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CONCORDANCE AMONG HOLDOUTS

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Concordance among Holdouts*

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Abstract

Holdout problems prevent decentralized aggregation of complementary goods, as in the assembly of land or transfer of corporate ownership. Therefore, as Mailath and Postelwaite (1990) formalized, some coercion is needed to enable the assembly of complements in large populations. We propose an *approximate individual rationality* condition under which individuals are guaranteed to receive as compensation for taking at least the best estimate of their value possible from others' information. Unlike strict individual rationality, this constraint is consistent with the *bilateral efficiency* achieved by a take-it-or-leave-it offer to the aggregate seller community. We propose a class of *Concordance* mechanisms which includes the maximally attractive mechanisms achieving these objectives together with, depending on the implementation considered, either dominant-strategy incentive compatibility or Bayesian incentive compatibility and budget balance.

JEL classification: D40, D70, D81, L10

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I Introduction

The right of each agent to withdraw her consent from a public project creates market power that demands a rent. Thus, in rich environments, private (voluntary and self-financing) provision of public goods—or bads such as land assembly—to a large number of self-interested agents is impossible, as the demand for individual rents overwhelms the value of the project. This *holdout* problem is ubiquitous in economics. First formalized by Cournot (1838), it takes its precise modern form in the work of Mailath and Postelwaite (1990).[1] Holdout concerns have informed wide-ranging policy decisions, including eminent domain[2] and corporate takeover laws.[3] Yet formal market design work aimed at coping with holdout has been circumscribed by its apparently sharp boundaries. This paper shows that relaxing the notion of individual consent, in a fashion that arguably brings it closer to the original motivation for its imposition, allows the construction of simple incentive compatible mechanisms that overcome the holdout problem and thus allow substantial efficiency.

We study holdout in settings where a good owned by a disparate community of sellers is desired by a buyer only in its entirety; for concreteness, we focus on the salient application of land assembly, but our mechanism may be applied to any binary holdout or public goods problem. The theorem of Mailath and Postelwaite (1990) implies that no incentive compatible mechanism can simultaneously achieve full efficiency and individually rational voluntary participation. However, as we show, it is possible to strike a balance between these two goals. Namely, we propose a class of mechanisms achieving

1. *bilateral efficiency* – outcomes are always as efficient as a bilateral bargain between the prospective buyer and a single agent representing the community of sellers in its entirety – and

2. *approximate individual rationality* – each individual is assured of receiving, if her prop-

[1] This problem is often known as "double marginalization" (Spengler, 1950) or "anticommons" (Michelman, 1967).

[2] To help alleviate the holdout problem that (especially public) developers face in assembling land (Posner, 2005), the policy of "eminent domain" in the Fifth Amendment to the United States Constitution allows the government to take "private property [...] for public use," but only after "just compensation" has been paid.

[3] When one individual or corporation seeks a controlling share in a public firm, most countries require that it make a bid for all shares (Kirchmaier et al., 2009). These regulations are designed to protect minority shareholders' interests in the case of take-overs by other firms whose interests do not concord with strict divisional profit maximization and to help ameliorate free-riding on corporate efficiency improvements by corporate "raiders" (Grossman and Hart, 1980). However, because individuals have heterogeneous risk-aversion and belief-driven infra-marginal utility from investing in the to-be acquired firm, it is nearly impossible for a prospective buyer to voluntarily purchase all shares. Thus to allow acquisitions to take place, nearly every jurisdiction allows consent by some super-majority of share-holders to squeeze-out (Croft and Donker, 2006) or overrule (Armour and Skeel, 2007) the remaining holdouts.

erty is taken, an unbiased estimate of her value based on any aggregate information about the value of her property.[4]

The first of these conditions weakens efficiency to account for the inherent limits on the efficiency of any bilateral bargain (Myerson and Satterthwaite, 1981). The second relaxes individual rationality following continental property law traditions: a reasonable community estimate determines compensation.

Our mechanisms are inspired by Cournot's theory of collaboration—*concours*—among producers and thus we label them *Concordance mechanisms*. Cournot's solution to the *concours* problem is for the sellers of complementary products to merge and determine prices as a collective. Similarly, our Concordance mechanisms treat the entire group of sellers as a "community," which bargains with the buyer and divides proceeds according to exogenously-specified shares. To incentive-compatibly determine the reserve price in the bargaining, sellers who influence the sale decision are forced to internalize their externalities through a Pigouvian tax.

Concordance mechanisms are asymptotically efficient under truthful reporting by sellers. Specific Concordance mechanisms exhibit tradeoffs between incentive compatibility and budget-balance familiar from auction design. The *Straightforward Concordance* mechanism maximizes seller surplus among all self-financing, approximately individually rational mechanisms which are strategy-proof for sellers. The Bayesian incentive compatible *Bayes-Nash Concordance* mechanism is the only incentive compatible, budget-balanced mechanism that divides revenues according to shares. Thus these Concordance mechanisms are the most desirable incentive compatible mechanisms achieving our desiderata.

To implement Concordance mechanisms, the government needs only an estimate of each seller's share of the total community value—it does not need to know the aggregate level of sellers' subjective valuations. The quality of the government's estimate of relative shares determines the degree to which approximate individual rationality resembles full individual rationality. If shares are perfectly assessed, then Concordance mechanisms are fully individually rational.

[4]Note that our strategy—shifting the relevant notion of property rights—is similar to that of Cramton et al. (1987). However, in the setting of Cramton et al. (1987), each individual is happy to accept the entire community plot, while in ours each individual gains value only from her own plot. Thus, the incentive problem in our setting is significantly different from that of Cramton et al. (1987); we therefore impose a different property rights adjustment.

Importance of the holdout problem

Back-of-the-envelope calculations indicate that holdout problems cost the economy a large part of global gross domestic product. In the United States alone, there are nearly 6000 active takings (Berliner, 2006) per year; these likely represent only a small fraction of all land assembly in the United States. Supposing an average stake of $10 million, these represent $60 billion annually; similar guesses for México and Brazil (see our online appendix) indicate together at least $20 billion dollars annually. Thus global land assembly activity is likely on the order of hundreds of billions of dollars each year. According to Dealogic, corporate acquisitions amounted to $972 billion or 5.5% of global GDP in the first quarter of 2008 alone (Twaronite, 2009). When the economy is weak, reduced acquisitions are compensated by debt settlements; in 2008, according to BankruptcyData.com, the assets of United States firms filing for bankruptcy amounted to more than a trillion dollars. Aggregating these and the other standard examples above gives a ballpark estimate of many trillions of dollars for the annual volume of transactions subject to holdout problems.[5]

Supposing an average of 20% potential gains from trade (equivalently a 10% monopoly mark-up under linear demand), and assuming that one quarter of these are lost to deadweight from holdout, this amounts to 5% of transaction volume. This is linear demand monopoly deadweight loss, much smaller than for most other demand functions. Furthermore, monopoly deadweight loss is modest compared to what one would expect from holdout. A high 5% real interest rate would roughly indicate that the discounted NPV of social gains from an efficient mechanism for holdout is on the order of many trillions of dollars or double digit percentages of global annual GDP. Thus we believe our estimates indicate that holdout is not just a problem of theory, but also a pressing social challenge of practical importance.

Relation to the literature

The previous literature on holdout has predominantly followed four trajectories: First, a series of papers has documented the inefficiencies created by holdout, both theoretically (Menezes and Pitchford, 2004; Miceli and Segerson, 2007) and empirically (Sorensen, 1999). Second, a small literature has sacrificed the richness of the environment to propose mecha-

[5]Rules in most countries require the consent of a supermajority of creditors to a debt renegotiation outside of bankruptcy, with thresholds differing across countries (La Porta et al., 1998). Following Federal Communications Commission (FCC) auctions, radio spectrum has become fragmented, inhibiting efficient high-speed wireless internet (Hazlett, 2005); reassembling this spectrum is a top priority of the FCC. Investors commonly assemble pools of complementary patented innovations and license them jointly, but difficulty forming pools can be a drag on innovation (Heller and Eisenberg, 1998). Class action legal settlements are often plagued by holdouts (Rob, 1989). Heller (2008) surveys a variety of other examples, from post-Communist property transitions in eastern Europe to share-cropping relations in the post-Bellum South.

nisms which, in restricted settings, achieve efficiency (Bagnoli and Lipman, 1988; Hellwig, 2003). A large literature on mechanism design for the provision of public goods is in the same spirit: many mechanisms for this problem Nash-implement Lindahl allocations (Groves and Ledyard, 1977; Hurwicz, 1977; Walker, 1981; Tian, 1989).[6] However, all of these mechanisms either have impractically large multiplicities of equilibria or assume complete information—and thus effectively rely, like Bagnoli and Lipman (1988), on common knowledge of values to achieve efficiency (Bailey, 1994). Third, a large literature (Andreoni, 2007) explores the role that altruistic preferences can play in allowing provision. And fourth, a small literature proposes mechanisms, which (like eminent domain) impose no requirement of community consent (Plassmann and Tideman, 2009).[7]

All of this work has remained loyal to the mechanism design criteria of the Mailath and Postelwaite (1990) negative result, enriching our understanding of its boundaries and implications. We build on these insights by following an approach that has proven effective in recent work on market design (Budish, 2010): we relax the design criteria, in a manner hopefully consistent with the spirit that motivated them, so as to escape the pessimistic restrictions and conclusions of the main holdout literature.

Organization of the paper

We begin by illustrating our approach with a simple example, in Section II. Then, in Section III, we develop a general mechanism design framework for studying the holdout problem, formalize and motivate our relaxed efficiency and individual rationality conditions. Next, in Section IV, we introduce *Concordance mechanisms* and show that any mechanism in that class satisfies our desired conditions (under truthful reporting by sellers). Specific Concordance mechanisms, discussed in Section V, exhibit tradeoffs between incentive compatibility

[6]Note that all of these papers formally consider public goods, but (binary) public goods settings are equivalent to land assembly: "seller values" become willingnesses to pay and the "buyer offer" becomes the (exogenous) cost of the project. Individual rationality become voluntary participation, perfect complementarity non-excludability and single-mindedness non-rivalry. True shares are Lindahl prices and actual (approximate) shares are closely connected to the *pseudo-Lindahl* prices of Bergstrom (1979b), a public authority's closest approximation to Lindahl based on public information. Thus our mechanism's guarantee of approximate individual rationality implies a corresponding binary public goods mechanism that implements the *pseudo-Lindahl* (efficient) pseudo-Lindahl equilibrium. As far as we know, ours is the first mechanism that does so in general.

In fact, a perfect example of public goods is the reverse of land assembly—land reform. All of our mechanisms apply just as easily there: the seller makes a take-it-or-leave-it offer to a community of tenant famers who are coerced to participate in a Concordance mechanism for purchasing the land.

[7]Additionally, various authors have proposed individually rational mechanisms for land assembly that eliminate strategic misrepresentation of valuations (Grossman et al., 2010). Unfortunately, these mechanisms do not solve the more fundamental holdout problem—they generally yield no-trade equilibria in large markets, even when trade is efficient (Shavell, 2007).

and budget-balance familiar from auction design. In Section VI, we conclude by discussing directions for future research. Additionally, on our websites, we provide a more extensive appendix discussing applications and the connections between holdout problems and Cournot's theory of *concours*, as well as software (designed by William Weingarten) which implements and simulates all the mechanisms described in this paper.

II An Illustrative Example

We begin with a simple example illustrating our main contribution, using the language of our land assembly application and the notation established more formally in Section III.

We suppose that $N = 10$ (potential) sellers $i = 1, \dots, 10$ own privately-valued pieces of a contiguous plot of land. Seller i has a share s_i of the total land (or more generally assessed value of land) in the plot, which has a market price of 10. However, by revealed preference, the land also has some subjective value to the sellers above its market value, which is on average $\gamma \geq 1$, but has some spread. In particular, the value v_i of seller i is drawn independently and uniformly from the interval $[s_i\gamma \cdot 10 \cdot (1-\zeta), s_i\gamma \cdot 10 \cdot (1+\zeta)]$, where $0 \leq \zeta \leq 1 - \frac{1}{\gamma}$, but γ is not known by the social planner. A buyer, who knows γ, has private value b for the collective plot.

Collective sale to the buyer is efficient if and only if $b > V \equiv \sum_i v_i$. Since all values are private, a bargaining challenge arises: how are the parties to identify and reach an efficient outcome?

Self-assessment mechanisms ask sellers to name reserve prices r_i, with the buyer acquiring the plots at those prices if and only if $b \geq \sum_i r_i$. However, when a seller is asked to name her price, she is incentivized to holdout by reporting $r_i > v_i$.

One solution to the problem of value-shading is for the buyer to make identical (share-weighted) take-it-or-leave-it offers to each seller, acquiring the plots if and only if all sellers accept. But under this approach, the probability of trade quickly goes to 0. To illustrate this, consider what happens when $\gamma = \frac{11}{2}$ and $\zeta = 1 - \frac{1}{\gamma} = \frac{9}{11}$, so that $[10\gamma(1-\zeta), 10\gamma(1+\zeta)] = [10, 100]$ and $s_i \equiv \frac{1}{10}$ for all i. Here, the expected total community value is given by $E[V] = (\sum_i s_i) \cdot E[v_i] = 55$, so that a buyer with value $b = 75$ expects total surplus of $b - E[V] = 75 - 55 = 20$. Even if the buyer were to take no surplus and offer each seller $\frac{75}{10} = 7.5$, the take-it-or-leave-it offer that maximizes the probability of sale, the probability of a sale taking place is $\left(1 - \frac{25}{90}\right)^{10} < .04$, although the probability that sale is efficient is greater than .99. We see that although there is no value-shading, the buyer again faces a "holdout" problem: with high probability, some seller will refuse sale, even if the buyer gives up a large fraction of the surplus.

A natural solution to holdout is eminent domain: the buyer, with government assistance, takes the plots and compensates the sellers according to the standard of (exogenously determined) "just compensation." This procedure typically involves the (local) government asking for an assessment of land values by a real estate expert. Given limited legal recourse for takees following recent Supreme Court decisions (*Kelo v. City of New London, Connecticut*, 2005), however, this "just compensation" is typically set at the minimal possible market value for the land—in this case, $s_i \cdot 10$. Eminent domain guarantees that all efficient trades occur—any buyer with value $b > 10$ will be willing to buy the land for 10, the lowest possible value for V. Unfortunately, even when trade under eminent domain is efficient, eminent domain does not incorporate the buyer's information about γ, and hence may drastically under-compensate sellers: $s_i \cdot 10$ is less than v_i with probability 1 by revealed preference. Moreover, when $55 > b > 10$, *inefficient* sales will often occur. Indeed, in our previous example, sale always occurs when $b = 20$, although the probability of such sale being efficient is less than .01.

Nonetheless, one of the basic ideas of eminent domain—that an individual's share of compensation should be rendered independent of her behavior—is sound. Our Concordance mechanisms build upon this principle, using $s_i o$ as baseline for the compensation T_i: an offer o is obtained from the buyer, and each seller i receives $s_i o$ whenever trade occurs. However, Concordance mechanisms link the sale decision directly to seller-reported reserve values r_i; trade occurs if and only if the offer exceeds the collective reserve, $o \geq R \equiv \sum_i r_i$. This prevents inefficient sales.

If the shares of the land perfectly reflect shares of subjective value ($s_i = \frac{v_i}{V}$ for each i) then this structure is all that is required to solve holdout. Sellers cannot affect the aggregate sale price o, and each seller i favors sale if and only if $\frac{v_i}{V} o = s_i o \geq v_i$, that is, when $o \geq V$. Thus, it is in the best interest of i to report $r_i = v_i$. But if shares are imperfectly assessed, then sellers with shares lower (higher) than average may seek to prevent (encourage) a sale by over-(under-)stating their values.

A Concordance mechanism is obtained by combining the Concordance compensation scheme ($s_i o$ in case of sale) and decision rule (sale if and only if $o \geq R$) with a tax scheme designed to discourage dishonest value reports. The simplest procedure to illustrate is based upon the externality-tax scheme of the VCG mechanism.

In particular: A seller is *pivotal in the sale decision* if the community would have supported a different decision were i replaced by a seller indifferent to the sale (i.e. one with value $s_i o$). Following VCG, our Straightforward Concordance (SC) mechanism forces all pivotal sellers to pay for the externalities they cause. Thus it is a dominant strategy for each seller to report her value truthfully. Thus, under SC it is also optimal for the buyer to make the monopsonist-optimal offer against V.

Suppose that b induces the offer $o = 56$ (conditional upon γ). We again consider the case in which $\gamma = \frac{11}{2}$ and $\zeta = 1 - \frac{1}{\gamma} = \frac{9}{11}$, and suppose the following seller shares and values:

$$\begin{array}{c|cccccccccc} i & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \hline s_i & \frac{1}{55} & \frac{2}{55} & \frac{3}{55} & \frac{4}{55} & \frac{5}{55} & \frac{6}{55} & \frac{7}{55} & \frac{8}{55} & \frac{9}{55} & \frac{10}{55} \\ v_i & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 10 & 9 \end{array} \tag{1}$$

Here all sellers except 9 and 10 have the mean valuation implied by their shares, while 9 has a (slightly) above-average valuation and 10 has a (slightly) below average valuation. Thus, relative to their private valuations, 9 is assigned too low a share, while 10 is assigned too high a share. Seller 10 is pivotal, since $\frac{\sum_{j \neq 1} r_j}{1 - s_{10}} = \frac{506}{9} \approx 56.22 > 56$; she pays a tax equal to her externality, $(1 - \frac{10}{55})|\frac{506}{9} - 56| = \frac{10}{55}$. Even with this tax, seller 10 still receives more than her value in total compensation: $s_i o - \frac{10}{55} = \frac{560}{55} - \frac{10}{55} = 10 > 9 = v_{10}$. On the other hand, non-pivotal seller 9 receives less than her valuation: $\frac{9}{55} \cdot 56 \approx 9.16 < 10$.

Each non-pivotal seller $i = 1, \ldots, 9$ receives at least

$$\frac{s_i \sum_{j \neq i} v_j}{1 - s_i}, \tag{2}$$

the (share-weighted) average (implied) value of sellers $j \neq i$ for the plot of seller i. This value (2) equals seller i's share of the collective reserve that would have obtained were i absent, and is an unbiased estimate of v_i conditional upon γ. For example, while seller 9 does not receive her full valuation of 10, she receives at least $\frac{9}{55} \cdot \frac{45}{1 - \frac{9}{55}} \approx 8.8$, which is much larger than $\frac{9}{55} \cdot 10 \approx 1.63$, her compensation under eminent domain. Because any seller i can opt to exert no influence on the collective decision by stating $r_i = s_i o$, this outcome provides a lower bound on seller payoffs in Concordance mechanisms. The sellers are thus paid according to the "community consensus on the severity of the harm inflicted," which some legal scholars (Ellickson, 1973) believe to be the appropriate for "just compensation."

As the assessment of the shares s_i becomes more accurate (i.e. as $s_i \to \frac{v_i}{V}$ for all i), (2) converges to v_i, hence the compensation guarantee of Concordance mechanisms is an *approximation to individual rationality.* The quality of the approximation is mediated by the noise level of the share assessment. In our example, this is mediated by how far ζ is from 0. To see this, suppose that γ is fixed at $\frac{11}{2}$, shares are fixed at $s_i \equiv \frac{1}{10}$ and the offer is fixed at $o = 56$. Figure 1 shows how average (over 1000 simulation trials) total undercompensation (measured as the total level of individual rationality violation, $\sum_i \max\{0, v_i 1_{\text{sale}} - T_i\}$) declines with ζ. While total undercompensation is slightly above 5 when $\zeta = \frac{9}{11}$, it declines

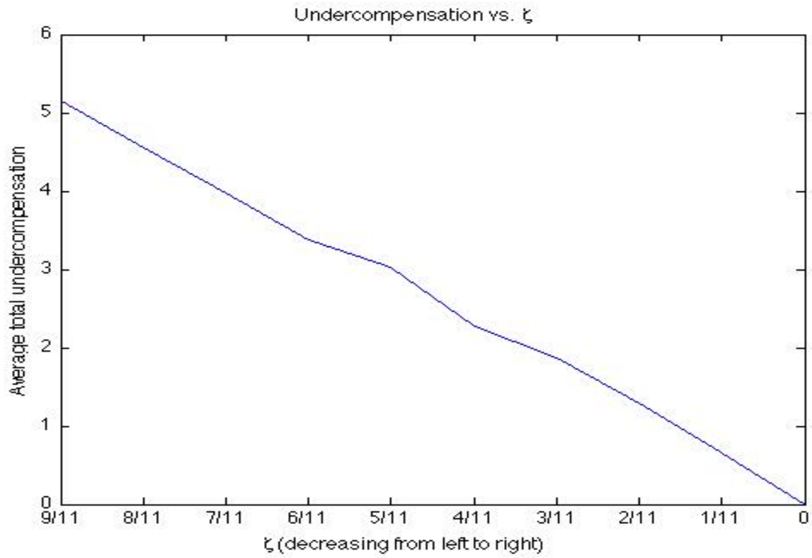

Figure 1: Average total undercompensation diminishes to 0 as share assessment becomes perfect ($\zeta \to 0$).

to 0 roughly linearly with ζ as the share estimate becomes perfect.

As with any other VCG implementation, the SC mechanism is not *budget-balanced*—some of the buyer's offer is lost in taxes. Nonetheless, in our example, total community compensation is $55.82 > 55 = V$, hence the community together receives at least its aggregate value, V.[8] Since sale in a Concordance mechanism occurs if and only if $o \geq R$, this *collective rationality* can be guaranteed in general. Indeed, in SC the community receives at least V in the case of sale if each seller i reports $r_i = s_i V$ (whence $R = \sum_i r_i = \sum_i s_i V = V \sum_i s_i = V$), as this reporting strategy never generates SC taxes. Unfortunately, in SC this behavior does not arise under truthful reporting unless shares are perfectly assessed. Our alternative mechanisms are budget-balanced and thus ensure collective rationality when sellers are truthful, but are not dominant-strategy incentive compatible.

As we see in our example, sale occurs in SC if and only if the monopsonist-optimal offer for the whole plot exceeds the community value ($o \geq R = V$). Thus, the sale decision is *bilaterally efficient*, i.e. it mimics a bilateral bargain between the buyer and a single seller—the community—with value V. Since the seller values v_i are drawn independently, (by the law of large numbers) the uncertainty regarding the efficiency of trade decreases as N grows large. When $N = 10$, $b = 75$, share are uniform and o is monopsonist-optimal at 59, the probability that $o < V < b$ (trade is efficient, but the offer is too low) is about .30—inefficiency obtains in less than a third of cases. As N grows, the probability of inefficiency

[8]This is a consequence of our distribution and value assumptions.

drops: for $N = 5$ it is about .38, while for $N = 25$, it is about .19, for $N = 100$ it is about .08 and for $N = 500$ it is about .03.[9] As a result, it follows that SC is *asymptotically efficient*, i.e. it becomes fully efficient in the limit as $N \to \infty$.

III The Model

III.A Basic framework

For concreteness, we present our model in terms of a land assembly example: there are $N > 0$ (potential) *sellers* i, each of whom owns a piece of a plot of land, privately valuing it at v_i. There is a single (potential) *buyer*, β, only interested in buying the entire plot of land, which she values at b.

We assume that there may be an aggregate shock to the value of the land, γ, known to the buyer but not to verifiable by the planner. We assume that γ is drawn independently from b, but make no other assumptions about it as they are not necessary in what follows. We assume that $v \equiv (v_1, \ldots, v_N)$ is drawn from $[\underline{v}, \infty)^N$ according to a smooth, full support joint probability density function g_γ conditional on γ and that b is independent of v. We let $V \equiv \sum_i v_i$ denote the total value of the seller *community.* The *share* of seller i is defined by

$$s_i(\tilde{\gamma}) \equiv E\left[\frac{v_i}{V} \mid \gamma = \tilde{\gamma}\right].$$

We assume that $s_i(\gamma) \equiv s_i$ is constant in γ and write $s \equiv (s_1, \ldots, s_N)$. Equivalently, we assume that a best guess of each seller's share of the total land value can be made independent of the actual aggregate value, and that the planner may condition the mechanism on this guess.[10]

Transactions are structured by a *mechanism* consisting of a collection of *offers* $\mathcal{O} \subseteq \mathbb{R}$ specifying actions that buyers can take, *reserve values* $\mathcal{R} \subseteq \mathbb{R}$ specifying actions that sellers can take, a *purchase rule* $P : \mathcal{B} \times \mathcal{R}^N \to \{0, 1\}$ specifying the actions of buyers and sellers following which a sale of the community plot takes place, and a *transfer rule* $T : \mathcal{B} \times \mathcal{R}^N \to \mathbb{R}^{N+1}$ specifying transfers to (or from) buyers and sellers.[11] The mechanism may also have a pair of *suggested strategies* $(o^\star, r^\star)$ that, while not technically part of the mechanism *per se*, help in structuring thought about the mechanism. Here, $r^\star : [\underline{v}, \infty) \to \mathcal{R}$ is a *suggested*

[9] The last three of these calculations were approximated using the normal distribution, rather than the uniform sum distribution.

[10] In the case of land assembly, individuals' shares of the total assessed market value of to-be-assembled land can be used to determine shares; in the case of a corporate acquisition, shares in the corporation play the same role.

[11] We write $T_\beta(o, r)$ for the transfer to the buyer and write $T_i(o, r)$ for the transfer to the seller i.

seller reserve function, and $o^\star$ is a *suggested buyer offer function*.

Throughout, we use the stylistic convention that offers and reserve values are reported simultaneously. However, this simultaneity is not strictly necessary—in all of our mechanisms, the buyer pays her full offer in the case of sale, hence this offer may be made and revealed to sellers before sellers' reserves are reported.

In the remainder of this section, we outline mechanism properties which may be treated as potential market design goals. First, we review standard financing and incentive properties. Then, we discuss efficiency and individual rationality properties, including our novel bilateral efficiency and approximate individual rationality conditions.

III.B Financing and incentive properties

We say that a mechanism $\mathscr{M}$ is *self-financing* if $T_\beta + \sum_i T_i \leq 0$, and *budget-balanced* if $T_\beta + \sum_i T_i = 0$. In general, self-financing mechanisms are desirable, as mechanisms that are not self-financing may be open to fraudulent exploitation. Budget-balance is also desirable, but is generally difficult to guarantee.

Small corporate investors and land owners in assembly are often inexperienced, under-resourced or otherwise ill-equipped to make complex calculations. It is therefore desirable that optimal strategies be dominant whenever possible. Formally, we say that a mechanism $\mathscr{M}$ is *straightforward for sellers (buyers)* if the suggested seller (buyer) strategy $r^\star$ ($o^\star$) is dominant. A weaker requirement is *(Bayes-Nash) implementability*, i.e. that the suggested strategies form a Bayes-Nash equilibrium of the game defined by the mechanism.

III.C Efficiency

A natural goal is the maximization of allocative *efficiency*, defined as

$$e(\mathscr{M}) \equiv \frac{E\left[(b-V)P\left(o^\star[b], r^\star\left[v\right]\right)\right]}{E\left[(b-V)1_{b>V}\right]},$$

the fraction of total possible gains from trade realized. A mechanism achieves this goal perfectly when it implements trade exactly when trade is beneficial. Formally, a mechanism is *fully efficient* if $e(\mathscr{M}) = 1$, i.e. if for all $(b, v) \in \mathcal{B} \times \mathcal{R}^N$, we have $P\left(o^\star[b], r^\star[v]\right) = 1_{b \geq \sum_i v_i}$.

The results of Myerson and Satterthwaite (1981) imply that even if the community of sellers could act in concert, then they would still face the distortions associated with a bilateral bargain with the buyer, as b and V are private information. A natural goal, embodied in our *bilateral efficiency* condition, is to recover at least as much efficiency as could be obtained in the underlying buyer–community bargain.

Definition 1. *A mechanism $\mathscr{M}$ is* bilaterally efficient *relative to another mechanism $\mathscr{M}'$ with $N = 1$ if $e(\mathscr{M}) \geq e(\mathscr{M}')$ when the (b-conditional) distribution of the single seller valuation under $\mathscr{M}'$ is the same as that of V under $\mathscr{M}$.*

Most mechanisms we discuss have natural analogs for collections of sellers of any size. We can therefore think of these mechanisms as forming a series $\{\mathscr{M}^n\}_{n=1}^{\infty}$ of n-agent mechanisms, and ask that efficiency increase as the number of sellers grows large.

Definition 2. *A series of mechanisms $\{\mathscr{M}^n\}_{n=1}^{\infty}$ is* asymptotically efficient *relative to a series of joint probability density functions if $\lim_{n\to\infty} e\left(\mathscr{M}^n\right) = 1$ under the induced measures.*

Because the seller has no aggregate uncertainty about valuations, bilateral efficiency typically leads to asymptotic efficiency. When seller values are independent and n is large, uncertainty about V vanishes as the number of sellers grows large. In that case, the efficiency (or inefficiency) of trade is known with near-certainty conditional upon b, hence the bilateral trade distortion vanishes (Myerson and Satterthwaite, 1981) and full efficiency obtains.[12]

Lemma 1. *A sequence $\{\mathscr{M}^n\}_{n=1}^{\infty}$ of mechanisms $\mathscr{M}^n$ bilaterally efficient relative to a take-it-or-leave-it offer to a single seller is asymptotically efficient if*

1. *there exists an $M > 0$ such that $ns_i^n < M$ for all i and n,*
2. *$\left\{\frac{v_i^n}{s_i^n}\right\}_{i=1}^{n}$ are distributed, conditional on γ, independently and identically (across n and i) according to some distribution with full support on $[\underline{V}, \infty)$.*

Proof. Given the independence of γ from b and that γ is known to the buyer, we consider the analysis for any given γ, suppressing the dependence thereon. (Given that the result holds for all γ, it must hold on average across γ values.)

We let $\{x_i^n\}$ represent the i.i.d. process in the lemma statement, and write μ and σ^2 for the mean and variance of this process, respectively. As $V^n = \sum_{i=1}^n v_i^n = \sum_{i=1}^n x_i^n s_i^n$ and $\sum_{i=1}^n s_i^n = 1$, we have $E[V^n] = \mu$ and $\text{Var}[V^n] < \frac{M^2\sigma^2}{n}$, by the share bound and i.i.d. hypotheses.

Because the gains from trade are bounded away from zero, it suffices to demonstrate that the total inefficiency of $\mathscr{M}^n$ vanishes as $n \to \infty$.

Now, a buyer's offer choice is equivalent to the selection of a probability of sale. Thus, we may interpret the buyer's maximization problem as the selection of probability of sale q in

$$q(b - S_n(q)), \tag{3}$$

[12] It is clear from this intuition and from the proof of Lemma 1 that full independence of seller values is not needed; weak mixing conditions would suffice. Indeed, the independence assumption is used in the proof only for a variance bound and an application of Chebyshev's inequality.

where $S_n(q)$ denotes inverse supply. We let $\tilde{q}_n(b)$ be the optimal choice of q in (3), for a buyer with value b.

The buyer always offers $o \leq b$, hence inefficient sales will never occur. Thus, by Harberger's inequality, the total inefficiency of $\mathscr{M}^n$ is given by

$$\int_0^\infty \int_{\underline{v}}^\infty (b - V)1_{o_n(b)<V} f_n(V)h(b)\,dV\,db \leq \int_0^\infty b[F_n(b) - \tilde{q}_n(b)]h(b)\,db, \tag{4}$$

where $o_n(b)$ is the offer of a valuation-b buyer facing n sellers, and f_n and h respectively represent the densities of V (in the presence of n sellers) and b.

By the one-sided Chebyshev inequality, we have for any $\alpha > 0$,

$$\text{Prob}[V^n - \mu \geq \alpha] \leq \frac{M^2\sigma^2}{M^2\sigma^2 + n\alpha^2},$$

which vanishes as $n \to \infty$. It follows that $S_n(q) \to \mu$ as $n \to \infty$ (pointwise). Thus, for any fixed $b > \mu$ and (sufficiently small) $\epsilon > 0$, we have

$$(q + \epsilon)\,(b - S_n(q + \epsilon)) > q(b - S_n(q)),$$

for n sufficiently large (as $(S_n(q + \epsilon) - S_n(q)) \to 0$). Thus, $\tilde{q}_n(b) \to 1$; it then follows that the right side of (4) vanishes as $n \to \infty$ for all $b > \mu$, as we always have $F_n(b) \geq \tilde{q}_n(b)$.[13]

Meanwhile, again by the Chebyshev inequality, we have $F_n(b) \to 0$ for any fixed $b < \mu$. It then follows that the right side of (4) vanishes as $n \to \infty$ for all $b < \mu$; combining this with our previous observations (and the fact that $b = \mu$ with probability 0) proves the result.

III.D Fairness properties

A mechanism $\mathscr{M}$ is *individually rational (for sellers)* if, for each $i = 1, \ldots, N$, $(b, v_{-i}) \in \mathcal{B} \times \mathcal{R}^{N-1}$ and $v_i \in [\underline{v}, \infty)$, there exists $r \in \mathcal{R}$ such that

$$T_i\left(o^\star[b], r, r^\star[v_{-i}]\right) \geq v_i P\left(o^\star[b], r, r^\star[v_{-i}]\right);$$

$\mathscr{M}$ is *strictly individually rational* if $r^\star$ has this property.

Individual rationality requires that every seller receive compensation that makes her "whole"; that is, she must have the option to be compensated at a level at least equivalent to her subjective valuation. Requiring individual rationality is essentially equivalent to im-

[13]This follows from an application of the dominated convergence theorem to (4); such an application is valid because $\int_0^\infty bh(b)\,db < \infty$ (in order for the efficiency of $\mathscr{M}^n$ to be well-defined) and $0 \leq F_n(b) - \tilde{q}_n(b) \leq 1$.

posing "perfect preservation of property rights," using the absolutist conception of property established in the Anglo-Saxon legal tradition (Dana and Merrill, 2002).

However, as the results surveyed in Section II illustrate, preserving such extreme individual property rights is inconsistent with ex-post social efficiency. As a result, practical mechanisms for solving the holdout problem must abrogate some property rights. Many would nonetheless argue that current compensation is too meagre to be fair. To limit the violation of individual rationality while allowing efficient assemblies, we propose an *approximate individual rationality* condition. This condition's definition draws upon a continental tradition which emphasizes the importance of fair sharing of the burdens of social projects, and thus argues that compensation should mirror "a community consensus on the severity of harm inflicted" (Ellickson, 1973).

Economists often informally justify the preservation of individual rationality constraints (i.e. property rights) by the necessity of preserving incentives for investments (Williamson, 1979; de Soto, 2003).[14] However, a large literature has found that preservation of individual property rights, in the penumbra of potential assembly, creates systematically *wrong* incentives, as individuals are spurred to over-invest to extract higher payments from purchasers (Blume et al., 1984). Especially given that shares in approximately individually rational compensation schemes may be adjusted to counteract these harmful incentives (Innes, 1997), it seems unlikely that preserving investment incentives provides a case for preferring strict to approximate individual rationality.

Definition 3. *A mechanism $\mathscr{M}$ is* approximately individual rational (for sellers) *if, for all $i = 1, \ldots, N$, and $(b, v) \in \mathcal{B} \times [\underline{v}, \infty)^N$, there exists $r \in \mathcal{R}$ such that*

$$T_i\left(o^\star[b], r, r^\star[v_{-i}]\right) \geq \frac{s_i \sum_{j \neq i} v_j}{1 - s_i} P\left(o^\star[b], r, r^\star[v_{-i}]\right).$$

While approximate individual rationality does not ensure individuals are compensated at their subjective valuation levels, it guarantees each seller an unbiased estimate of her valuation based on others reported valuations. To see this note that regardless of γ, the value $\frac{\sum_{j \neq i} v_j}{1 - s_i}$ is an unbiased estimator of V by our assumption that s is constant in γ:

$$E\left[s_i \frac{\sum_{j \neq i} v_j}{1 - s_i} \middle| \gamma \right] = E\left[s_i \frac{\sum_{j \neq i} s_j V}{1 - s_i} \middle| \gamma \right] = E\left[s_i V \middle| \gamma \right] = E\left[v_i \middle| \gamma \right].$$

Thus approximate individual rationality guarantees that each individual will receive at least a γ-conditional, unbiased estimate of her value. As there is uncertainty about the the

[14]The many philosophical, legal, and economic reasons for protecting property rights are discussed more extensively in our the online appendix.

aggregate shock γ, this estimate is more appealing than one which neglects the buyer's and sellers' information about γ. For any precise set of beliefs, the social planner might be able to obtain a more precise estimate of v_i based on v_{-i} and b. However, the frequentist estimate $\frac{\sum_{j\neq i} v_j}{1-s_i}$ provides a simple and robust guarantee: under approximate individual rationality each individual may choose to receive in compensation at least a reasonable community consensus regarding the harm inflicted on her. In a corporate acquisition, this particular guarantee elegantly corresponds to the requirement that all shares be treated equally.

A third legal standard of property is that of *collective property*, i.e. community ownership. This corresponds to individual rationality at the community level. As all Concordance mechanisms have this property, we introduce it formally.

Definition 4. *A mechanism $\mathscr{M}$ is* collectively rational *if for all $v \in [\underline{v}, \infty)^N$, there exists $r \in \mathcal{R}^N$ such that $\sum_i T_i(o,r) \geq V \cdot P(o,r)$ for each $(b,v) \in \mathcal{B} \times \mathcal{R}^N$. This condition holds* strictly *if $r^\star$ satisfies this property.*

Collective rationality is natural in joint-ownership systems (such as the Mexican *ejido* system) where land is legally owned by a community. Collective rationality also seems like a sensible goal for corporate acquisition settings, in which the protection of collective property rights seems important from an investment perspective.[15]

IV The Concordance Principle

Our basic approach, of which all of the mechanisms we propose can be seen as applications, is inspired by Cournot's solution to the collaboration problem. Cournot argued that the collaborating firms should merge so as to fairly share in—and hence internalize—each others' profits.[16] We see this suggestion, as it applies to holdout, as consisting of two parts:

1. Sellers should divide profits from a sale according to a pre-specified formula, just as a merger divides stock in the conglomerate among the shareholders of the merging firms.

2. Sellers should be incentivized to share information by paying for externalities caused by moving the group decision towards her preference, just as divisions of a firm (Groves and Loeb, 1979) are incentivized to communicate with headquarters.

Our *Concordance mechanisms* implement these ideas, basing the decision on the seller community interest and requiring payments only when sellers influence this decision.

[15]See our online appendix for further discussion.

[16]Lehavi and Licht (2007) suggest the notion of a "merger" as well in abstract terms, but without any explicit decision procedure.

Defining Properties of Concordance Mechanisms

1. Sellers are asked to report their values truthfully and buyers are asked to make the monopsonist-optimal offer to the seller community.
2. The buyer's offer is accepted when it exceeds the total reported reserve.
3. Each seller has the option to *exert no influence*, in which case the (share-scaled) reserve of all other sellers determines whether a sale occurs. If no seller exerts influence, then the sale proceeds.
4. Sellers exerting no influence and isolated sellers receive at least their share of the offer if a sale occurs and never pay anything; sellers potentially causing externalities may be required to pay a Pigouvian tax.

Definition 5. *A mechanism $\mathscr{M}$ is a* Concordance mechanism *if $P(o,r) = 1_{o \geq R}$,*

$$[r_i = s_i o] \implies T_i(o,r) \geq s_i o P(o,r), \quad r_j = s_j o \;\forall\; j \neq i \implies \sum_j T_j(o,r) \geq oP(o,r),$$

$T_\beta(o,r) = -oP(o,r)$, $r^\star(v) \equiv v$ and $o^\star(b) \equiv \operatorname{argmax}_o (b-o)\overline{G}_\gamma(o)$, where $\overline{G}_\gamma$ is the γ-conditional cumulative distribution function of V.

We henceforth denote the set of all Concordance mechanism by $\mathfrak{C}$. Bilateral and asymptotic efficiency, and collective and approximate individual rationality (under truthful play) follow almost immediately from Definition 5, as we now show.

IV.A Efficiency guarantees

The outcome of any Concordance mechanism is identical to that of a bilateral bargain between the buyer and a single "community seller," with the distribution of the community seller's value being that of $V = \sum_i v_i$. From this observation, we see that Concordance mechanisms are bilaterally efficient relative to their own bilateral forms (take-it-or-leave-it offers to single sellers). Combining this observation with Lemma 1 shows that Concordance mechanisms are also asymptotically efficient, under the conditions of Lemma 1.

Theorem 1. *Every mechanism $\mathscr{M} \in \mathfrak{C}$ is bilaterally efficient relative to its own bilateral form (a take-it-or-leave-it offer to a single seller). Furthermore, under the conditions of Lemma 1, a sequence $\{\mathscr{M}^n\}_{n=1}^\infty$ with $\mathscr{M}^n \in \mathfrak{C}$ for all n is asymptotically efficient.*

Theorem 1 shows that Concordance mechanisms alleviate holdout: Even as the number of sellers grows large, the efficiency of Concordance mechanisms does not deteriorate below that of bilateral trade. Moreover, Concordance mechanisms become fully efficient in the limit, provided that seller values are somewhat independent.

IV.B Fairness guarantees

Concordance mechanisms also satisfy our relaxations of individual rationality. Each seller not exerting influence receives her share of the buyer's offer, which is (if accepted) at least

$$s_i \frac{\sum_{j \neq i} r_j}{1 - s_i}.$$

It follows that approximate individual rationality is guaranteed for i when agents $j \neq i$ play the suggested (truthful) strategy. Meanwhile, Concordance mechanisms use the same decision rule for accepting offers as the community would, so they preserve collective rationality: the community can avoid aggregate tax payments if one seller i submits her share $s_i V$ of the community valuation and each other seller chooses not to exert influence.

Theorem 2. *All $\mathscr{M} \in \mathfrak{C}$ are approximately individually rational and collectively rational.*

IV.C Other properties

Additional details distinguishing Concordance mechanisms relate to how they use taxes to encourage truthfulness. The Concordance mechanisms we present in Section V are intuitive Concordance implementations of standard auction-theoretic incentive enforcement methods: VCG, expected externality, all-pay, and first-price.

V Concordance Mechanisms

V.A Straightforward Concordance (SC)

The simplest Concordance implementation uses the mechanism of Vickrey (1961), Clarke (1971) and Groves (1973) (VCG) to enforce truthful revelation of values. This mechanism incentivizes truthful revelation through Pigouvian externality taxes assessed on the base of other sellers' valuation reports.

Straightforward Concordance, outlined informally in the box above, is the Concordance

Straightforward Concordance

1. SC is a Concordance mechanism with taxes on sellers who are pivotal in the sense that R and $R_i \equiv \frac{\sum_{j \neq i} r_j}{1-s_i}$ are on different sides of o.
2. Pivotal sellers i pay a tax equal to the harm caused: $(1-s_i)|o-R_i|$.

mechanism with $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$, $P(o,r) = 1_{o \geq R}$, $T_\beta(o,r) = -oP(o,r)$, and

$$T_i(o,r) = \underbrace{s_i o P(o,r)}_{\text{transfer value}} - \underbrace{1_{(R_i-o)(R-o)<0}(1-s_i)|o-R_i|}_{\text{VCG tax}},$$

where $R_i \equiv \frac{\sum_{j \neq i} r_j}{1-s_i}$. SC is a Concordance mechanism because only sellers who are pivotal pay a tax. The crucial advantage of SC over other implementations is that it is straightforward for sellers (hence the mechanism's name). This implies that stated benefits associated with Concordance apply whenever sellers act rationally in their own interests.

Proposition 1. *SC is self-financing, straightforward for sellers, and implementable.*

It is clear from its construction that SC is a *Groves holdout mechanism*, i.e. a mechanism in which the buyer always pays his offer in case of sale, and which is equivalent to a Groves mechanism among sellers (given the buyer's offer o). Well-known results for Groves mechanisms yield Proposition 1 directly.

Applying the Holmström (1979) extension of the main result of Green and Laffont (1977) shows that only Groves holdout mechanisms are simultaneously straightforward for sellers and bilaterally efficient. This characterization leads to a uniqueness result for SC, as SC is the mechanism which maximizes seller surplus among all self-financing, approximately individually rational Groves holdout mechanisms.

Proposition 2. *Suppose that mechanism $\mathscr{M}$ is self-financing, straightforward for sellers, bilaterally efficient relative to a take-it-or-leave-it offer, and approximately individually rational. Then, the total seller surplus under $\mathscr{M}$ is bounded above by that of SC.*

If $\mathscr{M}$ is an approximately individually rational Groves holdout mechanism, then the transfer function $T_i(o,r)$ of $\mathscr{M}$ can be decomposed in the form

$$T_i(o,r) = s_i o P(o,r) + 1_{(R_i-o)(R-o)<0}(1-s_i)|o-R_i| + \hat{h}_i(o, r_{-i}), \tag{5}$$

with $\hat{h}_i(o, r_{-i}) \geq -s_i(o - R_i)1_{R_i > o}$. Indeed, as $\mathscr{M}$ is approximately individually rational, there must be some $r_i \in \mathcal{R}$ such that

$$T_i\left(o^\star(b), r_i, r^\star(v_{-i})\right) \geq \frac{s_i \sum_{j \neq i} v_j}{1 - s_i} P\left(o^\star(b), r_i, r^\star(v_{-i})\right) = \frac{s_i \sum_{j \neq i} r_j}{1 - s_i} P(o, r_i, r_{-i}) = s_i R_i P(o, r);$$

reorganizing this expression and substituting in (5) gives the bound on $\hat{h}_i(o, r_{-i})$. Lemma 3, Theorem 5, and Proposition 1 of Cavallo (2006) all extend to the case of Groves holdout mechanisms, hence Proposition 2 follows directly from the preceding observations because each seller has *potential for universal relevance nullification* in the sense of Cavallo (2006) (as sellers' valuations may be arbitrarily large). Thus in no self-financing mechanism of this form may $\hat{h}_i(o, r_{-i})$ be strictly positive.

Unfortunately, SC is highly vulnerable to collusion. In one form of collusion effective against SC, coalitions can avoid tax payments by share-weightedly averaging their values and reporting shares of this average. This leads to exactly the same sales rule as when members of the community act non-cooperatively, and additionally, collective rationality is strictly preserved. Since we are primarily concerned with achieving efficiency and protecting individual rationality, rather than raising revenue, collusion in this fashion in our view *improves* outcomes. Of course, collusion among sub-groups of sellers, or imperfect collusion among all sellers, can be harmful to efficiency and individual rationality.[17] Concerns about the possibility of such manipulation and budget balance are the primary motivation behind our other Concordance mechanisms.

V.B Other Concordance mechanisms

We now discuss three alternatives to SC that, by using other standard auction concepts, sacrifice straightforwardness to improve budget balance and, potentially, reduce collusion.

V.B.1 Bayes-Nash Concordance (BNC)

As Arrow (1979) and d'Aspremont and Gérard-Varet (1979) point out, collusion may be deterred by making each seller pay her expected externality. However, implementing an "expected externality mechanism" violates the Wilson (1987) doctrine: it requires knowledge of the distribution of seller valuations and depends heavily on the beliefs of agents. Nonetheless,

[17] This problem is well-known to be particularly severe if, as seems likely in applications like corporate acquisitions, there is a very large number of sellers and sellers can easily "de-merge," splitting one individual into two, each with half the share, who can then express identical, extreme preferences (Ausubel and Milgrom, 2005).

> ### Bayes-Nash Concordance
>
> 1. BNC is the Concordance mechanism with taxes equal to expected externalities, conditional on the reported reserve and the offer.
> 2. Sellers receive refunds equal to her share of others' expected externalities.

the expected externality Concordance mechanism, which we call the *Bayes-Nash Concordance (BNC)* mechanism, is of intellectual interest and helps frame the connection between SC and the mechanisms we describe below.

We assume for the purposes of defining BNC here that the planner can verify γ, and that the valuations v_i are independent of b, and of one another (conditional on γ). However, as Cremer and Riordan (1985) show, in the case where γ is only known to the buyer, the implementation of BNC can be incentive-compatibly delegated to her. Given this assumption, we can calculate for any reported v_i and offer o the expected Pigouvian tax the seller would have to pay under SC. This is just

$$\mathrm{EE}_i(r_i) \equiv (1-s_i)E_{v_{-i}}\left[|V_i - o| 1_{[V_i-o][r_i+(1-s_i)V_i-o]<0} \mid \gamma\right].$$

It is well-known that a mechanism in which sellers pay their expected externalities will be Bayesian incentive compatible.

This intuition leads to the Bayes-Nash Concordance (BNC) mechanism described informally in the box above and defined formally as the mechanism with Concordance suggested strategies, $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$, $P(o,r) = 1_{o \geq R}$ and $T_i(o,r)$ given by

$$\underbrace{s_i o P(o,r)}_{\text{transfer value}} - \underbrace{\mathrm{EE}_i(r_i)}_{\text{Pigouvian tax}} + \underbrace{s_i \sum_{j \neq i} \frac{\mathrm{EE}_j(r_j)}{(1-s_j)}}_{\text{tax refund}}.$$

The fact that BNC is actually a Concordance mechanism is immediate because if $r_i = s_i o$ then $(V_i - o)(r_i + (1-s_i)V_i - o) = (1-s_i)(V_i - o)^2 \geq 0$ and, given that all taxes collected are returned to the community, it is always the case that $\sum_j T_j \geq oP$. While BNC is not straightforward for sellers, it is implementable: so long as sellers $j \neq i$ report truthfully, seller i is incentivized to do so.

Proposition 3. *BNC is budget-balanced, implementable and strictly collectively rational.*

The implementability of BNC is immediate. Budget balance follows because

$$\sum_j \mathrm{EE}_j(r_j) = \sum_j \left(\sum_{i \neq j} \frac{s_i}{1 - s_j} \right) \mathrm{EE}_j(r_j) = \sum_i s_i \sum_{j \neq i} \frac{\mathrm{EE}_j(r_j)}{1 - s_j}.$$

The fact that BNC is strictly collectively rational is immediate because sale occurs under BNC only when $o \geq V$, and all tax revenues collected are shared amongst the sellers.

Relative to SC, BNC trades straightforwardness for budget balance. Like SC, BNC essentially uniquely achieves its package of properties.

Proposition 4. *BNC is the unique mechanism which is budget-balanced, implementable, bilaterally efficient relative to a take-it-or-leave-it offer, approximately individually rational and guarantees that* $\frac{T_i(o,r)}{T_j(o,r)} = \frac{s_i}{s_j}$ *when* $r_i = s_i o$ *and* $r_j = s_j o$.

The main result of Williams (1999) implies that an implementable, Pareto-optimal among sellers, and bilaterally efficient mechanism must be interim-equivalent to a Groves holdout mechanism.[18] This observation narrows the space of mechanisms so that for each i,

$$T_i(o, r) - s_i o P(o, r) = -\mathrm{EE}_i(r_i) + \hat{h}_i(o, r_{-i}),$$

for some $\hat{h}_i(o, r_{-i})$ depending only on o and r_{-i}. Budget-balance then implies that $\sum_i \mathrm{EE}_i(r_i) = \sum_i \hat{h}_i(o, r_{-i})$; hence, the only flexibility in a budget-balanced, implementable, and bilaterally efficient (relative to a take-it-or-leave-it offer) mechanism for the holdout problem is in the form of the tax refunds $\hat{h}_i(o, r_{-i})$. The requirement that $\frac{T_i(o,r)}{T_j(o,r)} = \frac{s_i}{s_j}$ when $r_i = s_i o$ and $r_j = s_j o$ pins down these refunds in a natural way: the tax $\mathrm{EE}_i(r_i)$ of each seller i is divided among the other sellers, in proportion to those sellers' respective shares.

V.B.2 All-Pay Concordance (APC)

BNC is difficult to implement because it requires that the social planner determine the expected externality payments appropriate for each valuation. These payments can be described by a function $f(v_i - s_i o)$ (possibly idiosyncratic across sellers) with $f(0) = 0$, $f'(x)x > 0$ for all x, because sellers with larger announced surplus are pivotal more often (and by larger amounts). The implementability problem of BNC can therefore be seen as arising from the fact that the appropriate functional form of f is unknown. A natural way to address this problem is to assume a simple functional form for f and hope that this roughly approximates the correct form.[19] One natural candidate is $f(x) = |x|$; this choice gives rise

[18] Technically, a minor modification of the Williams (1999) argument is required, in order to account for the presence of the buyer.

[19] In work in progress, Weyl (2011) is exploring the shape of these externalities.

All-Pay Concordance

1. APC is the Concordance mechanism with taxes equal to the surplus the seller would obtain from her desired outcome, given her announced reserve.

2. The sellers' taxes are redistributed, with each seller i receiving fraction $\frac{s_i}{1-s_j}$ of seller j's payments.

to the *All-pay Concordance (APC)* mechanism.

In APC each seller pays her full announced surplus; the outcome which creates greater (announced) net surplus is selected. Therefore, this procedure is equivalent to one in which each seller announces a preferred decision and an amount she is willing to pay to obtain this decision and whichever option has greater monetary support is chosen. Formally, the APC mechanism is given by $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$, $P(o,r) = 1_{o \geq R}$,

$$T_i(o,r) = \underbrace{s_i o P(o,r)}_{\text{transfer value}} - \underbrace{|s_i o - r_i|}_{\text{tax}} + \underbrace{s_i \sum_{j \neq i} \frac{|s_j o - r_j|}{1 - s_j}}_{\text{budget-balancing transfer}},$$

with Concordance suggested strategies. APC is perfectly budget-balanced, as

$$\sum_i s_i \sum_{j \neq i} \frac{|s_j o - r_j|}{1 - s_j} = \sum_j (1 - s_j) \frac{|s_j o - r_j|}{1 - s_j} = \sum_j |s_j o - r_j|.$$

This, in addition to the independence of taxes from other sellers' behavior that APC shares with BNC, seems to indicate that APC may also be more resilient to collusion than SC. Just as in the BNC mechanism, APC is strictly collectively rational. Furthermore, APC is a Concordance mechanism by construction, hence the results of Theorems 1 and 2 apply *if buyers and sellers follow suggested strategies.* However, like core-selecting package auctions (Day and Milgrom, 2008), APC is not implementable: suggested strategies for APC cannot form an equilibrium, as this would give sellers negative surplus with near certainty. Thus the relevance of APS's properties under truthfulness is unclear.

> **First-Price Concordance**
>
> 1. FPC is the Concordance mechanism where all positive surplus gained by sellers relative to their disfavored outcome is taxed away.
> 2. These taxes are then redistributed as in APC.

V.B.3 First-Price Concordance (FPC)

The standard first-price auction has bidders declare a value for the object being sold; winners are forced to pay their stated values. This gives (well-known) incentives for bidders to understate their valuations. The natural Concordance analog of this approach is to have sellers announce a (false) value and then have them pay the associated surplus as a result of obtaining the outcome (sale or no sale) they desire, if this outcome is indeed selected. This mechanism is described in the above box, and given defined formally by taking $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$, $P(o, r) = 1_{o \geq R}$, and $T_i(o, r)$ given by

$$\underbrace{s_i o P(o,r)}_{\text{transfer value}} - \underbrace{\max\left(0, [s_i o - r_i]\, 1_{\text{sale}}, [r_i - s_i o]\, 1_{\text{no sale}}\right)}_{\text{tax on desired outcome}} + \underbrace{s_i \sum_{j \neq i} \frac{\max\left(0, [s_j o - r_j]\, 1_{\text{sale}}, [r_j - s_j o]\, 1_{\text{no sale}}\right)}{1 - s_j}}_{\text{budget-balancing transfer}},$$

and imposing Concordance suggested strategies. Like APC, FPC is not implementable, but is budget-balanced and strictly preserves collective rationality.

V.C X-Plurality Mechanisms

In contrast to Concordance mechanisms is a class of voting-based mechanisms we call the X-*plurality* class. Like Concordance mechanisms, X-plurality mechanisms suggest that sellers report truthfully and divide the buyer's offer according to shares (conditional on sale). However, no taxes are paid. Moreover, rather than accepting the buyer's offer if it exceeds the sum of seller valuations, X-plurality mechanisms accept the offer if at least fraction $X \in [0, 1]$ of shares would "vote for" a sale. That is, sale occurs if there is a collection of sellers which have value-to-share ratios $\frac{v_i}{s_i}$ below the offer o and constitute at least fraction X of shares.

Standard mechanisms for complement aggregation can be interpreted as special cases of the X-plurality rule: As we explain below, typical rules for corporate acquisitions, as well as the related rules used for land assembly, are X-plurality rules with various thresholds X.

X-Plurality

1. The buyer is asked to submit the monopsonist-optimal offer against the distribution of minimum offers needed to persuade X percent of the shares to consent and sellers are asked to truthfully report their values.

2. If $\sum_i s_i 1_{s_i o \geq r_i} \geq X$, where X is a pre-specified value, and $o \geq \underline{V}$ where $\underline{V}$ is the lowest possible total community valuation, then the plot is sold. In this case, the buyer pays o and each seller receives $s_i o$. Otherwise no transaction takes place and no money changes hands.

The general X*-plurality mechanism* is formally implemented as $\mathcal{B} = \mathcal{R} = \mathbb{R}_{++}$,

$$P(o, r) = 1_{X \leq \sum_i s_i 1_{s_i o \geq r_i}},$$

$T_i(o, r) = s_i o P(o, r)$, $r^\star(v) = v$ and $o^\star \equiv \text{argmax}_o (b - o) \widetilde{G}_\gamma^X(o)$. Here $\widetilde{G}_\gamma^X$ is the γ-conditional cumulative distribution function of the $\underline{o}_{N,X} \equiv \text{argmin}_o 1_{X \leq \sum_i s_i 1_{s_i o \geq r_i}}$. In the case of equal shares, $\underline{o}_{N,X}$ is just the $\lceil N(1-X) \rceil$-th order statistic of the distribution of $\frac{v_i}{s_i}$. The case $X = 1$ encompasses the mechanisms of Bagnoli and Lipman (1988), Shavell (2007), and Grossman et al. (2010); the case $X = \frac{1}{2}$ is equivalent to majority share rule; and the case $X = 0$ corresponds to the typical application of eminent domain.[20]

The strength of X-plurality, and the reason it has likely been used so widely, is that it combines the straightforwardness of SC with the budget balance of the other Concordance mechanisms, while being eminently practical and simple. In fact, Bierbrauer and Hellwig (2011) show that in large-market holdout problems, only voting-based mechanisms like X-plurality are straightforward and coalition-proof for sellers.

Proposition 5. *For all X, the X-plurality mechanism is budget-balanced, straightforward for sellers, and implementable.*

Budget balance is immediate under X-plurality, as all revenues generated by the offer are disbursed to the sellers, and no taxes on sellers are assessed. Straightforwardness follows from the fact that the mechanism has sellers voting in favor of the (generically) unique outcome that yields them weakly positive surplus and thus truthfulness can only increase the probability of this outcome realizing. Implementability follows from the fact that the

[20]Here, we have assumed that the minimal value $\underline{V}$ is that which is assessed as compensation in a taking. This appears to be reasonable, as in practice takings are often compensated at or below market value—and therefore at the lower bound of possible subjective property valuations (Radin, 1982).

minimum successful offer that is just sufficient to achieve a sale is that equal to that which is just sufficient to have fraction X of shares consent. Consequently, the seller finds it optimal to make the monopsonist's optimal offer against the distribution of this.

Despite the advantages highlighted in Proposition 5, the X-plurality class of mechanisms suffers from two pervasive deficiencies: its inefficiency, potentially both encouraging inefficient sales and discouraging efficient ones, and its complicated relationship to individual rationality. We discuss these in turn.

The X-plurality mechanism leads to efficient community decision-making given a buyers' offer when the X-th percentile of the share-weighted empirical distribution of value-to-share ratios coincides with the share-weighted average of that distribution.[21] Thus any efficiency guarantee for X-plurality mechanism would rely on (the social planner) having a clear sense of the distribution of valuations. When such information is unavailable, the X-plurality mechanisms can be highly inefficient. If the true, properly-weighted distribution is such that the mean is consistently above (below) the X-th quantile and the buyer's value lies between these, most sales (failures to make a sale) will be inefficient except in small communities of sellers. Thus X-plurality seems likely to be inefficient, especially in large communities.

By construction, X-plurality is individually rational for a set of sellers constituting a fraction X of all shares. It also preserves approximate individual rationality and is collective rationality *to the extent that the X-th empirical quantile is weakly above the mean.*[22] To the extent it is below, collective and approximate individual rationality is violated. In practice this suggests that when X is large, X-plurality mechanisms will tend to preserve individual rationality at least as well as Concordance mechanisms do, if not better, although precise guarantees will depend on assumptions about distributions. However, raising X also inefficiently reduces the number of sales. Thus the class of X-plurality mechanisms, in practice, seems to embody a tradeoff between inefficiency and violation of individual rationality.

V.D Comparison of holdout mechanisms

Table 1 presents a comparison among the holdout mechanisms.

[21] Bergstrom (1979a,b) extensively developed in the context of public goods games (effectively) the theory of efficiency (among sellers) of the response to a buyer's offer in the case of equal (rather than share-based) voting weights and $X = \frac{1}{2}$. In that case in large communities efficiency results if and only if the median of the distribution of value-to-share rations coincides with its mean. Efficiency would require the social planner setting X at the quantile of the distribution corresponding to its mean as suggested by Ledyard and Palfrey (2002). Of course a number of additional complexities arise in our setting: share-weighting in voting, finite populations and optimal thresholds taking into account shading by the buyer. Careful analysis of these issues is beyond the scope of our paper but an interesting direction for future research.

[22] Except that the guarantee is $s_i V$ not $s_i V_i$.

	Finances	Simplicity	Efficiency	Individual Rationality	Share incentive	Collusion	Practical Issues
SC	Self-financing, asymptotically balanced	Straightforward for sellers, implementable	Bilateral, asymptotic	Collective, asymp. strict collective, approx. individual	Yes	Very high	
BNC	Balanced budget	Implementable	Bilateral, asymptotic	Strict collective, approximate individual	Yes	None	Requires detailed knowledge of valuations
APC	Balanced budget	Complex, possibly unimplementable	Bilateral, asymptotic	Same as BNC	Yes	None?	
FPC	Balanced budget	Very complex, likely unimplementable	Bilateral, asymptotic	Same as BNC	Yes	Very low?	
X-plurality (low X)	Budget balanced	Like SC	Too many sales	None	Mostly	None	
X-pluarlity (mid X)	Budget balanced	Like SC	If percentile matches mean	X of shares, approximate individual if efficient	No	None	
X-plurality (high X)	Budget balanced	Like SC	Holdout: no asymp. gains	Near-perfect individual	Mostly	None	

Table 1: Comparison of holdout mechanisms

The table contains one dimension not discussed above: *share incentive compatibility*, the incentive a seller has to disclose information regarding true share values. We omitted this concern above because for almost all mechanisms we have discussed, misreporting shares does not benefit the buyer in any way because the seller decision is independent of shares.[23] However, especially for moderate X-level X-plurality mechanisms, the seller may gain by allocating all shares to buyers with low valuations. Furthermore, although this is not reported in the table, under eminent domain (0-plurality) the buyer may have an incentive to distort down the "minimum seller valuation" so as to pay compensation below market prices. This has been a common concern in the controversy over eminent domain.

Two comparisons are clear. BNC nearly dominates SC, but sadly seems unimplementable. Moderate X-plurality mechanisms dominate those with low X, although what constitutes "low" and "moderate" is ambiguous.

Narrowing our focus to undominated mechanisms, we have a number of interesting but difficult-to-quantify trade-offs. BNC seems the best of the Concordance mechanisms when it is feasible, while the tradeoffs between the straightforwardness of SC and the other benefits APC and FPC are subtle. APC shows more promise than FPC, but without better theoretical knowledge of their equilibria, as well as the ability of communities to reach equilibrium without infeasible training, SC seems the more attractive alternative at present. However, we suspect that in the long term some variation on APC may be superior.

Among values for the X in the X-plurality mechanism above the best guess of that optimal for efficiency, a trade-off between efficiency and individual rationality appears. Opinions about the appropriate stand on this trade-off are likely to differ; however, allowing persistent holdout through a very high choice of X seems unlikely to be a widely accepted.

Comparing X-plurality to the Concordance mechanisms is difficult. An X-plurality mechanism is easy to explain and does not require any tax payments, which might conflict with the sellers' budget constraints. Furthermore, the X-fraction-of-shares individual rationality assurance protection may be attractive to some, though clearly Concordance mechanisms will, under appropriate comparisons of quantiles to means, also satisfy similar properties. However, if one is willing to put faith in complicated institutions, Concordance mechanisms offer a much more attractive set of guarantees about combinations of efficiency, individual rationality and incentives than any practical X-plurality mechanism could.

To summarize, we believe potential implementers' preferences likely fall into three camps:

1. Those strongly opposed to individual rationality violation, who will favor X-plurality

[23] In fact, the buyer may generally have an interest in reporting shares truthfully as this tends to reduce individual rationality violations (for Concordance mechanisms), or bring down minimum sale prices (for high X-level X-plurality mechanisms).

with a high X.

2. Those primarily interested in efficiency and simplicity, likely working in contexts where budgets and stakes are very small, who would tend to favor an X-plurality mechanism with an X chosen to roughly maximize efficiency.

3. Those interested in a mix of efficiency and individual rationality in high-stakes environments where they are willing to expend and require of sellers the resources (material and intellectual) necessary to implement the sophisticated Concordance mechanisms. It is in such contexts that our approach will be most valuable.

VI Conclusion

This paper makes two contributions. First, we bring the holdout problem to the attention of the market design community, emphasizing it as an important open question and synthesizing related literatures. Second, we introduce a potential framework for solving the holdout problem, balancing efficiency and fairness. We expect—and hope—that our work will leave many directions for future research, providing neither the most definitive comparison among the approaches we discuss, nor the final answer to the holdout problem.

Our analysis could naturally be extended: More analytical, computational and experimental work is needed to understand the behavior of the APC and FPC mechanisms, as well as the incentives for and impact of collusion in various mechanisms discussed above. More thought should be given to precisely implementing BNC.[24] It would be useful to understand better the efficiency-optimal choice of X for X-plurality mechanisms, particularly how this varies with community size and distributions. Fully understanding the reasons—philosophical, legal, and economic—why individual rationality protections are desirable properties of a holdout mechanism would help clarify what compromises on these rights are reasonable. Extension of the mechanisms and related individual rationality guarantees to broader public goods games is an interesting theoretical design problem.

Many relatively minor extensions of our mechanisms could expand their ranges of applicability: Concordance mechanisms place full property rights into community hands, but it would be simple, and natural in many eminent domain contexts, to place property rights partially into the buyer's hands; it is known that this helps mitigate the residual Cournot-Myerson-Satterthwaite distortion (Segal and Whinston, 2011). Our Concordance mecha-

[24]Fine-tuning the Concordance mechanisms and their explanation to sellers will require experimental research. A field implementation of the system will be a crucial test of concept. We plan to explore both of these directions in future work.

nisms all require sellers to make tax payments, which may be partially refunded, to enforce true community revelation about the preference for sale. In the real world, sellers often face budget constraints that may make this feature unattractive.[25]

Finally, we note that we restricted our attention to a case of perfect complements, assuming away competition between aggregate land plots. In many practical settings a contiguous block of land must be assembled, but several such blocks may compete to host such a project; some of the competing collections may even overlap. This problem of collaboration nested within competition raises a number of interesting questions: Both in Cournot's collaboration-competition model and in mechanism design, how fast must competition grow relative to collaboration for efficiency to improve (or worsen) with size?[26] What are natural mechanisms for a setting of holdout combined with competitive procurement? How does our approach extend to the case of imperfect complements?

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[25]Designs (similar to those of Pai and Vohra (2009) for auctions) that come close to preserving the properties of Concordance mechanisms while accommodating bidders with privately known budget constraints would be a challenging but practically important extension of our work.

[26]Kominers (2011) is considering this question and related issues in work in progress.

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