The Cost of Complying with the SEC's Regulations for Registered Investment Advisers: Evidence From Revealed Preferences^{*}

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Abstract

This paper examines the extent to which private fund advisers limit their assets under management (AUM) to delay or avoid registering with the Securities and Exchange Commission (SEC). I find that a substantial number of advisers bunch beneath the \$150 million mandatory registration threshold. I estimate that advisers prefer not to bunch only when their AUM exceed \$179 million. This implies that the annual dollar cost to registered advisers of meeting their regulatory obligations is approximately \$54,000. This estimate lies at the top of the SEC's estimated range and at the bottom of the range provided by industry participants. In contrast, I find no evidence of bunching at AUM thresholds that trigger additional reporting obligations for "large hedge fund advisers" and "large private equity advisers." These results suggest that the direct cost of the SEC's current regulatory regime is significant for private fund advisers with AUM near \$150 million.

Keywords: SEC regulation; private equity funds; hedge funds; investment advisers; regulatory costs; bunching

JEL Codes: G20; G23; G28; K22

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1 Introduction

For decades before the 2008 – 09 Financial Crisis, the private fund industry successfully avoided oversight by the Securities and Exchange Commission. Each time the Commission moved to increase regulation, the industry fought back with lobbying and litigation. One of the private fund industry's main economic arguments against regulation was a cost-benefit analysis. The limited benefits of regulating private funds, the industry argued, would be more than offset by what would certainly be substantial costs. After all, private fund advisers and investors are sophisticated and well-informed and could be expected to look out for their own interests, unlike the typical retail investors the SEC has always been keen to protect.

However, when the Dodd-Frank Act passed in 2010, it marked a regulatory sea change for the private fund industry. After the Financial Crisis, the public gained a renewed appreciation for problems that can crop up in private funds, and Congress responded by placing the private fund industry squarely within the SEC's supervision. The Dodd-Frank Act eliminated the key exemption in the Investment Advisers Act of 1940 ("Advisers Act") that advisers to hedge funds, private equity funds, and other private investment vehicles had long used to avoid registering with the SEC. It also directed the SEC to set up a comprehensive monitoring and reporting system for the industry, with just a handful of narrow exemptions. The SEC quickly obliged, setting up the regulatory framework that governs private fund advisers today. The framework requires private fund advisers to register with the SEC, file reports, develop internal compliance programs, and submit to periodic examinations.

In the decade since the passage of the Dodd-Frank Act, we have learned a great deal more about the private fund industry. The SEC has acquired considerable experience examining private fund advisers and has identified many instances where misleading practices have become widespread.¹ Scholars have also joined in documenting problems in the industry including bargaining problems between investors and fund managers (Clayton 2020), agency issues (Phalippou and Brown 2022), excessive fees (Ben-David, Birru, and Rossi 2020), and

¹See, for example, many of the Risk Alerts published at https://www.sec.gov/exams.

disappointing performance (Phalippou 2020).

With mounting evidence of problems in the industry, the SEC has proposed a spate of new rules to tighten up regulation of private fund advisers. The new rules would increase reporting to the SEC; mandate performance, fee, and expense disclosures to investors; force advisers to disclose preferential treatment they give to certain investors; and prohibit certain activities.² Predictably, the industry has come out against the SEC's proposals, and as before, a main argument in many comment letters consists of a cost-benefit analysis. The commenters project limited benefits and substantial costs from the SEC's proposal and conclude that the new rules should not be implemented.³

With this debate as a backdrop, this paper examines how private fund advisers have responded to the original regulatory framework set up by the SEC in 2011. It then uses their response to back out the direct cost imposed by the regulations. I take advantage of the fact that the Dodd-Frank Act directed the SEC to provide a registration exemption for advisers who manage only private funds with assets under management ("AUM") less than \$150 million. Because fund advisers can control their AUM to some extent, they can avoid or delay registering with the SEC by "bunching" beneath this threshold. Using the "fuzzy bunching" estimator developed by Alvero and Xiao 2023, I can estimate the extent to which advisers bunch beneath this threshold, which then allows me to infer the direct cost of registration.

To motivate my estimation procedure and connect the extent of bunching to the cost of registration, I develop a model for advisers' choice of AUM. The model follows closely the model used by Alvero, Ando, and Xiao 2022 to model banks' choice of size. Under the model, in the absence of regulation, advisers choose AUM to maximize a profit function. But when faced with a fixed regulatory cost that kicks in at a certain threshold, advisers

²See the SEC's published Fact Sheet (https://www.sec.gov/files/ia-5955-fact-sheet.pdf) and Press Release (https://www.sec.gov/news/press-release/2022-141) for additional details.

³See, for example, the comment letter from Citadel's Global Head of Government and Regulatory Policy, which predicts that the proposed changes will "increase advisers' costs, reduce investor returns, and create new barriers to entry." https://www.sec.gov/comments/s7-03-22/s70322-20127838-289028.pdf.

who would have chosen AUM just above the threshold are incentivized to bunch beneath the threshold. The "marginal adviser" in the model is one who is indifferent between bunching and choosing their undistorted optimal size. Following Alvero, Ando, and Xiao 2022, I obtain a formula from the model that uses the marginal adviser's AUM to calculate the direct cost of registration as a percentage of the marginal adviser's profits.

Next, I document evidence that private fund advisers do in fact bunch beneath the \$150 million threshold. I use AUM data from private fund advisers' SEC filings to construct a size distribution. The distribution shows a much larger number of advisers with AUM just below \$150 million than just above. I also construct separate AUM distributions for advisers who advise primarily private equity funds and hedge funds and confirm that bunching is not limited to one type of private fund. Advisers to both types of funds appear to bunch beneath the regulatory threshold to take advantage of the exemption.

To supplement this evidence, I also document several differences in the distributions of year-to-year asset growth for exempt advisers with AUM just beneath the threshold and advisers of the same size who have nonetheless chosen to register. The data show that exempt funds with AUM between \$135 and \$150 million are more likely than similarly sized, registered funds to grow by 0 to 11% but are substantially *less* likely to grow by 11 to 60%. This is consistent with the theory that exempt advisers are managing their growth to avoid tipping over the \$150 million threshold. I also estimate an event study-style regression specification to assess asset growth around the time advisers register. I find that advisers' AUM grow significantly more in the year they register than in the years before or after. This finding is also consistent with the theory that advisers' decisions to register are deliberately timed and accompanied by large inflows in assets.

Finding that the data are so far consistent with my model, I turn to estimating the marginal adviser's AUM. Using the fuzzy bunching procedure, I estimate the marginal adviser's AUM to be \$179 million, with a standard deviation of \$4.5 million. This means that, in order to avoid registering with the SEC, private fund advisers are willing to forgo man-

aging up to 16% of the assets they would choose to manage in the absence of regulation. I obtain similar estimates when I use samples of only private equity fund advisers or hedge fund advisers.

Plugging this estimate into the formula derived from my model, I next estimate the direct cost of being registered as a percentage of the marginal advisers' profits. I estimate that the cost is equal to 1.4% of profits. I then assume a typical fee structure and gross-of-fees return and estimate the annual dollar cost to be approximately \$54,000. Interestingly, this estimate closely matches the high end of the estimate range provided by the SEC at the time it implemented its new rules for private fund advisers. It also sits toward the low end of estimates provided by industry participants in their comment letters.

To assess whether my estimates are sensitive to my estimation procedure and assumptions, I also include several robustness checks. First, I test my estimate's sensitivity to the procedure I use for constructing the "counterfactual distribution." The fuzzy bunching procedure uses a smooth polynomial to construct a counterfactual distribution that approximates what the AUM distribution would be in the absence of regulation. I rerun the estimation several times varying the degree of the polynomial and the size of the sample used to fit the counterfactual. I find that these changes do not significantly alter my estimate of the marginal adviser's AUM. Second, I recalculate the dollar cost using different assumptions about the marginal adviser's expected gross-of-fees return. While the estimated dollar cost varies with different assumptions, the estimates are all within the same order of magnitude.

I conclude with several extensions. In the first, I look for evidence of bunching at two additional regulatory thresholds: the \$1.5 billion threshold for "large hedge fund advisers" and the \$2 billion threshold for "large private equity advisers." Both of these thresholds trigger additional reporting obligations. In contrast to the \$150 million threshold, however, there is no detectable bunching at either threshold. This suggests that the additional costs triggered above these thresholds are not material enough to affect the behavior of large advisers. These two tests also function as a sort of "placebo" test, confirming that the fuzzy bunching estimator is not prone to detecting false positives even when there is no actual bunching.

In a second extension, I explore the relationships between advisers' decisions to claim an exemption, their subsequent disciplinary reports, and the strength of their internal controls. I find evidence that — holding constant other factors, including internal controls — previously exempt advisers are no more likely than other first-time registrants to report disciplinary proceedings after their first year of registration. However, I also find evidence that exempt advisers are somewhat less likely than similar registered funds to implement strong internal controls, for example by obtaining annual audits and distributing financial statements to investors, and that advisers with weak internal controls are more likely to report being subject to disciplinary proceedings.

This paper makes two main contributions to the literature. First, it contributes to the literature on cost-benefit analysis in financial regulation. Legal scholars and economists have hotly debated the feasibility and wisdom of using cost-benefit analysis to guide future financial regulation.⁴ This paper is part of a growing subliterature that uses a more tractable "retrospective analysis approach" to assess the costs and benefits of already implemented regulation (Jackson and Rothstein 2019, page 212 n.41). It joins a number of papers — including Alvero, Ando, and Xiao 2022 and Ewens, Xiao, and Xu 2023 — that demonstrate that the revealed preferences of regulated parties can be used to infer the direct cost of financial regulations after they are implemented. These papers also provide direct cost estimates that regulators can use to inform their decisions moving forward about whether to tighten or loosen various regulations. This paper is the first to use a revealed preferences approach to back out direct regulatory costs in the private funds context.

Second, this paper contributes directly to the law and finance literature on private equity and hedge funds. Numerous recent studies have been devoted to identifying bargain problems

⁴For a classic discussion of the merits of cost-benefit analysis in the context of regulating systemic risk, see the debate in the Yale Law Journal and Yale Law Journal Forum in 2015 between Coates 2015, E. A. Posner and Weyl 2015, Sunstein 2015, and Kraus 2015.

(Clayton 2020) and agency costs (Phalippou and Brown 2022) in the private fund industry, suggesting that tighter regulations could correct these problems and produce significant *benefits*. Other studies have attempted to document these benefits, showing for example that registration requirements reduce financial misreporting by hedge funds (Dimmock and Gerken 2016; Honigsberg 2019). This paper takes a step toward completing the picture by assessing the *costs* of regulation. In doing so, I show that regulatory costs are economically significant for smaller advisers, enough so that they are willing to substantially limit their AUM to avoid registering with the SEC. On the other hand, I show that enhanced disclosure obligations are relatively insignificant to the largest private fund advisers.

2 Institutional Background

This section provides a brief history of attempts to regulate the private fund industry. It focuses on the passage of the Dodd-Frank Act (the first successful attempt at regulation) and its attendant debates. It also summarizes the current regulatory scheme for private funds, which provides the setting for this paper.

2.1 Early Attempts at Regulating Private Funds

Historically, private funds and their advisers managed to stay out of the SEC's sight by taking advantage of exemptions to the Investment Company Act of 1940 and the Advisers Act. The funds avoided being designated as an "investment company" by not offering their securities to the public and making sure that either (1) they had fewer than one hundred investors or (2) their investors were all "qualified purchasers," a designation generally reserved for very high net worth individuals and institutional investors. Their advisers, on the other hand, met the Adviser Act's definition of "investment adviser," so they were always subject to the Act's generally anti-fraud provisions. However, they were able to avoid registering with the SEC under a "private adviser exemption," which exempted any adviser from registration who had fewer than fifteen clients. From at least 1985 onward, the SEC maintained the position that each private fund counted as one client.

The SEC changed course, however, in 2004 (Zaun 2007). Motivated by the spectacular collapse of Long-term Capital Management in 1998 and reports showing private funds' increasing relevance in U.S. capital markets, the SEC proposed a rule to increase its oversight of private funds. The rule redefined "client" in the private adviser exemption to include investors in private funds, eliminating the exemption's availability for most private fund advisers.

Private fund advisers vehemently opposed the proposed rule. They argued that the costs they would face by being forced to register would outweigh whatever benefits the SEC would gain from increased insight into their businesses. The SEC voted to approve the rule over these objections in December 2004, but the new rule was short-lived. Less than two years later, in *Goldstein v. Securities and Exchange Commission*,⁵ the D.C. Circuit struck down the registration rule as "arbitrary," and the private fund advisers who had registered with the SEC quickly deregistered (Kaal and Oesterle 2017).

2.2 The Dodd-Frank Act

Following the financial crisis in 2008 - 09 — which some commentators blamed at least partly on hedge funds — there were renewed calls for the SEC to step up its oversight of private funds. Congress agreed and passed the Dodd-Frank Act. The Act was not focused primarily on private funds, but it nonetheless made several important changes to how the SEC oversees private funds. Most importantly, the Act repealed the private adviser exemption and directed the SEC to replace it with two much narrower ones: one for "venture capital fund advisers" and another for "private fund advisers" with less than \$150 million in assets under management.⁶ The SEC quickly implemented the new exemptions. As before, advisers who fall under one of these exemptions (called "exempt reporting advisers" (ERAs)) are still

⁵451 F.3d 873 (D.C. Cir. 2006).

⁶15 U.S.C. §80b-3(l), (m) (Advisers Act Section 203)

subject to the Advisers Act's general anti-fraud provisions⁷ and owe fiduciary duties to their clients.⁸ Exempt advisers are also required to fill out parts of Form ADV annually, providing the Commission with contact information and some basic details about their operations. They are not, however, required to register with the SEC.⁹

The SEC's implementing regulations also placed new requirements on registered investment advisers (RIAs). Among other things, registered advisers are required to

- Keep certain books and records,¹⁰
- Designate a chief compliance officer,¹¹
- Adopt, implement, and annually review compliance policies and procedures,¹²
- Provide clients and prospective clients with a brochure meeting certain requirements,¹³
- Fill out Form PF and Form ADV (in full) annually,¹⁴
- Create and enforce a code of ethics meeting certain requirements,¹⁵ and
- Use advertising materials that fall only within certain parameters.¹⁶

The new reporting structure also included additional requirements for the largest private funds. For instance, "large hedge funds advisers" — defined as those managing more than \$1.5 billion in hedge fund assets — are required to file Form PF *quarterly* and report additional information about their exposures across asset classes, leverage, liquidity, and

⁷15 U.S.C. §80b-6 (Advisers Act Section 206)

⁸Sec. & Exch. Comm'n v. Capital Gains Rsch. Bureau, Inc., 375 U.S. 180 (1963).

⁹See the Morgan Lewis client memo "Consequences of Registration Under the Investment Advisers Act of 1940" for more information (https://www.morganlewis.com/-/media/files/special-topics/vcpefdeskbook/regulation/vcpefdeskbook_consequencesofregistration.pdf).

¹⁰17 CFR §275.204-2

¹¹17 CFR §275.206(4)-7(c)

 $^{^{12}17}$ CFR §275.206(4)-7(a), (b)

¹³17 CFR §275.204-3

¹⁴17 C.F.R. §275.204(b)-1 ¹⁵17 CFR §275.204A-1

¹⁶17 CFR <u>9275.204A-1</u>

 $^{^{16}17\ {\}rm CFR}$ §275.206(4)-1

risk profile. "Large private equity advisers" — those managing more than 2 billion in private equity fund assets — are only required to file annually, but must provide additional information about their portfolio companies, leverage, and geographic exposure.¹⁷

As private fund advisers started to register, the SEC began regularly examining registered advisers to assess compliance with the new regulations. The SEC could not examine all advisers every year, so it complemented its examination program with regularly published "Risk Alerts" to "raise awareness of compliance issues observed by the staff."¹⁸ At the outset, SEC Chair Mary Shapiro advised that, although the SEC has authority to examine all investment advisers, it did not intend to perform regular examinations of exempt advisers.¹⁹

2.3 Data

The data used for this paper come from data provided by both exempt and registered private fund advisers on Form ADV. As discussed previously, the SEC requires all private fund advisers to file some basic information every year on Form ADV. Form ADV includes information about each fund adviser's total assets under management, private fund assets under management, types of private funds, and home state. It also includes some basic information about whether each adviser has been subject to civil, criminal, or regulatory sanctions in the past 10 years.

Data from Form ADV are freely available on the SEC's website.²⁰ For this paper, I use Form ADV data from Q1 2013 through Q2 2022 for registered and exempt investment advisers. I limit my sample to advisers whose assets are more than 90% invested in private

¹⁷See Form PF (https://www.sec.gov/files/formpf.pdf) and Cadwalader's client memo "The SEC Approves Final Version of Form PF" (https://www.cadwalader.com/resources/ clients-friends-memos/the-sec-approves-final-version-of-form-pf).

¹⁸SEC Risk Alert, Nov. 9, 2015 (https://www.sec.gov/files/ ocie-2015-risk-alert-cco-outsourcing.pdf).

¹⁹See the Cadwalader client memo "SEC Adopts Dodd-Frank Act Investment Adviser Rules and Delays Implementation of Some Deadlines (https://www.cadwalader.com/resources/clients-friends-memos/ sec-adopts-dodd-frank-act-investment-adviser-rules-and-delays-implementation-of-some-deadlines).

²⁰The data can be downloaded from https://www.sec.gov/foia/docs/form-adv-archive-data. I gratefully acknowledge Professor Colleen Honigsberg's work in persuading the SEC to make Form ADV data freely available (Honigsberg 2019).

funds, and I exclude advisers who rely on the venture capital exemption. Where a single adviser has filed Form ADV multiple times in a single calendar year (perhaps to correct an error or because its AUM crossed the \$150,000,000 threshold), I use only the latest filing in the year.

Tables 1, 2, and 3 and Figure 10 summarize some aspects of the Form ADV data provided by private fund advisers. Table 1 highlights the remarkable growth in the number of private fund advisers in the ten years after the Dodd-Frank Act's passage. The number of exempt advisers nearly doubled, growing from 688 in 2012 to 1,368 in 2021. This growth was uniform across different types of exempt advisers. The number of registered advisers also increased, though not as dramatically, dampened by the fact that the number of registered advisers that only advise hedge funds remained flat over this period.

Table 2 sheds light on the relative size of registered and exempt advisers. The difference is stark: the median registered fund adviser in 2021 managed nearly \$700 million, while the median exempt adviser managed less than a tenth of that amount. This difference is explained by the regulatory structure, which limits the private fund adviser exemption to advisers with less than \$150 million in AUM.

Table 3 displays the number of private fund advisers headquartered in the ten states with the most private fund advisers. Unsurprisingly, New York is home to the most private fund advisers, and other major population and economic centers rank high on the list. (California, Texas, and Florida are all in the top five.) Particularly noteworthy is the degree to which fund advisers cluster in New York; New York has more than twice as many private fund advisers as the second most popular state (California). New York occupies a unique space in the regulatory scheme for investment advisers because advisers in New York are allowed to report to the SEC with fewer assets under management than if they were in other states. Typically, investment advisers with less than \$100 million in AUM are not allowed to report to the SEC and fall instead under the supervision of their state securities authority. But in New York, advisers are required to report to the SEC if they have AUM over \$25 million. This raises the possibility that smaller private fund advisers are flocking to New York, not just because of its importance in the finance industry, but also because they see some benefit to being supervised by a federal rather than state regulator.

Finally, Figure 10 provides some insight into how advisers are using the private fund exemption. It tracks the reporting status of private fund advisers in the years after they first report to the SEC as an exempt adviser. A few features of the graph are worth highlighting. First, the majority of funds that start reporting as exempt advisers are no longer reporting to the SEC seven years after they start reporting. This could be because their assets drop below the \$100 million (or \$25 million in New York) or because they fail or are acquired. Second, of the approximately 25% of advisers who eventually register, nearly all have registered within just three years. Only around 20% of advisers are still claiming the exemption nine years after their first report. Overall, this picture is consistent with the story that most private fund advisers see exempt reporting status as a temporary stop. Most exempt advisers will either grow quickly and register, or disappear after a few years of reporting.

3 Model and Estimation Procedure

3.1 Model of AUM Choice

In this section, I describe a simple model of private fund advisers' choice of AUM. This model very closely follows the model Alvero, Ando, and Xiao 2022 use to describe banks' decisions about how many deposits to take on. Advisers' primary goal in this model is to maximize their profit, which equals their earned fees minus their expenses. In the absence of regulation, fund advisers aim to manage log AUM q^* that solves:

$$\max_{q} \pi(q|z) \exp(q) \tag{1}$$

Here, $\pi(q|z)$ is the fund adviser's profit, expressed as a percentage of AUM. The fund

adviser's profit is equal to the fees charged by the fund adviser minus the cost of operating the fund. Each adviser's profit depends on the fund's log AUM (q) and a vector of other characteristics (z), which could include the adviser's skill, expertise, and reputation. Private fund advisers are typically compensated through a combination of two fees: a management fee based on assets under management and a performance fee. The management fee is traditionally intended to cover the adviser's operating costs, while the performance fee is intended to align the adviser's and investors' incentives. Therefore, as a first approximation, equation (1) says that fund advisers choose log AUM to maximize the performance fee portion of their compensation.

Going forward, I assume that $\pi(q|z)$ is decreasing in q and increasing z. The intuition behind the second assumption is simply that more skilled fund advisers can earn higher returns and thus charge higher fees. The first assumption can be explained by either (or both) of two theories about how advisers operate. The first is that, as advisers take on more assets, they find it more and more difficult to invest those assets profitably. Thus, as advisers increase their AUM, their overall gross return decreases, as does their performance fee (as a percentage of AUM). The second is that potential investors differ in how much they are willing to pay for an adviser's services. For example, the investors may have different beliefs about the adviser's skill. This means that an adviser may be able to charge high fees if it accepts only the investors with the highest willingness to pay. However, to attract more investors and increase AUM, an adviser would have to lower its fees.²¹

Once regulation is introduced, the adviser's objective changes. The adviser now seeks to manage log AUM \tilde{q} that solves:

$$\max_{q} \pi(q|z) \exp(q) - \mathbb{1}(q > \underline{q}) \cdot \kappa \tag{2}$$

In this new objective function, q represents the regulatory threshold, or the log AUM

²¹Note that the first assumption also implicitly assumes that economies of scale do not overwhelm either of the two effects I have just described.

above which the fund adviser must register with the SEC. In our context, $\underline{q} = \log(15000000)$. κ represents the regulatory costs borne by registered advisers that are not borne by exempt ones. It includes the extra money registered advisers spend to hire outside counsel or additional employees to meet their heightened reporting obligations, and it also includes the cost associated with being prepared for regular SEC examinations. These latter costs may not show up as clearly on fund advisers' financial statements, but they are a real cost of submitted to more intense oversight.

Using Equation (2), we can show that, in the presence of regulation, fund advisers will aim to manage log AUM of either q^* (their optimal size) or \underline{q} (the regulatory threshold). Fund advisers whose optimal size is below the threshold don't have an incentive to change their size. Funds whose optimal size is above the threshold face a tradeoff: they can accept fewer AUM and avoid the regulatory costs, but in doing so, they forfeit the fees they could have earned by managing additional assets. Some of the funds whose optimal size is just above the threshold will choose to "bunch" beneath the threshold, managing log AUM of \underline{q} . But advisers whose optimal size is above an indifference point \overline{q} will choose *not* to bunch and will instead reach their optimal size q^* . We can express this result as follows:

$$\tilde{q} = \begin{cases} q^*, \quad q^* \notin (\underline{q}, \overline{q}) \\ \underline{q}, \quad q^* \in (\underline{q}, \overline{q}) \end{cases}$$
(3)

For some fund adviser, the additional profit from managing more assets is exactly equal to the regulatory cost κ . This "marginal adviser" is indifferent to bunching and has optimal size $q^* = \overline{q}$. If we can identify this firm's optimal size, then we can back out the regulatory cost. To do this, we can first rewrite the regulatory cost as a percentage τ of the marginal adviser's profits: $\kappa = \pi(\overline{q}|z) \exp(\overline{q}) \cdot \tau$. Next, because the marginal adviser is indifferent to bunching, their profit at \underline{q} must be equal to their profit at \overline{q} , minus the regulatory cost. This allows us to write the following indifference condition:

$$\pi(\overline{q}|z)\exp(\overline{q})(1-\tau) = \pi(q|z)\exp(q) \tag{4}$$

Using this condition, we can then derive a formula, following Alvero, Ando, and Xiao 2022, that approximates the percentage regulatory cost τ (see Appendix A for proof):

$$\tau \approx 1 - (\overline{q} - q + 1) \exp(q - \overline{q}) \tag{5}$$

This formula shows that we can learn about regulatory cost if we can estimate the marginal fund's optimal size \bar{q} . I will discuss a procedure for estimating \bar{q} in the next subsection. The figure τ is itself meaningful because it allows us to interpret the regulatory cost as being equivalent to a τ percent tax on the marginal adviser's profits.²² Additionally, we can estimate the dollar cost of becoming a registered adviser by multiplying τ by an estimate of the marginal adviser's expected profits.

To provide some intuition for how it is possible to infer the dollar cost of regulation from the indifference point \bar{q} , it is useful to consider a simple, numerical example. Suppose that we estimate the indifference point to be \$180 million and that, at that level, the marginal adviser can earn a gross return of 11%, charges a performance fee of 20% of its gross return, and charges a management fee of 1.5% of assets under management, which exactly offsets its operating expenses. Further suppose that, if the marginal adviser were to accept only \$150 million from investors, it could earn a gross return of 11.2% and charge a management fee of 2.1%, and its operating costs would increase to only 1.75% of AUM. (The performance fee would remain unchanged.) For the marginal adviser to be indifferent between managing \$180 million and \$150 million, the regulatory cost must be equal to the difference in expected profit between these two scenarios. Under these facts, we would estimate the regulatory cost to be \$75,000. The details of this calculation are reported in Table 4.

 $^{^{22}\}mathrm{This}$ way of thinking about regulatory costs channels the spirit of R. A. Posner 1971).

3.2 Fuzzy Bunching Estimation

To estimate the optimal AUM for the marginal fund adviser, I use the "fuzzy bunching" estimator developed by Alvero and Xiao 2023. This estimator relies on the fact that when advisers bunch beneath the regulatory threshold (\underline{q}), they create a "bulge" in the cumulative distribution function (CDF) for adviser AUM. Using the area of the bulge, this estimator backs out the size of the "bunching range" (Δq), which is defined as the difference between the marginal firm's optimal size and the regulatory threshold: $\overline{q} - \underline{q}$.

Alvero and Xiao 2023's fuzzy bunching estimator is a good fit in this context because it estimates the bunching range accurately even when there is considerable "noise" in the bunching variable. Even though a fund adviser may try to target a certain AUM, at any give time, their AUM might differ significantly from the optimal level because asset prices fluctuate. This means that we can expect the AUM distribution to have a very diffuse bulge around the regulatory threshold, making the bunching range difficult to estimate using more traditional "sharp" bunching estimators.

Alvero and Xiao 2023 provide the following formula for estimating the size of the bunching range:

$$\widehat{\Delta q} = \sqrt{\frac{2A}{\overline{f_0}}} \tag{6}$$

A represents the area of the bulge, which can be calculated by integrating the area between the actual AUM CDF (F) and a counterfactual CDF (F₀): $A = \int (F(q) - F_0(q)) dq$. The counterfactual CDF is an approximation of what the AUM distribution would be in the absence of regulation. It can be constructed by fitting a smooth polynomial to the actual distribution, excluding the bunching bulge. $\overline{f_0}$ is "the average density around the [regulatory] threshold" (Alvero and Xiao 2023). I estimate $\overline{f_0}$ using the average slope of the counterfactual CDF between q and \overline{q} .

Alvero and Xiao 2023 also include an adjustment to the fuzzy bunching estimator to ac-

count for "optimization frictions."²³ The reason for such an adjustment is that some advisers might choose not to bunch beneath the threshold, even though it would seem optimal to do so. This might be due to a careless omission or some other friction. For example, a fund adviser whose optimal AUM falls beneath \bar{q} due to economic conditions, operational changes, or bad luck might be reluctant to return assets to investors to drop beneath the regulatory threshold. Doing so could send a negative signal to investors and create additional problems for the fund.

To account for such imperfect optimization, Alvero and Xiao 2023 introduce a new variable α that represents the fraction of "non-optimizing agents." α can be estimated based on the probability mass in the bunching range (i.e., between \underline{q} and \overline{q}) using the following equation:

$$\hat{\alpha} = \frac{2(F(\overline{q}) - F(\underline{q}))}{\overline{f_0}\Delta q} - 1 \tag{7}$$

Then, using this estimate for alpha, the bunching range can be calculated using the following adjusted version of equation (6):

$$\widehat{\Delta q} = \sqrt{\frac{2A}{(1-\alpha)\overline{f_0}}} \tag{8}$$

Since Δq is an input in equation (7), equations (7) and (8) need to be estimated simultaneously.

4 Results

4.1 Preliminary: Evidence of Bunching

If the model described in Section 3 correctly describes private fund advisers' behavior, then we would expect to see more advisers with AUM just below \$150 million than just above.

 $^{^{23}\}mathrm{Kleven}$ and Waseem 2013 introduced the idea of making such an adjustment and applied it to a sharp bunching estimator.

Figure 1 displays the AUM distribution for all private fund advisers. As expected, there is a sharp drop in the number of advisers at the \$150 million threshold. This drop is clearly visible in the histogram displayed in the bottom panel. This drop shows up in the red CDF displayed at the top of the page as a bulge at the threshold. Here, the bulge is clearly visible when plotted next to the blue, dashed "counterfactual" distribution, which was constructed by fitting a smooth polynomial to the empirical CDF, excluding the region containing the bulge.

Figures 2 and 3 similarly show the AUM distributions of advisers whose AUM are >90% invested in hedge funds and private equity funds respectively. Once again, we see a sharp drop in the number of advisers at the \$150 million threshold and an accompanying bulge in the CDFs. This confirms that the bunching visible in the full sample is not being driven by one type of private fund adviser.

4.2 Preliminary: Evidence of Growth Management

Next, I examine the behavior of AUM growth for fund advisers with AUM close to the \$150 million threshold. I also look at AUM growth around the time advisers cross the regulatory threshold. The purpose of this analysis is to provide support for the conclusion that the bunching identified in Figures 1, 2, and 3 is not random, but the product of advisers' deliberate efforts to delay or avoid registration.

Figure 6 shows the distribution of single-year AUM growth for two different groups of advisers. The red distribution plots the distribution of AUM growth for advisers with AUM between \$135 and \$150 million who were registered at the beginning of the year. The blue distribution plots the growth distribution for advisers of the same size who were unregistered at the beginning of the year. These distributions show that exempt advisers with AUM just beneath the threshold were more likely than similarly sized, registered advisers to grow by less than 11 percent: 18.7% of unregistered advisers grew between 0 and 11% compared to only 11.1% of registered advisers. It is worth noting that an adviser that starts with \$135 million in AUM that grows AUM by 11% or less will stay beneath the \$150 million threshold. In contrast, already-registered funds in this sample were much more likely to grow AUM by 11 to 60%: 30.1% of registered funds experienced growth within this range versus only 18.2% of unregistered funds.

Because the distributions in Figure 6 could impacted by year-to-year fluctuations in the number and type of registered firms with AUM between \$135 and \$150 million, I estimate the following quantile regression specification:

$$Growth_{i,t} = \beta_0 Exempt_{i,t} + \beta_1 Below_{i,t} + \beta_2 (Exempt_{i,t} \times Below_{i,t}) + \mathbf{X}'_{i,t}\gamma + \epsilon_{i,t}$$

In this specification, $Growth_{i,t}$ is equal to AUM at the end of year t divided by AUM at the end of year t - 1, minus 1, for adviser . $Exempt_{i,t}$ and $Below_{i,t}$ are both indicator variables: $Exempt_{i,t}$ is equal to one if adviser i was unregistered at the beginning of year t and zero otherwise, and $Below_{i,t}$ is equal to one if adviser i had AUM between \$135 million and \$150 million at the beginning of year t. $\mathbf{X}_{i,t}$ is a vector of controls and includes year fixed effects and indicators for whether adviser i manages solely private equity or hedge funds. The results of estimating this specification at each decile from 0.1 to 0.9 are reported in Table 7. Additionally, Figure 7 plots the coefficient on the $Exempt_{i,t} \times Below_{i,t}$ interaction term (β_2) across all deciles.

The results from this estimation tell the same story as the distributions plotted in Figure 6. The coefficient on the interaction term is negative and statistically significant in the 0.6 quantile estimate, indicating that, on average, the 60th percentile growth rate for advisers who are unregistered and manage just under \$150 million in assets is lower than the 60th percentile growth rate for other similar advisers. In other words, this suggests that these advisers are more likely to experience modest growth than advisers who are either registered or manage AUM outside the \$135 to \$150 million range. While this result does not prove that advisers are managing their growth to delay or avoid registration, it is consistent with

that theory.

An additional implication of the model presented in Section 3 is that unregistered advisers should register only when they can increase their AUM to a level above some indifference point (\bar{q}). If the model is correct, we should see abnormally high growth in AUM around the time an adviser first registeres with the SEC. On the other hand, if being registered is essentially costless, average AUM growth around the time of registration should be indistinguishable from average AUM growth at any other point in time. To test this implication, I estimate the following event study specification (using the adjustment from Sun and Abraham 2021 because registration dates are staggered):

$$Growth_{i,t} = \mu_i + \gamma_t + \sum_{j=-9, j \neq -1}^{8} \mathbb{1}\{t - Year \ registered_i = j\}\delta_j + \epsilon_{i,t}$$

As before, $Growth_{i,t}$ is the change in AUM for adviser *i* during year *t*. μ_i and γ_t are adviser and year fixed effects respectively, and *Year registered*_i is the first year in which adviser *i* reports as a registered adviser. Note that this specification is not causal: it is *not* intended to show that registering *causes* an large inflow of assets. Rather, its aim is to discover whether advisers time their registrations to coincide with large increases in AUM.

The results of estimating this specification are reported in Table 6 and the δ_j coefficients are plotted in Figure 8. Here again, the results are consistent with the model's predictions. While the lead and lag coefficients are negative and mostly insignificant, the coefficient for the year of registration is positive, very large, and statistically significant. In fact, the estimate is large enough to suggest that many advisers may be deliberately registering during major fundraising years instead of allowing themselves to slip over the registration threshold during years with modestly positive returns. Importantly, these estimation results are consistent with the theory that advisers try to avoid operating in "no man's land," with AUM between the registration threshold (\underline{q}) and some indifference point (\overline{q}). Figure 9 highlights this suboptimal size range graphically by plotting the AUM distribution for advisers in the years before and after they register with the SEC. The plots show that the vast majority of advisers avoid operating with AUM just above \$150 million (represented by the lower dashed line in the Figure).²⁴

4.3 Main Result: Estimate of Regulatory Cost

Table 5 displays results from using the fuzzy bunching procedure to estimate the level of AUM at which advisers are indifferent to bunching (\overline{q}). The first column of Table 5 provides an estimate of the indifference point using the sample of all private fund advisers. Columns 2 and 3 provide alternate estimate from samples limited to advisers hedge funds and private equity funds respectively.

Using the full sample, I estimate the indifference point to be \$179 million. The estimates from the smaller samples are similar: \$171 million from the hedge fund sample and \$181 million from the private equity fund sample. Additionally, the fuzzy bunching procedure estimates α , the fraction of advisers who do not bunch due to optimization frictions. For the full sample, I estimate α to be 54%. For the hedge fund and private equity fund samples, I obtain estimates of 56% and 38% respectively.

Then, plugging estimates for \overline{q} into Equation (5), I obtain estimates for the cost of registration as a percentage of profits for an adviser with AUM equal to $\exp(\overline{q})$: 1.4% of profits for the full sample and 0.8% and 1.6% of profits for the hedge fund and private equity fund samples. These estimates are all statistically significant. As a comparison, Alvero, Ando, and Xiao 2022 estimate the cost of complying with certain provisions of the Dodd-Frank Act for banks with around \$10 billion in assets to be "0.41% of average annual profits."

Finally, we can translate this percentage cost into an approximate dollar cost by estimating the marginal adviser's average annual return, gross of fees. For simplicity, I assume that advisers operating at their undistorted optimal size charge management fees that just cover

 $^{^{24}{\}rm The}$ upper dashed line is at approximately \$180 million, which corresponds to the indifference point estimated in the following subsection.

their operating costs, so their profits are equal to their performance fees. I also assume that average annual returns before fees are 11% of AUM. This estimate seems sensible given other estimates of private fund performance in the finance literature. For example, Ben-David, Birru, and Rossi 2020 estimate that from 1995 - 2016, hedge funds collected performance fees equal to 1.9% of AUM per year on average, which implies a 9.7% annual return before fees (assuming a standard 20% performance fee). Phalippou and Gottschalg 2009 estimate that gross-of-fees returns for private equity funds exceeded the return on the S&P 500 by 2.96% per year on average over their sample period (funds raised between 1980 and 1993). Based on data from Professor Ken French's website,²⁵ U.S. equities have returned on average 11.3% per year over the past 30 years, which would imply a gross return for private equity funds of 14.2%.²⁶

Under these assumptions, a private fund adviser with AUM of \$179 million would earn (on average) \$3.9 million per year in performance fees, which implies an approximate annual cost of \$54,000. The cost estimates for hedge funds and private equity funds are similar: \$30,000 for hedge funds and \$63,000 for private equity funds, with standard deviations of \$15,000 and \$23,000 respectively.

4.4 Comparison to Other Estimates

In 2011, the SEC issued its final rule implementing the Dodd-Frank Act's registration exemption for private fund advisers with AUM less than \$150 million. In the release accompanying its final rule, the SEC provided several estimates of the annual, ongoing costs that would be incurred by newly registering private fund advisers.²⁷ First, the SEC reported its own estimate from its Proposing Release that "annual costs of compliance and examination would

 $^{^{25}{\}rm The~data}$ are available for download at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

 $^{^{26}}$ It is possible that average private equity returns may be even higher. In a more recent paper, Phalippou 2020 reports that net-of-fees private equity returns have recently been around 11%: roughly in line with public equity returns since "at least 2006." Under a typical fee structure (e.g., 1.5% management fee and 20% performance fee) this suggests an average annual gross return in recent years of 15.6%.

²⁷The SEC's rule release is available at https://www.sec.gov/rules/final/2011/ia-3221.pdf.

range from \$10,000 to \$50,000." Second, it reported projections from private fund advisers who predicted that their "ongoing annual compliance costs [would] rang[e] from \$50,000 to \$500,000." Interestingly, the cost estimates derived from fuzzy bunching estimation, which is based on advisers' revealed preferences, lie on the border between the SEC's range and the industry's range.

As an additional data point, a February 18, 2012 article in The Economist reported that an "informal survey of hedge-fund managers" put the annual cost of filling out required paperwork at around \$40,000.²⁸ This estimate is quite close the full-sample, fuzzy bunching estimate, albeit a bit lower.

Professor Wulf Kaal has also attempted to estimate the cost to private fund advisers of registering with the SEC using survey data (Kaal 2016). Professor Kaal's survey results in 2012 and 2015 lined up with the cost estimates provided by private fund advisers in comment letters to the SEC. His respondents reported registration costs ranging from \$50,000 to "More than \$400k," with over 70% of respondents reporting costs between \$50,000 and \$200,000. Once again, the fuzzy bunching estimates fall at the bottom of the range of industry estimates.

There are several possible explanations for the huge discrepancy between some industry participants' projections of registration costs and the costs implied by their behavior. A cynical explanation might be that private fund advisers are prone to exaggerating regulatory costs in an attempt to persuade the SEC to lighten up on regulations. Another possibility is regulatory costs increase substantially as advisers increase their AUM. This could be because larger advisers need to hire additional employees to keep up with their compliance obligations. Based on the discussion in the SEC's final rule release, it appears that the highest cost estimates from fund advisers assumed that they would hire a dedicated "chief compliance

²⁸The article can be found at https://www.economist.com/briefing/2012/02/18/ too-big-not-to-fail?utm_medium=cpc.adword.pd&utm_source=google&ppccampaignID=17210591673& ppcadID=&utm_campaign=a.22brand_pmax&utm_content=conversion.direct-response.anonymous& gclid=CjOKCQjwtsCgBhDEARIsAE7RYh1oDDZiJlf6vrmy3thG9WLMP011UySBQ6TkE6iXbuLJgbMn7n74SwwaAn1PEALw_ wcB&gclsrc=aw.ds. Unfortunately, the article is not clear about which forms it is referring to.

officer" with a salary in the hundreds of thousands of dollars. The SEC affirmed in its release that its compliance rule "does not require advisers to hire a new individual to serve as a full-time CCO," but it is possible that some large advisers decided to anyway. Unfortunately, it is not possible to verify this empirically with data from Form ADV. Registered advisers do report how many employees the have on Form ADV, but private fund advisers were not required to file Form ADV before 2012, so it is impossible to do a pre- and post-registration comparison. If it is true that larger private fund advisers are effectively required to hire additional compliance personnel, then the fuzzy bunching cost estimate is only valid for advisers with AUM near the \$150 million threshold.

5 Robustness and Extensions

5.1 Sensitivity Analyses

Because AUM data for private fund advisers are not available from before Dodd-Frank's passage, using the fuzzy bunching estimator to estimate the marginal adviser's AUM requires estimating what the AUM distribution would be in the absence of regulation. I rely on the assumption that this counterfactual distribution would be regular enough that I can approximate it well with a polynomial.

The counterfactual distribution I use for the estimation depends on the procedure I use to construct it. To construct the counterfactual distribution here, I fit a smooth polynomial to the empirical AUM CDF, excluding the region with the bulge. The counterfactual thus depends on the degree of the fitted polynomial and the size range of the sample I use to estimate the fit. To test whether my estimates are robust to my choice of polynomial degree and sample range, I repeat the estimation procedure using several alternatives.

Table 8 reports the results of re-estimating the marginal adviser's AUM using three sample ranges (\$100 to 300 million, \$100 to 500 million, and \$100 million to 1 billion) and three choices of polynomial degree (4, 5, and 6). I do not extend the sample below \$100

million because advisers with AUM in that range are generally prohibited from registering with the SEC.²⁹ These results are generally consistent with my original estimates. The full sample estimates range from \$176 million to \$188 million, but are mostly clustered between \$175 and \$180 million. The hedge fund and private equity fund adviser estimates exhibit somewhat greater variation but are still clustered around the same range.

Next, I assess the sensitivity of my dollar cost estimate to changes in assumption about the average gross-of-fees return of the marginal adviser and under different estimates of the marginal adviser's AUM. Table 9 reports the results from this sensitivity analysis. I vary the gross return assumption from 10% to 14% and vary the marginal adviser's AUM from \$175 million to \$180 million. This produces dollar cost estimates ranging from around \$38,000 to around \$74,000, with most of the estimates falling between \$45,000 and \$60,000. The estimates are all of a comparable magnitude, which confirms that my dollar cost estimate is not overly sensitive to changes in these parameters.

5.2 Bunching at Other Thresholds

In addition to looking at the \$150 million threshold, I also looked for bunching at two higher thresholds. As mentioned in Section 2 above, the SEC uses a tiered reporting system for registered advisers, and large private equity and hedge fund advisers with AUM above certain thresholds are required to provide substantially more information in their regular filings. Large hedge fund advisers are also required to file Form PF quarterly, instead of merely annually. It is possible that these heightened reporting obligations are costly enough to induce bunching below their related threshold.

Figures 4 and 5 display the AUM distributions around these two thresholds. Figure 4 shows the distribution of hedge fund AUM around the \$1.5 billion threshold for "large hedge fund advisers," and Figure 5 shows the distribution of private equity fund AUM

²⁹Advisers headquartered in New York are a noteworthy exception. They are allowed to register with the SEC as long as their AUM exceed \$25 million. See the SEC's outline "Regulation of Investment Advisers by the U.S. Securities and Exchange Commission," page 9 for more information (https://www.sec.gov/about/offices/oia/oia_investman/rplaze-042012.pdf).

around the \$2 billion "large private equity adviser" threshold. Interestingly, there is no evidence of bunching at either of these thresholds. In contrast to the \$150 million threshold, neither of these higher thresholds appears with a sharp drop in the number of advisers or an accompanying bulge in the CDF. Consistent with this visual evidence, the fuzzy bunching estimator does not generate a positive estimate for the marginal adviser's AUM in either of these samples. This suggests that the heightened reporting requirements for large private equity and hedge fund advisers are not substantial enough to trigger detectable bunching.

The fact that the fuzzy bunching estimator does not provide an estimate of \bar{q} at these thresholds has the added benefit of lending credibility to the estimates obtained using the \$150 million threshold. It suggests that they are not accidental. These additional tests confirm that the estimator is not prone to generating "false positives" but rather detects actual distortions in the distribution.

5.3 Internal Governance and the Benefits of Registration

So far, this paper has focused on estimating the direct cost of the SEC's requirements for registered private fund advisers. While cost estimates are certainly useful, in isolation they cannot prove whether the current regime is optimal. An important question, then, is whether these costs are offset by benefits that flow from the regulatory requirements. While a full assessment of benefits is beyond the scope of this paper, this subsection sheds light on potential benefits by examining some of the differences between private fund advisers that register with SEC and those that report to the SEC but claim a registration exemption. This builds on prior studies such as Dimmock and Gerken 2016 and Honigsberg 2019 that show that registration reduces the incidence of misreporting among hedge funds (compared to a baseline of no regulation).

To explore the difference between basic reporting and full-blown registration, I estimate two regression specifications. The first tests whether newly registered advisers are more likely to report disciplinary proceedings in the year following their initial registration if they claimed an exemption for at least one year before registering. It takes the following form:

$$DRP_{i,t+1} = \beta_0 First \ Reg_{i,t} \times Exempt_{i,t-1} + \beta_1 First \ Reg_{i,t} + \beta_2 Exempt_{i,t-1} + \mathbf{X}'_{i,t}\gamma + \epsilon_{i,t}$$

 $DRP_{i,t+1}$ is an indicator that takes the value 1 if adviser *i* reported disciplinary proceedings in year t+1 and 0 otherwise. These disciplinary proceedings could include regulatory or civil enforcement actions or criminal charges. *First Reg_{i,t}* is also an indicator variable and is equal to 1 if adviser *i* registered for the first time in year *t*. Similarly, *Exempt_{i,t-1}* is equal to 1 if adviser *i* reported to the SEC but claimed the private fund adviser exemption in year t-1. $X_{i,t}$ is a vector of controls, and includes a polynomial control for log AUM (recentered at \$150 million), year and state fixed effects, indicators for the type of funds advised, and several "internal controls" variables that indicate, among other things, whether the adviser obtains annual audits and regularly distributes financial statements to investors.

Table 10 reports the results of estimating this specification. The results suggest that advisers that were previously exempt are no more likely than other first-time registrants to report disciplinary proceedings after one year of registration. First-time registrants are, on average, 2.5% (β_1) less likely to report disciplinary proceedings than advisers with similar characteristics,³⁰ and first-time registrants who were previously exempt are an additional 1.34% ($\beta_0 + \beta_2$) less likely to make such a report. While inconclusive by themselves, these figures seem inconsistent with the theory that advisers that present a high-risk of fraud or other poor behavior are more likely to claim an exemption in order to limit SEC oversight and avoid getting into trouble.

This specification also provides estimates of the relationship between various internal controls and the likelihood of reporting disciplinary proceedings. The results suggest that advisers whose financial statements are prepared in accordance with GAAP are 2.5% less likely to report disciplinary proceedings than similar funds, and advisers who receive a qual-

³⁰This is not surprising because advisers are required to report disciplinary proceedings for the 10 years prior to each filing of Form ADV. Thus, advisers who have been around for longer are more likely to have accumulated a record.

ified opinion following an audit are 2.4% more likely to do so. These estimates are both statistically significant. Interestingly, funds that obtain annual audits and distribute audited financial statements to investors do not seem any more or less likely to report disciplinary proceedings than those who do not.

Next, I estimate a second regression specification to explore whether exempt funds are more or less likely than others to implement strong internal controls. The specification takes the following form:

$$Y_{i,t} = \beta_0 First \ Reg_{i,t} \times Exempt_{i,t-1} + \beta_1 First \ Reg_{i,t} + \beta_2 Exempt_{i,t-1} + \mathbf{X}'_{i,t}\gamma + \epsilon_{i,t}$$

I estimate this specification four times using different outcome variables for $Y_{i,t}$: (1) an indicator for whether the adviser obtains an annual audit, (2) an indicator for whether the adviser's financial statements are prepared in accordance with GAAP, (3) an indicator for whether the adviser distributes its audited financial statements to investors, and (4) an indicator for whether the adviser has recently received a qualified opinion from its auditor. The vector of controls $X_{i,t}$ is the same set of controls as the previous specification but without the internal controls variables.

Table 11 reports the estimation results. The results are consistent for each outcome variable. In every instance, advisers that previously claimed, and are still claiming, an exemption are significantly less likely to have implemented strong internal controls than similar registered advisers. They are less likely to obtain an annual audit, prepare financials in accordance with GAAP, and distribute audited financials to investors. They are also more likely to have obtained a qualified opinion for their auditor.

Together with the results from the first specification, these estimates point toward a few conclusions about the behavior of fund advisers. First, exempt reporting advisers seem more likely, on average, to have weak internal controls than registered advisers. However, it is unclear what is driving this divergence. On the one hand, it may be the case that unscrupulous advisers choose to avoid registration because they do not care about compliance and do not want to be told off by a regulator. On the other hand, it may be that advisers that are growing choose to temporarily take advantage of lighter oversight to give them time to scale up their compliance systems. These advisers might not have strong controls at the moment they start reporting but plan to build compliance systems quickly and then register.

Second, these results suggest that exempt reporting advisers who choose to implement strong internal controls despite their lack of regulatory oversight pose an especially low risk for problematic conduct. The fact that some advisers choose to implement strong controls without regulatory pressure is intriguing and could have several possible explanations. One is that the advisers adopt strong controls due to pressure from investors. Another is that these exempt advisers are incentivized by the mere fact that they have to report some of their controls on Form ADV.

What do these results tell us, then, about the benefits of registration? Most clearly, these results seem to suggest that registration pushes more advisers to adopt strong internal controls than would otherwise choose to do so. And to the extent registration leads to stronger controls, it would also seem to reduce fraud, misreporting, and other bad behaviors. However, it also seems to be the case that a combination of investor pressure and basic reporting is enough to incentive many (if not most) advisers to adopt basic internal controls. Thus, additional work is needed to determine whether the incremental benefit of full-blown registration is worth the additional cost.

6 Conclusion

This paper investigates the response of private fund advisers to increased regulatory scrutiny following the passage of the Dodd-Frank Act. Specifically, I examine the extent to which advisers limit their AUM to stay below the \$150 million threshold that triggers a requirement to register with the SEC. I find evidence that a significant number of advisers bunch beneath

the threshold to delay registration or avoid it entirely. Using Alvero and Xiao 2023's fuzzy bunching estimator, I estimate that advisers prefer not to bunch only when their assets under management exceed the threshold by 19%. This estimate implies that the annual dollar cost to a registered adviser of complying with regulatory requirements is approximately \$54,000. This estimate lies at the top of the range of estimates provided by the SEC and at the bottom of the range provided by industry participants. In contrast to the \$150 million registration threshold, I find no significant bunching at higher AUM thresholds that trigger heightened reporting obligations for "large hedge fund advisers" and "large private equity advisers." Together, these results suggest that the direct cost of the SEC's current regulatory regime is significant for private fund advisers with AUM near \$150 million, though perhaps less so for the largest advisers.

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Tables

| Year: | | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 |
|----------------|----------------------|--|---|--------------|--------------|--|--|--|--------------|--------------|---|
| Hedge | Registered Exempt | 852 364 | $\begin{array}{c} 856 \\ 380 \end{array}$ | 943 423 | 929 441 | $\begin{array}{c} 918\\ 452 \end{array}$ | 894 482 | 896 494 | 849 505 | $852 \\ 509$ | $873 \\ 547$ |
| Private Equity | Registered Exempt | $\begin{vmatrix} 465 \\ 207 \end{vmatrix}$ | $446 \\ 225$ | $489 \\ 259$ | $546 \\ 265$ | $582 \\ 285$ | $\begin{array}{c} 616\\ 310 \end{array}$ | $\begin{array}{c} 652\\ 342 \end{array}$ | $714\\384$ | 746 415 | $831 \\ 465$ |
| Other | Registered Exempt | $395 \\ 117$ | $\frac{380}{136}$ | 407 154 | 418 163 | 475 181 | $515\\210$ | $543 \\ 250$ | $583 \\ 261$ | 640 298 | $\begin{array}{c} 692\\ 356\end{array}$ |
| Total: | | 2400 | 2423 | 2675 | 2762 | 2893 | 3027 | 3177 | 3296 | 3460 | 3764 |

Table 1: Summary Statistics

| Assets Under Management $(\$m) - 2021$ | | | | | | | |
|--|------|---------|---------|---------------|-------------|-----------|--|
| | Min. | 0.25 | Median | 0.75 | Max | Mean | |
| Registered | 0 | 265.151 | 698.216 | $2,\!334.037$ | 295,029.150 | 4,284.348 | |
| Exempt | 0 | 22.953 | 45.968 | 85.894 | 4,121.272 | 58.763 | |

Table 2: Summary Statistics (cont.)

 Table 3: Summary Statistics (cont.)

| States with the Most Private Fund Advisers — 2021 | | | | | |
|---|-------|-----|-------|-------|--|
| State | Hedge | PE | Other | Total | |
| NY | 539 | 355 | 276 | 1170 | |
| CA | 186 | 167 | 202 | 555 | |
| ΤХ | 100 | 163 | 86 | 349 | |
| CT | 99 | 72 | 49 | 220 | |
| FL | 94 | 70 | 55 | 219 | |
| MA | 78 | 79 | 59 | 216 | |
| IL | 50 | 92 | 53 | 195 | |
| PA | 33 | 28 | 25 | 86 | |
| NJ | 52 | 11 | 20 | 83 | |
| CO | 14 | 25 | 22 | 61 | |

| Scenario: | (1) | (2) |
|----------------------|-------------------|-------------------|
| Parameters: | | |
| AUM | $180 \mathrm{mm}$ | $150 \mathrm{mm}$ |
| Gross Return | 11% | 11.2% |
| Performance Fee | 20% | 20% |
| Management Fee | 1.5% | 2.1% |
| Operating Costs | 1.5% | 1.75% |
| Calculation: | | |
| Performance Fee | $3,\!960,\!000$ | 3,360,000 |
| + Management Fee | 2,700,000 | $3,\!150,\!000$ |
| - Operating Expenses | 2,700,000 | $2,\!625,\!000$ |
| Profit | 3,960,000 | 3,885,000 |
| Difference: | | \$ 75,000 |

Table 4: Example — Calculating the Dollar Cost of Regulation

| Threshold | \$150 mil | \$150 mil | \$150 mil |
|---|------------------|------------------|---------------------|
| | | Danal A. Estimat | |
| | | Panel A. Estimat | Jes |
| Marginal Adviser $(\exp(\overline{q}))$ (\$m) | 178.926 | 171.103 | 181.276 |
| | [4.503] | [5.600] | [5.622] |
| Tax-equivalent Rate (τ) (%) | 1.384 | 0.794 | 1.582 |
| | [0.373] | [0.361] | [0.493] |
| Regulatory Costs (κ) (\$m) | 0.054 | 0.030 | 0.063 |
| | [0.016] | [0.015] | [0.023] |
| Non-bunching Fraction (α) | 0.540 | 0.560 | 0.380 |
| | [0.065] | [0.101] | [0.104] |
| | | | |
| | | Panel B. Paramet | ers |
| Expected Gross Return (%) | 11% | 11% | 11% |
| Performance Fee $(\%)$ | 20% | 20% | 20% |
| | | Panel C. Sample | es |
| Sample Period | Q1 '13 - Q2 '22 | Q1 '13 - Q2 '22 | Q1 '13 - Q2 '22 |
| Counterfactual Polynomial Degree | 4 | 4 | 4 |
| Adviser Type | Any Private Fund | Hedge Fund | Private Equity Fund |

Table 5: Bunching Estimation of the Cost of Becoming an RIA

Ben-David, Birru, and Rossi 2020 provide summary statistics on hedge fund fee structure. The 50th percentile performance fee is 20%, and the 50th percentile management fee is 1.5%. I also assume that the management fee covers the adviser's operating costs. Bootstrapped standard errors with 5,000 resamples are reported in brackets. In calculating the standard errors the private equity fund sample, I restricted α to vary between 0 and 0.85. This prevented unreasonably high estimates for \overline{q} that occasionally cropped up in the bootstrap samples.

| Dependent Variable: | growth |
|--|------------------------------|
| Model: | (1) |
| Variables | |
| yr = -9 | -0.4717^{**} |
| | (0.2048) |
| yr = -8 | -0.1434 |
| | (0.2024) |
| yr = -7 | -0.2985^{*} |
| | (0.1565) |
| yr = -6 | -0.2801^{*} |
| | (0.1624) |
| yr = -5 | -0.2877** |
| | (0.1320) |
| yr = -4 | -0.2064 |
| | (0.1284) |
| yr = -3 | -0.1032 |
| | (0.0925) |
| yr = -2 | -0.1642*** |
| _ | (0.0623) |
| yr = 0 | 1.638*** |
| | (0.1749) |
| yr = 1 | -0.2841** |
| _ | (0.1391) |
| yr = 2 | -0.0863 |
| | (0.0850) |
| yr = 3 | -0.0320 |
| | (0.1043) |
| yr = 4 | -0.1733* |
| _ | (0.0919) |
| yr = 5 | -0.0867 |
| | (0.1353) |
| yr = 6 | -0.1361 |
| _ | (0.1633) |
| yr = 7 | -0.2452* |
| 0 | (0.1489) |
| yr = 8 | -0.2782 |
| | (0.1973) |
| Fixed-effects | |
| erd | Yes |
| yr | Yes |
| Fit statistics | |
| Observations | 25,263 |
| \mathbb{R}^2 | 0.22475 |
| Within \mathbb{R}^2 | 0.00483 |
| Tristatistics Diservations R^2 Within R^2 | 25,263 0.22475 0.00483 |

Table 6: Event Study of Registration Using Sun and Abraham 2021

Clustered (crd) standard-errors in parentheses Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

| Dependent Variable: | growth | | | | | | | | |
|---------------------|-----------------|-----------------|-----------------|-----------|-----------------|-----------------|----------------|----------------|----------------|
| Quantile | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| Variables | | | | | | | | | |
| exempt | -0.0506*** | -0.0105 | -0.0042 | -0.0080** | -0.0146^{***} | -0.0145^{***} | 0.0026 | 0.0481^{***} | 0.2500^{***} |
| | (0.0116) | (0.0069) | (0.0040) | (0.0032) | (0.0039) | (0.0051) | (0.0083) | (0.0145) | (0.0330) |
| below | -0.0416 | -0.0587^{***} | -0.0722^{***} | -0.0310 | 0.0020 | 0.0367 | 0.0686 | 0.0744^{**} | 0.1870^{**} |
| | (0.0278) | (0.0207) | (0.0123) | (0.0303) | (0.0276) | (0.0245) | (0.0497) | (0.0317) | (0.0784) |
| ре | 0.0091 | 0.0058 | -0.0016 | -0.0001 | -0.0049 | -0.0095 | -0.0197^{**} | -0.0127 | 0.0372 |
| | (0.0105) | (0.0065) | (0.0040) | (0.0035) | (0.0042) | (0.0060) | (0.0093) | (0.0146) | (0.0271) |
| hedge | -0.0704^{***} | -0.0443*** | -0.0224^{***} | -0.0048 | 0.0047 | 0.0079 | -0.0072 | -0.0115 | -0.0141 |
| | (0.0116) | (0.0066) | (0.0045) | (0.0040) | (0.0044) | (0.0057) | (0.0084) | (0.0131) | (0.0257) |
| exempt:below | 0.0781* | 0.0702*** | 0.0631*** | 0.0099 | -0.0311 | -0.0761** | -0.0620 | 0.0307 | -0.0404 |
| - | (0.0441) | (0.0244) | (0.0147) | (0.0313) | (0.0292) | (0.0303) | (0.0664) | (0.0862) | (0.1160) |
| Fixed Effects | | | | | | | | | |
| yr | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |

 Table 7: Quantile Regression Output

Standard-errors in parentheses Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

| Sample Size Range (\$m): | (100 - 300) | (100 - 500) | (100 - 1,000) | | |
|--------------------------|----------------------|-----------------|---------------|--|--|
| | Panel A. Full Sample | | | | |
| Polunomial Degree: | | | | | |
| 4 | 178.926 | 180.828 | 187.588 | | |
| | [4.471] | [4.981] | [4.059] | | |
| 5 | 177.353 | 176.126 | 185.220 | | |
| | [4.737] | [4.487] | [5.221] | | |
| 6 | 179.972 | 177.167 | 178.382 | | |
| | [4.628] | [4.729] | [5.184] | | |
| |] | Panel B. Hedge | e Fund | | |
| Polynomial Degree: | | | | | |
| 4 | 171.103 | 178.773 | 187.824 | | |
| | [5.648] | [9.341] | [9.748] | | |
| 5 | 175.512 | 169.172 | 184.499 | | |
| | [6.116] | [5.462] | [10.713] | | |
| 6 | 179.097 | 170.757 | 175.902 | | |
| | [6.468] | [5.892] | [8.442] | | |
| | Pane | l C. Private Ec | quity Fund | | |
| Polynomial Degree: | | | | | |
| 4 | 181.276 | 170.568 | 182.603 | | |
| | [5.378] | [4.407] | [4.539] | | |
| 5 | 165.770 | 177.474 | 171.513 | | |
| | [6.488] | [5.451] | [4.793] | | |
| 6 | 179.832 | 184.104 | 171.440 | | |
| | [11.655] | [5.622] | [4.874] | | |

Table 8: Bunching Estimation with varying Polynomial Degree and Sample Size

Bootstrapped standard errors with 1,000 resamples reported in brackets.

| Gross Return: | 10% | 11% | 12% | 13% | 14% |
|------------------------|------------|------------|------------|------------|------------|
| Marginal Adviser (\$m) | | | | | |
| 175 | $37,\!548$ | 41,303 | $45,\!058$ | 48,812 | $52,\!567$ |
| 177.5 | 44,994 | $49,\!493$ | $53,\!993$ | $58,\!492$ | $62,\!992$ |
| 180 | $53,\!035$ | $58,\!339$ | $63,\!642$ | 68,946 | 74,249 |

Table 9: Dollar Cost of Regulation with varying Gross Returns and Marginal Adviser AUM

| Dependent Variable: | | | DRP_{t+1} | | |
|---|----------------|----------------|--------------|--------------|---------------|
| Model: | (1) | (2) | (3) | (4) | (5) |
| Variables | | | | | |
| Constant | 0.0873^{***} | 0.0873^{***} | | | |
| | (0.0047) | (0.0049) | | | |
| First Reg. \times Exempt _{t-1} | -0.0101 | 0.0232^{*} | 0.0232^{*} | 0.0214^{*} | 0.0243^{**} |
| 0 101 | (0.0121) | (0.0121) | (0.0121) | (0.0121) | (0.0121) |
| First Reg. | -0.0260*** | -0.0266*** | -0.0266*** | -0.0263*** | -0.0259*** |
| <u> </u> | (0.0080) | (0.0079) | (0.0079) | (0.0079) | (0.0079) |
| Exempt_{t-1} | -0.0102** | -0.0364*** | -0.0363*** | -0.0341*** | -0.0377*** |
| | (0.0048) | (0.0049) | (0.0049) | (0.0048) | (0.0049) |
| PE | -0.0345*** | -0.0326*** | -0.0326*** | -0.0320*** | -0.0272*** |
| | (0.0051) | (0.0051) | (0.0051) | (0.0051) | (0.0051) |
| Hedge | 0.0006 | -0.0027 | -0.0029 | -0.0081 | 0.0005 |
| 0 | (0.0051) | (0.0051) | (0.0051) | (0.0053) | (0.0055) |
| log AUM | 0.0196*** | -0.0014 | -0.0013 | -0.0015 | -0.0014 |
| | (0.0014) | (0.0021) | (0.0022) | (0.0021) | (0.0021) |
| $(\log AUM)^2$ | (0.0022) | 0.0027*** | 0.0028*** | 0.0026*** | 0.0024*** |
| () | | (0.0004) | (0.0004) | (0.0004) | (0.0004) |
| $(\log AUM)^3$ | | 0.0009*** | 0.0009*** | 0.0010*** | 0.0009*** |
| () | | (0.0001) | (0.0001) | (0.0001) | (0.0001) |
| Annual Audit | | (0.0001) | (0.0001) | (0.0001) | -0.0011 |
| | | | | | (0.0061) |
| GAAP | | | | | -0.0249*** |
| | | | | | (0.0079) |
| F.S. Distributed | | | | | -0.0105 |
| | | | | | (0.0070) |
| Qualified Opinion | | | | | 0.0243^{**} |
| gaamoa opinion | | | | | (0.0095) |
| Fired officets | | | | | (0.0000) |
| r incu-ejjecis | | | Vog | Vog | Voc |
| yi stato | | | 162 | Vos | Vog |
| SLAUC | | | | 165 | 162 |
| Fit statistics | | | | | |
| Observations | $21,\!687$ | $21,\!687$ | $21,\!687$ | $21,\!687$ | $21,\!687$ |
| \mathbb{R}^2 | 0.02676 | 0.03887 | 0.03902 | 0.05067 | 0.05271 |
| Within \mathbb{R}^2 | | | 0.03887 | 0.03584 | 0.03792 |

Table 10: Likelihood of Discipline Based on Adviser Characteristics

Heteroskedasticity-robust standard-errors in parentheses Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

| Dependent Variables: | Annual Audit | GAAP | F.S. Distributed | Qualified Opinion |
|---|--------------------|-----------------|------------------|-------------------------|
| Model: | (1) | (2) | (3) | (4) |
| Variables | | | | |
| $\operatorname{Exempt}_{t-1}$ | -0.1443*** | -0.0619*** | -0.0484*** | 0.0530*** |
| | (0.0077) | (0.0057) | (0.0068) | (0.0051) |
| First Reg. | 0.0239** | 0.0168** | -0.0320*** | -0.0133** |
| | (0.0103) | (0.0082) | (0.0113) | (0.0065) |
| First Reg. \times Exempt _{t-1} | 0.1137^{***} | 0.0510*** | 0.0447^{**} | -0.0457*** |
| | (0.0192) | (0.0144) | (0.0193) | (0.0112) |
| $\rm PE$ | 0.1716^{***} | 0.0939^{***} | 0.0830*** | -0.0542^{***} |
| | (0.0078) | (0.0063) | (0.0068) | (0.0051) |
| Hedge | 0.2974^{***} | 0.1690^{***} | 0.1591^{***} | -0.0962*** |
| | (0.0070) | (0.0058) | (0.0063) | (0.0048) |
| $\log AUM$ | 0.0132^{***} | -0.0019 | 0.0111^{***} | -0.0033** |
| | (0.0024) | (0.0018) | (0.0022) | (0.0016) |
| $(\log AUM)^2$ | -0.0063*** | -0.0021^{***} | -0.0035*** | 0.0016^{***} |
| | (0.0006) | (0.0004) | (0.0005) | (0.0004) |
| $(\log AUM)^3$ | $-5.9	imes10^{-5}$ | -0.0002 | -0.0005*** | 6.51×10^{-5} |
| | (0.0002) | (0.0001) | (0.0001) | (9.68×10^{-5}) |
| Fixed-effects | | | | |
| yr | Yes | Yes | Yes | Yes |
| state | Yes | Yes | Yes | Yes |
| Fit statistics | | | | |
| Observations | $23,\!366$ | $23,\!366$ | 23,366 | $23,\!366$ |
| \mathbb{R}^2 | 0.13201 | 0.07939 | 0.05727 | 0.06508 |
| Within \mathbb{R}^2 | 0.12099 | 0.05790 | 0.04407 | 0.03345 |

Table 11: Strength of Internal Controls Based on Adviser Characteristics

Heteroskedasticity-robust standard-errors in parentheses Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

Figures



Figure 1: CDF and Histogram of log AUM: All Private Fund Advisers

This Figure plots the AUM distribution for all private fund advisers with AUM between \$100 million and \$500 million. In both panels, the dashed blue line is an estimated counter-factual distribution. In the top panel, the counterfactual is estimated by fitting a polynomial to the empirical CDF, excluding the region containing the bulge. In the bottom panel, the counterfactual is calculated analytically by taking the first derivative of the counterfactual estimated for the top panel.



Figure 2: CDF and Histogram of log AUM: Hedge Fund Advisers



Figure 3: CDF and Histogram of log AUM: Private Equity Fund Advisers



Figure 4: CDF and Histogram of log Hedge Fund AUM: Large Hedge Fund Advisers



Figure 5: CDF and Histogram of log Private Equity AUM: Large Private Equity Advisers







Figure 7: Quantile Regression Coefficients on Exempt-Size(\$135 - \$150mm) Interaction







Figure 9: AUM Boxplot Around Time of Registration



Figure 10: Adviser Reporting Status in Years After First Exempt Filing

Appendix A

A.1 Derivation of Regulatory Cost Formula

This derivation is nearly identical to the derivation of the regulatory cost formula used in Alvero, Ando, and Xiao 2022. Because a thorough proof is provided in that paper's appendix, I will provide only an abbreviated proof here.

First, note that if we take the first order condition of Equation (1) by differentiating with respect to q, we obtain the following:

$$\frac{\partial}{\partial q} \pi(q|z) \exp(q) = 0$$

$$\frac{\partial \pi(q^*|z)}{\partial q} \exp(q^*) + \pi(q^*|z) \exp(q^*) = 0$$

$$\frac{\partial \pi(q^*|z)}{\partial q} = -\pi(q^*|z)$$
(9)

Second, returning to the indifference condition in Equation (4), we can replace $\pi(\underline{q}|z)$ with an approximation using a first-order Taylor expansion around $\pi(\overline{q}|z)$:

$$\pi(\overline{q}|z) \exp(\overline{q})(1-\tau) = \pi(\underline{q}|z) \exp(\underline{q})$$
$$\pi(\overline{q}|z) \exp(\overline{q})(1-\tau) \approx [\pi(\overline{q}|z) + \frac{\partial \pi(\overline{q}|z)}{\partial q}(\underline{q}-\overline{q})] \exp(\underline{q})$$
(10)

Third, because $\overline{q} = q^*$ for the marginal adviser, we can use Equation (9) to replace the derivative in Equation (10) and simplify, as follows:

$$\begin{aligned} \pi(\overline{q}|z) \exp(\overline{q})(1-\tau) &\approx [\pi(\overline{q}|z) - \pi(\overline{q}|z)(\underline{q}-\overline{q})] \exp(\underline{q}) \\ \pi(\overline{q}|z) \exp(\overline{q})(1-\tau) &\approx [\pi(\overline{q}|z) + \pi(\overline{q}|z)(\overline{q}-\underline{q})] \exp(\underline{q}) \\ \pi(\overline{q}|z) \exp(\overline{q})(1-\tau) &\approx \pi(\overline{q}|z) \exp(\underline{q}) + \pi(\overline{q}|z) \exp(\underline{q})(\overline{q}-\underline{q}) \\ \pi(\overline{q}|z) \exp(\overline{q})(1-\tau) &\approx \pi(\overline{q}|z) \exp(\underline{q})(1+(\overline{q}-\underline{q})) \\ 1-\tau &\approx \frac{\pi(\overline{q}|z) \exp(\underline{q})(1+\overline{q}-\underline{q})}{\pi(\overline{q}|z) \exp(\overline{q})} \\ 1-\tau &\approx \exp(\underline{q}-\overline{q})(1+\overline{q}-\underline{q}) \\ \tau &\approx 1-(1+\overline{q}-\underline{q}) \exp(\underline{q}-\overline{q}) \blacksquare \end{aligned}$$