tilde{x})\right]-C\left(\hat{x}-\tilde{x},\hat{y}-\underline{y}\right).$$

Trades that could result in $\theta^*(\hat{x},\hat{y})\notin\left[0,F\left(\hat{x}-\tilde{x}\right)\right]$ cannot be part of an equilibrium, for as already shown, equilibrium requires $\theta=\theta^*$ and $\theta\in\left[0,F\left(\hat{x}-\tilde{x}\right)\right]$. It remains to be shown that θ_{eq} can also be used off equilibrium. This is left for the appendix. ■

Given the inference function θ_{eq}, the hedge fund's optimization problem is trivial and leads immediately to

Proposition 1 *The hedge fund's equilibrium (expected) profits are* $\max\{0,\pi^*\}$*, where*

$$\pi^* \equiv \max_{\omega}\pi(x,y), \tag{3}$$

$$\omega \equiv \left\{(x,y)\,|x>\underline{x},\ y\in[x-\delta,x]\right\}, \tag{4}$$

$$\pi(x,y) \equiv \frac{F(x)\lambda(1-\lambda)(x-y)(y+\delta-x)}{\lambda(x-y)+(1-\lambda)(y+\delta-x)} - \frac{\lambda(x-y)C(x,y+\delta)+(1-\lambda)(y+\delta-x)C(x,y)}{\lambda(x-y)+(1-\lambda)(y+\delta-x)}. \tag{5}$$

[9]The equation takes into account that the hedge fund will always incur trading costs $C(x,y)$, but will earn a trading profit only when the noise realization is favorable, i.e., hides its long or short trade, as the case may be. This happens with probability $\lambda(1-\lambda)$ for the long (short) trade.

The hedge fund's equilibrium strategies depend on π^**:*

(a) If $\pi^* < 0$*, the unique equilibrium is for the hedge fund not to trade at all* ($x = y = 0$)*.*

(b) If $\pi^* > 0$*, any strategy such that* $\sum_{(x^*,y^*)\in \arg\max_\omega \pi(x,y)} [\sigma(x^*, y^*) + \sigma(x^*, y^* + \delta)] = 1$ *and* $\sigma(x^*, y^*) > 0 \Rightarrow \frac{\sigma(x^*,y^*)}{\sigma(x^*,y^*+\delta)} = \frac{1-\lambda}{\lambda}\left[\frac{F(x^*)}{\theta^*(x^*+\tilde{x},y^*+\bar{y})} - 1\right] \; \forall (x^*, y^*) \in \omega$ *is an equilibrium; the equilibrium is unique if and only if* $\arg\max_\omega \pi(x, y)$ *is unique.*

(c) If $\pi^* = 0$*, any linear combination of (a) with strategy profile (b) is an equilibrium.*

Proof. The hedge fund will trade (if and) only if the maximum expected profits π^* from doing so are (strictly) positive. Only long trades $(x, y) \in \omega$ and associated short trades $(x, y + \delta)$ can achieve positive trading profits, as the hedge fund needs the power to influence the vote ($x > \underline{x}$) and the ability to hide its short/long position behind the noise some of the time ($y \in [x - \delta, x]$). For all other trades, market makers can infer security and derivative payoffs with certainty, erasing any trading profits. Given positive trading cost, the hedge fund will never place such other trades.

Expected hedge fund profits for the long trade $(x, y) \in \omega$ are $\lambda\theta_{eq}(x + \tilde{x}, y + \bar{y})(x - y) - C(x, y)$. For purposes of determining the trade that yields maximum non-negative profits, if any, one can simplify this expression by substituting θ^* for θ_{eq}, yielding $\pi(x, y)$. The reason is that expected profits are negative anyway at both θ_{eq} and θ^* for any trade that can result in $\theta^*(x + \tilde{x}, y + \tilde{y}) \notin [0, F(x)]$ (see the appendix, continuation of the proof of Lemma 1). By construction of θ^*, $\pi(x, y)$ is also equal to expected profits from the associated short trade $(x, y + \delta)$, provided they are non-negative.

The hedge fund will be indifferent between any long trades $(x^*, y^*) \in \arg\max_\omega \pi(x, y)$ and their associated short trades $(x^*, y^* + \delta)$. Equilibrium only obtains, however, if the relative frequency of any long trades and associated short trades actually placed matches that implicit in θ and Bayes' Rule, namely $\theta_{eq}(x^* + \tilde{x}, y^* + \bar{y}) \equiv \Pr(v = 0|\hat{x}, \hat{y}) = \frac{F(x^*)(1-\lambda)\sigma(x^*,y^*+\delta)}{(1-\lambda)\sigma(x^*,y^*+\delta)+\lambda\sigma(x^*,y^*)}$.

Rearranging the latter expression yields the restriction on $\frac{\sigma(x^*,y^*)}{\sigma(x^*,y^*+\delta)}$ in part (b) of the proposition. ■

Corollary 1 *There always exists a non-zero cost function $C(\cdot,\cdot)$ such that a mixed equilibrium exists.*

Proof. If $C(x,y)=0\ \forall(x,y)$, then $\pi^*=\max_\omega \frac{F(x)\lambda(1-\lambda)(x-y)(y+\delta-x)}{\lambda(x-y)+(1-\lambda)(y+\delta-x)}>0$. The proof then follows by continuity of $\pi(\cdot,\cdot;C(\cdot,\cdot))$ in $C(\cdot,\cdot)$. ■

The equilibrium of the model depends principally on the hedge fund's trading cost relative to the value impact of the decision and the hedge fund's ability to influence it. If the costs are large, they outweigh any trading gains, such that abstention ($x=y=0$) is the hedge fund's only viable strategy.

On the other hand, if the hedge fund's costs are low, it always pays for the hedge fund to try its luck. To the extent noise trades camouflage the hedge fund's trade, market makers cannot be sure if the hedge fund is long or short. The market makers must therefore choose some intermediate price. At this intermediate price, the hedge fund can turn a trading profit. Of course, this only works because the hedge fund sometimes votes for the "bad" decision. The upshot is that negative voting will be a problem whenever trading cost are low. This is true regardless of the number of hedge funds in the market:

Corollary 2 *Regardless of the number of hedge funds, the equilibrium $x=y=0$ exists if and only if $\pi^*\leq 0$.*

Proof. If $\pi^*\leq 0$ and market makers' inference function is θ_{eq}, no individual hedge fund can profitably deviate by trading, while market makers correctly infer that the possibility of the "wrong" decision being adopted is zero. Conversely, if $\pi^*>0$, then any one hedge fund would be better off trading regardless of the inference function (recall that θ_{eq} minimizes the maximum possible trading profit), so $x = y = 0$ cannot be an equilibrium. The presence of other non-trading hedge funds is irrelevant to this argument. ■

4.2 Equilibrium with quadratic cost, uniform voting threshold distribution, and symmetric liquidity trades

To gain further insight into the properties of the model's equilibrium, consider the special case

$$C(x,y) = \frac{c}{2}\left(x^2 + y^2\right), \tag{6}$$

$$F(x) = \max\{0, \min\{x, 1\}\}, \tag{7}$$

$$\lambda = \frac{1}{2}, \tag{8}$$

where $c > 0$. Using Proposition 1, it is easy to verify that the hedge fund's optimal securities trade in this case is

$$x^* = \frac{\delta}{16c} \tag{9}$$

together with either of

$$y_1^* = x^* - \frac{\delta}{2}, \text{ or} \tag{10}$$

$$y_2^* = x^* + \frac{\delta}{2}, \tag{11}$$

provided that $0 < \delta \leq 16c \leq 2\sqrt{2}$ (for larger c, expected profits from trading would be negative, so abstention would be optimal; for larger $\delta \leq \frac{1+\sqrt{1-32c^2}}{2c}$, the corner solution

$x^* = 1$ and $y^*_{1,2} = 1 \pm \frac{\delta}{2}$ entails).

Not surprisingly then, the hedge fund becomes more aggressive (x^* increases) as the market becomes noisier and hence provides more camouflage (δ increases), and as the costs of trading decrease (c decreases). This translates into a higher unconditional probability that the "wrong" decision will be adopted. With symmetric noise ($\lambda = 1 - \lambda = \frac{1}{2}$) and a unique trading equilibrium, this probability is

$$\begin{aligned} \Pr(v = 0) &= F(x^*)\,\sigma(x^*, y_2^*) \\ &= \frac{F(x^*)(1-\lambda)\,\sigma(x^*, y_2^*)}{(1-\lambda)\,\sigma(x^*, y_2^*) + \lambda\sigma(x^*, y_1^*)} \\ &= \theta_{eq}\left(x^* + \tilde{x}, y_2^* + \underline{y}\right) \\ &= \frac{\delta}{16c}\left(\frac{1}{2} - 2c\right), \end{aligned} \tag{12}$$

where the third step follows from the equilibrium condition for σ in Proposition 1(b).

This is increasing in the amount of "noise" or demand fluctuation δ, and decreasing in the trading cost c. The liquidity traders' trading losses $\frac{\delta^2}{128c}(1 - 4c)$ are also increasing in the amount of "noise" or demand fluctuation δ, and decreasing in the trading cost c. At least in this special case, the model therefore shows that increasing market volume (δ) and decreasing trading costs (c), while generally viewed as positive, aggravate the problem of over-hedging and negative voting.

5 Discussion

The model abstracts from many economic and legal constraints that curtail over-hedging and negative voting. In fact, nothing in the mathematics of the model distinguishes the "derivative" y from other negatively correlated investments. There are real world constraints, however, that restrict the model mostly to derivatives, in particular in connection with out-of-bankruptcy restructurings and freezeouts.

5.1 Derivatives vs. other hedges

The first question to ask is why over-hedging is specifically a problem of derivatives. In principle, over-hedging can occur with any investment that is negatively related to the shares or debt at issue. Some examples include parallel investments in competing firms, parallel investments in both the acquiror and the target of a merger transaction, parallel investments in different securities of the same firm, or selling short some amount of a security while holding on to a smaller amount. These other investments, however, are either not perfectly correlated with the shares or debt and hence represent higher risk, or they are only available in particular situations, or they are available only in small quantities or at higher cost, or all of the above. These shortcomings severely limit the facility, frequency, and extent to which these other investments could enable over-hedging.

By contrast, derivatives are designed to be perfectly (negatively) correlated with the payoffs of shares or debt. Many derivatives markets, such as those for equity options, are highly liquid at all times. Even those that are not, such as single-name CDS, exhibit liquidity spikes around key events when over-hedging is most profitable, such as changes in credit outlook for CDS (Chen et al. 2011). In general, the rapid growth of derivatives markets over the last decade or two means that derivatives are in principle available in high volumes at low prices (spreads). It is not unusual that the face amount of derivatives written on the shares or debt of an individual company exceeds the amount of shares or debt issued by that company (Stulz 2010).

5.2 Required control stakes

Even if derivatives are available, it might seem an implausible proposition to acquire and over-hedge a voting majority (51%) of a corporation's shares or publicly traded debt. Such quantities of shares/debt and derivatives may not even be available on the market, and if they were, could hardly be acquired in secret and without strongly affecting prices. For shares, acquiring such quantities would also trigger disclosure and other obligations under corporate and securities laws and, in most U.S. corporations, the "poison pill."[10]

Many relevant decisions, however, can be affected by much smaller percentages of shares or debt. One possibility is that an over-hedged shareholder or creditor joins forces with some other constituency pursuing interests other than maximizing share or debt value, such as a corporate insider.

More importantly, some corporate decisions provide blocking power to relatively small minorities. In particular, out-of-bankruptcy restructurings tend to set acceptance thresholds around 95%, providing blocking rights to 5% or even less of the outstanding debt. Importantly, restructurings that do not bind all holders, such as a standard debt exchange, do not constitute a credit event under the prevailing CDS documentation and hence do not trigger settlement of the CDS.[11] Practitioners suspect that over-hedging and negative voting are common in out-of-bankruptcy restructurings.[12] In addition to restructurings, small stakes

[10] See in particular section 13(d) of the Exchange Act, which requires disclosure of equity ownership stakes above 5% and arguably of any hedges relating thereto (cf. discussion in the next section), and section 16 of the Exchange Act, which forces 10% shareholders to disclose their hedges (sec. 16(a)) and disgorge short-swing trading profits (sec. 16(b)). Moreover, section 16(c) prohibits 10% shareholders from engaging in short sales, and rule 16c-4 explicitly extends this to over-heding using puts, *while* they are a 10% shareholder, thus outlawing any strategy of acquiring a voting stake first and over-hedging it later (but not the other way around).

[11] Cf. Art. 4.7(a) of the 2003 ISDA Credit Derivative Definitions, as amended by the "Small Bang Protocol," available at http://www.isda.org/publications/pdf/July-2009-Supplement.pdf.

[12] Author's conversation with the head of the restructuring practice of a major New York law firm.

may be sufficient to affect freeze-out mergers. Majority-of-the-minority conditions in freeze-outs can give blocking rights to as little as a few percent of the corporation's outstanding equity.[13]

5.3 Legal constraints

At least in the U.S., current law only provides incomplete protection against over-hedging and negative voting. With respect to formal voting, U.S. law arguably provides some protection, but enforcement may be hindered by a lack of disclosure. Outside of formal voting, negative voting and over-hedging are arguably entirely unregulated.

Under §1126(e) of the U.S. Bankruptcy Code, bankruptcy judges have the power to disallow votes by a creditor "whose acceptance or rejection of [a reorganization] plan was not in good faith." In a recent decision, the U.S. Bankruptcy Court for the Southern District of New York held, obiter, that this provision would justify disqualification of votes by over-hedged creditors.[14] Bankruptcy courts will generally not know, however, if creditors are over-hedged. Current bankruptcy rules do not require disclosure of hedging transactions relating to debt claims filed in the bankruptcy.

For shares, the Delaware Supreme Court recently recognized "[a] Delaware public policy of guarding against the decoupling of economic ownership from voting power."[15] There is thus reason to believe that Delaware courts would at least seriously consider a remedy against voting by over-hedged shareholders. Section 13(d)(1)(E) of the Exchange Act arguably requires that owners of 5% or more of a corporation's stock disclose hedging transactions,

[13] Such majority-of-the-minority conditions have been imposed by Delaware courts as a condition for obtaining favorable review of the consideration paid to the minority, see In re Cox Communications Inc. Shareholders Litigation 879 A.2d 604 (2005); In re CNX Gas Corp. Shareholders Litigation, 4 A.3d 397 (Del. Ch. 2010).

[14] In re DBSD North America, Inc., 421 B.R. 133, 143 n. 44 (Bankr. S.D.N.Y. 2009).

[15] Crown Emak Partners, LLC v. Kurz, 992 A.2d 377, 387 n. 17 (Del. 2010).

but in practice market participants have not done so effectively. To address the enforcement problem, commentators have advocated stricter disclosure obligations. For example, Hu and Black (2006, 885) argue that voting by over-hedged shareholders or creditors above a threshold of 0.5% of a company's shares or debt should be reported.

Neither of these rules or proposals, however, deals with the exercise of control rights other than formal voting rights. In particular, no rule forces an over-hedged creditor to participate in a debt exchange, even if the over-hedging were publicly known. In freeze-out tender offers, the Delaware Chancery Court has excluded votes by hedged shareholders for purposes of a majority-of-the-minority condition.[16] These decisions are based on fiduciary duties of the board and parent shareholders, however, and it is not clear that they would extend to situations in which the hedged shareholder stands in opposition to the board and the parent. In particular, the Court has affirmed that even controlling shareholders are under no obligation to sell their shares, even if doing so might be beneficial to other shareholders or the corporation.[17]

6 Conclusion

This paper has shown theoretically that derivatives can create opportunities for purely value-reducing activity (over-hedging and negative voting) if derivative traders can conceal their overall positions from their counterparties. It has also argued that the institutional and legal conditions in the US are such that the threat of such parasitic activity seems real at least in out-of-bankruptcy restructurings and freezeout mergers. The threat is particularly acute

[16] See In re CNX Gas Corp. Shareholders Litigation, 4 A.3d 397, at 418 (Del. Ch. 2010); In re Pure Resources, Inc., Shareholders Litigation, 808 A.2d 421, at 426 and 446 (Del. Ch. 2002).

[17] Cf. In re Digex, Inc. Shareholders Litigation, 789 A.2d 1176, 1189-91 (Del. Ch. 2002) (noting that a controlling shareholder is free to block the sale of the controlled corporation to another bidder by not selling).

because the activity requires no superior information or abilities beyond private information about one's own trades.

This assessment of the role of derivatives is considerably less benign than that of other papers that have assumed no asymmetric information in the relationship between derivative counterparties, in particular Bolton and Oehmke (2011) and Campello and Matta (2012).[18] To be sure, all three papers agree that voting while net short is harmful and should be curtailed through regulation or contractual design of control rights. But such measures are more urgent in the anonymous trading environment of the present paper, which is the environment that regulatory reforms have been promoting.

[18] Brav and Mathews (2011) also explore a trade-off. But their trade-off is between having or not having an informed trader, rather than allowing or not allowing negative voting.

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A Proof of Lemma 1 (continued): Off Equilibrium

To complete the proof of lemma 1, it remains to be shown that $\theta_{eq}(\hat{x}, \hat{y})$ can w.l.o.g. be used also off-equilibrium when $\hat{x} - \tilde{x} > \underline{x}$ and $\hat{y} - \underline{y} \geq \hat{x} - \tilde{x} \geq \hat{y} - \bar{y}$ (in the other cases, θ_{eq} is unique for the reasons given in the main proof). In particular, the claim to be established here is that any equilibrium that can be supported by some inference $\theta_{off}(\hat{x}, \hat{y})$ can also be supported by $\theta_{eq}(\hat{x}, \hat{y})$.

To support the equilibrium, the inference $\theta_{off}(\hat{x}, \hat{y})$ must "deter" (i.e., make unprofitable) both off-equilibrium trades that could generate such $(\hat{x}, \hat{y})$. These trades are the long and short trades associated with $(\hat{x}, \hat{y})$: $\left(\hat{x} - \tilde{x}, \hat{y} - \underline{y}\right)$ and $(\hat{x} - \tilde{x}, \hat{y} - \bar{y})$. At θ^*, the long and short trades are equally (un-)profitable in expectation. The expected profit from the long trade is strictly increasing in θ, while that of the short trade is strictly decreasing in θ. It follows that if neither deviation is profitable at $\theta_{off}(\hat{x}, \hat{y}) \neq \theta^*(\hat{x}, \hat{y})$, then they are also both unprofitable at $\theta^*(\hat{x}, \hat{y})$.

Truncating $\theta^*(\hat{x}, \hat{y})$ such that θ_{eq} lies in $[0, F(\hat{x} - \tilde{x})]$ does not generate any profitable deviations. If $\theta^*(\hat{x}, \hat{y}) \notin [0, F(\hat{x} - \tilde{x})]$, then both long and short trades generating $(\hat{x}, \hat{y})$ are unprofitable in expectation even at the boundaries of the interval. Consider first $\theta^*(\hat{x}, \hat{y}) < 0$. Expected hedge fund profits for the long trade $\pi_L(\hat{x}, \hat{y}; \theta)$ are negative at $\theta = 0$. So we have for $\theta^*(\hat{x}, \hat{y}) < 0$

$$0 > \pi_L(\hat{x}, \hat{y}; 0) > \pi_L(\hat{x}, \hat{y}; \theta^*(\hat{x}, \hat{y})) = \pi_S(\hat{x}, \hat{y}; \theta^*(\hat{x}, \hat{y})) > \pi_S(\hat{x}, \hat{y}; 0),$$

where π_S is expected hedge fund profit for the short trade. The argument for $\theta^*(\hat{x}, \hat{y}) > F(\hat{x} - \tilde{x})$ is symmetric.