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DISTRIBUTION OF INCOME

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Abstract

This article addresses conceptual issues concerning the distributive incidence of public goods. Solutions depend on the specific purposes for asking the question of distributive incidence – notably, assessing the extent to which various public goods should be provided, determining how the provision of public goods affects the desirability of income redistribution, and providing a meaningful description of the distribution of well-being. In the course of the analysis, a simple and intuitive version of the benefit principle of taxation (qualitatively different from those commonly advanced in pertinent literatures) is presented, and some of the problems confronting empirical attempts to measure the distributive incidence of public goods are resolved.

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1. Introduction

How does the provision of public goods and services affect the distribution of income?¹ This question is important in light of the significant fraction of GDP accounted for by government provision and, in particular, the large impact such provision may have on the well-being low-income individuals. However, answering this question has proved difficult, both on account of practical challenges in measurement and due to theoretical quandaries, notably involving the benchmark for comparison.² (Is the value of police and national defense measured against a baseline of anarchy? And, whatever the baseline, how are we to determine the distribution in a hypothetical state so far removed from our present situation?) As well, there has been debate about the appropriate formulation of benefit taxation, which proves to raise related questions involving the distributive incidence of public goods.³

This article addresses some conceptual issues concerning the distributive incidence of public goods. The approach adopted here assumes that sensible answers can only be derived if one specifies more precisely the purposes for undertaking the inquiry. This article considers a number of important purposes and in the process seeks to further the understanding of the optimal provision of public goods, the relationship between the distributive incidence of public goods and optimal redistribution policy, and also the more purely descriptive task of identifying the distributive incidence of public goods. Furthermore, this investigation is of relevance to empirical work on the subject, particularly in suggesting that some of the greatest obstacles confronted in the existing literature might be avoidable. Finally, a particular notion of benefit taxation, which differs from those in the literature, is argued to be the most useful construct in the contexts considered herein.

Section 2 considers the question of distributive incidence as it relates to the optimal provision of public goods. The objective is to identify the approach to measurement that is most useful in addressing how the distributive incidence of a public good affects the extent to which the good should be provided. A useful benchmark that has emerged in the pertinent literature (which is surveyed briefly in this section) is that distributive incidence is irrelevant to optimal public good provision, which should reflect efficiency considerations. A central construct in such analysis is a benefit-absorbing tax adjustment, that is, a tax adjustment that, for each level

¹In this article, “public goods” refers to all governmentally provided goods and services, regardless of the extent to which they are public goods in the technical sense. Furthermore, the present analysis is largely applicable to issues involving the distributive incidence of government regulations as well, even though the pertinent literatures do not usually consider the subject. Exceptions include Kaplow (1996) and Page (1983).

²See, for example, Aaron and Maguire (1970), de Wulf (1975), Dodge (1975), Maital (1973), Musgrave, Case, and Leonard (1974), Page (1983), Reynolds and Smolensky (1977), and Ruggles and O’Higgins (1981).

³See, for example, Hines (2000) and Musgrave (1959).

of income, fully absorbs the benefits of the public good, leaving each individual indifferent between not having the good and having the good while being subject to the foregoing tax adjustment (in addition to the preexisting tax system, taken to be a nonlinear income tax). This tax adjustment is a benefit tax of sorts, one having a distributive incidence that mirrors that of the public good, wherein distributive incidence is measured by the impact of the public good on the utility level of individuals. As will be discussed, this formulation of a benefit tax – reflecting a particular notion of the distributive incidence of public goods – is related but not equivalent to the idea of Lindahl pricing, and it differs more substantially from some other formulations of benefit taxation.

Section 3 examines a second purpose, concerning how the distributive incidence of a newly provided public good affects the desirability of income redistribution. That is, if more public goods are provided, does the marginal impact on social welfare of further income redistribution rise or fall, and how does this depend on the distributive incidence of the public goods in question? To illustrate, suppose that there was in place an optimal income tax consisting of a \$5000 per capita grant and a roughly linear income tax. Against this background, the government decides to provide a public good that costs \$1000 per capita and produces benefits slightly exceeding that amount. Is the optimal per capita grant still the same? Somewhat lower because higher marginal tax rates are needed to finance the public good? Lower by approximately \$1000, because \$4000 plus \$1000 for the public good yields the original \$5000? Lower by less than \$1000, because reducing the grant raises individuals' marginal utility of income, especially that of the poor? And, in particular, how does this answer depend on the distributive incidence of the public good and on the tax adjustments made to finance the public good? An alternative and useful way to formulate this problem is to ask, when deciding how to finance the public good, what is the distributive character of the optimal adjustment to the original tax system and how does that adjustment relate to the distributive incidence of the public good.

In general, the answers to these questions will depend on how the public good affects the *level* of utility at different income levels, how it affects the *marginal utility of income* at different income levels, and how it affects *revenues*, which in turn depends on how it affects labor supply. Section 3 begins by considering one of the special cases that has received significant attention in the literature on public goods and distortionary taxation, namely, that in which the public good is a perfect substitute for consumption. In a standard model, it turns out that the optimal means of finance involves the previously described benefit-absorbing tax adjustment. Furthermore, if the public good is thus financed in a manner that is overall distribution neutral – so that the combined effect of the public good and the tax adjustment leaves the distribution of utility unaffected – the gain or loss to social welfare from more or less redistribution will be the same as it was before the public good was provided. In this instance, keeping utility levels constant at each income level also keeps constant everyone's marginal utility of income and, furthermore, results in labor supply remaining the same at every level of income.

Section 3 then considers the same question concerning how the provision of public goods affects the marginal welfare impact of further redistribution in a more general range of settings, including another special case commonly examined in various literatures, in which the public good provides benefits that are additively separable from private consumption. It is natural to

consider again whether a benefit-absorbing tax adjustment is optimal. In general, it is not. For example, in the case in which the public good benefits higher-income individuals to an extent disproportionate to income (that is, individuals with twice the income benefit more than twice as much from the public good), a benefit-absorbing tax adjustment would involve a progressive adjustment to the preexisting tax system. However, when such a tax adjustment is employed, in conjunction with provision of the public good, it can be shown that the ratio of the marginal utility of income of the rich to that of the poor rises, which reduces the marginal social welfare gain from redistribution. Furthermore, because the tax system has higher marginal rates, there is a greater marginal revenue cost from the adverse labor supply effect of redistributive taxation. Finally, the labor supply effect of further redistribution will also differ from what it was before providing the public good and implementing the benefit-absorbing tax adjustment. Although general results cannot be obtained, there is some basis for the conjecture in this case that the optimal tax adjustment would not result in a tax system that is as redistributive as that associated with implementation of the benefit-absorbing tax adjustment (which, as noted, is progressive in this instance).

Whereas section 3 considers how changing the level of public goods affects the desirability of further redistribution of income, section 4 asks how if at all the distributive incidence of the preexisting level of public goods – now taken as given – affects the desirability of income redistribution. For example, if careful analysis reveals that roads are relatively more beneficial to the poor than previously thought (say, by reducing the prices of goods that are disproportionately consumed by the poor), is the optimal degree of redistribution through taxes and transfers less than otherwise? It is clear that the distributive incidence of public goods is in principle relevant to optimal distribution policy because this incidence affects utility levels and the marginal utility of income. The analysis in the preceding section, including the view that benefits should be measured by public goods' effects on individuals' utility, without regard to the costs of provision, proves to be directly applicable. However, it turns out that, for purposes of assessing the desirability of redistribution, it is not necessarily true in theory and one suspects often unwise in practice to measure directly the distributive incidence of many preexisting public goods. Moreover, this recognition dissolves familiar conundrums about the baseline problem (comparison to a state of anarchy?) and renders unimportant some obstacles to the effective measurement of the distributive incidence of certain types of public goods.

Section 5 briefly examines how the distributive incidence of public goods should be assessed for purely descriptive purposes. This inquiry is, on its face, problematic, unless the purposes of the description are set forth. Some descriptive purposes have, in essence, already been considered. For example, if the purpose of describing the distributive incidence of public goods was to help determine how they should be financed, then the approach set forth in section 3 should govern. Additional descriptive purposes can be identified, and these too seem to require an assessment of levels of individuals' well-being. For example, in attempting to identify the median voter and predict his behavior with regard to public expenditure decisions, the concept of a benefit-absorbing tax adjustment – which is defined as the income-equivalent of the effect of a public good on utility, as a function of income – is again a useful concept, certainly including situations in which such tax adjustments are not actually employed (in which case the median voter may well not prefer the efficient outcome).

Before proceeding, it should be noted that, following the convention in much of the conceptual literature on public goods, this article will ignore not only serious practical difficulties in the measurement of benefits (exacerbated by the nonmarket nature of public goods provision and information revelation problems) but also other significant issues involving such matters as the time frame for analysis (capital goods, intergenerational questions), definition of households, and heterogeneity of preferences. It is hoped, however, that some of these subjects might also be illuminated by the approach offered here.

2. Optimal Provision of Public Goods

One longstanding approach to incorporating the distributive incidence of public goods into the analysis of optimal provision thereof is to use distributive weights in cost-benefit analysis, to take into account that a dollar of cost or benefit has a different effect on social welfare depending on who pays or receives it. See, for example, Weisbrod (1968), Drèze and Stern (1987). Another, parallel literature, beginning with Pigou (1947), considers how the distortionary cost of finance affects the optimal provision of public goods. See, for example, Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971), Atkinson and Stern (1974), Fullerton (1991), Ballard and Fullerton (1992), Auerbach and Hines (2002). Although not expressly concerned with the distributive incidence of public goods, this latter literature studies distortionary labor taxation rather than uniform lump-sum taxation, the implicit motivation being that the former is appropriate on distributive grounds.

Subsequently, these literatures have been synthesized. Hylland and Zeckhauser (1979) showed that, if public goods are financed by an appropriate adjustment to the income tax, then in certain special cases no distributive weighting is appropriate. Subsequently, Christiansen (1981) and Boadway and Keen (1993) demonstrated that, when an optimal nonlinear income tax is employed, the Samuelson (1954) rule provides the right test for public goods provision in certain cases (notably, weak separability of leisure in the utility function). Kaplow (1996) extended this result to cases in which the preexisting income tax is not optimal and explicitly considered the relationship between the distributive incidence and labor supply impact of public goods and the incidence and labor supply impact of the tax adjustment used to finance the goods, focusing on a benchmark case in which the Samuelson rule held. Slemrod and Yitzhaki (2001) provide a more general formula for adjusting the Samuelson rule when public goods are financed by a specified (although arbitrary) tax adjustment.⁴

The present discussion follows that in this last set of literature, focusing on the relationship between the distributive incidence of the public good and of the tax adjustment used to finance it. The analysis will also illuminate the concept of benefit taxation.

2.1. *The model*

Utility (U) will be taken to be a function of consumption (c), the level of government expenditure on public goods (g), and labor supply (l). In particular, it is assumed that utility is

⁴See also the extensions and qualifications in Ng (2000) and Slemrod and Yitzhaki (2001).

weakly separable in labor supply, so we can write $U(v(c,g), l)$, where v is a subutility function.⁵ Individuals differ in their ability or wage (w), which has density function $f(w)$.

Individual's consumption is given by

$$(1) c = wl - T(wl, g),$$

where T is a (possibly nonlinear) income tax function; moreover, T is permitted to be a function of g because it will be useful to consider how the tax schedule is adjusted as the level of public goods is changed. Individuals choose their labor supply to maximize their utility.⁶

The social objective is taken to be maximization of a separable, concave social welfare function

$$(2) SW = \int W(U(v(c, g), l))f(w)dw.$$

Government revenue (R) is given by

$$(3) R = \int T(wl(w), g)f(w)dw,$$

where labor supply is written as $l(w)$ to denote that each individual of type (ability or wage) w chooses a level of l that in general depends on his/her w . The government's problem is to choose the tax function T and the level of public goods g to maximize (2) subject to the constraints that individuals choose l to maximize their utility (taking T and g as given) and that the government's budget balances, i.e., $g = R$.

2.2. Benefit-absorbing tax adjustment

Define a benefit-absorbing tax adjustment as a shift in the schedule T as g changes such that $\Delta U / \Delta g = 0$ for all types w and at every level of l that each type might supply. (It will momentarily be established that this is possible given the assumption of weak separability of labor.) That is, holding l constant, we seek to identify the change in T necessary to offset the effect of the change in g on each individual's utility. For the stated utility function, this partial derivative is

$$(4) \frac{\partial U}{\partial g} = \frac{\partial U}{\partial v} (v_c c_g + v_g),$$

⁵As is familiar, when labor supply is not weakly separable, it would be optimal to supply more or less of the public good than otherwise depending on whether the public good is a substitute or complement for leisure.

⁶To keep the exposition clear, a number of technical assumptions and qualifications will be employed without explicit mention. For example, it will be assumed that individuals supply a positive level of labor and that the solution to their first-order condition is unique. To that extent, the exposition should be taken as heuristic.

where v_i denotes the derivative of v with respect to its i th argument and c_g denotes the (here partial) derivative of c with respect to g . From the budget constraint (1),

$$(5) \quad c_g = - \frac{\partial T(wl, g)}{\partial g}.$$

Now, for any level of before-tax income wl , set the tax adjustment to equal the marginal rate of substitution, as follows:

$$(6) \quad \frac{\partial T(wl, g)}{\partial g} = \frac{v_g}{v_c}.$$

Using equations (5) and (6) to substitute into (4), we can see that $dU/dg = 0$ for any given w and l . Thus, in this case it is possible to construct a benefit-absorbing tax adjustment in a straightforward manner.

It is useful to comment briefly on some of the properties of this tax adjustment. First, this tax adjustment, as its name suggests, precisely absorbs the benefits from raising g . More generally, it exactly offsets the utility effect of changing g , allowing as well for cases in which a public good reduces utility at some income levels and for cases in which g is reduced.

Second, the partial derivative in (4), as noted, holds l constant. But would individuals in fact change their labor supply in response to a change in g financed by the specified adjustment to the tax schedule T defined in (6)? On reflection, it should be apparent that, indeed, individuals of all types (w) would not change their labor supply. The reason is that, although (4) holds for a given l , it holds *for all* l . Hence (for each type w), if l^* was superior to all $l \dots l^*$ before g was changed, this will continue to be so afterwards because the utility *at each and every* l is unaltered by the change in g (combined with the accompanying adjustment to T).

This explanation of why labor supply is unaffected can usefully be expressed another way. In essence, for each type of individual w , it is possible to write the subutility v in reduced form as $v(l)$. The preceding analysis shows that this function $v(l)$ is unaffected by the change in g . Hence, if a given l maximizes $v(l)$ initially, that same l will still maximize $v(l)$ after g is changed. Alternatively, and more conventionally, one can derive the first-order condition for l for a given type w and differentiate that first-order condition with respect to g , making the appropriate substitutions using (4) - (6). This derivation (which appears in the appendix) shows that $dl/dg = 0$.

2.3. *Optimal provision and the benefit-absorbing tax adjustment*

Consider when it is optimal to provide more (or less) of a public good when a benefit-absorbing tax adjustment is employed. By construction, each individual's utility is held constant. However, we have yet to determine the effect of changing g , with the corresponding adjustment to T , on revenue (3).

$$(7) \frac{dR}{dg} = \int \frac{dT(wl(w), g)}{dg} f(w) dw = \int \frac{v_g}{v_c} f(w) dw.$$

The right side of (7) is simply the sum (integral) of individuals' marginal rates of substitution.⁷ If this term exceeds one – which is to say, if the Samuelson rule is satisfied – there will be a surplus. Hence, it will be possible to produce a Pareto improvement (such as by rebating the surplus pro rata, raising the level of the rebate until the budget balances). Likewise, if the net benefits are less than one, there will be a deficit. In such cases, decreasing g would yield a surplus, again making possible a Pareto improvement.

The foregoing analysis can, therefore, be summarized by a very simple rule for this case: *When finance is by an offsetting tax adjustment, it is optimal to raise (lower) the level of the public good if and only if the total benefits are greater (less) than the cost of provision.*

Put another way, the analysis demonstrates that there is an interesting relationship between a particular measure of distributive incidence, a particular tax adjustment, and the question of optimal provision. Distributive incidence may be measured by willingness to pay (individuals' marginal rates of substitution), which in general varies with income. One can construct a tax adjustment – which may be understood as a form of benefit taxation – that simply mirrors this distributive incidence. Now, when this benefit tax is in fact employed, there is no net distributive effect of public good provision (the impact of the public good and of the tax are precisely offsetting). Moreover, it turns out that there is also no net effect on labor supply (because any effects of providing the public good are precisely offset by the effects of the tax). Hence, since there is no effect on distribution or on labor supply, the desirability of providing the public good depends only on efficiency considerations, namely the costs and benefits of the public good itself.

2.4. Discussion

The analysis in section 2.3 establishes that a particular manner of finance – corresponding to a particular notion of benefit taxation, reflecting a particular view of distributive incidence – may be employed and that, when it is, the Samuelson rule provides proper guidance. It is useful to consider how this result relates to various literatures on benefit taxation, the distributive incidence of public goods, the optimal provision of public goods, and optimal income taxation.

Benefit taxation. – Should this benefit-absorbing tax adjustment be defined as *the* correct principle of benefit taxation? Hardly. Indeed, the question is not entirely meaningful, for definitions themselves have no necessary implications. What has been established is that this formulation of benefit taxation constitutes *a* useful definition, for the specified purpose. Likewise, it should be apparent that no materially different definition of benefit taxation would

⁷In (7), in the first integrand there appears dT/dg instead of M/M_g because it is the total effect on tax revenue that matters. However, the values of the total and partial derivatives are the same in this instance because, as demonstrated in section 2.2, labor supply (l) is unaffected in this case.

have the stated property. (Sections 3-5 further suggest that the proposed formulation is also the most useful one for the other purposes considered in this article.)

Also note that the benefit-absorbing tax adjustment differs from prior understandings of benefit taxation and of distributive incidence in important ways. Regarding benefit taxation, the posited tax adjustment is equivalent to Lindahl pricing at the margin, but it is not equivalent for a discrete change. That is, if g is increased by a definite amount, the benefit-absorbing tax adjustment would correspond to an area under an implicit demand curve, absorbing the full benefits of provision. Thus, if marginal benefits are declining, the average rate of tax would exceed the marginal rate at the final point of the increase in g . The stated tax adjustment also differs from many notions of benefit taxation because it is based entirely on the benefits of the public good without regard to its cost. (To be sure, the benefit-absorbing tax adjustment must, in general, be further altered to produce budget balance, but how this is done is not part of the underlying concept.) In a sense, it does seem natural for a benefit tax to be defined more directly in terms of benefit rather than cost, although such an approach is not conventional.⁸

Various authors have proposed a number of candidates for benefit taxation, each of which differs from the present formulation in other ways as well. See, for example, Hines's (2000) proposal and his review of Lindahl pricing and related alternatives, such as that advanced in Moulin (1987). Such work usually presents as its objective the derivation of a benefit measure that has certain properties in common with market pricing of private goods or that meets other a priori criteria.⁹ By contrast, the present investigation (in this section and those that follow) focuses instead on what concept best illuminates particular policy problems or most effectively serves descriptive purposes relevant to empirical work that attempts to understand

⁸One rationale for the conventional approach – see, for example, the Aaron-McGuire/Brennan debate, discussed in note 9, and also Maital (1973) – is that a benefit tax or other method of determining distributive incidence is supposed to allocate national income, and thus consumer surplus is irrelevant. Yet no purpose for performing such an accounting allocation is offered. The other most commonly invoked justification – often implicit – is that a motivation for formulating a benefit tax is to offer a manner to finance public goods. Yet when there is also a system of redistributive taxation in place, the purpose of isolating the benefit tax component is not made clear in this literature. Moreover, when a public good does generate a surplus (or deficit), there is no a priori unambiguous way of determining how it should be allocated – which is why the question of how budget balance is to be achieved if a change in public goods is financed as an initial matter by a benefit-absorbing tax adjustment is left as a separate question, one properly resolved using principles of optimal redistributive taxation rather than using some stipulated principle of allocation.

⁹The Lindahl approach to benefit taxation well illustrates the point that the appropriate definition or concept depends on the purpose. Lindahl's solution answers a specific question, namely, what hypothetical pricing scheme for public goods could mimic competitive market prices for private goods to support a decentralized equilibrium in a particular game that corresponds to the operation of a market economy in all goods. See, for example, Lindahl (1919) and Foley (1970). Aaron and McGuire (1970) endorse a benefit measure based on Lindahl pricing because multiplying an individual's marginal rate of substitution by the quantity of the public good is precisely analogous to how the income value of a private good is determined. Brennan (1976a) disagrees with this approach to allocating the benefits of public goods because he favors a different private goods benchmark. (As is clear from Aaron and McGuire's (1976) reply and Brennan's (1976b) rejoinder, the dispute revolves entirely around which private good analogy is most apt in a context in which the goal is to find an appropriate measure of the overall distribution of income, but the purpose for describing the distribution in one or another manner is not specified by either side in the debate.) Hines's (2000) proposed variation, in turn, is motivated by the desire to better capture yet other aspects of private good pricing, such as the feature that every individual faces the same price.

various economic relationships.

Another common feature of many prior analyses of benefit taxation is the interest in whether such taxation, once properly defined, would be progressive, regressive, or proportional. See, for example, Hines (2000) and Snow and Warren (1983). But why this feature should be of interest also is unclear. Moreover, as the preceding discussion suggests, it is misleading to consider the progressivity of a benefit tax in isolation from the distributive incidence of the public goods it finances. Thus, as noted, the incidence of the benefit-absorbing tax adjustment will be progressive (or otherwise) precisely to the extent that the incidence of the public good being financed is progressive (or otherwise), with the net incidence of the tax and the public good, taken together, being distribution neutral in all cases.

Distributive incidence of public goods. – The present approach also differs from that in some of the literature on the distributive incidence of public goods, much of which makes no reference to benefit taxation. Notably, the benefit-absorbing tax adjustment described here is relative to some posited tax schedule, most naturally, the status quo. Hence, in principle it is only necessary to measure the incidence of the *change* in public goods relative to that same reference point. Never is there any need to ask what is the distributive incidence of the entire system of government, compared perhaps to anarchy or some hypothetical state of nature. (This difference in information requirements will also be seen in the sections that follow.)

Optimal provision of public goods and other means of finance. – Initially, it should be observed that the conclusions of the present analysis do not depend at all on whether the initial or final tax system is optimal. Rather, the only assumptions made about the tax system concern how it is to be adjusted when the level of the public good is changed. To be sure, the conclusions hold when the system is optimal. (Furthermore, at the optimum, for small changes it does not matter how the tax system is adjusted to maintain budget balance, as long as the adjustments themselves are small). As will now be explored, however, significant illumination is provided for the more general case in which optimality (of the initial system and of the tax adjustment) cannot be assumed. This can be seen by considering more generally how public goods might be financed.

What level of provision of public goods is optimal when the method of finance does not consist of a benefit-absorbing tax adjustment? This question is addressed most directly by Slemrod and Yitzhaki (2001), who consider the effects of public goods and of changing the tax system on both distribution and the deadweight loss of taxation. However, as suggested in Kaplow (1996), the present framework can also be used to illuminate the more general case because, for any arbitrary method of financing a change in the level of public goods, it is possible (in the present simple setting) to imagine it as a combination of two components: (1) finance by a benefit-absorbing tax adjustment, followed by (2) an additional, purely redistributive tax adjustment. (Step (2) simply consists of the difference between the benefit-absorbing tax adjustment and the actual tax adjustment.) The proper analysis of the first component is that given above. For the second component, we have a purely redistributive tax change, which we can consider in a fairly general manner. Notably, if step (2) increases redistribution, the (un)desirability of this deviation will reflect the extent to which the status quo redistributes too little (much). Furthermore, it is always possible to implement adjustments like

the second component independently of whether a public good is provided. Finally, it is likely to be difficult in the long run to associate a given tax change with a given public good.¹⁰

Other considerations beyond the scope of this article are obviously pertinent in determining the optimal provision of public goods. See, for example, Ng (2000) and Slemrod and Yitzhaki (2001). All that is claimed at present is that a particular form of benefit taxation – the benefit-absorbing tax adjustment – constitutes a useful measure of the distributive incidence of public goods for the purpose of thinking about the question of optimal provision.

Optimal income taxation and revenue requirements. – As already suggested, there is no necessary connection between the present analysis and the results on optimal income taxation because the conclusions obtained here do not depend on the tax system or on the tax adjustment being optimal. There is one respect, however, in which the present analysis illuminates the literature on optimal income taxation, namely, the familiar result that, the larger the revenue requirement, the less redistribution is optimal.¹¹

This result is normally obtained by specifying a given revenue requirement, where the revenue in a sense vanishes from the model, which is to say that it does not finance public goods that enter individuals' utility functions. Now, it is true that if more revenue is indeed required for some exogenous reason (say, due to higher costs of collecting taxes per se) and this revenue does not independently affect utility, a greater revenue need results in higher distortionary costs at the margin and thus less scope for redistribution (when the system is optimized).

Suppose instead that the revenue requirement is used to finance a public good, as in the present model. Now, begin with some initial level of the public good, g^0 , and assume that the nonlinear income tax has been set optimally for a given social welfare function. Next, consider raising the level of the public good to a higher level, g^1 . To focus purely on any effects due to the scope of government, suppose further that the benefits of this increase in the public good precisely equal the costs of the increase. What is the implication for the optimal extent of redistribution? As a first approximation, none. To see this, suppose that the increase from g^0 to g^1 is financed by a benefit-absorbing tax adjustment. When this is done, everyone's utility level remains exactly the same; hence, the distribution of well-being is unaffected. Furthermore, the revenue raised by the benefit-absorbing tax adjustment precisely equals the cost of the increase in the public good in the present case. Hence, the now-higher revenue requirement can be met while redistributing to precisely the same extent as before (taking into account the distributive incidence of the public good as well as the distributive effect of the income tax).

As will be seen in the next section, this is the complete story only in special cases and

¹⁰Even if the two are linked at implementation, the posited source of revenue might otherwise have been used to finance the next public good proposal to come along; also, both the tax system and level of public good are likely to be adjusted in various ways in the future. What is necessary, in principle, is a political economy model that links the average long-run difference in the tax system to a change in a particular type of public good. For example, a very simple model that assumed that the political equilibrium regarding the extent of redistribution was independent of whether it turned out to be efficient to supply one or another public good is one in which the benefit-absorbing tax adjustment would in fact be the incremental source of finance over the long run.

¹¹See, for example, Mirrlees (1971) and Tuomala (1990).

must be supplemented more generally. Nevertheless, it will remain true that there is no simple, general tendency of a higher revenue requirement – reflecting a higher level of provision of public goods – to be associated with the optimality of less overall redistribution.

3. Optimal Redistribution of Income and Changing the Level of Public Goods

3.1. Preliminaries

The question considered here is how providing more of a public good affects the desirability of income redistribution and, in particular, how this effect depends on the distributive incidence of the public good. It is useful to begin by being more precise about what it might mean to say that further income redistribution has become more or less desirable on account of providing more of a public good.

Consider a modification to the tax schedule T that itself is a linear income tax:

$$(8) \tau(wl) = -I + twl,$$

where I is the lump-sum grant and t is the marginal tax rate. The entire tax schedule can now be expressed as $T + eJ$, so that increasing e , beginning at $e = 0$, corresponds to an unambiguous increase in the extent of redistribution (more precisely, it would constitute an unambiguous increase in progressivity, where progressivity is defined in terms of the average tax rate). Note that with this formulation of the entire tax schedule, t can be any positive number. The grant I is set at the level that continues to balance the budget as e is increased slightly from zero, that is, I is chosen such that

$$(9) \frac{d}{de} \left[\int (T(wl, g) + e\tau(wl))f(w)dw \right]_{e=0} = 0.$$

Thus, in raising e slightly from zero, we are considering a redistributive adjustment in the tax system that continues to balance the budget and hence is feasible.¹²

Observe that if the tax schedule T is chosen optimally, then it must also be true that, at $e = 0$, $dSW/de = 0$. In other words, it must be that the disincentive effect of increasing the extent of redistribution is sufficiently great so as to just offset the gain in social welfare from further redistribution.¹³

¹²Also feasible are redistributive adjustments that produce a surplus; thus, I could be lower than the level that satisfies equation (9). But welfare would be increased if the surplus from such an adjustment were rebated (rather than implicitly destroyed).

¹³This condition is familiar from optimal income taxation analysis. The intuition is from the calculus of variations: It must be that no feasible variation from the postulated optimum raises the value of the objective function – or lowers it, for one could always move in the opposite direction (here, choosing an e that was slightly less than zero). For present purposes, it is illuminating to focus on this simple variation because it can be so readily interpreted. If $dSW/de \neq 0$ at $e = 0$, then it is clear that a different level of redistribution is desirable; of course, if it does equal zero, one may not be at an optimum because some other, more subtle variation might raise welfare.

It is now possible to return to our central question, whether changing the level of public goods affects the desirability of redistribution. Using the foregoing apparatus, we can now ask whether, after raising g , financed by some appropriate tax adjustment, it continues to be true that $dSW/de = 0$ (at $e = 0$). If, for example, it were now the case that $dSW/de > 0$, then further redistribution would appear to be desirable. (Note that, if one were not initially at an optimum, one could similarly ask whether dSW/de was higher or lower than before g was adjusted; if it were higher, for example, then the relative desirability of redistribution would have increased.)

To complete the analysis, we must consider whether, at $e = 0$, $dR/de = 0$ (expression 9) continues to hold when g is changed and a corresponding tax adjustment to finance it is made. This condition may no longer be satisfied for two reasons. First, holding the labor supply effect (dl/de , or l_e) constant, a given change in labor supply from raising e may now have a different effect on tax revenue. The reason is that T depends on g ; after whatever tax adjustment is made to finance the change in g , it may be that some individuals' marginal tax rates differ, so the change in labor supply from a tax variation (here, raising e) will in general have a different effect on revenue. Second, the labor supply effect from raising e may differ because, with a different level of the public good, a different level of consumption, and a different tax system in place, a given variation in the tax system need not have the same effect on labor supply as before.

If the revenue effect from the variation becomes positive ($dR/de > 0$), this would involve a benefit relative to before g was increased (the added revenue could be rebated pro rata, for example), whereas if the revenue effect becomes negative, this would be a detriment relative to before g was raised. This effect, combined with the direct effect on social welfare (given by the sign of dSW/de), determines the overall effect of changing g on the relative desirability of income redistribution.

It should be obvious that whether redistribution becomes more or less advantageous, after changing the level of the public good and making a tax adjustment to finance it, will depend on what tax adjustment is made. If the public good were, for example, financed by raising taxes exclusively on the poor (rich), greater (less) redistribution may then become desirable. But this conclusion would be more directly a product of the chosen tax adjustment than of the distributive incidence of the public good.

Accordingly, it is again useful to consider the benefit-absorbing tax adjustment examined in section 2. When that tax adjustment is used, the combined effect of the public good provision and the tax adjustment is distribution neutral. From this benchmark, one might conjecture that the relative desirability of further redistribution would be unaffected by the combined change. After all, the extent of aggregate redistribution is the same as before, and (as explained earlier) the net effects of the public good and the tax adjustment on labor supply are offsetting.

It turns out that this conjecture is not true in general; the actual result depends on many subtleties that vary depending on how the public good enters into individuals' utility functions (even preserving the assumption of weak separability of leisure). The analysis to follow begins with a special case in which the conjecture is true and then continues with a broader exploration. It will be seen that even when the conjecture is false, the benchmark provided by a benefit-absorbing tax adjustment helps us understand the relevant effects. Furthermore, whatever is to

be said about whether further redistribution becomes more or less desirable, any such statement can only be meaningful relative to some baseline, and as explained earlier a baseline of distribution-neutral finance seems helpful in this context.

3.2. *Public goods that are perfect substitutes for private consumption*

A case sometimes considered in the literature on public goods is that in which a public good is a perfect substitute for private consumption.¹⁴ More precisely, suppose that we can write the subutility function v as follows:

$$(10) \quad v(c, g) = c + b(g),$$

where the function b indicates, in a sense, how public goods are converted into dollars.¹⁵

In this special case, it is straightforward to show that changing the level of public goods has no effect on the desirability of further redistribution. Specifically, this is true when the public good g is financed by the benefit-absorbing tax adjustment described previously. That is, if the combined impact of changing g and the adjustment to the tax system have no effect on the distribution of income, then the marginal welfare effect of any redistributive adjustments to the income tax remains the same. A corollary is that if the method of finance is chosen such that more (less) overall redistribution results, then it will be the case that reducing (increasing) redistribution from that now higher (lower) level would be beneficial if the previous extent of redistribution was optimal at the previously given level of public goods provision.

The intuition behind this result can be understood by reflecting on what it means for a public good to be a perfect substitute for private consumption. Giving individuals more of the public good is identical – in terms of effects on individuals' utility – to giving them more cash, in an amount equal in value to the additional public good. The level of the public good can be seen as an in-kind element of the lump-sum component of a tax and transfer scheme.

To state this more formally, one can use the subutility function defined by (10) to determine that the benefit-absorbing tax adjustment (6), at the margin, is

$$(11) \quad \frac{\partial T(wl, g)}{\partial g} = \frac{v_g}{v_c} = \frac{b'}{1} = b'.$$

For a discrete change in g , say from g^0 to g^1 , the benefit-absorbing tax adjustment is simply $b(g^1) - b(g^0)$. Hence, for an individual of type w , for any chosen level of l , consumption in the regime with g^1 will be given by

¹⁴For empirical evidence on the plausibility of this assumption, see Ahmed and Croushore (1996) and Aschauer (1985).

¹⁵It might have seemed more natural to write $v(c, g) = v(c + b(g))$, but expression (10) is without loss of generality because overall utility is expressed as $U(v(c, g), l)$, and any nonlinearity in v may simply be incorporated into U .

$$(12) c = wl - T(wl, g^0) - (b(g^1) - b(g^0)).$$

Combining expressions (10) and (12), we can now write an individual's subutility when public goods are supplied at the level g^1 as

$$(13) v(c, g^1) = [wl - T(wl, g^0) - (b(g^1) - b(g^0))] + b(g^1) \\ = wl - T(wl, g^0) + b(g^0).$$

From expression (13), it is apparent that, for any individual (that is, any type w), any choice of l in the initial regime (g^0) will yield the identical level of subutility (v) and utility (U) as will that same level of l under the new regime (g^1 , financed by the benefit-offsetting tax adjustment). Furthermore, the analysis in section 2 indicates that every individual will choose the same level of l under both regimes and will achieve the same level of utility.

It now should be clear that the desirability of redistribution is unaffected in this case, when the change in the level of the public good is financed by the benefit-absorbing tax adjustment. Derivations appear in the appendix, but the intuition behind them can be stated simply. Initially, $dSW/de = 0$ when evaluated at $e = 0$ (assuming that this held before) because, in essence, nothing has changed – more precisely, the only changes have directly offsetting effects within the subutility function v and no other effects. Furthermore, it is straightforward that $dR/de = 0$, when evaluated at $e = 0$ (assuming that it did before).¹⁶ First, there is no change in anyone's labor supply l or in anything else affecting labor supply, so that, whatever is the effect of changing e on labor supply (l_e), this effect will remain the same. Second, whatever is l_e for any type (w), it will have the same effect on tax revenue as before because no one's marginal tax rate has changed. (Notice that the benefit-absorbing tax adjustment, $b(g^1) - b(g^0)$, is simply a change in the fixed, grant component of the tax schedule; that is, the benefit-absorbing tax adjustment does not depend on one's income level.)

In sum, in the case in which a public good is a perfect substitute for private consumption, changing the level of the public good, when financed in a distribution-neutral manner, does not alter the relative desirability of income redistribution. Whatever distribution was optimal before will continue to be optimal. Likewise, if the level of redistribution was previously insufficient or excessive, then this will continue to be so – and to the same extent – if the public good is financed by the benefit-absorbing tax adjustment.

The only caveat is that, as discussed in section 2, the benefit-absorbing tax adjustment does not, in general, itself result in a balanced budget. If changing the level of the public good passes (fails) the Samuelson test, there will be a surplus (deficit), which would have to be rebated (financed). The effect of having such a surplus (deficit) on the optimal income tax problem is identical to the effect of changing the exogenous revenue requirement in the standard

¹⁶These claims obviously hold for other variations as well.

formulation (in which public goods are not introduced explicitly).¹⁷

3.3. General case

As in section 3.2, the approach will be to suppose that a change in the level of the public good is financed by a benefit-absorbing tax adjustment and then to inquire whether the desirability of further redistribution is affected. (Again, whether a benefit-absorbing tax adjustment produces any surplus or deficit – which depends in this case on whether providing the public good is efficient – will be set aside in most of the discussion.¹⁸) Section 3.1 explains that such an effect may arise through two channels, changes in dSW/de (changes in the direct effect on social welfare of further redistribution) and changes in dR/de (changes in the revenue impact of further redistribution).

Direct effect on social welfare. – The derivative of the social welfare function, SW , in (2) with respect to e is:

$$(14) \frac{dSW}{de} = \int W' \left[U_1 v_c (w l_e (1 - T' - e \tau') - \tau) + U_2 l_e \right] f(w) dw = - \int W' U_1 v_c \tau f(w) dw,$$

where W' is the derivative of the welfare function, U_1 is the derivative of an individual's utility function with respect to its first argument (the subutility function v), v_c (as before) is the partial derivative of the subutility function with respect to consumption, and primes on T and J indicate derivatives with respect to wl (i.e., marginal income tax rates). The final component of the right integrand (and the negative sign) reflect that the derivative of consumption with respect to e , if l is held constant, simply equals $-J$. All the terms in the left integrand involving changes in labor supply, l_e , net to zero on account of the envelope theorem: Any change in l induced by a small change in e do not affect individuals' utility – and thus social welfare – because l is assumed to be at a maximum, from which changes have no first-order effect. (There may be indirect effects of a change in labor supply, notably on revenue, but these will be captured in the subsequent discussion.)

The question, recall, is how (14) changes as g is varied, accompanied by a benefit-

¹⁷To elaborate, one could, say, rebate a surplus pro rata, which is to say, raise everyone's lump-sum grant. (The amount of increase that could be financed will tend to be less than the pro rata surplus because the income effect of the rebate will reduce labor supply, which in turn will absorb some of the surplus revenue.) Relative to this further adjustment, it will tend to be desirable to cut back on the extent of the rebate (that is, the net adjustment will involve rebating less of the surplus than was feasible because, as the surplus is rebated, marginal utilities fall, which tends to make redistribution less desirable than otherwise). In other words, it will tend to be optimal to use some of the surplus to reduce marginal tax rates. Whatever the optimal set of adjustments, there may be no unambiguous way to characterize them as involving more or less redistribution, when combined with the effect of the public good, which itself is like a lump-sum grant in the present case.

¹⁸One can think of the present analysis as directed to the case of a public good that produces benefits (measured in total dollars) just equal to its cost. Hence, we can focus purely on distributive issues, namely, whether provision of such a public good, which is otherwise a neutral matter on efficiency grounds, and which is also neutral as a first approximation on distributive grounds when financed by a benefit-absorbing tax adjustment that by construction is net distribution neutral, makes the case for further redistribution stronger or weaker.

absorbing tax adjustment. Intuitively, it should be apparent that, when a change in the level of the public good is financed by a benefit-absorbing tax adjustment, the only possible effect is through v_c . The reason is that, with such a tax adjustment, each individual's level of subutility v is unaffected; hence, U_1 does not change, and this in turn implies that W does not change. And J does not change. Furthermore, these results are true independently of how public goods enter individuals' (sub)utility functions. To show this, we can evaluate the derivative:

$$(15) \frac{d^2 SW}{dedg} = - \int \left[W' U_1 v_c w \tau' l_g + W' U_1 \tau (v_{cc} c_g + v_{cg}) + W' v_c \tau (U_{11} (v_c c_g + v_g) + U_{12} l_g) \right. \\ \left. + U_1 v_c \tau (W'' (U_1 (v_c c_g + v_g) + U_2 l_g)) \right] f(w) dw.$$

Because there is a benefit-absorbing tax adjustment, $l_g = 0$, and, using (5) and (6), $v_c c_g + v_g = 0$. Thus, each term except the second equals zero. Furthermore, using (5) and (6) to substitute $-v_g/v_c$ for c_g , we have:

$$(16) \frac{d^2 SW}{dedg} = - \int W' U_1 \tau \left(- \frac{v_g v_{cc}}{v_c} + v_{cg} \right) f(w) dw.$$

Focus, then, on the term in parentheses, which is dv_c/dg , indicating how individuals' marginal (sub)utility of consumption will change. In standard cases, this will rise. Specifically, in the case of a public good that is valuable at all income levels, everyone's benefit-absorbing tax adjustment will be positive: They will all pay some additional tax to finance the increase in the level of the public good. As a consequence, their disposable income will fall. For typical formulations of utility functions with public goods, this implies that individuals' marginal (sub)utility of consumption will increase.

How does this affect the value of expression (16)? Intuitively, one can see that this will depend on whether the increase in individuals' marginal utility of consumption is itself proportional to consumption or, if not, whether the relative increases are greatest for those with high or low levels of consumption. To understand why this is so, note that J is negative for low-income individuals (those for whom the lump-sum grant exceeds the additional tax) and positive for high-income individuals. (Specifically, this term is increasing monotonically, indeed linearly, with income.) Hence, if the value of (14) equals zero initially, it must be that if the only change caused by raising g (and financing this with a benefit-absorbing tax adjustment) is that v_c rises for all individuals by the same proportion, the value of (14) will continue to equal zero.¹⁹

However, if v_c were to rise relatively more for the poor than for the rich, there would be more weight at the low end of the income distribution, where the value of J is negative (and is greatest in magnitude for those with the lowest income), and less weight at the high end, where

¹⁹Similar logic is applicable if it was not initially the case that $dSW/de = 0$. The sign will be preserved and the magnitude will be increased when everyone's marginal utility increases in the same proportion.

the value of J is positive (and largest for those with the highest income). Thus, further redistribution would, on this account, be more valuable than before (recalling the negative sign preceding the integral). Contrariwise, if v_c were to rise relatively more for the rich, further redistribution would be less valuable. These results follow because the social value of further redistribution depends on the *relative* values of individuals' marginal (sub)utilities of consumption (among other factors, but they are being held constant here).²⁰

To make the foregoing analysis more concrete and precise, consider the special case (often examined in the literature) in which public goods enter utility in a manner that is additively separable from private consumption. Specifically, write the subutility function v as follows:

$$(17) v(c, g) = z(c) + b(g),$$

where z is the (sub)utility of consumption. A direct consequence of additive separability is that a given level (or change in the level) of the public good is equally valuable to all individuals, *where this value is measured in (sub)utility rather than in dollars*. (In the special case examined in section 3.2, in which $z(c) = c$, the public good had equal value measured in dollars as well.) To know how changing the level of the public good affects the marginal (sub)utility of consumption – when the change is financed by a benefit-absorbing tax adjustment – it is necessary to determine the consumption-equivalent of this common (sub)utility value of the public good for individuals with different levels of consumption.

To further explore the issue, suppose that z is a constant-relative-risk-aversion utility function of the form

$$(18) z(c) = \frac{c^{1-\alpha} - 1}{1-\alpha}, \text{ for } \alpha \geq 0, \alpha \neq 1, \text{ and}$$

$$= \ln c, \text{ for } \alpha = 1,$$

where α is the coefficient of relative risk aversion ($\alpha = -cz''/z'$).

Now we can return and evaluate the change in the direct welfare effect of redistribution given by (16). For the additive separability case, expression (17) implies that $v_c = z'$, $v_g = b'$, $v_{cc} = z''$, and $v_{cg} = 0$. Thus, the terms in parentheses in (16) together equal $-b'z''/z'$. To evaluate this further, we will consider different possible values of α , the coefficient of relative risk aversion.

First, consider the case in which $\alpha = 1$, so that marginal (sub)utility is inversely

²⁰Specifically, if W itself is concave – so that the welfare function is more redistributive than would be a utilitarian one – relative utility *levels* will matter in addition to the marginal effects examined in the text. But, as noted, a benefit-absorbing tax adjustment keeps utility levels constant for all individuals, so relative utility levels are unaffected.

proportional to consumption. Then, at the margin, when more of the public good is provided, the benefit-absorbing tax adjustment will also be proportional to consumption: From (6), $\frac{\partial v_g/\partial g}{\partial v_c/\partial c} = \frac{v_g/v_c}{b} = \frac{1/c}{b}$. Accordingly, everyone's consumption will be reduced by the same proportion. Notice that, before the change, the ratio of any two individuals' marginal (sub)utility, say, that of a rich and a poor person, was simply given by the inverse of the ratio of their consumption because each person's marginal (sub)utility is simply $1/c$. (A person with twice the consumption level has half the marginal (sub)utility of consumption.) After the change, everyone's consumption is reduced by the same factor, say, θ , so each person's remaining consumption is $(1-\theta)c$. As a result, the ratio of any two individuals' marginal (sub)utilities is precisely the same as before.

To see this more formally, for the present case we have $v_c = 1/c$ and $v_{cc} = -1/c^2$. Thus, the terms in parentheses in (16), when combined, simply equal b/c . Now, comparing (14) and (16), we can for this case write:

$$(19) \quad \frac{d^2 SW}{dg dg} = b' \frac{dSW}{de}.$$

Thus, with additively separable consumption and a constant-relative-risk-aversion (sub)utility function, when the coefficient of relative risk aversion is 1 (the logarithmic case) we indeed have an instance in which the direct effect on social welfare of further redistribution is unchanged if, initially, the extent of redistribution was optimal ($dSW/de = 0$).²¹

Second, consider the more general case in which $\alpha \neq 1$. This case can be analyzed formally as before. Now, we have $v_c = c^{-\alpha}$ and $v_{cc} = -\alpha c^{-\alpha-1}$. Thus, the terms in parentheses in (16), when combined, equal $\alpha b/c$. Letting $\int_0^1 f(w) dw$ equal the integrand in (14), we therefore can write

$$(20) \quad \frac{d^2 SW}{dg dg} = -\alpha b' \int_0^1 c^{\alpha-1} f(w) dw.$$

(The explanation for $c^{\alpha-1}$ being part of the integrand in (20) is that, in (14), $v_c = c^{-\alpha}$, whereas, as just noted, for the corresponding component in (16), we have $1/c$ (in addition to $\alpha b/c$ which appears before the integral sign.) Focusing on the case in which $dSW/de = 0$ when evaluated at $e = 0$, we can show that the sign of (20) will be determined by whether $\alpha > 1$ or $\alpha < 1$. This is because $d(c^{\alpha-1})/dc = (\alpha - 1)c^{\alpha-2}$, which is positive when $\alpha > 1$ and negative when $\alpha < 1$. In the former case, the weight is rising with c (and, it will be assumed, as is standard in optimal income tax analysis, by type w). As a consequence, there is relatively more (less) weight on high (low) types than there was before, making further redistribution less attractive. Conversely, when

²¹If instead the extent of redistribution was too small (great), expression (19) indicates that the social welfare gain (loss) from further redistribution will be proportional to what it was initially (and, in the case where additional public good supply is neutral on efficiency grounds, so that $b = 1$, the magnitude of the social welfare effect will be the same).

" < 1, redistribution becomes relatively more attractive.²²

To explore the intuition behind these results, begin with the (probably more realistic) case in which the coefficient of relative risk aversion exceeds 1.²³ In this case, the benefit-absorbing tax adjustment is not proportional but progressive. (If, for example, " = 2, then an individual with twice the consumption level will have one fourth the marginal (sub)utility of consumption and thus will be subject to four times as high a tax adjustment as will an individual with pre-reform consumption half as great.) The preceding formal demonstration in essence shows that, as a consequence of this differential, the ratio of the marginal (sub)utility of consumption of higher-income individuals to that of lower-income individuals rises as the level of the public good (financed by this benefit-absorbing tax adjustment) is increased. This is what makes further redistribution less attractive than before. Note, however, that redistribution has become relatively less desirable *from a benchmark in which the public good is financed with a benefit-absorbing tax adjustment, which itself is progressive*. Indeed, the identified effect on relative marginal (sub)utilities was due to this progressivity.

To complete the discussion, suppose that the coefficient of relative risk aversion is less than 1. In this case, the benefit-absorbing tax adjustment is regressive. (Note, however, that it is only regressive in a vacuum, for combined with the change in the level of the public good, the net effect is, by construction, distribution neutral.) Paralleling the foregoing analysis, raising the level of the public good reduces the ratio of the marginal (sub)utility of consumption of the rich relative to that of the poor as the level of the public good is increased, which makes further redistribution relatively more desirable than it was beforehand – again, more desirable relative to a benchmark in which the public good is financed with a benefit-absorbing tax adjustment that, although rising with income, is regressive.

In summary, the optimal overall tax adjustment – the benefit-absorbing tax adjustment plus any further adjustment to the extent of redistribution that becomes optimal – is (in the special case under consideration) more moderate, in a sense, than the benefit-absorbing tax adjustment considered in isolation (that is, in isolation from the distributive incidence of the public good itself). When the benefit-absorbing tax adjustment is progressive or regressive, the subsequent modification is in the direction of proportionality.²⁴

Effect on revenue. – To determine the revenue effect of the perturbation e , we need to examine the derivative in (9) to see whether, assuming that it equals zero (as stated) before changing g , it still equals zero afterwards.

²²If one does not assume that initially the extent of redistribution was optimal, analogous statements can be made.

²³See, for example, Barsky et al. (1997), Campbell (2003), Kocherlakota (1996), and Maitel (1973).

²⁴It should be clear that one would not on this account want to overshoot the point of proportionality in either case, for it is precisely the progressive or regressive nature of the benefit-absorbing tax adjustment that produces the effect on relative marginal (sub)utilities that makes some contrary modification optimal. Also note that the “overall” adjustment described in the text still leaves out the modification necessary to balance the budget in light of any surplus or deficit arising from use of the benefit-absorbing tax adjustment to fund the change in the level of the public good.

$$(21) \frac{dR}{de} = \int (wT'l_e + we\tau'l_e + \tau)f(w)dw.$$

Since we are evaluating this derivative at $e = 0$, we can ignore the second term in the integrand. To determine how (21) changes with g , when the change is financed by a benefit-absorbing tax adjustment, we can evaluate:

$$(22) \frac{d^2R}{dedg} = \int (wT'l_{eg} + wl_e(wT''l_g + T'_g) + w\tau'l_g)f(w)dw$$

$$= \int (wT'l_{eg} + wT'_g l_e)f(w)dw,$$

where the second equality follows from the fact that $l_g = 0$ (because of the use of a benefit-absorbing tax adjustment).

The discussion of (22) that follows is heuristic and incomplete because no precise characterization is possible without further specification of functional forms. Begin with the second term in parentheses on the right side of (22) (in the second line, that is). The effect of changing e on labor supply, l_e , is plausibly negative on average.²⁵ To see this, recall from (8) that J is a redistributive linear income tax, with the grant I set to break even. As e is increased from zero, individuals become subject to this tax. The substitution effect in all standard cases will be negative. The income effect will be mixed: Raising e increases the disposable income of the lower part of the income distribution, which tends to further reduce their labor supply, but it reduces the disposable income of the upper part of the income distribution, which tends to mitigate their reduction in labor supply due to the substitution effect (and, if strong enough, could cause them to work more). Overall, the average income effect is, by construction, zero (or, as we change g , nearly so); hence, it seems most plausible that, overall, labor supply will tend to fall.

This effect is multiplied by wT'_g the change in the marginal tax rate as a consequence of the benefit-offsetting tax adjustment, weighted by the wage rate. For public goods that produce benefits that rise as a function of consumption (measured now in *dollars*, not (sub)utility), raising g will produce a tax schedule with a greater marginal tax rate T'_g . As a result, if the overall (average, weighted) effect of raising e on labor supply is as conjectured, the effect on revenue will be more negative – that is, relative to what the effect was before g was raised. In this case, therefore, there is a revenue effect that implies that the optimal extent of redistribution falls as a consequence of raising the level of the public good (under the assumption that the increase is financed with a benefit-absorbing tax adjustment, which in the case under

²⁵The first (upper) right side of expression (A10) in the appendix is the expression for l_e in the present case. The substitution effect referred to in the text that follows is the first term in the numerator (with JN which just equals the marginal tax rate t of the perturbation), and the income effect is the second term in the numerator (the sequence of sub-terms that are all multiplied by J , which, as the text explains, is positive for high-income individuals and negative for low-income individuals).

consideration involves higher marginal tax rates). Similarly, if one were to consider a public good whose benefits (measured in dollars) were falling with the level of consumption, there would be a contrary effect. Notice that both of these effects are relative to the use of a benefit-absorbing tax adjustment that itself is a function of income. Again, we have a result in the direction of moderation, in this instance favoring tax adjustments closer to lump-sum adjustments than is a pure benefit-absorbing tax adjustment.²⁶

There is another revenue effect in (17) to consider, corresponding to the first term, namely, the possibility that the effect of the perturbation on labor supply, l_e , may differ when more of the public good is provided; that is, it may be that $l_{eg} \dots 0$. Having undertaken some exploration of both the general case and some special cases (notably, additive separability of consumption, public goods, and labor supply), I have been unable to sign this effect. (In particular, there are multiple nonzero terms; some cannot readily be signed and, of those that can, there are opposing effects.) Hence, further illumination of this revenue effect is not possible short of significant further assumptions (that may well involve making stipulations about particular parameters as well as about functional form). Furthermore, the sign and magnitude of l_{eg} need not be the same at different levels of income (more precisely, for different types w), further complicating the matter of determining the net effect on the value of (22).

Summary. – From the foregoing, it is apparent that no simple statement may be made about the effect of changing the level of public goods supply on the optimality of further redistribution. As emphasized previously, any such statement can only be meaningful relative to some benchmark. That used here is finance provided by a benefit-absorbing tax adjustment (and setting aside any surplus or deficit that results).

Regarding direct effects on social welfare – arising from differences in individuals’ marginal (sub)utilities at different consumption levels – raising g is neutral when the utility of consumption and public goods is additively separable and (sub)utility is logarithmic in consumption, so that when a benefit-absorbing tax adjustment (itself proportional to income in this case) is employed, the case for further income redistribution is unaffected on this account. But when (sub)utility exhibits constant relative risk aversion greater (less) than one, the benefit-absorbing tax adjustment – which then is progressive (regressive) – should be moderated in the direction of proportionality.

Revenue effects were more difficult to pin down. To the extent that a public good produces benefits that, measured in dollars, rise (fall) with income, it is optimal to reduce (increase) redistribution relative to a benefit-absorbing tax adjustment, which involves higher (lower) marginal tax rates in this case. An additional revenue effect due to changes in how further redistribution influences labor supply was indeterminate in the general case and could not be signed without substantial further specification.

²⁶As the explanation makes clear, this factor would not lead one to overshoot because the moderating factor is itself a product of the distributive effect of the benefit-absorbing tax adjustment differing from that of a lump-sum adjustment.

4. Optimal Redistribution of Income and the Preexisting Level of Public Goods

The question considered here is how the distributive incidence of the preexisting level of public goods – now taken as given, in contrast to section 3 – affects the desirability of income redistribution. The social problem in this case is to choose the tax/transfer schedule to maximize social welfare (2) subject to the constraint that revenue (3) is sufficient to fund extant public goods. Because social welfare is taken to be a function of each individual's utility U , which in turn depends on public goods g , it seems clear that in principle one needs to know how public goods affect utility. Furthermore, because one needs to know the utility of each type (w) and thus at different levels of disposable income (consumption), the distributive incidence of public goods should be of particular interest. Note that the present discussion of distributive incidence refers to the direct effects on utility U of public goods g . The cost of provision will also be relevant indirectly because the disposable income of each type of individual will depend in part on the tax schedule, and as noted taxes must be set to raise revenue to finance the public goods. But, regarding the public goods themselves, it is individuals' benefits, not some theoretical allocation of their costs (which may or may not be reflected in the actual tax schedule), that matters.

As suggested by much of the analysis in section 3, of particular relevance will be individuals' *marginal* utilities of consumption; indeed, if the social welfare function were utilitarian, this information would be sufficient to conduct the optimization. More generally, utility *levels* will be relevant as well. For example, if the social welfare function weights lower levels of well-being more heavily, one would need to know just how low each individual's utility is. Thus, it is necessary to consider how individuals' marginal utilities and utility levels might be ascertained.

Observe that full knowledge of the distributive incidence of public goods is equivalent to knowledge of the functional form (and parameters) of individuals' utility functions: If one knows individuals' utility functions – and, what is taken for granted here, the levels of each type of public good (note that g can readily be interpreted as a vector for present purposes) – it is obvious that one can determine both individuals' utility levels and their marginal utilities of consumption. Contrariwise, if one knows individuals' utility levels for all possible combinations of levels of public goods and consumption, one will by definition know their utility functions.²⁷ This latter requirement, however, is quite demanding because it supposes not only substantial knowledge of what exists but also of scenarios that may differ greatly from what may plausibly be observed.

Fortunately, however, one does not need full knowledge of distributive incidence – and, in particular, of what individuals' utility levels would hypothetically be in other states of the world – in order to determine individuals' utility levels and marginal utilities in a given, existing circumstance. It is sufficient (although hardly easy in practice) to understand individuals' present situation. Accordingly, many of the conceptual and some of the practical difficulties faced in the literature that attempts to measure the distributive incidence of public goods need

²⁷Knowing utility levels at all possible levels of consumption implies, of course, that one knows individuals' marginal utilities as well.

not be confronted for present purposes.

First, consider the familiar baseline problem: In inquiring into the distributive incidence of public goods, are we asking how much individuals are better off compared to a state of anarchy? What would that really be like? And if we are not comparing the status quo to anarchy, what hypothetical regime are we implicitly using as our benchmark? Furthermore, how do we deal with the fact that the value of some public goods (say, parks) depends greatly on the level of others (say, police protection)? It would seem that, in order to answer these questions, one would first need to choose a baseline, next determine utility levels for all individuals in that baseline state, and then determine how each person's utility changes as each public good is raised from its baseline level (zero?) to the present level. Alternatively, one might be able to measure everyone's utility in the status quo and in the baseline state; then simple subtraction would reveal the distributive incidence of public goods. But, as just explained, for present purposes all that we really need to know is the utility level in the existing state – and how this changes with changes in consumption – which seems to be a far simpler (if nevertheless daunting) task. Thus, at least for the presently stated purpose, the baseline problem need not be addressed.²⁸

Second, there is a potential double-counting problem that arises when multiple inputs, at least one of which is a public good, together contribute to utility – notably, when public goods are intermediate goods in production. When we measure the utility due to police protection, are we not implicitly including at least some of that contribution when we determine how much an individual's expenditures on housing enhance his utility? Or when we determine that existing inter-city highways lower freight costs and thus the price of various consumer goods, have we not already included this benefit in assessing the marginal utility of consumer expenditures, which must (at least implicitly) make some reference to a price vector that itself will reflect the transportation costs of consumer goods? And how should these effects of public goods be allocated between public expenditures on their provision and the private consumption that occurs with the public goods in the background? It would seem that the foregoing discussion suggests that, with regard to the issue under consideration – determining how redistributive the tax/transfer system should be – these questions too need not be answered. We do need to have an estimate of the utility achievable at different levels of consumption, but we do not need to decompose that utility by source.

Indeed, it is not even obvious a priori that one needs to know *anything* about the distributive incidence of public goods in order to determine optimal tax/transfer policy. The problem is purely a pragmatic one concerning the relative cost and reliability of various forms of empirical evidence on individuals' utility levels and marginal utilities. For example, subjective evidence (such as happiness surveys), evidence of components of well-being (such as data on

²⁸Other reasons have also been offered for ignoring the baseline problem. Bird (1980) suggests that the effect of a reform on the observed distribution may be policy-relevant, but that this does not require knowing what the relationship between the existing distribution and a hypothetical no-government baseline. Piggott and Whalley (1987) show that aggregate fiscal incidence can be a misleading guide to policy because marginal incidence – the effect of most contemplated reforms – can be qualitatively different from the average incidence of the existing regime.

life expectancy), and evidence drawn from much marketplace behavior (such as locational and occupational choices, purchases of insurance, and portfolio allocations) may illuminate these matters without ever having to make direct reference to the effects of public goods. Furthermore, even when one wishes to adjust for differences in levels of public goods – say, between localities – direct measurement of the distributive incidence of public goods may not be the best approach. For example, examination of land values and mobility is more likely to indicate net differences in utility (attributable to differences in local public goods as well as to differences in natural amenities and other factors); further disentanglement may be quite difficult but, more to the point, unnecessary for present purposes. (Of course, it will be necessary to estimate the marginal value of particular public goods in order to address the different question of determining the optimal level of provision. For example, there may be little value in knowing how much the prices of various goods would be higher if there were no highways, but decisions regarding highway improvement do depend on how various changes would affect the production costs of consumer goods.²⁹)

To be sure, there may be instances in which measuring the benefits of public goods for individuals with different levels of income will be helpful in determining individuals' utility levels and marginal utilities. But in guiding empirical research, it will be helpful to keep in mind the purpose of the inquiry and whether attempts to measure the distributive incidence of public goods is likely to be one of the more useful techniques for assessing different individuals' well-being in a particular context.

5. Descriptive Uses

Much of the literature that analyzes or measures the distributive incidence of public goods is on its face purely descriptive. It seems to be a natural and important adjunct to work measuring the distribution of income, much of which focuses on income or disposable income, ignoring the significant portion of GDP expended on public goods and services.

Nevertheless, it appears that many uses of such descriptive measures are for the normative purposes identified in the previous sections: Distributive incidence might be thought relevant to whether more of a public good should be provided, to how it should be financed, to how its provision affects the desirability of further redistribution, and to assessing the existing extent of overall inequality which in turn is relevant to the strength of the case for redistribution, taking as given the current level of public goods. For such purposes, the foregoing remarks

²⁹Furthermore, the distributive incidence of changes in the level of public goods will be indirectly relevant to policy analysis in two respects. First, to know the aggregate benefit of changing the level of public goods, one needs to take account of the possibility (often realized) that the dollar benefit of public goods is not identical for individuals at all income levels. Indeed, if one is not attempting to allocate the costs of public goods – the focus of much distributive analysis as well as many formulations of benefit taxation – but to determine their benefits, one must measure them, period. And if, upon more careful analysis, it turns out that some income groups benefit significantly more or less than previously appreciated, such findings could have an important influence in determining how much of the public goods should be provided. Second, as sections 2 and 3 indicate, the question of how best to finance changes in the level of public goods and how one might wish further to adjust redistribution policy when changing the level of public goods does depend on knowing how public goods enter individuals' utility functions.

concerning the need for measurement (or lack thereof) and the appropriate approach to measurement (generally, measuring individuals' benefits rather than allocating costs) are pertinent.

For example, suppose that one wished to compare the extent of absolute poverty across countries in order to set aid priorities. Perhaps in country *X* a greater portion of the population lives on less than \$1 per day than in country *Y*, but this measure may be misleading if publically provided goods and services – the value of which may be omitted from such measures – are relatively greater in country *X*. In this case, what is relevant is the contribution of the public goods to individuals' utility, not their cost or how that cost might be allocated according to some benefit principle of taxation. Moreover, a better estimate of relative need might be obtained from more direct attempts to assess poor individuals' overall well-being in each country (perhaps using measures of infant mortality, longevity, and literacy) rather than attempting to determine income-equivalent valuations of public goods. This suggestion is reinforced by the existence of other welfare-relevant variations between countries, such as in climate; indeed, some public provision (greater health expenditures directed at communicable diseases) might exist precisely to counteract such differences, so valuing the public goods while omitting the welfare-reducing conditions to which they respond would produce misleading advice in the determination of relative need.

One can also consider a different type of descriptive purpose in measuring the distributive incidence of public goods, notably, in attempting to understand the causes and consequences of inequality. Thus, regressions might be run to determine how inequality affects growth or crime or to evaluate political economy models that predict the extent of redistribution.³⁰ In most such instances, it would seem that individuals' level of well-being would be most relevant, so again approaches to measurement that focus on benefits at different income levels rather than allocating costs seem most sensible.

Similarly, if one were attempting to predict whether the median voter will favor an increase in the level of a public good, one would presumably compare the additional tax he would pay with the incremental benefit he would receive, where the benefit refers to his actual utility gain (converted into dollars, to be comparable with his additional tax obligation) rather than to some hypothetical allocation of the costs of the public good. For a public good that is efficient, it is easy to imagine a favorable vote even if standard methods of allocating costs might assign a lower "value" to this voter, possibly lower than his share of the tax increase used to finance the public good – and conversely for an inefficient good. Since a significant amount of government activity involves decisionmaking with regard to the provision of public goods and services, understanding their distributive incidence in terms of their contribution to individuals' utilities is likely to be important in providing empirical illumination in the field of political economy.

In other areas of inquiry, especially when poverty is thought to be important (perhaps in

³⁰See, for example, Alesina and Rodrik (1994), Barro (1999), Bénabou (1996), Fajnzylber, Lederman, and Loayza (2002), Forbes (2000), and Welch (2001). For further discussion of descriptive uses of inequality measures, see Kaplow (2002).

predicting crime), understanding the distributive incidence of public goods is likely to be crucial because public goods arguably have a relatively greater impact on the well-being of low-income individuals. However, as discussed in section 4, in many instances it may not be necessary to measure directly the distributive incidence of public goods; measuring overall well-being, whatever its source, may be more apropos in principle and less difficult in practice. In any event, it would appear that greater attention to the purposes of an inquiry into the distributive incidence of public goods is likely to provide important guidance regarding whether and how the inquiry might best be conducted.

6. Conclusion

There are many reasons that one might wish to inquire into the distributive incidence of public goods. This article explores several. The distributive incidence of a public good might be thought relevant to whether more of a public good should be provided. Drawing on prior work, this turns out not to be so in a simple benchmark case. This benchmark, which included the assumption that the public good is to be financed by a benefit-absorbing tax adjustment – a benefit tax of sorts – helps to illuminate the relationship between efficiency and distributive considerations, namely, involving the combined distributive incidence of the public good and the tax adjustment used to finance it.

Second, how does the distributive incidence of a public good affect the optimality of further redistribution when more of a public good is provided? No meaningful analysis is possible without some specification of how the public good itself is to be financed. Again, a convenient baseline involves a benefit-absorbing tax adjustment, which is to say, distribution-neutral finance. In the special case in which public goods are a perfect substitute for private consumption, providing more of a public good, thus financed in a distribution-neutral manner, has no effect on the optimality of further redistribution. More generally, however, this is not the case. A number of subtle effects were identified, and those that could be signed tended to counsel moderation (thus, if the benefit-absorbing tax adjustment would be progressive, the optimal overall tax adjustment would be less progressive).

Third, the distributive incidence of preexisting public goods is in principle relevant to how much redistribution through the tax/transfer scheme is optimal. This is because public goods affect both individuals' utility levels and marginal utilities of consumption. However, the relevant data concerns these utility levels and marginal utilities themselves, without requiring that one identify how they would differ were it not for the prevailing level of public goods and services. Hence, many of the conceptual difficulties as well as some of the practical challenges confronted by those attempting to measure the distributive incidence of public goods may not have to be faced, at least for the purpose under consideration.

Finally, there are some purely descriptive uses of information about the distributive incidence of public goods. The main observation is that most uses – just as with the normative purposes considered previously – seem concerned with benefits measured in utility terms rather than allocations of cost that underlie most prevailing notions of benefit taxation. Furthermore, for many descriptive purposes, it again may be unnecessary to know the actual effect of public goods rather than merely determining utility levels (whatever their source); for others (such as to

test political economy models), it would be necessary to identify the distributive incidence of proposals under consideration but not of the background level of public goods.

This article's inquiry into the distributive incidence of public goods – with its emphasis on identifying the purposes of the inquiry – is of value in a number of ways. Most important, in the areas considered, this focus helps to provide some conceptual illumination of the proper analytical framework for policy assessment with regard to public goods provision and income redistribution. In the process, the analysis suggests a simple approach to benefit taxation that differs from existing formulations. In addition, this investigation can help guide empirical efforts; clarifying the purposes of measurement is useful generally, and in the present setting it appears that some of the greatest conceptual and practical obstacles raised by the literature on the subject may not, on reflection, need to be overcome.

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Appendix: Various Derivations

Demonstration that $dl/dg = 0$ [page 6].

The first-order condition for the choice of l for an individual of type w is:

$$(A1) \quad U_1 v_c w(1 - T') + U_2 = 0,$$

where U_i is the derivative of U with respect to its i th argument and T' (the marginal income tax rate) is the derivative of T with respect to its first argument (wl). Differentiating (A1) with respect to g yields:

$$(A2) \quad U_1 v_c w(-wT''l_g - T'_g) + U_1 w(1 - T') \left(v_{cc}(wl_g - wT'l_g - T_g) + v_{cg} \right) \\ + v_c w(1 - T') \left[U_{11} \left(v_c(wl_g - wT'l_g - T_g) + v_g \right) + U_{12} l_g \right] \\ + U_{21} \left(v_c(wl_g - wT'l_g - T_g) + v_g \right) + U_{22} l_g = 0,$$

where subscripts of U , v , and T indicate partial derivatives as before and $l_g = dl/dg$. Solving (A2) for l_g produces a fraction, the denominator of which is simply the terms of the second-order condition for l (the derivative of the left side of (A1) with respect to l), which is taken to be strictly negative (to avoid complications of multiple optima, which add technical qualifications that do not affect the substance of the argument), and the numerator of which is the following expression:

$$(A3) \quad U_1 v_c wT'_g - U_1 w(1 - T')(-v_{cc}T_g + v_{cg}) - v_c w(1 - T')U_{11}(-v_c T_g + v_g) \\ - U_{21}(-v_c T_g + v_g).$$

From (6), $T_g = v_g/v_c$, so each of the last two terms equals zero. Next, consider T'_g

$$(A4) \quad T'_g = \frac{d(T_g)}{d(wl)} = \frac{d\left(\frac{v_g}{v_c}\right)}{d(wl)} = \frac{v_c(v_{cg}(1 - T') + v_{gg} \cdot 0) - v_g(v_{cc}(1 - T') + v_{cg} \cdot 0)}{v_c^2} \\ = \frac{(1 - T')(v_c v_{cg} - v_g v_{cc})}{v_c^2}.$$

Using (A4), we can now rewrite the numerator of l_g (the first two terms in A3), again using $T_g = v_g/v_c$ from (6), as:

$$(A5) \quad \frac{U_1 v_c w (1 - T') (v_c v_{cg} - v_g v_{cc})}{v_c^2} - U_1 w (1 - T') \left(v_{cg} - v_{cc} \frac{v_g}{v_c} \right)$$

$$= U_1 w (1 - T') \left[\frac{v_c v_{cg} - v_g v_{cc}}{v_c} - \left(v_{cg} - \frac{v_g v_{cc}}{v_c} \right) \right] = 0.$$

Hence, $l_g = 0$, as claimed in the alternative demonstration in the text.

Demonstration for case in which public goods are perfect substitutes for private consumption [page 14].

Begin with the direct effect on welfare, which involves differentiating (2) with respect to e :

$$(A6) \quad \frac{dSW}{de} = \int W' (U_1 v_c c_e + U_2 l_e) f(w) dw.$$

Next, we differentiate (A6) with respect to g , to determine how this condition is affected by changing the level of the public good.

$$(A7) \quad \frac{d^2 SW}{dedg} = \int [W' (U_1 v_c c_{eg} + U_1 c_e (v_{cc} c_g + v_{cg}) + v_c c_e (U_{11} (v_c c_g + v_g) + U_{12} l_g)$$

$$+ U_2 l_{eg} + l_e (U_{21} (v_c c_g + v_g) + U_{22} l_g))$$

$$+ (U_1 v_c c_e + U_2 l_e) W'' (U_1 (v_c c_g + v_g) + U_2 l_g)] f(w) dw.$$

To evaluate (A7), we need to take advantage of the assumed formulation of v and the fact that a benefit-absorbing tax adjustment is being employed to finance the change in g . Recalling from (10) that $v(c, g) = c + b(g)$, it follows that $v_c = 1$, $v_g = b'$, $v_{cc} = 0$, and $v_{cg} = 0$. In addition, recall that $l_g = 0$ (because a benefit-absorbing tax adjustment leaves labor supply unchanged). Furthermore, in this case, modifying (12), consumption is given by

$$(A8) \quad c = wl - T(wl, g^0) - e\tau(wl) - (b(g) - b(g^0)),$$

where g^0 refers to the initial level of the public good and c is evaluated at the new level of the public good g , where the change in g is financed by a benefit-absorbing tax adjustment. From

(A8), we can derive that $c_g = -bN$ (using the fact that $l_g = 0$); $c_e = wl_e(1 - TNeJN) - J$; and, from the latter, we have $c_{eg} = wl_e(-wTQ_g - TNg - weJQ_g) - wJNg + wl_{eg}(1 - TNeJN)$, which reduces to $c_{eg} = wl_{eg}(1 - TNeJN)$ because in this case $l_g = 0$ and $TNg = 0$ (the benefit-absorbing tax adjustment is a change in the lump-sum component, as reflected in (A8)).

Finally, we need to evaluate l_{eg} . Individuals' first-order condition for this case is:

$$(A9) \quad U_1 v_c w(1 - T' - e\tau') + U_2 = 0.$$

Differentiating (A9) with respect to e and rearranging terms gives the expression:

$$(A10) \quad l_e = \frac{\tau' U_1 w v_c + \tau (U_1 w(1 - T' - e\tau') v_{cc} + U_{11} w(1 - T' - e\tau') v_c^2 + U_{21} v_c)}{SOC_l}$$

$$= \frac{\tau' U_1 w + \tau (U_{11} w(1 - T' - e\tau') + U_{21})}{SOC_l},$$

where SOC_l denotes the terms in the second-order condition for individuals' maximization (A9), and the latter equality makes use of the above facts that $v_c = 1$ and $v_{cc} = 0$. Next, we differentiate (A10) with respect to g ; the denominator of the resulting expression is SOC_l^2 , which (as noted above) is taken to be nonzero. The numerator is

$$(A11) \quad SOC_l \left[\tau' w (U_{11} (v_c c_g + v_g) + U_{12} l_g) + U_1 w^2 \tau'' l_g + w \tau' l_g (U_{11} w(1 - T' - e\tau') + U_{21}) \right.$$

$$+ \tau (w(1 - T' - e\tau') (U_{111} (v_c c_g + v_g) + U_{112} l_g)$$

$$+ U_{11} w (-wT'' l_g - T'_g - we\tau'' l_g) + U_{211} (v_c c_g + v_g) + U_{212} l_g \left. \right]$$

$$+ \left[\tau' U_1 w + \tau (wU_{11} (1 - T' - e\tau') + U_{21}) \right] \frac{dSOC_l}{dg}.$$

We know, however, that $l_g = 0$, $v_c c_g + v_g = 0$ (from (5) and (6), characterizing the benefit-offsetting tax adjustment), and $TNg = 0$ (another, above-noted property of the benefit-offsetting tax adjustment for this special case in which the public good is a perfect substitute for private consumption). Thus, each of the terms in the first pair of square brackets, which multiplies SOC_l , equals zero. A further (quite tedious) calculation shows that $dSOC_l/dg = 0$ (making use of the same facts as well as some of the earlier-noted properties for this case). Hence, we have $l_{eg} = 0$.

Finally, if one uses all of the properties noted or derived following (A7) and then reviews (A7) term by term, it can be seen that each term of (A7) equals zero; hence, we have that $d^2SW/dedg = 0$, as explained intuitively in the text. Note further that this derivative equals zero regardless of whether, before changing g , we had $dSW/de = 0$, which means that the direct welfare effect from redistribution is unchanged when g is varied, accompanied by a benefit-offsetting tax adjustment, in this case.

Now, consider the revenue effect. Just as for the general case in the text (21), we have:

$$(A12) \quad \frac{dR}{de} = \int (wT'l_e + we\tau'l_e + \tau)f(w)dw.$$

Differentiating (A12) with respect to g :

$$(A13) \quad \frac{d^2R}{dedg} = \int (wT'l_{eg} + wl_e(wT''l_g + T'_g) + we\tau'l_{eg} + w^2el_e\tau''l_g + w\tau'l_g)f(w)dw.$$

We have previously shown that $l_g = 0$, $l_{eg} = 0$, and $T'_g = 0$ in the special case under consideration, so $d^2R/dedg = 0$, which was also explained intuitively in the text. And, as above, this expression holds regardless of whether $dR/de = 0$ initially.