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Bundling and Quality Assurance*

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Abstract

With imperfect private monitoring, a firm selling two experience goods can increase both producer and consumer surplus by bundling. Bundling constrains consumers to buy two products, making consumers better informed and ensuring that they use tougher punishment strategies. Both increased monitoring and increased punishment benefit other consumers, so bundling overcomes a free-rider problem. The social value of bundling is even larger if consumers cannot attribute a negative signal to the specific product that generated it, or if one of the two goods is a durable and the other is a complementary nondurable. Our results are robust to mixed bundling.

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1 Introduction

Although tying and bundling have been viewed with suspicion by US courts – and are per se illegal in some circumstances – it is common for quality assurance to be used as an explicit efficiency defense in antitrust cases.[1] In the 1930s, General Motors successfully invoked quality assurance to defend their business practice of requiring their dealers to use only General Motors parts in the aftermarket service and repairs of their cars.[2] In the 1980s, Mercedez-Benz successfully argued that if their dealers could procure parts directly from independent suppliers rather than through Mercedes-Benz of North America (MBNA), then dealers would shirk on quality-control testing, free riding on the efforts of others, and deliver a substandard product to consumers. The court found "ample evidence to support a finding that the tying arrangement is a legitimate means of maintaining the quality of Mercedes replacement parts supplied by dealers, and thereby protecting the reputation of the MBNA product."[3]

This article argues that bundling may be necessary to assure the quality of experience goods when monitoring is private and imperfect. We show that a multiproduct firm that bundles its products together can assure quality more effectively than a multiproduct firm that sells its products separately and unbundled. Intuitively, consumers who purchase the full product line from one firm develop a deeper personal experience with that firm's products and capabilities than consumers who mix-and-match their purchases across different firms. Purchasing a bundle of products makes consumers better private monitors and better private enforcers of product quality. With bundling, consumers detect product quality deviations more readily and retaliate with greater force by boycotting all of the bundled products.[4] Because private monitoring and private enforcement are public goods, product bundling raises social welfare.

We consider a formal model in which a multiproduct branded firm competes with a fringe of competitive manufacturers who produce unbranded, low-quality

[1]Tying is permissible "if implemented for a legitimate purpose and if no less restrictive alternative is available." *Phonetele, Inc. v. American Tel. & Tel. Co.* (664 F.2d 716, 738-39 [9th Cir. 1981]). Kaplow (1985, p.545 at N. 121) provides discussion of additional cases.

[2]*Pick Mfg. Co. v. General Motors Corp. et al.* (80 F. 2d 641 [7th Cir. 1935]). "Defective parts, preventing efficient operations of cars, bring dissatisfaction with automobiles themselves. The material result is blame of the manufacturer and consequent loss of sales."

[3]*Mozart Co. v. Mercedes-Benz of North America Inc.* (833 F. 2d 1342 [9th Cir. 1987]). Although this example is significantly more complex than our model because MBNA is a reseller, not a manufacturer, of automobile parts, it nevertheless clearly illustrates that quality assurance is facilitated by constraints on buyers who would otherwise free ride on other buyers' behavior.

[4]Bundled pricing makes such enforcement rational. Following a bad experience with just one product, consumers are not typically willing to pay the bundled price for the remaining products.

products. Consumers do not directly observe product quality; instead, consumers receive imperfect private signals of quality after making their purchase decisions. The branded firm is (almost surely) capable of producing high-quality products, but can also produce low-quality products. In each period, the branded firm decides whether or not to invest in product quality for each of its two products and sets the prices for those products. Consumers have heterogeneous preferences for the branded firm's products. Consumers purchase each period, and those that buy from the branded firm observe imperfect private signals of the branded firm's quality which they then use to update their beliefs about the branded firm's product quality over time.

We begin by considering a technology where a negative private signal about one product is informative about the future quality of both products. In this case, a consumer would naturally stop purchasing both branded products after observing a negative signal about just one of them. Bundling is valuable in this setting because it forces consumers to experience both products instead of just one, increasing the chance that low quality will be detected. Specifically, we show that the range of parameter values for which high quality is sustainable as a perfect Bayesian equilibrium is larger with bundling than without bundling. For parameters in this range, bundling increases consumer surplus as well as producer surplus.

We then extend the model to consider a technology where a negative signal about one product is informative about the future quality of only that product. In this case, a consumer would naturally stop purchasing a product after observing a negative signal about that particular product, but may rationally continue to purchase the firm's other product. Bundling creates value in this setting in two ways. As before, bundling constrains consumers to experience multiple products and to be better monitors. But in this case bundling can also make the punishments more severe. Many consumers would balk at paying a high bundled product price for just one high-quality product, and therefore many consumers would stop buying the bundle after just one negative signal. Again, more intense monitoring and more severe punishments benefit consumers as well as the firm.

Taken together, the analysis shows that bundling can solve two free-rider problems. First, without bundling, some consumers shirk on monitoring by purchasing only one of the firm's products. Bundling increases the proportion of consumers purchasing two products and this improves the quality of their private monitoring. Second, without bundling, a consumer may shirk on punishing the firm by continuing to purchase one of the branded firm's products even after observing a negative signal about the other product. Bundling makes more severe punishments credible because consumers must pay the bundled-product price even when they want to consume just one of the firm's products.[5]

Importantly, our analysis provides a robust demonstration of the superiority of bundling over umbrella branding when monitoring is private and imperfect. Umbrella branding is never as effective as bundling at assuring the high quality of experience goods. When a negative signal about one product is informative about the future quality of the other product, then consumers will free ride on the monitoring of other consumers, and so bundling increases quality more than umbrella branding because it increases the proportion of consumers who purchase two products instead of just one. When a negative signal about one product is uninformative about the future quality of the other product, then consumers will shirk on punishment, and so bundling increases quality because it increases the proportion of consumers who punish the firm by not purchasing either product. In this case, umbrella branding does nothing to increase product quality relative to the single-product firm benchmark, whereas bundling does increase product quality relative to the same benchmark.

We present several additional empirically-relevant extensions. First, we extend the model to consider imperfect attribution, which arises when consumers cannot attribute a negative signal to the specific product that generated it. Second, we extend the model to consider the tying of a nondurable good, or aftermarket service, to a durable good. In both cases we show that sustaining high quality is more difficult without bundling, but no more difficult with bundling. Thus, bundling adds even more social value in these two settings. Finally, we show that our results are robust to mixed bundling, where the branded firm may sell the products separately as well as discounted in a bundled version. Although the branded firm's profits are necessarily higher when mixed bundling is allowed, the range of parameter values for which high quality is sustainable with mixed bundling is exactly the same as the range of parameter values for which high quality is sustainable with pure bundling.

Our article is organized as follows. Section 2 discusses some of the related literature. Section 3 describes the model and characterizes the optimal pricing and product quality for the benchmark case in which the firm sells a single product. Section 4 compares the multiproduct firm with and without product bundling when product quality is unobservable and the firm's types are perfectly correlated. Section 5 extends the model. Section 5 considers the case where the firm's types are independently distributed. Section 5 explores the value of bundling when consumers cannot attribute negative quality signals to specific products, only to the pair of products. Section 5 considers the value of bundling when one

[5] Of course, the latter is relevant only when consumers don't view a negative signal about one product as informative about the quality of the other product.

of the two goods is an infinitely-lived durable good. Section 5 allows for mixed bundling and shows that our main insights continue to hold. Section 6 concludes.

2 Literature

Although bundling and tying have been widely studied in the legal and economic literatures, the relationship between bundling and quality assurance has received relatively little attention. Bork (1978) and Posner and Easterbrook (1981) argue informally that when consumers use low-quality products along with high-quality products, then low overall performance may be erroneously attributed to the producer of the high-quality product and that by tying or bundling the products together, the seller can protect its reputation.[6] Bork (1978, p. 380) and Nalebuff (2009, p. 377) both observe that purchasing multiple products improves monitoring, but they question why firms would need to use bundling to constrain consumers to purchase multiple products because better monitoring is in consumers' self interest. Our article rigorously demonstrates that bundling may be necessary to assure product quality when consumer monitoring is private and imperfect and consumers have an incentive to free ride.

Bundling as a means of price discrimination has been studied by Adams and Yellen (1976) and McAfee, McMillan and Whinston (1989) who argue that bundling allows a monopolist to extract greater rents from consumers.[7] Other articles, including Whinston (1990) and Nalebuff (2004), argue that bundling can be used by an incumbent to foreclose competition in the market (see the survey by Nalebuff, 2008). Salinger (1995) emphasizes that bundling may also reduce the costs of production and distribution of products; for example, it is more efficient for automobile manufacturers to bundle cars with tires than for consumers to do it themselves.[8]

Our work is also related to the literature on umbrella branding, some of which considers multiproduct versions of the repeated moral hazard model of Klein and Leffler (1981). In a perfect public monitoring model related to Bernheim and Whinston's (1990) model of multimarket contact, Andersson (2002) shows that as long as the products are asymmetric and at least one of the constraints for

[6] Iacobucci (2003) refined this argument further and highlighted the importance of the attribution problem which we discuss in Section 5.3. See also Bar-Gill (2006) and additional references in Iacobucci (2003).

[7] Fang and Norman (2006) generalize these results. Elhauge and Nalebuff (2014) (and others) argue that bundling or tying facilitates price discrimination because firms can use consumption of tied nondurable goods as a proxy for consumers' valuation of a durable good.

[8] Dana and Fong (2011) and Baranes, et al. (2011) argue that bundling facilitates tacit collusion.

producing high quality binds, joint production is profitable for a monopolist. Choi (1998) shows that the firm can leverage its future product introduction rents to credibly reduce the cost of signaling high quality in the present.[9] Cabral (2009) and Cai and Obara (2006) show that joint production can reduce equilibrium-path punishments when product markets are symmetric and there is imperfect public monitoring.[10] These articles in the umbrella branding literature emphasize the advantages that multiproduct firms have over single-product firms. Our analysis delves deeper and emphasizes the advantages of product bundling over simple umbrella branding.

Some articles are more closely related to our durable-good extension. Schwartz and Werden (1996) consider a model in which a privately-informed firm uses tying to signal the quality of a durable good. By tying the sale of the durable good to a nondurable one, the firm can shift the rents from the durable to the nondurable and help overcome the hidden-information problem. In contrast, our model is a hidden-action model and is the only article to formally demonstrate that the advantage of bundling is greatest when there is asymmetric information about the quality of both the durable good and the nondurable good.

Although our emphasis is not on reputations and reputation building *per se*, our model is nevertheless closely related to Mailath and Samuelson (2001) and other articles in the reputation literature.[11] Like Mailath and Samuelson (2001), we assume that a positive fraction of the firms are inept (we call these firms incapable), which implies that consumers in our model will attribute negative signals to inept firms and hence punish firms that generate negative signals.

Finally, Dana and Spier (2015) present a simple model in which consumers are heterogeneous and demand different amounts of a single experience good. That article shows that quantity forcing, or by analogy volume discounts, can be used to improve consumer monitoring by limiting the number of consumers who purchase too few units of the good. Although some of the intuitions in the two articles are similar, the analysis in this article is significantly richer. First, we allow the firm to choose prices and product qualities every period, not just at the beginning. Second, we show that bundling not only improves monitoring but also increases the credibility and severity of punishment. Third, we endogenize

[9]Other articles on umbrellay branding include Wernerfelt (1988), Cabral (2000), Montgomery and Wernerfelt (1992), Pepall and Richards (2002), and Rasmusen (2015).

[10]Hakenes and Peitz (2008) also consider imperfect public monitoring, but they assume (as we do) that there is a positive probability of accurately observing that the firm produced a low-quality product.

[11]See also Bar-Isaac and Deb (2014) and Bar-Isaac, Caruana and Cuñat (2012). These articles consider firm reputations when consumers ignore the impact of their monitoring and information gathering on the hidden actions of the firm (or agent).

the firm's prices and highlight the important role that price plays in motivating the firm to produce high quality. Fourth, we explicitly compare bundling and umbrella branding and demonstrate the fundamental advantages of the former. And finally, we show that our results generalize to models with mixed bundling.

3 Model

A unit mass of infinitely-lived, risk-neutral consumers demand a single unit of each of two experience goods, product 1 and product 2, in each period of their lives, $t = 1, 2, \dots, \infty$. The two products are supplied by a competitive fringe and by a branded multiproduct firm, which we sometimes refer to as simply the firm or the monopolist. As both products are experience goods, the quality of each product, low or high, is not directly observed by consumers at the time of purchase. The consumers and the firms share a common discount factor, δ.

Consumers. A consumer's willingness to pay for a low-quality product is normalized to zero, as is the marginal cost of production of low-quality goods. The willingness to pay for a high-quality product varies across the population of consumers. Specifically, consumers' valuations for the high-quality products, v_1 and v_2, are jointly distributed with symmetric probability density $f(v_1, v_2)$ on $[0, \bar{v}] \times [0, \bar{v}]$. We assume that the density $f(v_1, v_2)$ is strictly positive on its support, which implies that v_1 and v_2 are not perfectly correlated. We also assume that $\bar{v} > c$, where $c > 0$ is the marginal cost of producing a high-quality product, which we assume is the same for both goods.[12] Each consumer's purchase decision maximizes his or her consumer surplus.

Firms. The firms in the competitive fringe are incapable of producing high-quality products and supply only low-quality products. Because the consumers' willingness to pay and the marginal cost of low-quality products are both normalized to zero, it follows that the price, consumer surplus, and producer surplus associated with the competitive fringe are all zero as well. In contrast, the branded firm is almost surely capable of producing high-quality products. More specifically, the branded firm privately observes whether it is capable or incapable, and with probability $1-\epsilon$ it is a capable type, and with probability ϵ it is an incapable type, where ϵ is a small, but strictly positive probability.[13] For convenience, we

[12]For simplicity, in our main model we ignore any complementarities in the consumption of the two goods. In particular, a consumer's willingness to pay for a high-quality product 1 does not depend on whether the consumer also purchases a high-quality product 2, and vice versa.

[13]Mailath and Samuelson (2001) use the terms inept and competent instead of incapable and capable to describe the low-quality commitment type and the type that is free to choose

assume that ϵ is vanishingly small which allows us to ignore consumer learning in our expressions for demand and profits, and in our calculation of optimal prices.[14]

If the branded firm is capable, it can choose to produce high-quality products by paying a cost of c per unit or low-quality products by paying 0 per unit. If the branded firm is incapable, then it produces low-quality products at zero cost (as does the competitive fringe). All firms maximize their profits.

Signals. After making their purchase decisions, consumers receive imperfect private signals of the quality of each product purchased. Conditional on purchasing just one low-quality product, the probability that the consumer privately observes a negative signal is $1 - \pi \in (0, 1)$, and the corresponding probability that no negative signal is observed is π.[15] Conditional on purchasing two low-quality products, the consumer may receive zero, one, or two negative signals. We let $1 - \pi_b$ denote the probability that the consumer observes at least one negative signal. If the signals are independently distributed, then $\pi_b = \pi^2$. We allow for either positive or negative correlation between the signals and assume only that $\pi > \pi_b > 2\pi - 1$.[16] We simplify our analysis by assuming that high-quality products do not generate negative signals. This is a strong assumption, and we discuss ways that it can be relaxed in the conclusion.

Timing. At the beginning of the game, $t = 0$, the branded firm chooses whether or not to bundle its products. Without bundling, consumers are free to mix-and-match between the branded firm's and competitive fringe's products. With bundling, the consumer must purchase both products from the branded firm or both products from the competitive fringe. Next, consumers learn their valuations, v_1 and v_2, and the branded firm learns its type, capable or incapable. In each subsequent period of the game, $t = 1, 2, \ldots, \infty$, the branded firm chooses the price and quality for each of its two products, then consumers observe the prices of the two products (but not their qualities) and decide whether to purchase from the branded firm or from the competitive fringe, and then, finally, consumers receive imperfect private signals of product quality and update their beliefs.

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product quality.

[14]Without this assumption demand would grow as consumers became more and more confident that the firm was capable, and the equilibrium prices would change over time. Also, consumers' beliefs depend on the purchase history and therefore are heterogeneous.

[15]The simplest interpretation of our assumptions is that the signal is purely information. Alternatively, the signal can be interpreted as a product failure, so the expected valuation of product i would be equal to v_i when quality is high and 0 when quality is low.

[16]With perfect positive correlation, $\pi_b = \pi$; with perfect negative correlation, $1-\pi_b = 2(1-\pi)$ or $\pi_b = 2\pi - 1$.

Throughout the article, we focus on high-quality perfect Bayesian equilibria, that is, perfect Bayesian equilibria (PBE) in which the capable branded firm produces only high-quality products on the equilibrium path. Within this set of equilibria, we focus on the PBE that delivers the highest expected profit to the branded firm.[17] In this PBE, the capable branded firm has an incentive to choose high quality over low quality because negative signals are punished. Specifically, consumers believe that the branded firm is almost surely capable and will continue to produce high-quality products as long as they have not observed any negative private signals in the past. If a consumer sees a negative private signal from the branded firm, then the consumer believes that the branded firm must be incapable and so prefers to purchase both products from the competitive fringe.

Preliminaries

We begin by defining some important notation.

Demand. Suppose that, in equilibrium, the branded firm sells the two high-quality products separately (unbundled) at prices p_1 and p_2, allowing consumers to purchase one, both, or neither. Consumers purchase if their expected valuation exceeds the price, which formally means that $(1-\epsilon)v_i \geq p_i$, or $v_i \geq p_i/(1-\epsilon)$, because their valuation is zero if the firm is the incapable type.[18] Because we assumed ϵ is vanishingly small, the demand for product i from the branded firm can be approximated by

$$d_i(p_i) = \int_0^{\bar{v}} \int_{p_i}^{\bar{v}} f(v_i, v_j) dv_i dv_j. \tag{1}$$

When the two products are sold separately, some consumers mix-and-match, buying one product from the branded firm and the other from the fringe, while other consumers purchase both products from the branded firm and still others purchase both products from the competitive fringe. Because we assumed ϵ is vanishingly small, the mass of consumers who purchase both products from the branded firm can be approximated by

$$\phi_{12}(p_1, p_2) = \int_{p_1}^{\bar{v}} \int_{p_2}^{\bar{v}} f(v_1, v_2) dv_1 dv_2, \tag{2}$$

[17] The expected profit is a weighted average of the capable type's profit and the incapable type's profit, but the weight on the incapable type's profit is zero because ϵ is assumed to be vanishingly small. So we pick the PBE that maximizes the capable type's profit.

[18] Here we are assuming consumers ignore the impact of their consumption on their future beliefs about ϵ, the probability that the firm is incapable. This is reasonable as we are assuming that ϵ is vanishingly small.

and the mass of consumers who buy product i, but not product j, can be approximated by

$$\phi_i(p_1, p_2) = d_i(p_i) - \phi_{12}(p_1, p_2). \tag{3}$$

When the two products are bundled together and sold at price $2p_b$, the demand for the bundle can be approximated by

$$d_b(p_b) = \iint\limits_{v_1+v_2 \geq 2p_b} f(v_1, v_2)dv_1dv_2. \tag{4}$$

Expressing the bundled price as $2p_b$ facilitates comparison of the per unit bundled price, p_b, to the single-product prices, p_1 and p_2.

Full Information Prices. Suppose that product quality is observed by consumers at the time of purchase. The branded firm would earn no profits from selling low-quality products because these are already supplied at cost by the competitive fringe. So, the capable branded firm would sell only high-quality products.

Suppose that the branded firm sells the two high-quality products separately. The prices that maximize the firm's profits, using the demand in (1), are

$$p_1^* = p_2^* = p_s^* = \arg\max_{p_i}(p_i - c)d_i(p_i).$$

The prices for the two products are the same, $p_1^* = p_2^*$, because we assumed that the distribution of consumer valuations $f(v_1, v_2)$ is symmetric and the costs are the same. If the firm bundles the two products, then the bundled price that maximizes the firm's profits, using the demand in (4), is

$$p_b^* = \arg\max_{p_b}(2p_b - 2c)d_b(p_b).$$

We assume that these profit-maximizing prices are uniquely defined by the first-order conditions and that the branded firm earns strictly higher profits selling the two products separately than by selling them as a bundle, or

$$\sum_{i=1,2}(p_s^* - c)d_i(p_s^*) > 2(p_b^* - c)d_b(p_b^*). \tag{5}$$

That is, when product quality is known at the time of purchase, the monopolist prefers to sell the products separately rather than as a bundle. This assumption underscores that the role of bundling in our model is solely to assure product quality when quality is unobservable.

Numerical Example. Suppose quality is observable, $c = \frac{1}{4}$, and $f(v_1, v_2)$ is uniformly distributed on $[0,1] \times [0,1]$. The individual product demands are given by $d_i(p_i) = 1 - p_i, i = 1,2$, and the bundled product demand is given by $d_b(p_b) = 1 - 2p_b^2$ if $p_b \in [0, \frac{1}{2}]$ and $d_b(p_b) = 2(1-p_b)^2$ if $p_b \in (\frac{1}{2}, 1]$. If the firm does not bundle, then $p_s^* = \frac{5}{8}$ and $d_i(p_s^*) = \frac{3}{8}$. Firm profit is $2(p_s^* - c)d_i(p_s^*) = \frac{9}{32}$. If the firm bundles, then $p_b^* = \frac{1}{2}$, so the price of the bundle is $2p_b^* = 1$ and $d_b(p_b) = \frac{1}{2}$. Firm profit is $(2p_b^* - 2c)d_b(p_b^*) = \frac{1}{4}$. The profit from bundling is smaller than the profit from selling the products separately, so bundling is not profitable for the branded firm in this example.[19]

Single-Product Benchmark

Suppose the branded firm produces just one of the two products, product i, and the other product is available only from the competitive fringe (and is a low-quality product). The quality of the branded product is not observed at the time of purchase, but consumers receive imperfect private signals as described above. In a high-quality PBE, consumers believe that the capable branded firm always produces high-quality products and, if a negative signal is observed, then consumers attribute the negative signal to the branded firm being incapable (rather than having shirked) and switch to the competitive fringe.

Consider a high-quality PBE where the branded firm charges price p in every period along the equilibrium path. A necessary condition for choosing high quality to be optimal is:

$$d_i(p)\frac{p-c}{1-\delta} \geq d_i(p)\left[p + \pi\delta\left(\frac{p-c}{1-\delta}\right)\right]. \tag{6}$$

The left-hand side of (6) is the present discounted value of the firm's profit if it produces a high-quality product. The right-hand side is the present discounted value of the firm's profit if it makes a one-time deviation to low quality, and then reverts to high quality afterwards. The benefit to the firm of deviating to low quality is that it can lower its production cost from c to zero in the deviation period, giving it a per-unit profit of $p > p - c$. The cost of deviating is that a subset of consumers will receive negative signals and stop purchasing from the firm, so its demand will shrink to $\pi d_i(p)$ starting in the period after the deviation.

Note that the demand $d_i(p)$ appears on both sides of equation (6). Thus, the level of demand for the product does not affect the firm's incentive to choose

[19] Although profits are lower with bundling in this example, note that the consumer surplus is $.5(\frac{3}{8})^2 + .5(\frac{3}{8})^2 = (\frac{3}{8})^2$ without bundling and $\int_0^1\int_0^1 \max\{0, v_1+v_2-1\}dv_1dv_2 = \frac{1}{6}$ with bundling, so consumers are better off with bundling.

high quality. Regardless of how many consumers remain, those consumers that do remain believe that the firm is capable and is still on the equilibrium path, so if equation (6) is satisfied, the firm will not find it profitable to deviate to low quality at any of its on or off-the-equilibrium-path decision nodes. In other words, by the one-shot deviation principle (see, for example, Tadelis (2013), p. 284), equation (6) is not just necessary, but also sufficient for a high-quality PBE to exist.

Equation (6) may be rewritten as:

$$S_i(p, \delta) = \left(\frac{1-\delta\pi}{1-\delta}\right)(p-c) - p \geq 0. \tag{7}$$

When δ is sufficiently large, then (7) is slack, and the firm will charge the full-information monopoly price, $p_s^* = \arg\max_p (p-c)d_i(p)$. For $\delta < \overline{\delta}_s$, where

$$\overline{\delta}_s = \frac{c}{\pi c + (1-\pi)p_s^*}, \tag{8}$$

equation (7) cannot be satisfied at the full-information price, p_s^*, but might be satisfied at a higher price. From (7), the constraint is satisfied when

$$p \geq \frac{c(1-\delta\pi)}{\delta(1-\pi)}. \tag{9}$$

Finally, for $\delta \leq \underline{\delta}_s$, where

$$\underline{\delta}_s = \frac{c}{\pi c + (1-\pi)\bar{v}}, \tag{10}$$

the constraint in (9) cannot be satisfied without setting $p \geq \bar{v}$, which implies that sales and profits are zero. This is summarized in the following proposition. All proofs are in the Appendix.

Proposition 1. *When product quality is unobservable, and the branded firm sells a single product, then high quality is sustainable with positive sales if and only if* $\delta > \underline{\delta}_s$. *The price charged in the highest-profit perfect Bayesian equilibrium is*

$$p_s(\delta) = \begin{cases} \frac{c(1-\delta\pi)}{\delta(1-\pi)} & \text{if } \delta \in (\underline{\delta}_s, \overline{\delta}_s) \\ p_s^* & \text{if } \delta \in [\overline{\delta}_s, 1], \end{cases}$$

where $\lim_{\delta \downarrow \underline{\delta}_s} p_s(\delta) = \bar{v}$, *and profits go to zero, as* δ *approaches* $\underline{\delta}_s$ *from above.*

When the discount factor is sufficiently high, $\delta \geq \overline{\delta}_s$, then there exists a PBE in which the capable branded firm produces a high-quality product and charges the full-information price, $p_s(\delta) = p_s^*$, in every period of the game. The capable firm does not deviate to low quality, because this would induce a sufficiently large subset of consumers to buy from the competitive fringe (thinking the branded firm is incapable) making the deviation unprofitable.

When $\delta < \overline{\delta}_s$, there does not exist a high-quality PBE in which the branded firm charges $p_s(\delta) = p_s^*$. The profit margins from maintaining high quality are insufficient to prevent the firm from shirking. However, the proposition shows that when $\delta \in (\underline{\delta}_s, \overline{\delta}_s)$, there does exist a high-quality equilibrium with $p_s^* < p_s(\delta) < \bar{v}$. The higher profit margins associated with this price allow the branded firm to maintain incentive compatibility.

The firm has no incentive to deviate and charge a different price. A lower price is not profitable because, by construction, consumers believe the capable firm would choose low quality at any price below the highest-profit equilibrium price. So a price cut would lead to zero sales and zero profits.[20] Finally, because $p_s(\delta) \geq p_s^*$ for all δ, deviating to a higher price would strictly lower the firm's profits, regardless of the consumers' beliefs.

Remarks

This section provides further discussion of several modeling assumptions.

Imperfect Private Monitoring. The assumption that monitoring is private and imperfect is critical for our analysis. If the branded firm deviates in quality, then a fraction $1-\pi$ of consumers privately receive a negative signal and stop purchasing the product. The other consumers remain unaware of this and continue to purchase the product. If monitoring were public, then consumers would see the negative signals received by others and so everyone would stop purchasing the product. That is, when $\pi = 0$, the incentive constraint in equation (7) becomes the standard constraint with perfect public monitoring, $(p-c)/(1-\delta) - p \geq 0$ which is easier to satisfy.[21]

Multiproduct firms – especially those that bundle – have a profound advantage over single product firms when monitoring is imperfect and private. In a world of imperfect private monitoring, the average consumer of a multiproduct firm is better informed than the average consumer of a single-product firm. Consumers who buy both branded products observe two imperfect private signals of

[20] When $\delta \in (\underline{\delta}_s, \overline{\delta}_s)$, if the firm were to cut the price below $p_s(\delta)$ then the incentive constraint would be violated so the consumers' beliefs are sensible.

[21] So a policy change which makes consumer learning public, or more generally raises $1 - \pi$, such as the law requiring restaurants to post cleanliness ratings in their windows, which was analyzed by Jin and Leslie (2003), can increase product quality and social welfare.

quality instead of just one. These consumers detect deviations faster, and punish the firm more effectively, than consumers who buy one branded product only. Conversely, in a world with public monitoring, where all information is shared among consumers, a consumer sees the same public signals whether he or she bought one or two products.[22]

Incomplete Information. Including a small amount incomplete information about the branded firm's type is important in our model. The assumption that $\epsilon > 0$ puts every observable history for consumers on the equilibrium path and implies that in a high-quality PBE, consumers will attribute negative signals from a branded product to the incapable type. That is, a consumer who privately observes a negative signal from one of the branded firm's products will clearly stop all future purchases (punish the branded firm) because he or she believes that the firm is incapable of producing high-quality products. So if the firm deviates and produces a low-quality product, it will lose a fraction $1 - \pi$ of its customers, and this loss is permanent. Knowing that consumers will react in this way gives the capable firm an incentive to produce high-quality products in every period.

The assumption that ϵ is arbitrarily small is made for analytical convenience and greatly simplifies the characterization of demand. Specifically, this assumption allows us to ignore the changes in demand, and associated changes in price, over time that would otherwise occur as consumers learn about the likelihood that the firm is capable.[23]

The characterization in Proposition 1 is not a PBE when $\epsilon = 0$. To see why, suppose that $\delta > \overline{\delta}_s$ so the firm charges $p_s(\delta) = p_s^* > c$. Now consider a deviation where the firm shirks in some period t and produces a low-quality product. This deviation is privately observed by fraction $1 - \pi$ of the consumers. because $\epsilon = 0$, the subset of consumers who receive the negative signal believe (correctly) that the firm cheated. However, it is not sequentially rational for this subset of consumers to punish the firm. With private monitoring, a fraction π of consumers did not observe the negative signal and therefore expect the quality of the branded product to be high in the future, and these consumers are still willing to pay the price $p_s^* > c$. The firm, being rational, should revert to produce high quality at $t + 1$ (and all future periods). The subset of consumers who saw the negative signal should anticipate that quality will be high in the future, and hence they are not willing to punish the firm. When $\epsilon > 0$, however, consumers

[22] However, under public monitoring, bundling might still lead to better informed consumers by increasing aggregate consumption, and bundling could eliminate some equilibria in which consumers use less severe punishment strategies (see Section 5).

[23] It also allows us to ignore forward-looking behavior by consumers — if ϵ is strictly positive consumers might purchase more in order to learn more quickly which could increase their expected consumer surplus in the future.

attribute negative signals to the incapable type (rather than cheating by the capable type), and so punishment is sequentially rational.

Quality and Pricing Decisions. We have assumed that the branded firm chooses its prices and qualities in each period, $t = 1, 2, \ldots, \infty$. Similar results would be obtained in a static model where the firm commits to price and quality at the beginning of the game at $t = 0$. To see why, consider the single product benchmark above where quality is chosen once-and-for-all at $t = 0$. A necessary and sufficient condition for high quality to be chosen is

$$d_i(p)\frac{p-c}{1-\delta} \geq d_i(p)\frac{p}{1-\delta\pi}.$$

The left-hand side is the present discounted value of the firm's profit if it produces a high-quality product, and the right-hand side is the profit from a deviation to low quality. If the firm deviates and produces a low-quality product, it saves the product cost and continues to sell the product to those consumers who have not yet seen the negative signal, a population that is shrinking in proportion to π each period. This expression is equivalent to equation (7) above so the results in Proposition 1 apply in this case as well.

The next section of the article fully characterizes the high-quality PBE for the multiproduct model using our general dynamic framework where prices and qualities are chosen in every period of the game. The results we derive there are unchanged if prices and qualities were chosen once-and-for-all at time $t = 0$ as well. Later, when we extend the model to consider independent types, the attribution problem, durable goods, and mixed bundling, we will exploit the relative simplicity of the static approach and assume that the firm commits to prices and qualities at the beginning of the game.

4 The Multiproduct Firm

In this section we analyze the full model described above, in which the branded firm produces two products, the quality of each product is unobservable, and the branded firm has private information about its type. First, we consider the case in which the branded firm produces both products and bundles them so that consumers are constrained to buy both or neither. Second, we consider the case in which the branded firm sells the two products separately and unbundled, so that consumers are free to purchase just one product from the firm and the other from the competitive fringe. A comparison of these two cases demonstrates the incremental value of product bundling over umbrella branding.

Bundled Products

We now characterize the highest profit PBE of the game when the branded firm sells the two products as a bundle. The consumer beliefs that support a high-quality equilibrium are analogous to those for the single-product benchmark. Consumers believe that the capable firm always produces high-quality products, and after observing one (or two) negative signals, a consumer believes that the firm must be incapable of producing high-quality products and stops purchasing both products from the firm.

A necessary condition for a high-quality PBE to exist when the firm bundles its products is that deviating to two low-quality products is not profitable, or

$$2d_b(p_b)\frac{p_b - c}{1-\delta} \geq 2d_b(p_b)\left[p_b + \delta\pi_b\left(\frac{p_b - c}{1-\delta}\right)\right]. \tag{11}$$

The left-hand side is the present discounted value of selling a bundle of high-quality products, and the right-hand side is the present discounted value from a one-time deviation to two low-quality products where the firm reverts to high quality afterwards.

In the proof of Proposition 2 below, we show that (11) is also sufficient for a high-quality PBE to exist. In particular, we show that deviating to one low-quality product is also not profitable as long as (11) holds. We also show that (11) guarantees that choosing high quality for both products is optimal for the capable type at all of the firm's decision nodes, not just on the equilibrium path.

Equation (11) may be rewritten as:

$$B(p_b, \delta) = \left(\frac{1-\delta\pi_b}{1-\delta}\right)(2p_b - 2c) - 2p_b \geq 0. \tag{12}$$

Because $\pi_b < \pi$, the bundled-product firm's constraint is easier to satisfy than the single-product firm's constraint in (7). This is intuitive. Consumers who observe even one negative signal stop purchasing the bundle, so the punishment for deviating is more severe – the firm loses future sales of both products.

When δ is sufficiently large, (12) is slack and the firm will charge the full-information bundled price, $p_b^* = \arg\max_{p_b}(2p_b - 2c)d_b(p_b)$. When delta is small, that is $\delta < \overline{\delta}_b$, where

$$\overline{\delta}_b = \frac{c}{\pi_b c + (1-\pi_b)p_b^*}, \tag{13}$$

then (12) is not satisfied at p_b^*, but may be satisfied at a higher price. Using (12), the price must satisfy

$$p_b \geq \frac{c(1-\delta\pi_b)}{\delta(1-\pi_b)}. \tag{14}$$

However for $\delta \leq \underline{\delta}_b$, where

$$\underline{\delta}_b = \frac{c}{\pi_b c + (1-\pi_b)\bar{v}}, \tag{15}$$

the constraint (14) cannot be satisfied without setting $p_b \geq \bar{v}$, which implies that the sales and profit of the firm are zero.

It is interesting to compare the lower bound for the discount factor in (15) to the lower bound from the single-product benchmark $\underline{\delta}_s$ defined in (10). Because $\pi_b < \pi$, we can see immediately that $\underline{\delta}_b < \underline{\delta}_s$. With bundling, high quality is sustainable for a broader range of discount factors.

This is summarized in the following proposition, which holds whether price and quality are chosen every period or only at the start of the game.

Proposition 2. *When product quality is unobservable, and the two branded products are bundled, then high quality is sustainable with positive sales in a perfect Bayesian equilibrium if and only if $\delta > \underline{\delta}_b$. The price charged in the highest-profit perfect Bayesian equilibrium is*

$$p_b(\delta) = \begin{cases} \frac{c(1-\delta\pi_b}{\delta(1-\pi_b)} & \text{if } \delta \in (\underline{\delta}_b, \overline{\delta}_b) \\ p_b^* & \text{if } \delta \in [\overline{\delta}_b, 1], \end{cases}$$

where $\lim_{\delta \downarrow \underline{\delta}_b} p_b(\delta) = \bar{v}$, and profits go to zero, as δ approaches $\underline{\delta}_b$ from above.

Unbundled Products

Next suppose that the branded firm sells the two products separately. Consumers can buy one, both, or neither of the branded firm's products. As in the bundled-product analysis, we will focus our discussion in the body of the article on deviations along the equilibrium path in the quality of both products.

Using the definitions of $d_i(p_i)$, $\phi_i(p_1, p_2)$, and $\phi_{12}(p_1, p_2)$ in (1), (2), and (3) above, a necessary condition for any high-quality PBE to exist is

$$\sum_{i=1,2} d_i(p_i)\frac{p_i - c}{1-\delta} \geq \sum_{i=1,2} \phi_i(p_1, p_2)\left[p_i + \delta\pi\frac{p_i - c}{1-\delta}\right] + \phi_{12}(p_1, p_2)\left[p_1 + p_2 + \delta\pi_b\frac{p_1 + p_2 - 2c}{1-\delta}\right]. \tag{16}$$

The left-hand side of this condition is the present discounted value of the firm's profits when the firm chooses high quality. The right-hand side is the present discounted value of the firm's profits if the firm deviates to low quality for both products in the first period and then reverts to high quality every period thereafter. Clearly (16) is a necessary condition for a high-quality PBE to exist, however (16) may not represent a one-shot deviation. In particular, because consumers are heterogeneous, a PBE may exist in which following a deviation to low quality, continuing to produce low quality is optimal.

To better understand the deviation profit on the right-hand side of (16), recall that the subset of consumers who purchase exactly one product from the firm, $\phi_i(p_1, p_2)$, observe negative signals at a rate of $1-\pi$ per period, so a proportion π of these consumers continue to purchase from the branded firm. Consumers who purchase both products, $\phi_{12}(p_1, p_2)$, have two opportunities to observe a negative signal. When the firm deviates, these consumers observe negative signals more often, and so a smaller proportion, $\pi_b < \pi$, of these consumers continue to purchase from the branded firm following a deviation.

Because $d_i(p_i) = \phi_i(p_1, p_2) + \phi_{12}(p_1, p_2)$, equation (16) can be rewritten as

$$\sum_{i=1,2} \phi_i(p_1, p_2) \left[\left(\frac{1-\delta\pi}{1-\delta} \right) (p_i - c) - p_i \right] + \phi_{12}(p_1, p_2) \left[\left(\frac{1-\delta\pi_b}{1-\delta} \right) (p_1 + p_2 - 2c) - (p_1 + p_2) \right] \geq 0. \quad (17)$$

The incentive constraint in (17) is a linear combination of the single-product and bundled-product incentive constraints in (7) and (12):

$$M(p_1, p_2, \delta) = \sum_{i=1,2} \phi_i(p_1, p_2) S_i(p_i, \delta) + \phi_{12}(p_1, p_2) B\left(\frac{p_1 + p_2}{2}, \delta \right) \geq 0. \quad (18)$$

The fact that the incentive constraint for the unbundled-product firm is a linear combination of $S_i(\cdot) \geq 0$ and $B(\cdot) \geq 0$ makes sense. With unbundled products, some consumers will purchase both branded products and other consumers will choose to purchase just one. It is straightforward to show that $\partial S_i(\cdot)/\partial\delta > 0$ and $\partial B(\cdot)/\partial\delta > 0$, so $M(\cdot, \cdot, \delta)$ is also increasing in δ. Thus, raising the discount factor makes it easier for the multiproduct firm to sustain high quality.

The following proposition shows that the incentive constraint in (18) is both necessary and sufficient for a high-quality PBE to exist.

Proposition 3. *When product quality is unobservable, and the two branded products are sold separately and unbundled by a multiproduct firm, then there exists a* $\underline{\delta}_m > 0$ *such that high quality is sustainable with positive sales if and only if* $\delta > \underline{\delta}_m$. *There exists a* $\overline{\delta}_m \in (\underline{\delta}_m, 1)$ *such that for all* $\delta \geq \overline{\delta}_m$ *the price charged in the highest-profit perfect Bayesian equilibrium is defined by*

$$p_m^* = \arg\max_{p_m} 2(p_m - c) d_i(p_m),$$

so $p_m^* = p_s^*$, and for all $\delta \in (\underline{\delta}_m, \overline{\delta}_m)$ the price charged in the highest-profit perfect Bayesian equilibrium is defined by $M(p, p, \delta) = 0$.

Because consumers choose to consume different numbers of products, they observe deviations at different rates. As a consequence the proof of sufficiency is more complex than in the analogous proofs of Propositions 1 and 2. The Appendix presents a proof of Proposition 3 that proves existence of a PBE by checking that simultaneous deviations in both product qualities are not profitable. The full proof, which includes checking deviations in the quality of just one of the two products, is available in a supplemental online appendix.

Comparison

We now present two results. First, without bundling, the multiproduct firm can never do worse — and will often do better — than the single-product firm. Second, the multiproduct firm may do even better by bundling its products together.

Proposition 4. *Absent bundling, the multiproduct firm is at least as profitable as the single-product firm. The multiproduct firm creates the same (per product) producer and consumer surplus as the single-product firm when $\delta \in (\overline{\delta}_s, 1)$, and creates strictly higher producer and consumer surplus than the single-product firm when $\delta \in (\underline{\delta}_s, \overline{\delta}_s)$.*

It is not hard to see why this is true. Recall that the full-information prices are the same for the single-product and multiproduct firms, $p_s^* = p_m^*$. According to Proposition 1, when $\delta \in (\overline{\delta}_s, 1)$ the single-product firm's incentive constraint (6) is slack when evaluated at the full-information price. Because consumers who happen to purchase two low-quality products from the multiproduct firm are even more likely to observe a negative signal, $\pi_b < \pi$, the multiproduct firm's incentive constraint in (17) is slack as well. Therefore the single-product firm and the multiproduct firm achieve the same full-information outcome when $\delta \in (\overline{\delta}_s, 1)$.[24]

Now suppose that the discount factor is in the middle range, $\delta \in (\underline{\delta}_s, \overline{\delta}_s)$. According to Proposition 1, the single-product firm cannot sustain high quality at price p_s^* and must instead distort the price to be $p_s(\delta) > p_s^*$. The equilibrium price, $p_s(\delta)$, is the price at which the single-product firm's incentive constraint (6) binds. Because $\pi_b < \pi$, the multiproduct firm's incentive constraint in (18) is slack when evaluated at $p_s(\delta)$. The multiproduct firm therefore can sustain high quality more easily than the single-product firm. This allows the multiproduct firm to charge lower prices, $p_m(\delta) < p_s(\delta)$, and raise its profits.[25] Because the multiproduct firm's prices are lower, consumer surplus is higher.

[24]Using the notation from (18), if $S_i(p_s^*, \delta) > 0$ then $B(p_s^*, \delta) > 0$ and so $M(p_s^*, p_s^*, \delta) > 0$.

[25]Formally, if $S_i(p_s(\delta), \delta) = 0$ then $B(p_s(\delta), \delta) > 0$, and so $M(p_s(\delta), p_s(\delta), \delta) > 0$.

We now state one of the main results of the article.

Proposition 5. *High quality can be supported for the greatest range of discount factors under bundling, that is:* $0 < \underline{\delta}_b < \underline{\delta}_m \leq \underline{\delta}_s < 1$. *When* $\delta \in (\underline{\delta}_b, \underline{\delta}_m)$, *a high-quality perfect Bayesian equilibrium exists if and only if the firm bundles its products, and bundling increases profits and consumer surplus.*

Proposition 5 shows that for a range of discount factors, bundling is necessary for the branded firm to sustain high quality and increases both producer and consumer surplus. When the discount factor is in $(\underline{\delta}_b, \bar{\delta}_m)$, there does not exist a high-quality PBE when the branded firm sells its two products unbundled and separately (Proposition 3). There does, however, exist a high-quality PBE in which the firm bundles the products and charges $p_b(\delta) \in (c, \overline{v})$ for the bundle (Proposition 2). In this interval, both producer surplus and consumer surplus are zero without bundling and strictly positive with bundling. Note also that because $p_b(\delta)$ is falling in δ, both the branded firm and the consumers are better off when the discount factor rises in this range.

When the discount factor is in the interval $(\underline{\delta}_m, \bar{\delta}_m)$, high quality is feasible both with and without bundling. If the discount factor is near the bottom of this range, $\delta = \underline{\delta}_m + \Delta$ where Δ is positive and small, then bundling delivers strictly higher producer and consumer surplus than umbrella branding. This is true by continuity because $p_b(\underline{\delta}_m) < \overline{v} = p_m(\underline{\delta}_m)$ so if $\Delta = 0$, then producer and consumer surplus are positive with bundling but identically equal to zero without it. When the discount factor is at the top of this range, $\delta = \bar{\delta}_m$, then the branded firm can achieve the full information outcome without bundling, charging p^*_m for each of the products (Proposition 3). Because we assumed that the firm strictly prefers not to bundle under full information (5), it follows that producer surplus is also higher without bundling when $\delta = \bar{\delta}_m - \Delta$ where Δ is positive and small.

Notice that for discount factors in the interval $(\underline{\delta}_m, \bar{\delta}_m)$, the firm's decision to bundle may either help or harm consumers. Let δ' be the highest value of δ at which bundling is more profitable. Clearly $\delta' \leq \bar{\delta}_m$ because for discount factors above $\bar{\delta}_m$ bundling is never more profitable. At δ' producer surplus is the same with or without bundling because the firm is indifferent, but the bundled product price is lower than the multiproduct firm's price.[26] So consumers who purchase two products are clearly better off, but some consumers who would have bought just one product are worse off. Thus, the impact of bundling on consumer surplus is ambiguous.

[26]This is because $B(p_b(\delta), \delta) = 0$ by definition; because $\pi_b > \pi$ implies $S(p_b(\delta), \delta) < 0$; and because by (18) M is a weighted average of B and S, so $M(p_b(\delta), p_b(\delta), \delta) < 0$.

Numerical Example

Our results can be illustrated using the simple numerical example that was introduced in Section 3 where $c = \frac{1}{4}$, $f(v_1, v_2)$ is uniformly distributed on $[0,1] \times [0,1]$, $\pi = \frac{2}{3}$, and $\pi_b = \pi^2 = \frac{4}{9}$ (the signals are independently distributed).

Consider first the single-product benchmark. Using (8) and (10) we have that $\underline{\delta}_s = \frac{1}{2}$ and $\overline{\delta}_s = \frac{2}{3}$. When $\delta \geq \overline{\delta}_s$, the firm charges the full-information price, $p_s^* = \frac{5}{8}$, and has profits of $(p_s^* - c)(1 - p_s^*) = \frac{9}{64}$. When $\delta \in (\underline{\delta}_s, \overline{\delta}_s)$, then the firm charges $p_s(\delta) > p_s^*$ as defined in Proposition 1. As δ approaches $\underline{\delta}_s$ from above, the price $p_s(\delta)$ approaches $\overline{v} = 1$, and profits converge to zero.

Next, consider a multiproduct firm that bundles its products. Using (13) and (15), we have $\underline{\delta}_b = \frac{3}{8}$ and $\overline{\delta}_b = \frac{9}{14}$. When $\delta \geq \overline{\delta}_b$, then the firm charges the full-information bundled price $p_b^* = \frac{1}{2}$ and earns profits $\frac{1}{4}$. When $\delta \in (\underline{\delta}_b, \overline{\delta}_b)$, then the firm charges $p_b(\delta) > p_b^*$ as defined in Proposition 2. As δ approaches $\underline{\delta}_b$ from above, the price $p_b(\delta)$ approaches $\overline{v} = 1$, and profits converge to zero.

Finally, consider a multiproduct firm that does not bundle its products. When the discount factor is sufficiently high, the multiproduct firm charges the full-information price, $p_m^* = p_s^* = \frac{5}{8}$ and earns profits $\frac{9}{64} + \frac{9}{64} = \frac{9}{32}$. The number of consumers who purchase product i by itself is $\phi_i(p_m^*, p_m^*) = p_m^*(1 - p_m^*) = \frac{15}{64}$, and the number of consumers who purchase both products is $\phi_{12}(p_m^*, p_m^*) = (1 - p_m^*)^2 = \frac{9}{64}$. Plugging these values and the formulas for $S_i(p_m^*, \delta)$ and $B(p_m^*, \delta)$ from (7) and (12) into the incentive constraint (18), and setting (18) equal to zero gives $\overline{\delta}_m = \frac{8}{13}$. Taking the limit as the prices approach $\overline{v} = 1$ in (18), one can show that $\underline{\delta}_m = \frac{1}{2}$, the same lower bound as for the single-product benchmark.

The result that $\underline{\delta}_m = \underline{\delta}_s$ is interesting. In the multiproduct environment, as δ falls and the optimal price increases, the proportion of units that are sold to consumers who are purchasing both products instead of just one is falling. In our uniform example, the mass of consumers who buy both products is $(1-p)^2$, while the total sales is $2(1-p)$. As prices approach 1, the proportion of consumers purchasing both products converges to zero, and the advantage the multiproduct firm has over the single-product firm vanishes, so $\underline{\delta}_m = \underline{\delta}_s$.

[Figure 1 about here.]

Figure 1 compares the prices, $p_s(\delta)$, $p_b(\delta)$, and $p_m(\delta)$, for the three regimes. Note that $p_m(\delta) \leq p_s(\delta)$ because incentive compatibility is easier when the firm sells multiple products.[27] Note also that $\overline{\delta}_m < \overline{\delta}_s$, so the full-information prices can be sustained for a larger range of parameter values for the multiproduct firm

[27]The closed-form solution for $p_b(\delta)$ is in Proposition 2. $p_m(\delta)$ is defined implicitly by $M(p_m(\delta), p_m(\delta), \delta) = 0$. Numerical solutions were found using the Solver feature of Excel.

as compared to the single-product firm. Finally, note that $\underline{\delta}_b < \underline{\delta}_m$. There is a range of parameter values where bundling is necessary to assure high quality.

Figure 2 compares the profits under the three regimes. For the single-product benchmark, we double the profits to maintain comparability with the multiproduct firm. First, note that the firm prefers not to bundle when the discount factor is sufficiently high. Note also that the multiproduct firm achieves strictly higher profits than the single-product benchmark when $\delta \in (\underline{\delta}_s, \overline{\delta}_s)$. Selling two products instead of just one relaxes the firm's incentive compatibility constraint. Finally, note the firm strictly prefers to bundle its products when $\delta \in (\underline{\delta}_b, \underline{\delta}_m)$. The firm's profits are strictly positive if it bundles, but zero otherwise because high quality is not sustainable. Note that there is a range of discount factors $\delta \in (\underline{\delta}_m, 0.54)$ where high quality is feasible without bundling, but the firm strictly prefers to bundle.

Figure 3 compares the total surplus – firm profits plus consumer surplus – for the three regimes. Simulations reveal that total surplus is highest with bundling if and only if $\delta < 0.57$. Taken together with the results in Figure 2, we see that when $\delta < .54$, the firm's private decision to bundle its products serves the interests of society.

[Figure 2 about here.]

[Figure 3 about here.]

5 Extensions

We now consider several extensions of the model. To simplify the presentation, we assume throughout the section that price and quality are chosen at time zero before the firm learns its type. As discussed in Section 3 for the single-product benchmark, and in the proof of Proposition 3 in the Appendix, this timing delivers similar results as the full dynamic model.[28]

Independent Types

Section 4 assumed that the branded firm's type – capable or incapable – was perfectly correlated across the two products. So, if the branded firm was incapable of producing a high-quality version of product 1, it was also incapable of producing a high-quality version of product 2. This assumption is realistic if we think of

[28]It is more natural to assume that the firm chooses its quality after it learns its type, but this action is unobservable, so the timing is easily generalized. The firm might also prefer to adjust its prices over time, but our conclusions are robust to relaxing the fixed price assumption.

an incapable type as being attributable to poor central management, low-quality inputs, or other problems that are likely to be common to different activities of the firm. This assumption implied that after observing just one negative signal, a consumer would stop purchasing both products from the firm, whether the products were bundled or unbundled.

We now assume that the branded firm's types for the two products are independently distributed, so the branded firm may be incapable for product 1, but capable for product 2. In a high-quality PBE, if a consumer observes a negative signal from product 1, the consumer will believe that the firm is incapable of producing product 1, but will not draw a negative inference about the firm's type for product 2. Because the types are independently distributed, a negative signal about one product does not change the consumer's posterior beliefs about the firm's type for other product. In this setting, we find that a multiproduct firm selling unbundled products has no advantage whatsoever over a single-product firm. In contrast, a multiproduct firm selling bundled products is at a significant advantage over a single-product firm. So once again, bundling helps the firm to assure quality.

To begin, suppose that the products are unbundled and sold separately. In a high-quality PBE, committing to high quality for two products dominates committing to high quality for just one product if

$$d_i(p_i)\frac{p_i - c}{1-\delta} \geq d_i(p_i)\frac{p_i}{1-\delta\pi}, \tag{19}$$

and similarly, committing to high quality for two products dominates offering two low-quality products if

$$\sum_{i=1,2} d_i(p_i)\frac{p_i - c}{1-\delta} \geq \sum_{i=1,2} d_i(p_i)\frac{p_i}{1-\delta\pi}. \tag{20}$$

Notice that (19) implies (20), and that (19) is exactly the same as our single-product incentive constraint in (6). It follows that a high-quality PBE with positive sales is sustainable if and only if $\delta > \underline{\delta}_s$, the same threshold as for the single-product benchmark defined in (10). Indeed, the results of Propostion 1 apply here as well. In the highest-profit PBE, the prices charged are the same as the prices in the single-product benchmark in Proposition 1, $p_m^I(\delta) = p_s(\delta)$.

With bundling, the multiproduct firm is in a significantly better position to assure high quality. To see why, consider a PBE in which the firm chooses high-quality for both products. If the consumer sees a negative signal for product 1 only, then the consumer believes that the firm is incapable for product 1, and expects the quality of product 1 to be low in the future, but the consumer still believes the firm is capable (almost surely) for product 2. This implies that a consumer who was initially indifferent between purchasing the bundle from the branded firm and purchasing two low-quality products from the fringe will stop buying the bundle, even if they expect that the firm will continue to choose high quality for product 2. Indeed, if we assume further that $p_b^* \geq \overline{v}/2$, so the price of the bundle exceeds $\overline{v}$, then not just this consumer, but all consumers, will abandon the branded firm (stop buying the bundle) after observing just one negative signal.[29] Punishment is more severe when the firm bundles the products because after observing one negative signal the consumer stops purchasing both products instead of just one.

To see this more formally, note that in a high-quality PBE with bundling (and assuming $p_b \geq \overline{v}/2$) the firm would not want to deviate and produce one low-quality product when

$$2d_b(p_b)\frac{p_b - c}{1-\delta} \geq d_b(p_b)\frac{2p_b - c}{1-\delta\pi}. \tag{21}$$

The left-hand side is the present discounted value of selling two high-quality products, and the right-hand side is the present discounted value from deviating and producing one low-quality product and one high-quality product. Similarly, the firm would not want to deviate and sell two low-quality products when

$$2d_b(p_b)\frac{p_b - c}{1-\delta} \geq 2d_b(p_b)\frac{p_b}{1-\delta\pi_b}, \tag{22}$$

where the right-hand side is now the present discounted value from deviating and producing two low-quality products. It is straightforward to show that (22) implies (21), because $\pi_b > \pi$. Notice also that (22) is identical to the incentive constraint for correlated types (12).[30] So if $p_b^* \geq \overline{v}/2$, then the equilibrium characterization in Proposition 2 also holds for independently distributed types.[31] High quality is sustainable with positive sales if and only if $\delta > \underline{\delta}_b$ and in the highest-profit PBE, $p_b^I(\delta) = p_b(\delta)$.

If price and quality were chosen every period, then the analysis for independently distributed types would be more complicated. In particular, a firm that learns that it is incapable of producing product 1 might prefer to adjust its price

[29] Our assumption that $p_b \geq \overline{v}/2$ holds for a broad range of demand specifications, including our numerical example discussed earlier, but even without this assumption, bundling would still induce some consumers to stop buying both products which implies that bundling does better.

[30] If $p_b < \overline{v}/2$, then consumers with valuations $v_i > 2p_b$ would continue to purchase the bundle even after receiving a negative signal, which adds profits to the right hand side of (22). In this case, (22) would be harder to satisfy than (12).

[31] Because $p_b(\delta) \geq p_b^*$ for all δ in Proposition 2, we have $p_b(\delta) > \overline{v}/2$.

downwards, and sell a high-quality product 2 bundled with a low-quality product 1. However when $\delta \in (\underline{\delta}_b, \underline{\delta}_s)$, high quality for just one product is not sustainable because $\delta < \underline{\delta}_s$. So in any perfect Bayesian equilibrium, when δ is in this range, consumers cannot believe that following any price change (a publicly observable event) they will be in a subgame in which the firm produces just one high-quality product. So our conclusion that bundling helps the firm to assure high quality under independently distributed types for a broader range of parameter values is robust to relaxing the assumption that price is set only at the start of the game.

This extension demonstrates that the quality-assuring advantage of bundling is stronger – and more profound – when the branded firm's type is uncorrelated across the two products. Without bundling, the multiproduct firm is no better at providing high quality than the single-product firm. Because the types are independently distributed, observing a negative signal for product 1 is uninformative about the firm's type for the other product 2. Because monitoring is private, the consumer would stop buying product 1 after a negative signal but would continue to purchase product 2, believing that the quality is (almost surely) high and will continue to be high in the future. This means that under imperfect monitoring, the independent-types model cannot explain the empirical evidence associated with chain restaurants in Jin and Leslie (2009) whereas the correlated-types model can.

When the products are bundled, consumers are constrained to purchase either both products from the branded firm or neither product from the branded firm. In equilibrium, if a consumer sees a negative signal for just one product, then the demand for the bundle will fall. Importantly, this does not happen because the consumer believes that both products are low quality. Instead, it happens because the presence of one low-quality in the bundle pulls down the overall value of the bundle, reducing consumer surplus. So by bundling the products together, the multiproduct firm induces consumers to punish its product quality deviations more severely, helping it to assure high quality.

The Attribution Problem

We now consider a variant of our model in which consumers cannot attribute negative signals to specific products. Consumers don't know which product or products caused the negative signal, or even how many signals were received. Consumers only observe whether or not there was a negative signal. For brevity, we present this extension in a simple setting where prices and qualities are chosen at the beginning of the game. The results extend to a more general setting in which prices and qualities are chosen every period.

The attribution problem has been invoked in a number of antitrust lawsuits.

In a landmark case from the 1930s, IBM argued that tying the sale of paper tabulating cards to the lease of tabulating machines was a necessary and legitimate business practice to assure quality. According to IBM, even small deviations in the size or thickness of the cards, or the presence of slime or carbon spots, "could cause inaccuracies in the function of the machine, serious in their consequences and difficult to trace to their source, with consequent injury to the reputation of the machines and the good will of the lessors."[32]

To begin, note that for sufficiently imperfect monitoring, a single-product firm cannot maintain a reputation for high quality because consumers will never punish the firm. In a high-quality PBE, a consumer who buys just one product from the branded firm and the other product from the fringe will attribute negative signals (almost surely) to the product purchased from the fringe. This is a rational inference in a high-quality equilibrium, because the fringe definitely produces low-quality products and the branded firm (almost surely) produces high-quality products. But if consumers always blame the fringe, then shirking by the branded firm is profitable. While consumers also learn from the rate of negative signals, it still follows that a high-quality PBE can fail to exist. That is, for sufficiently large ϵ, consumers will punish the firm (i.e., believe the firm is likely to be incapable) if the observed rate of negative signals is sufficiently high, but for any δ there exists an ϵ sufficiently small that the incidence of punishment is too low to sustain a high-quality PBE.[33]

Next, suppose that the firm bundles its products. A necessary and sufficient condition for sustaining a high-quality PBE with bundling is given by (12) in Section 4. Because the firm's types are correlated, a negative signal from the bundle is enough to convince consumers that the firm is incapable of producing a high-quality version of either product. Believing that the bundle is worthless, the consumer will stop buying the bundle. The conditions under which a high-quality PBE exists and the characterization of the highest-profit PBE is exactly the same as in Proposition 2.

Finally, suppose that the branded firm sells the two products separately. In a high-quality PBE, some consumers purchase just one product from the branded firm while others will purchase both branded products. The former group attributes negative signals to the competitive fringe, and do not punish the branded firm. The latter group attributes negative signals to the branded firm, concluding that the branded firm is incapable of producing high-quality products.

[32] *IBM v. United States* (298 U.S. 131 [1936]). While accepting the need for quality assurance, the Supreme Court observed that IBM could include contractual restrictions in its leases requiring lessees to only use cards that met the necessary specifications.

[33] Note however that high quality cannot be supported with probability one when quality is chosen every period.

A necessary and sufficient condition for the existence of a high-quality PBE without bundling is

$$\sum_{i=1,2} d_i(p_i)\frac{p_i}{1-\delta} \geq \sum_{i=1,2} \phi_i(p_1,p_2)\frac{p_i - c}{1-\delta} + \phi_{12}(p_1,p_2)\frac{p_1+p_2}{1-\delta\pi_b}. \qquad (23)$$

The proof that (23) is sufficient is identical to the proof of sufficiency in Proposition 2. The left-hand side of this expression is the present discounted value of profits when the product quality is high, and the right-hand side is the present discounted value of profits when the quality of both products is low. This incentive constraint will be more difficult to satisfy than before in (16). Consumers who purchase a single product from the branded firm continue to purchase from the branded firm even after they observe a negative signal, diluting the incentives of the branded firm to produce high-quality products.

We have just shown that bundling has even greater private and social value when the consumer cannot attribute negative signals to individual products. With imperfect attribution, consumers will rationally blame the fringe for negative signals. For the single-product benchmark, we showed that a high-quality PBE fails to exist. The attribution problem also reduces the firm's incentives to produce high quality when it sells two unbundled products. But the attribution problem does not change the firm's incentives when the products are bundled, which establishes that the incremental value of bundling is even greater under the attribution problem, at least when monitoring is sufficiently imperfect.

Bundling Durable and Nondurable Goods

Many examples of bundling, particularly those that are challenged in court, involve a durable good tied to a nondurable complementary good or service: IBM computers and punch cards, printers and ink, and automobiles and spare parts are all such examples. We now explore whether bundling can facilitate high quality when one good is an infinitely-lived durable good, and the other good is a nondurable good that is purchased every period.[34]

We explicitly assume that the goods are complementary, so a consumer derives no utility from the durable good in any given period unless he or she also consumes the nondurable good.[35] We use d and n (short for durable and nondurable) in place of the numbers 1 and 2 to denote the two products. So c_n is the cost of

[34]As discussed in Section 2, the quality-assurance advantages of bundling a durable and nondurable have been previously explored by Schwartz and Werden (1996) and by Dana and Spier (2015). But the analysis here is much more general.

[35]This is consistent with the main model, but was not explicitly stated earlier because consumers always bought both products every period.

supplying the high quality nondurable good, and the perpetuity cost of supplying one unit of the durable good is c_d, so $C_d = \frac{c_d}{1-\delta}$ represents the total production cost of the durable. Similarly, we define p_d to be the perpetuity payment for the durable, so $P_d = \frac{p_d}{1-\delta}$ is the price of the durable. Although symmetry may be less plausible in this setting, for convenience and brevity we nevertheless maintain our assumptions that $f(v_d, v_n)$ is symmetric, that $c_d = c_n = c$, and that $\pi_d = \pi_n = \pi$.

Suppose that the branded firm bundles the two products. Then a consumer who purchases a durable from the branded firm may not purchase nondurables from the fringe in the future. In this context, bundling can be thought of as either an enforceable contract that prevents consumers from mixing-and-matching the branded durable good with the nondurable good from the fringe, or a technological decision that makes the branded durable good incompatible with the fringe's nondurable good.

In a high-quality PBE, a consumer will purchase the bundled contract when the sum of the valuations is smaller than the sum of the perpetuity payments, $v_d + v_n \leq p_d + p_n$. A high-quality PBE exists if and only if

$$d_b(p_b)\left(\frac{p_d - c_d}{1-\delta}\right) + d_b(p_b)\left(\frac{p_n - c_n}{1-\delta}\right) \geq d_b(p_b)\left(\frac{p_d}{1-\delta}\right) + d_b(p_b)\left(\frac{p_n}{1-\delta\pi_b}\right), \quad (24)$$

where $p_b = (p_n + p_d)/2$. The proof that (24) is sufficient is identical to the proof of Proposition 2, so the proof is omitted. The left-hand side is the present value of the equilibrium profits from the sales of the durable, expressed as a stream, plus the present value of the profits from the sales of the nondurable. The right-hand side is the present value of the firm's profits if it deviates to low quality for both products.

It is not hard to see that (24) is just as easy to satisfy as (12), the incentive constraint for bundling two nondurable goods from Section 4. To see why, suppose the firm does not charge anything for the durable good and makes all of its profits through the continued sales of the nondurable good: $p_d = 0$ and $p_n = 2p_b$. With this transformation, along with our assumption that $c_d = c_n = c$, the incentive constraint in (24) is equivalent to that in (12).[36] Shifting all of the rents to the nondurable good allows the firm to maintain the same incentives to produce high quality as a firm that produces nondurable goods only.

Recall that we assumed the durable and nondurable goods are complements, so consumers derive no value from the durable good unless they also consume the nondurable good. If this were not true, then a consumer might purchase

[36] Note that this argument does not rely in any substantive way on the symmetry assumption.

the branded durable good by itself at price $p_d = 0$ and forego the use of the nondurable good altogether, depriving the branded firm of the stream of future profits. Similarly, it is critical that bundling can prevent consumers who purchase the branded durable good from using nondurables supplied by the fringe.

Finally, consider a multiproduct firm that sells the durable for $P_d = \frac{p_d}{1-\delta}$ at time $t = 1$ and the nondurable at a price p_n at $t = 1$ and in every period thereafter, but does not bundle the two goods. We can again represent the firm's per-period demand in three components: consumers who buy both products, $\phi_{dn}(p_d, p_n)$; consumers who buy only the durable good, $\phi_d(p_d, p_n)$; and consumers who buy only the nondurable good, $\phi_n(p_d, p_n)$. Finally, recall that $d_i(p_d, p_n) = \phi_i(p_d, p_n) + \phi_{dn}(p_d, p_n)$. A high-quality PBE exists if and only if

$$\begin{aligned} &d_d(p_d, p_n)\left(\frac{p_d - c_d}{1-\delta}\right) + d_n(p_d, p_n)\left(\frac{p_n - c_n}{1-\delta}\right) \\ &\quad \geq d_d(p_d, p_n)\left(\frac{p_d}{1-\delta}\right) + \phi_n(p_d, p_n)\left(\frac{p_n}{1-\delta\pi}\right) + \phi_{dn}(p_d, p_n)\left(\frac{p_n}{1-\delta\pi_b}\right) \end{aligned} \quad (25)$$

where the left-hand side is the present value of the equilibrium profits from the sales of the durable plus the nondurable and the right-hand side is the present value of the profits if the firm deviates to low quality at time 0. Clearly this constraint is more difficult to satisfy than (17). So, at least under our admittedly restrictive assumptions, bundling has even more added value when one product is a durable good.

Mixed Bundling

Our analysis has thus far considered pure bundling, where consumers could purchase either both products or neither product from the branded firm. We now extend our framework to consider mixed bundling and show that our core insight – bundling creates incentives for higher quality and can raise producer and consumer surplus – is robust to mixed bundling.

First, note that when quality is observable, then the firm will prefer mixed bundling to pure bundling (see Adams and Yellen, 1976, and McAfee, McMillan and Whinston, 1989). This is easy to see using our numerical example. If the firm used pure bundling it would set $p_b = 1/2$ and earn a profit of $1/2$ on each bundle. Offering a bundle price of $p_b = 1/2$ and single-product price of $p_1 = p_2 = 7/8$ clearly does better because some customers purchase the single product who would not have purchased otherwise and customers who would have bought the bundle otherwise now generate a profit of $5/8$, but they only generate a profit of $1/2$ when they purchase the bundle.

When quality is unobservable to consumers, mixed bundling is a double-edged sword. Mixed bundling is profitable for the firm insofar as it facilitates price discrimination. But on the other hand, single-product sales can compromise the branded firm's incentives to maintain high quality. Because consumers who buy single products are worse monitors, the branded firm will need to pay particular attention to the adverse incentives associated with the single-product sales when choosing the prices for the mixed bundle.

To highlight the incentive effects of mixed bundling, let p and p_b be the per unit prices for the single and bundled products. Let $\psi_i(p, p_b)$ be the demand for product $i = 1, 2$ alone, and let $\psi_b(p, p_b)$ be the demand for the bundle. A necessary condition for a symmetric high-quality PBE to exist is

$$\sum_{i=1,2} \psi_i(\cdot)\frac{p-c}{1-\delta} + \psi_b(\cdot)\frac{2p_b - 2c}{1-\delta} \geq \sum_{i=1,2} \psi_i(\cdot)\frac{p}{1-\delta\pi} + \psi_b(\cdot)\frac{2p_b}{1-\delta\pi_b}. \tag{26}$$

The left-hand side is the firm's equilibrium path profits, and the right-hand side is the profit earned if the firm deviates to low quality for both products.

Note that (26) strongly resembles the multiproduct firm's constraint when selling unbundled products in (16). The expressions differ because here p_b is a choice variable whereas in (16) the price p_b was defined to be the average of the single product prices. As was true before, (26) can be expressed as a linear combination of the single-product and bundled-product incentive constraints:

$$\sum_{i=1,2} \psi_i(p, p_b) S_i(p, \delta) + \psi_b(p, p_b) B\left(p_b, \delta\right) \geq 0, \tag{27}$$

where $B(\cdot)$ and $S(\cdot)$ are defined as they were in (18).

The incentive constraint in (26) reveals an important result: The range of discount factors δ for which high quality can be supported is exactly the same with mixed bundling as it was with pure bundling. To see why this is true, recall that $\underline{\delta}_b$ and $\underline{\delta}_s$ are the thresholds for which high-quality can be sustained in the bundled-product and single-product cases, respectively. When $\delta < \underline{\delta}_b < \underline{\delta}_s$, then $S_i(\overline{v}, \delta) < 0$ and $B(\overline{v}, \delta) < 0$, so high quality cannot be sustained in the single-product or bundled-product cases. It follows that when $\delta < \underline{\delta}_b < \underline{\delta}_s$, there does not exist a high-quality PBE with mixed bundling, either. Finally, when $\delta > \underline{\delta}_b$ then there is a high-quality PBE with pure bundling, so there is a high-quality PBE with mixed bundling as well (because the firm has the option to set single-product prices $p = \overline{v}$). Thus, although our main model focused on pure bundling, the core results are robust to mixed bundling.

6 Conclusion

Product bundling creates private and social value when consumers are small, heterogeneous, and receive imperfect private signals about product quality. When a multiproduct firm bundles its products, consumers are constrained to purchase both products instead of just one. This makes the firm's consumers into better monitors and tougher private enforcers, increasing both the accuracy with which consumers observe quality deviations and the severity of the punishments that they can and will impose. We proved that the quality-assuring advantages of bundling are even stronger when the firm's type (capable or incapable) is uncorrelated across products, when consumers are unable to attribute a negative result to the particular product that caused it, and when one product is a durable good. We also showed that our insights are robust to mixed bundling.

Several of our assumptions deserve some additional discussion.

First, we assumed that the probability that the branded firm is the incapable type is arbitrarily small. This greatly simplified the characterization of the strategies and demand both on and off of the equilibrium path. Without this assumption, the analysis would have been more complex for several reasons. The first reason is that because monitoring is private and consumers have heterogeneous experiences, the consumers' beliefs about the firm's type (capable or incapable) would evolve differently over time. Also, as consumers are long-lived, their optimal consumption strategies would no longer be characterized by simple static consumer surplus maximization. That is, consumers would internalize the impact of current consumption on their learning and on their expected consumer surplus from future consumption. And finally, because the distribution of consumers' beliefs would not be stationary over time, the firm's prices would evolve over time as well. Our assumption that the probability that the firm is incapable is arbitrarily small allows us to sidestep these issues. We do not think that relaxing this assumption would change the intuition or qualitative findings.

Second, we made the restrictive assumption that high-quality products do not generate negative signals. If this were relaxed, then negative signals would arise when the firm is capable as well as incapable. If the probability of the firm being capable (but unlucky) is very small compared to the probability of the firm being incapable, then consumers would rationally punish the firm following the observation of a negative signal. Consumers would infer that in all likelihood the firm is incapable, not just unlucky, and that product quality will be low in the future. So our qualitative results should continue to hold in this case.

If high-quality products generated negative signals with high enough probability, then the equilibrium of our game would be quite different. In a pure-strategy equilibrium, following a negative signal, a consumer believes it is more likely than not that the capable firm chose high quality, but was unlucky with the signal. Given this belief, the consumer would not punish the firm, so a pure-strategy equilibrium with high quality cannot exist. Although high-quality equilibria could exist in mixed strategies, a thorough analysis of the mixed strategy equilibrium would be particularly difficult in our setting because consumer heterogeneity implies that consumers learn at different rates.[37]

Perhaps a simpler way to introduce negative signals on the equilibrium path would be to assume that consumers observe a public signal in addition to their private signals. Following Cai and Obara (2006) and Green and Porter (1984), one could consider a public-trigger-strategy equilibrium where the capable firm produces high-quality products until a negative public signal is observed, after which the firm produces low-quality products and consumers stop buying from the firm. The negative public signal occurs even when quality is high, but is more likely when quality is low. Because consumers punish the firm on the equilibrium path, firms have an incentive not to choose low quality which would increase the likelihood of punishment.[38] Bundling would be valuable if it increases the informational content of the public signal which would be clearly be true if the public signal were a random sample of the consumers' private signals.

Third, we assumed that the two products are symmetric in cost, in demand, and in the monitoring technology. Symmetry simplified several of our proofs. For example, without symmetry, it may be the case that a firm may find it profitable to forego selling a bundle of two high-quality products and focus instead on selling a single high-quality product. That said, bundling should always make it easier to sustain high quality for both products relative to selling the two unbundled products. Bundling makes punishment more severe (never easier), and makes monitoring more informative (never less). The model and its insights should generalize to a variety of asymmetric settings that are important for antitrust applications. For example, if one product is purchased more frequently, or one product's quality is easier to observe, then for some range of discount factors bundling should still increase firm profits and consumer surplus. That said, the potential ways in which products can differ is very large, and so demonstrating robustness in each relevant dimension of asymmetry is beyond the scope of this article.

Finally, our model focused on the quality assurance rationale for bundling. In fact, there are many other rationales for bundling such as price discrimination and economies of scope in production and design as discussed in Section 2. There

[37]This is why Mailath and Samuelson's (2001) assumption that new inept types arrive at a rate that assures the stationarity of inept types in their model will not work in our context.

[38]Note that profits would be even higher in an equilibrium with finite-length punishment periods (as in Green and Porter, 1984).

may also be broader strategic effects, because product bundling can impact the intensity of competition between incumbent firms and may deter entry of potential competitors. The social costs of any anticompetitive effects from bundling would naturally need to be weighed against the social benefits of any quality improvements. Theoretical and empirical work combining these effects may be fruitful directions for future research.

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7 Appendix

Proof of Proposition 1. In the body of the article we showed that (6), or $\delta > \underline{\delta}_s$, was necessary for a high-quality PBE. Here we show that it is also sufficient.

Consider a PBE in which the firm, if capable, always produces a high-quality product (at all of its on and off-the-equilibrium-path decision nodes), and consumers believe the firm is the capable type (almost surely) unless they see a negative signal, in which case consumers believe that the firm is the incapable type and do not purchase the product again.

The number of remaining consumers at any decision node in this equilibrium depends only on the number of deviations in the past, not on when those deviations occurred, so the firm will prefer high quality to low quality at any decision node (a one-shot deviation) if

$$d_i(p)\pi^{k_i}\frac{p-c}{1-\delta} \geq d_i(p)\pi^{k_i}\left[p+\pi\delta\left(\frac{p-c}{1-\delta}\right)\right], \tag{28}$$

where k_i, $i = 1, 2$, is the number of prior deviations and π^{k_i} is the fraction of the consumers who have not observed a negative signal after k_i deviations, and when $k_i = 0$, this reduces to (6), the on-the-equilibrium-path constraint. (Recall that the branded firm produces only good i so consumers do not stop purchasing after negative signals about another product).

Both (28) and (6) are equivalent to (7), so high quality is optimal at every decision node if and only if (6) holds, and so by the one-shot deviation principle the firm has no profitable deviations in the proposed equilibrium, and (6), or $\delta > \underline{\delta}_s$, is sufficient for a high-quality PBE to exist. □

Proof of Proposition 2. In the body of the article we showed that (11), or $\delta > \underline{\delta}_b$, was necessary for a high-quality perfect Bayesian equilibrium. Here we show that it is also sufficient.

Consider a perfect Bayesian equilibrium in which the branded firm chooses high quality for both of its products at every decision node, both on and off the equilibrium path. If a consumer observes either one or two negative signals, then the consumer believes that the firm is the incapable type and does not purchase the product again, otherwise the consumer believes the firm is (almost surely) the capable type.

First, a deviation to low quality (one-shot) for the firm is not profitable if

$$2d_b(p_b)\pi^{k_1}\pi^{k_2}\pi_b^{k_{12}}\frac{p_b-c}{1-\delta} \geq 2d_b(p_b)\pi^{k_1}\pi^{k_2}\pi_b^{k_{12}}\left[2p_b-c+\delta\pi\left(\frac{p_b-c}{1-\delta}\right)\right], \tag{29}$$

where k_i, $i = 1, 2$, is the number of prior deviations in just the quality of product i, k_{12} is the number of prior deviations in the quality of both products, and π^{k_1} π^{k_2}

$\pi_b^{k_{12}}$ is the fraction of the consumers who have not observed at least one negative signal. When $k_1 = k_2 = k_{12} = 0$, this reduces to (11), the on-the-equilibrium-path constraint. But clearly both (29) and (11) are equivalent to (12), so if (11) holds then no deviation to two low-quality products is profitable at any on or off-the-equilibrium-path decision node.

We now prove that the strategies are also optimal with respect to one-shot deviations in the quality of just one product. That is, if deviations in both qualities are not profitable, or (12) holds, then deviations in just one quality are not profitable, or equivalently that (29), which can be rewritten as

$$\frac{p_b - c}{1-\delta} \geq \frac{p_b}{1-\delta\pi_b}, \tag{30}$$

implies

$$2d_b(p_b)\pi^{k_1}\pi^{k_2}\pi_b^{k_{12}}\frac{p_b - c}{1-\delta} \geq 2d_b(p_b)\pi^{k_1}\pi^{k_2}\pi_b^{k_{12}}\left[p_b + \delta\pi\left(\frac{p_b-c}{1-\delta}\right)\right]. \tag{31}$$

Equation (30) can be rewritten as

$$c \leq \left[\frac{1-\delta\pi_b}{1-\delta} - 1\right](p_b - c) = (1-\pi_b)\frac{\delta}{1-\delta}(p_b - c), \tag{32}$$

and (31) can be rewritten as

$$\frac{2p_b - 2c}{1-\delta} \geq \frac{2p_b - c}{1-\delta\pi}, \tag{33}$$

or

$$c \leq \left[\frac{1-\delta\pi}{1-\delta} - 1\right](2p_b - 2c) = 2(1-\pi)\frac{\delta}{1-\delta}(p_b - c), \tag{34}$$

so (30) implies (31) if $1 - \pi_b \leq 2(1-\pi)$, which is true because $\pi_b > 2\pi - 1$ by assumption. □

Proof of Proposition 3. In the body of the article we showed that (18), or $\delta > \underline{\delta}_m$, is necessary for existence of a high-quality PBE. We now show that (18) is also sufficient. Below, we prove that a simultaneous deviation in the quality of both products is not profitable. The proof that more general deviations are not profitable is in the supplemental online appendix.

Every decision node in the game can be characterized by k_1, k_2, and k_{12}, where k_i is the number of times the firm has deviated to low quality only for product i, and k_{12} is the number of times the firm has deviated to low quality simultaneously for both products. Let

$$\widehat{M}(p_1, p_2, \delta, k_1, k_2, k_{12}) = \sum_{i=1,2} \hat{\phi}_i(\cdot) S_i(p_i, \delta) + \hat{\phi}_{12}(\cdot) B\left(\frac{p_1 + p_2}{2}, \delta\right), \qquad (35)$$

where $\hat{\phi}_i(\cdot) = \phi_i(p_1, p_2)\pi^{k_i + k_{12}}$ and $\hat{\phi}_{12}(\cdot) = \phi_{12}(p_1, p_2)\pi^{k_1}\pi^{k_2}\pi_b^{k_{12}}$ are the number of remaining consumers purchasing just product i and purchasing both products respectively, and where the arguments of $\hat{\phi}_i(p_1, p_2, k_i, k_{12})$ and $\hat{\phi}_{12}(p_1, p_2, k_i, k_{12})$ are suppressed for brevity. Note that $\widehat{M}(p_1, p_2, \delta, 0, 0, 0) = M(p_1, p_2, \delta)$, so $\widehat{M}(\cdot) \geq 0$ is a generalization of $M(\cdot) \geq 0$ that describes every decision node of the game.

Note that condition (18) is equivalent to $M(p_1, p_2, \delta) \geq 0$ or equivalently $\widehat{M}(p_1, p_2, \delta, 0, 0, 0) \geq 0$. We now show that the following strategies and beliefs define a PBE if (18) holds. The capable firm produces high quality for both products if and only if $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}) \geq 0$. Consumers believe the firm is the incapable type if they ever observe a negative signal or if they ever observe a price below the equilibrium price, and consumers stop purchasing from the branded firm when they believe it is the incapable type.

Note that if $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}) > 0$ for any k_1, k_2, and k_{12}, then on the equilibrium path beginning at any decision node which follows exactly k_1, k_2, and k_{12} deviations, the capable firm will produce high quality every period because k_1, k_2, and k_{12} will not change so $\widehat{M}(\cdot)$ will not change. Similarly, if $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}) < 0$ for any k_1, k_2, and k_{12}, then on the equilibrium path beginning at any decision node following exactly k_1, k_2, and k_{12} deviations, the capable firm will produce low quality every period because $\widehat{M}(\cdot)$ will decline as k_{12} grows, so $\widehat{M}(\cdot)$ will remain negative.

Consider deviations in the quality of both products (deviations in just one product quality are considered in a supplemental online appendix). Such deviations can occur at three mutually exclusive types of decision nodes: First, nodes at which equilibrium quality is low and will stay low in subsequent periods (i.e., $\widehat{M}(\cdot) < 0$). Second, nodes at which equilibrium quality is high but a deviation in the quality of both goods moves the firm to a decision node at which its equilibrium quality is subsequently low (i.e., $\widehat{M}(\cdot) \geq 0$, and $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}+1) < 0$). And third, nodes at which equilibrium quality is high and a deviation in quality moves the firm to a node at which equilibrium quality remains high (i.e., $\widehat{M}(\cdot) \geq 0$ and $\widehat{M}(p, p, \delta, k_1, k_2, k_{12} + 1) > 0$).

By the one-shot deviation principle – see, for example, Tadelis (2013) – it is sufficient to consider one-shot deviations at each of these three types of decision nodes. First, consider deviations at nodes where $\widehat{M}(\cdot) < 0$. A one-shot deviation to high quality for both goods at this decision node is not profitable as long as

$$\sum_{i=1,2} \hat{\phi}_i(\cdot)\frac{p_i}{1-\delta\pi} + \hat{\phi}_{12}(\cdot)\frac{p_1+p_2}{1-\delta\pi_b} \geq \sum_{i=1,2} \hat{\phi}_i(\cdot)(p_i - c) + \hat{\phi}_{12}(\cdot)(p_1+p_2-2c) + \delta\left[\sum_{i=1,2} \hat{\phi}_i(\cdot)\frac{p_i}{1-\delta\pi} + \hat{\phi}_{12}(\cdot)\frac{p_1+p_2}{1-\delta\pi_b}\right], \quad (36)$$

where the right-hand side characterizes the profit from deviating to high quality for both goods at the decision node and reverting to low quality for both goods (because $\widehat{M}(\cdot) < 0$ at every period thereafter). This can be rewritten as

$$\left(\frac{1-\delta}{1-\delta\pi}\right)\sum_{i=1,2} \hat{\phi}_i(\cdot)S(p_i,\delta) + \left(\frac{1-\delta}{1-\delta\pi_b}\right)\hat{\phi}_{12}(\cdot)B\left(\frac{p_1+p_2}{2},\delta\right) \leq 0, \quad (37)$$

which is clearly satisfied because $\widehat{M}(\cdot) < 0$, $B(\frac{p_1+p_2}{2},\delta) > S(p_1,\delta) + S(p_2,\delta)$, and $1-\delta\pi < 1-\delta\pi_b$, so a one-shot deviation to high quality for both goods is not profitable.

Next consider nodes at which $\widehat{M}(\cdot) \geq 0$, and $\widehat{M}(p,p,\delta,k_1,k_2,k_{12}+1) < 0$, so following a deviation to low quality the firm continues to produce low quality. A deviation to low quality for both goods is not profitable as long as

$$\sum_{i=1,2} \hat{\phi}_i(\cdot)\frac{p_i-c}{1-\delta} + \hat{\phi}_{12}(\cdot)\frac{p_i-c}{1-\delta} \geq \sum_{i=1,2} \hat{\phi}_i(p_1,p_2)\frac{p_i}{1-\delta\pi} + \hat{\phi}_{12}(\cdot)\frac{p_1+p_2}{1-\delta\pi_b}, \quad (38)$$

which can be rewritten as

$$\frac{1}{1-\delta\pi}\sum_{i=1,2} \hat{\phi}_i(\cdot)S(p_i,\delta) + \frac{1}{1-\delta\pi_b}\hat{\phi}_{12}(\cdot)B(p_i,\delta) \geq 0. \quad (39)$$

Using $\pi_b < \pi$ and the definition of $\widehat{M}(\cdot)$ it is easy to see that $\widehat{M}(\cdot) \geq 0$ implies (39), so a one-shot deviation to low quality for both goods is not profitable.

Finally, consider nodes at which $\widehat{M}(\cdot) \geq 0$ and $\widehat{M}(p,p,\delta,k_1,k_2,k_{12}+1) \geq 0$. A deviation to low quality is not profitable if

$$\sum_{i=1,2} \hat{\phi}_i(\cdot)S(p_i,\delta) + \hat{\phi}_{12}(\cdot)B\left(\frac{p_1+p_2}{2},\delta\right) \geq 0. \quad (40)$$

which is clearly identical to $\widehat{M}(\cdot) \geq 0$, so it clearly holds.

It is also clear that no deviation in price is profitable. Lowering price causes consumers to expect low quality, and raising price lowers expected profits even if consumers expect high quality.

So the firm's strategies described above are optimal, and clearly consumers' strategies are optimal and consistent with Bayes' rule, and so the strategies constitute a high-quality PBE as long as $\widehat{M}(p, p, \delta, 0, 0, 0) \geq 0$, so high quality is optimal prior to any deviations.

Because $M(p_1, p_2, \delta)$ is continuous and increasing in δ, it is straightforward to show that for any p_1, p_2, there exists a unique $\delta(\cdot) > 0$ such that $M(p_1, p_2, \delta(\cdot)) = 0$. Clearly $\underline{\delta}_m = \min_{p_1,p_2} \delta(\cdot)$. So there exists $\underline{\delta}_m$ and associated prices, $\hat{p}_1, \hat{p}_2$, such that $M(\hat{p}_1, \hat{p}_2, \underline{\delta}_m) = 0$, such that $M(p_1, p_2, \delta) < 0$ for all p_1, p_2 and for $\delta < \underline{\delta}_m$, and such that $M(p_1, p_2, \delta) > 0$ (i.e., (18) is satisfied) for some prices p_1, p_2 for $\delta > \underline{\delta}_m$.

Similarly it is straightforward to show that there exists a unique $\overline{\delta}_m \in (0, 1)$ such that $M(p_i^*, p_i^*, \overline{\delta}_m) = 0$ and $M(p^*, p^*, \delta) \geq 0$ if only if $\delta \geq \overline{\delta}_m$. And because p_i^* is the unconstrained monopoly price, $M(p_i^*, p_i^*, \overline{\delta}_m)$ must be locally increasing in these prices – a small price increase has no effect on the left-hand side of (16) – the derivative with respect to prices is equal to zero – but the right-hand side of (16) is clearly decreasing in price because higher prices reduce the share of single product purchasers and so increase the average rate of negative signals following a deviation. So there exist prices such that $M(p_1, p_2, \overline{\delta}_m) > 0$, which implies that $\underline{\delta}_m < \overline{\delta}_m$. □

Proof of Proposition 4. Clearly when $\delta < \underline{\delta}_m$, then high quality cannot be sustained – single-product firms and the multiproduct firm produce low quality. When $\delta \in (\underline{\delta}_m, \underline{\delta}_s)$, then in the most profitable PBE, the multiproduct firm produces high quality and the single-product firms produce low quality – so producer surplus and consumer surplus is strictly higher with a multiproduct firm. When $\delta \in (\underline{\delta}_s, \overline{\delta}_m)$, then both single-product firms and the multiproduct firm produce high quality, but the prices charged are strictly lower for the multiproduct firm, so producer surplus and consumer surplus is strictly higher with a multiproduct firm. When $\delta \in (\underline{\delta}_m, 1)$, then both single-product firms and the multiproduct firm produce high quality, and the prices charged are the same, so producer surplus and consumer surplus are the same. □

Proof of Proposition 5. By definition $B(\bar{v}, \underline{\delta}_b) = 0$, and clearly $S_i(\bar{v}, \underline{\delta}_b) < 0$, $\forall i$ because $\pi_b > \pi$ which implies $\underline{\delta}_b < \underline{\delta}_s$. So (18) clearly implies that $M(\bar{v}, \bar{v}, \underline{\delta}_b) < 0$ in a neighborhood of $p = \bar{v}$. And because $S_i(p_i, \delta)$, $\forall i$ and $B((p_1 + p_2)/2, \delta)$ are strictly increasing in p_1 and p_2, $M(p_1, p_2, \delta)$ is strictly increasing in p_1 and p_2. This implies $M(p_1, p_2, \underline{\delta}_b) < 0$ for all $p_1 < \bar{v}$ and $p_2 < \bar{v}$, so $\underline{\delta}_b < \underline{\delta}_m$. And by definition $M(\bar{v}, \bar{v}, \underline{\delta}_m)$, and $B(\bar{v}, \bar{v}, \underline{\delta}_m) > 0$ because $\underline{\delta}_b > \underline{\delta}_m$. So by (18), $S(\bar{v}, \bar{v}, \underline{\delta}_m) \leq 0$, which implies $\underline{\delta}_s \geq \underline{\delta}_m$.

Profits and consumer surplus are clearly higher with bundling for $\delta \in (\underline{\delta}_b, \underline{\delta}_m)$ because they are both equal to zero when high quality cannot be sustained. □

8 Supplemental Online Appendix

In the proof of Propositon 3 in the article, we omitted part of the proof. Specifically, we did not show that condition (18) implied that deviations by the firm in just one product quality were not profitable. This supplemental appendix contains the omitted part of the proof.

Omitted Part of the Proof of Proposition 3

Proof. One-shot deviations in the quality of just one good can occur at three exclusive types of decision nodes (not necessarily the same as the types of decision nodes analyzed for deviations in both product qualities). The first type of decision node is one for which $\widehat{M}(\cdot) < 0$, and so in equilibrium low quality is produced forever.[39] The second type of decision node is one for which $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}) \geq 0$ and if the firm deviates then $\widehat{M}(p, p, \delta, k_1 + 1, k_2, k_{12}) \leq 0$ (or analogously, $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}) \geq 0$ and $\widehat{M}(p, p, \delta, k_1, k_2 + 1, k_{12}) \leq 0$). The third type of decision node is defined as a decision node at which $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}) \geq 0$ and $\widehat{M}(p, p, \delta, k_1 + 1, k_2, k_{12}) \geq 0$ (or analogously, $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}) \geq 0$ and $\widehat{M}(p, p, \delta, k_1, k_2 + 1, k_{12}) \geq 0$).

Consider the first of the three types of decision nodes, or subgames. At these decision nodes, a deviation to high quality for just one good (good 1) is not profitable as long as

$$\sum_{i=1,2} \hat{\phi}_i(p_1, p_2) \frac{p_i}{1 - \delta\pi} + \hat{\phi}_{12}(p_1, p_2) \frac{p_1 + p_2}{1 - \delta\pi_b} \geq \hat{\phi}_1(\cdot)(p_1 - c) + \hat{\phi}_2(\cdot) p_2$$
$$+ \hat{\phi}_{12}(\cdot)(p_1 + p_2 - c) + \delta\hat{\phi}_1(p_1, p_2) \frac{p_1}{1 - \delta\pi} + \delta\pi\hat{\phi}_2(p_1, p_2) \frac{p_2}{1 - \delta\pi}$$
$$+ \delta\pi\hat{\phi}_{12}(p_1, p_2) \frac{p_1 + p_2}{1 - \delta\pi_b}, \quad (41)$$

where the left-hand side shows the profits from low quality for both goods every period, and the right-hand side shows the profits from high quality for only good 1 in the first period, but then reverting to low quality for both goods every period thereafter.

[39] To see that $\widehat{M}(p, p, \delta, k_1, k_2, k_{12}) < 0$ implies both $\widehat{M}(p, p, \delta, k_1 + 1, k_2, k_{12}) < 0$ and $\widehat{M}(p, p, \delta, k_1, k_2 + 1, k_{12}) < 0$, first note that $S_i(p, \delta)$ must be negative or $\widehat{M}(\cdot) > 0$ which is a contradiction. And $B(p, \delta)$ must be positive because $B(p, \delta) < 0$ implies $M(\cdot) < 0$ which implies that (18) cannot hold. So $\widehat{M}(p, p, \delta, k_1 + 1, k_2, k_{12}) = \pi\widehat{M}(p, p, \delta, k_1, k_2, k_{12} + (1 - \pi)\hat{\phi}(\cdot) S_i(p_i, \delta)$ which must be strictly negative because it is a weighted average of two strictly negative numbers. And because $\widehat{M}(p, p, \delta, k_1 + 1, k_2, k_{12}) < 0$ the firm will produce two low-quality products in every subsequent period and so $\widehat{M}(\cdot)$ will remain negative in every subsequent period.

Gathering the terms with c in them on the left-hand side, this can be rewritten as

$$c\left[\hat{\phi}_1(\cdot)+\hat{\phi}_{12}(\cdot)\right] \geq \delta(1-\pi)\hat{\phi}_1(\cdot)\frac{p_1}{1-\delta\pi}+\delta(\pi-\pi_b)\hat{\phi}_{12}(\cdot)\frac{p_1+p_2}{1-\delta\pi_b}, \tag{42}$$

and using $p_1 = p_2$, this becomes

$$c \geq \frac{\delta(1-\pi)\hat{\phi}_1(\cdot)\frac{p_1}{1-\delta\pi}+\delta(\pi-\pi_b)2\hat{\phi}_{12}(\cdot)\frac{p_1}{1-\delta\pi_b}}{\hat{\phi}_1(\cdot)+\hat{\phi}_{12}(\cdot)}. \tag{43}$$

We know that (36) holds, or deviating by increasing both qualities is not profitable, and (36) can be rewritten as

$$c\left[\hat{\phi}_1(\cdot)+\hat{\phi}_2(\cdot)+2\hat{\phi}_{12}(\cdot)\right] \geq \delta(1-\pi)\hat{\phi}_1(\cdot)\frac{p_1}{1-\delta\pi} + \delta(1-\pi)\hat{\phi}_2(\cdot)\frac{p_2}{1-\delta\pi}+\delta(1-\pi_b)\hat{\phi}_{12}(\cdot)\frac{p_1+p_2}{1-\delta\pi_b}, \tag{44}$$

so using symmetry, i.e., $p_1 = p_2$ and $\hat{\phi}_1 = \hat{\phi}_2$, and dividing both sides by 2, it follows that

$$c \geq \frac{\delta(1-\pi)\hat{\phi}_1(\cdot)\frac{p_1}{1-\delta\pi}+\delta(1-\pi_b)\hat{\phi}_{12}(\cdot)\frac{p_1}{1-\delta\pi_b}}{\hat{\phi}_1(\cdot)+\hat{\phi}_{12}(\cdot)}. \tag{45}$$

But (45) clearly implies (43) because the denominators are the same and the numerator in (43) is smaller because $\pi_b > 2\pi - 1$, so $2(\pi-\pi_b) < (1-\pi_b)$. So at the first type of decision node, a deviation in the product quality of just one product is not profitable.

Next, consider the second type of decision node. At these decision nodes, it is the case that $\widehat{M}(p,p,\delta,k_1,k_2,k_{12}) \geq 0$ and $\widehat{M}(p,p,\delta,k_1+1,k_2,k_{12}) \leq 0$, or that $\widehat{M}(p,p,\delta,k_1,k_2,k_{12}) \geq 0$ and $\widehat{M}(p,p,\delta,k_1,k_2+1,k_{12}) \leq 0$, or both. Deviating by reducing the quality of one product (e.g., good 2) is not profitable as long as

$$\sum_{i=1,2}\hat{\phi}_i(p_1,p_2)\frac{p_i-c}{1-\delta}+\hat{\phi}_{12}(p_1,p_2)\frac{p_1+p_2-2c}{1-\delta} \geq \hat{\phi}_1(\cdot)(p_1-c)+\hat{\phi}_2(\cdot)p_2+\hat{\phi}_{12}(\cdot)(p_1+p_2-c) + \delta\hat{\phi}_1(p_1,p_2)\frac{p_1}{1-\delta\pi}+\delta\pi\hat{\phi}_2(p_1,p_2)\frac{p_2}{1-\delta\pi}+\delta\pi\hat{\phi}_{12}(p_1,p_2)\frac{p_1+p_2}{1-\delta\pi_b}, \tag{46}$$

where the right-hand side is the profit from lowering the quality of good 2 in the first period and choosing low quality for both goods every period thereafter. This can be rewritten as

$$\frac{c}{1-\delta}\left[\hat{\phi}_1(\cdot)+\hat{\phi}_2(\cdot)+2\hat{\phi}_{12}(\cdot)-(1-\delta)\hat{\phi}_1(\cdot)-(1-\delta)\hat{\phi}_{12}(\cdot)\right]\leq \hat{\phi}_1(\cdot)\frac{p_1}{1-\delta}+\hat{\phi}_2(\cdot)\frac{p_2}{1-\delta}+\hat{\phi}_{12}(\cdot)\frac{p_1+p_2}{1-\delta} -(1-\delta\pi+\delta)\hat{\phi}_1(\cdot)\frac{p_1}{1-\delta\pi}-\hat{\phi}_2(\cdot)\frac{p_2}{1-\delta\pi}-(1-\delta\pi_b+\delta\pi)\hat{\phi}_{12}(\cdot)\frac{p_1+p_2}{1-\delta\pi_b}, \quad (47)$$

or, using symmetry, as

$$\frac{c}{1-\delta}\left[(1+\delta)\hat{\phi}_1(\cdot)+(1+\delta)\hat{\phi}_{12}(\cdot)\right]\leq 2\hat{\phi}_1(\cdot)\frac{p_1}{1-\delta}+2\hat{\phi}_{12}(\cdot)\frac{p_1}{1-\delta} -(2+\delta(1-\pi))\hat{\phi}_1(\cdot)\frac{p_1}{1-\delta\pi}-(2+2\delta(\pi-\pi_b))\hat{\phi}_{12}(\cdot)\frac{p_1}{1-\delta\pi_b}. \quad (48)$$

After additional manipulation, (48) can be rewritten as

$$c\left[\hat{\phi}_1(\cdot)+\hat{\phi}_{12}(\cdot)\right]\leq \frac{1}{(1-\delta\pi)}\left[\delta(1-\pi)\right]\hat{\phi}_1(\cdot)p_1 +\frac{1}{(1-\delta\pi_b)}\left[\delta(1-\pi_b)+\frac{(1-\delta)}{(1+\delta)}\delta\left(1+\pi_b-2\pi\right)\right]\hat{\phi}_{12}(\cdot)p_1 \quad (49)$$

We know that deviating by producing two low-quality products is not profitable, that is (38) holds, or

$$\sum_{i=1,2}\hat{\phi}_i(\cdot)\frac{p_i-c}{1-\delta}+\hat{\phi}_{12}(\cdot)\frac{p_1+p_2-2c}{1-\delta}\geq \sum_{i=1,2}\hat{\phi}_i(p_1,p_2)\frac{p_i}{1-\delta\pi}+\hat{\phi}_{12}(p_1,p_2)\frac{p_1+p_2}{1-\delta\pi_b}, \quad (50)$$

which can be rewritten as

$$c\frac{1}{1-\delta}\left[\sum_{i=1,2}\hat{\phi}_i(\cdot)+2\hat{\phi}_{12}(\cdot)\right]\leq \sum_{i=1,2}\hat{\phi}_i(\cdot)\frac{p_i}{1-\delta}+\hat{\phi}_{12}(\cdot)\frac{p_1+p_2}{1-\delta} -\sum_{i=1,2}\hat{\phi}_i(p_1,p_2)\frac{p_i}{1-\delta\pi}-\hat{\phi}_{12}(p_1,p_2)\frac{p_1+p_2}{1-\delta\pi_b}, \quad (51)$$

or, using symmetry,

$$c\left[\hat{\phi}_1(\cdot)+\hat{\phi}_{12}(\cdot)\right] \leq \frac{1}{(1-\delta\pi)}\delta(1-\pi)\hat{\phi}_1(\cdot)p_1 + \frac{1}{(1-\delta\pi_b)}\delta(1-\pi_b)\hat{\phi}_{12}(\cdot)p_1 \quad (52)$$

And clearly (52) implies (49) because the left-hand sides of both equations are the same but the right-hand side of (49) is larger because $\pi_b > 2\pi - 1$ by assumption.

Finally, consider the third type of decision node. These are decision nodes at which $\widehat{M}(p,p,\delta,k_1,k_2,k_{12}) \geq 0$ and $\widehat{M}(p,p,\delta,k_1+1,k_2,k_{12}) \geq 0$, or alternatively, $\widehat{M}(p,p,\delta,k_1,k_2,k_{12}) \geq 0$ and $\widehat{M}(p,p,\delta,k_1,k_2+1,k_{12}) \geq 0$ (or both). At these decision nodes, a deviation to one low-quality product (e.g., good 2) is not profitable as long as

$$\begin{aligned}\sum_{i=1,2}\hat{\phi}_i(p_1,p_2)\frac{p_i-c}{1-\delta} + \hat{\phi}_{12}(p_1,p_2)\frac{p_1+p_2-2c}{1-\delta}\\ \geq \hat{\phi}_1(\cdot)(p_1-c)+\hat{\phi}_2(\cdot)p_2+\hat{\phi}_{12}(\cdot)(p_1+p_2-c)+\delta\hat{\phi}_1(p_1,p_2)\frac{p_1-c}{1-\delta}\\ +\delta\pi\hat{\phi}_2(p_1,p_2)\frac{p_2-c}{1-\delta}+\delta\pi\hat{\phi}_{12}(p_1,p_2)\frac{p_1+p_2-2c}{1-\delta}. \quad (53)\end{aligned}$$

Using symmetry ($p = p_1 = p_2$), this can be rewritten as

$$c \leq \delta\frac{(1-\pi)\hat{\phi}_2(\cdot)+2(1-\pi)\hat{\phi}_{12}(\cdot)}{\hat{\phi}_2(\cdot)+\hat{\phi}_{12}(\cdot)}\left(\frac{p-c}{1-\delta}\right). \quad (54)$$

Because $\widehat{M}(p,p,\delta,k_1,k_2,k_{12}) \geq 0$, using symmetry ($p = p_1 = p_2$ and $\hat{\phi}_1 = \hat{\phi}_2$) it follows that

$$c \leq \delta\frac{2(1-\pi)\hat{\phi}_2(\cdot)+2(1-\pi_b)\hat{\phi}_{12}(\cdot)}{2\hat{\phi}_2(\cdot)+2\hat{\phi}_{12}(\cdot)}\left(\frac{p-c}{1-\delta}\right), \quad (55)$$

and (55) implies (54) because $1-\pi_b < 2(1-\pi)$ by assumption, so a deviation in the product quality of just one product is not profitable at this type of decision node.

□

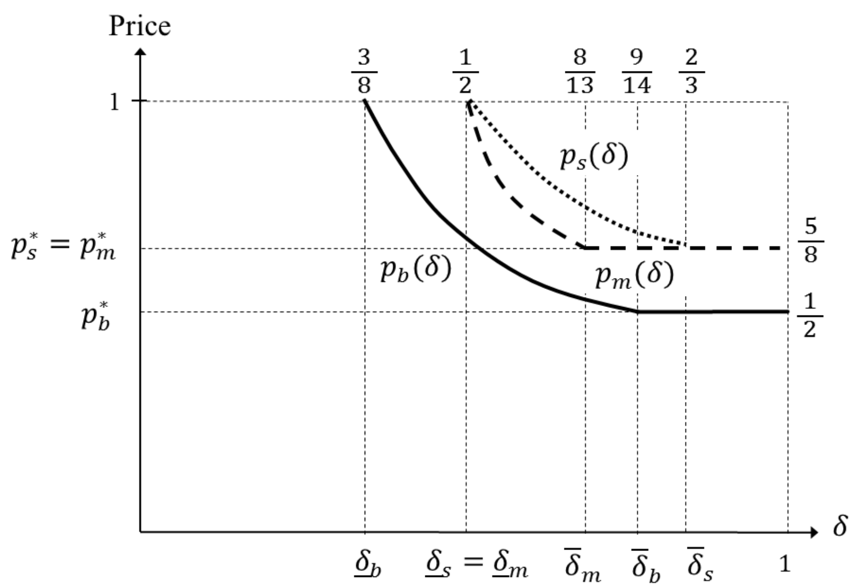

Figure 1: Prices with Unobservable Quality
$f(v_1, v_2)$ uniform, $c = \frac{1}{4}$, $\pi = \frac{2}{3}$, $\pi_b = \frac{4}{9}$

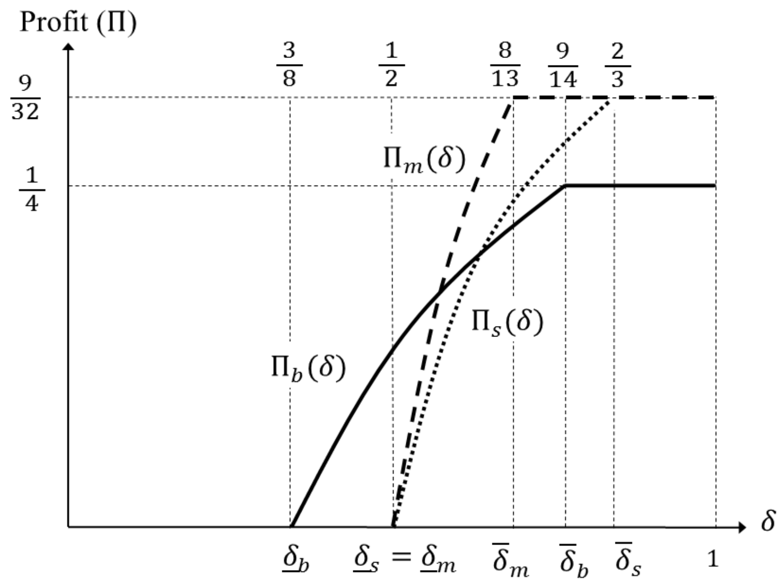

Figure 2: Profits with Unobservable Quality
$f(v_1, v_2)$ uniform, $c = \frac{1}{4}$, $\pi = \frac{2}{3}$, $\pi_b = \frac{4}{9}$

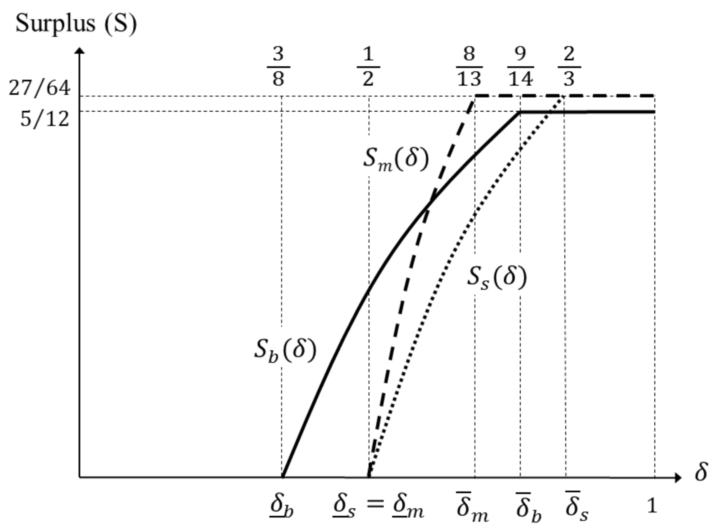

Figure 3: Total Surplus with Unobservable Quality
$f(v_1, v_2)$ uniform, $c = \frac{1}{4}$, $\pi = \frac{2}{3}$, $\pi_b = \frac{4}{9}$