

# **Any Non-Individualistic Social Welfare Function Violates the Pareto Principle**

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## *Abstract*

The public at large, many policymakers, and some economists hold views of social welfare that attach some importance to factors other than individuals' utilities. This note shows that any such non-individualistic notion of social welfare conflicts with the Pareto principle.

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## 1. Introduction

Economists usually evaluate policies with reference to individualistic notions of social welfare, under which assessment of policies depends exclusively upon their effects on individuals' utilities. In contrast, ordinary citizens and many government decisionmakers tend to believe that individuals' utilities are not the only relevant consideration in determining which policies society ought to adopt. They often regard other principles as important in their own right. Thus, individuals' rewards should be based on merit or desert, punishment should fit the crime, and so forth.<sup>1</sup> In addition, economists (such as Musgrave and Sen) sometimes advance social welfare functions that depart from purely individualistic ones.<sup>2</sup>

In this note, we show that, for any non-individualistic social welfare function, there always exist circumstances in which the Pareto principle is violated.<sup>3</sup> That is, any conceivable notion of social welfare that does not depend solely on individuals' utilities will sometimes require adoption of a policy that makes every person worse off. Thus, the tension between concern for individuals' well-being and conceptions of social welfare that give weight to factors apart from individuals' utilities is sharper than may have been apparent.

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<sup>1</sup>Lest we be misunderstood, we are stating that such principles are given normative weight independent of any instrumental value they might have (such as the effect of imposing punishment that fits the crime on deterrence). The view that certain principles are valued per se — regardless of their consequences for individuals' well-being — is, of course, that of many moral philosophers (deontologists).

<sup>2</sup>Musgrave (1959, 1990), among others, would take into account “horizontal equity,” and Sen (1985) would measure social welfare with reference to individuals' basic capabilities rather than their utilities.

<sup>3</sup>Sen (1970) showed that a particular non-individualistic social welfare function — namely, one under which the government is not allowed to interfere with a stated domain of individuals' activity, even though such activity may affect others' well-being — can conflict with the Pareto principle.

## 2. Analysis

Let  $x$  denote a complete description of the world. In particular,  $x$  includes a comprehensive account of each of  $n$  individuals' situation and of anything that might be relevant under any method of evaluating the state of the world. Let  $X$  be the set of all such states of the world.

A social welfare function,  $F$ , is a function from the set of states of the world,  $X$ , to the real line,  $R$ .

An individual  $i$ 's utility function,  $U_i$ , is also a function from  $X$  to  $R$ .

An individualistic social welfare function,  $W$ , is a social welfare function of the form  $W(U_1(x), \dots, U_n(x))$ .

*Observation:* A social welfare function  $F$  is not individualistic if and only if there exist  $x, x' \in X$  such that  $U_i(x) = U_i(x')$  for all  $i$  and  $F(x) \neq F(x')$ .

We remark briefly that familiar notions of fairness correspond to social welfare functions that are not individualistic. Consider, for example, the notion that the punishment should fit the crime. If a social evaluator attaches weight to this idea of fairness, he would prefer a state  $x$  in which punishment fits the crime to a state  $x'$  in which punishment does not fit the crime, if other things are equal (and thus if all individuals have the same level of utility in the two states). Because, then,  $F(x) \neq F(x')$  even though  $U_i(x) = U_i(x')$  for all  $i$ , the social welfare function that incorporates the notion of fairness is not individualistic.<sup>4</sup>

The (weak) Pareto principle is that if, for any states  $x, x' \in X$ , we have  $U_i(x) > U_i(x')$  for all  $i$ , then  $F(x) > F(x')$ .

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<sup>4</sup>To be concrete, consider the differentiable social welfare function  $F(U_1(y_1), \dots, U_n(y_n), z)$ , where  $y_i$  is the level of income of individual  $i$ ,  $z$  is a measure of the number of instances in which monetary punishments do not fit the bad act,

Let us make two assumptions.

(A1) There exists a good such that, if each person has  $\delta$  more of it, then each person is better off. Specifically, let  $m_i$  be individual  $i$ 's amount of the good. Then, if two states,  $x$  and  $x'$ , are identical except that  $m_i = m_i' + \delta$  for all  $i$  and  $\delta > 0$ , then  $U_i(x) > U_i(x')$  for all  $i$ .

(A2)  $F$  is continuous in the  $m_i$ .

Observe that A2 does not involve the stronger assumption that  $F$  is continuous in  $x$ , because some non-individualistic social welfare functions that we want to consider involve discontinuities. (For example, a social welfare function embodying the principle that promises should be kept might fall discontinuously if a promise is broken.) All that we require is that there is some good (satisfying A1) in which  $F$  is continuous. (We imagine that the normative appeal of promise-keeping would be unrelated to the social value of at least one ordinary consumption good.)<sup>5</sup>

*Proposition:* If a social welfare function  $F$  satisfies assumptions A1 and A2 and  $F$  is not an individualistic social welfare function, then  $F$  violates the Pareto principle.

*Proof:* Because  $F$  is not an individualistic social welfare function, we know from the observation that there exist  $x, x' \in X$  such that  $U_i(x) = U_i(x')$  for all  $i$  and  $F(x) \neq F(x')$ . Suppose, without loss of generality, that  $F(x) > F(x')$ . Construct  $x''$  from  $x'$  by increasing each  $m_i$  in  $x'$  by a small amount  $\varepsilon$ . By continuity (A2), we know that if  $\varepsilon$  is sufficiently small, then  $F(x) > F(x'')$ . By A1, we have  $U_i(x'') > U_i(x')$  for all  $i$ , and, because  $U_i(x') = U_i(x)$ , we know that  $U_i(x'') > U_i(x)$  for all  $i$ . Hence, if the Pareto principle is satisfied,  $F(x'') > F(x)$ . But  $F(x) > F(x'')$ , so  $F$  violates the Pareto principle.

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and  $\partial F/\partial z < 0$ . To show that this  $F$  is not individualistic, we only need to consider two states,  $x$  and  $x'$ , in which all individuals have the same level of utility (that is, income net of any punishments) but in which  $z$  is different.

The plausibility of the proposition is suggested by reflection on what it means for a social welfare function not to be individualistic. Such a social welfare function in essence gives weight to some factor independently of its effect on individuals' utilities. Therefore, a social state that is desirable with respect to the factor will be deemed superior to another state that is identical except that (1) it is inferior with respect to the (non-utility) factor and (2) all individuals are slightly better off.

### **3. Conclusion**

We have demonstrated that any social welfare function that differs from a purely individualistic social welfare function violates the Pareto principle. That is, policy evaluation that gives any weight to principles or factors independently of their effect on individuals' utility might lead to choices under which everyone is worse off. Therefore, if one adheres to the Pareto principle and believes that normative precepts must be logically consistent, one must reject any non-individualistic social welfare function. It is simply irrelevant to this point that Pareto dominance will be rare among actual policy alternatives. Belief in the Pareto principle not only rules out choice of Pareto-dominated policies; it also renders inadmissible certain criteria for assessing policy.<sup>6</sup>

Having stressed that social welfare functions must be individualistic if one believes in the Pareto principle, we comment on several senses in which certain apparently utility-independent principles, notably, commonly-held notions of fairness, may nevertheless be relevant when assessing policy under an individualistic social welfare function. First, individuals may have a taste for adherence to a principle of fairness, that is, their utilities might be higher if a policy

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<sup>5</sup>We note that assumptions A1 and A2 are stronger than necessary to prove our result.

<sup>6</sup>A similar point, that examining the case of identical individuals can lead one to reject certain normative approaches, is made by Mirrlees (1982).

embodies some notion of fairness (suppose that the punishment fits the crime). In this case, the taste for fairness would be relevant under an individualistic measure of social welfare, just as would any other taste. Second, a notion of fairness might be useful for policymaking if it serves as a good proxy for raising utilities when direct assessment of policies cannot be undertaken. (Punishing in proportion to the seriousness of crime may approximate optimal deterrence.) Third, some notions of fairness (such as rights of individuals against the government) might usefully be embodied in rules in order to constrain the behavior of agents who cannot be trusted to use their discretion to maximize social welfare.<sup>7</sup> Fourth, teaching and promotion of principles of fairness and everyday morality are entirely consistent with maximization of individualistic measures of social welfare, for belief in these principles (such as keeping promises) serves to induce individuals to refrain from behavior (breaking promises) that would harm others.<sup>8</sup>

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<sup>7</sup>The libertarian rights examined in Sen (1970) and the concern for “process” noted by Diamond (1967) seem appealing for this reason. It should be emphasized, however, that endorsing such notions is consistent with a view under which their justification is assessed entirely by reference to an individualistic social welfare function.

<sup>8</sup>In particular, individuals are led to adhere to the principles by a feeling virtue if they do so and by a fear of guilt or of social approval if they do not. Such instrumental views of morality are emphasized by, among others, Mill (1861) and Hare (1981).

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