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Abstract

Many consumer markets feature a multi-dimensional price. A policymaker – a legislator, a regulator or a court – concerned about the level of one price dimension may decide to cap this price. How will such a price cap affect other price dimensions? Will the overall effect be good or bad for consumers? For social welfare? Price caps can be beneficial when sellers set prices in response to consumer misperception. The scope for welfare-enhancing regulation depends on the type (and direction) of the underlying misperception, as well as on market structure.

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1. Introduction

The Credit Card Accountability, Responsibility and Disclosure Act of 2009 (the CARD Act), and its implementing regulations, imposed restrictions on certain dimensions of the credit card price. In particular, late fees were subjected to a de facto price cap. Other fees and interest rates were also curtailed. A few years later, the Dodd-Frank Act restricted the permissible magnitude of prepayment penalties in mortgage contracts. Other examples of price caps are easy to find. Usury laws cap interest rates. Courts applying the Penalty Doctrine imposed de facto caps on cellphone early termination fees. The European Union (EU) caps roaming fees and international calling rates. The Singapore Telecommunication Act of 2000 caps the price that hotels can charge for international phone calls. Etc.

In these examples, lawmakers, responding to concern about an excessively high price, resolved to cap the suspect price. The lawmakers did not fully account, however, for the possibility of unintended consequences. In particular, credit cards, mortgages, cellular service and hospitality services are all multi-dimensional products with multi-dimensional prices. When the law caps one price dimension, we cannot assume that other price dimensions will remain unchanged. If sellers react to the new law by increasing other prices, then it is no longer clear that the law will achieve its stated purpose.

Will the price cap increase social welfare? Will it make consumers better off? To answer these questions we need to first understand the forces driving the pre-cap pricing structure. If a well-functioning market produces efficient prices, then a price cap will likely reduce social welfare and hurt consumers. These distortions might be exacerbated in a multi-price market, where a price cap on one dimension can lead to adjustment away from the efficient level also on other price dimensions. If, on the other hand, pre-cap prices were designed to exploit consumer biases, then legal intervention may increase welfare and help consumers.

The scope of welfare-enhancing price regulation critically depends on the type and direction of consumer misperception. I consider two general categories of misperception: utility misperception and price misperception. And for each category, I consider both underestimation and overestimation. Take cellular roaming, for example. A consumer might underestimate (or overestimate) the utility of the roaming service, perhaps because she underestimates (or overestimates) how often she will travel abroad. The consumer might also underestimate (or overestimate) the roaming charges – the per-unit price charged by her cellphone company for the roaming service.

In the absence of a price cap, profit-maximizing sellers will adjust their pricing in response to consumer misperception, deviating from efficient, cost-based pricing (Bar-Gill, 2012). I show that the direction of the deviation depends on the direction of the misperception (under- vs. over-estimation), but not on the type of misperception (utility misperception vs. price misperception). Cellphone companies will set high roaming fees, when consumers underestimate the utility of the roaming service *and* when consumers underestimate the roaming fees themselves. Conversely, cellphone companies will set low roaming fees in response to overestimation of either utility or price.

Perhaps more surprising: given the existence of misperception, optimal prices also deviate from first-best, cost-based pricing. But here the direction of the deviation depends on the type of misperception, not on the direction of the misperception. When consumers misperceive the utility of the roaming service, the (second-best) optimal roaming fee is higher than the first-best, cost-based price, regardless of whether utility is under- or overestimated. And when consumers misperceive the roaming fees themselves, the (second-best) optimal roaming fee is lower than the first-best, cost-based price, regardless of whether utility is under- or overestimated.

The scope of welfare-enhancing price regulation is a function of the difference between the (second-best) optimal prices and the prices that profit-maximizing sellers would set in the absence of a cap. Accordingly, policymakers who are considering whether to impose a price cap should pay close attention to both the type and direction of the underlying misperception. More specifically, the analysis in this paper offers practical guidance to well-meaning, but imperfectly informed policymakers, about how to set price caps. For example, the results derived in this paper could assist EU lawmakers as they reconsider, or recalibrate, their cap on cellular roaming fees.

This paper shows that price caps can be beneficial. However, it should not be read as a general call for more price caps. While providing guidance about the information necessary to set a welfare-increasing price cap, the paper does not claim that regulators will always have the necessary information. Moreover, the domain of analysis, and the applicability of the results, is not without limits. The model studied in this paper captures the reality of important consumer markets, where sellers shift pricing across a limited number of plausible dimensions – some accurately perceived and others misperceived. In this setting, a cap on the misperceived price can help consumers and increase welfare. Such price regulation would be less effective in markets where sellers can easily “invent” additional price dimensions that would trigger similar misperceptions.

Related Literature. Markets with multi-dimensional products, and multi-dimensional prices, have been studied in the Industrial Organization (IO) literature. Products with an aftermarket – for parts or service – provide a key example. See Farrell and Klemperer (2007) and Farrell (2008). In the behavioral IO literature, several papers study multi-dimensional pricing. See, e.g., Gabaix and Laibson (2006), Grubb (2009), Spiegler (2011) and Heidhues, Koszegi and Murooka (2012). These papers by and large do not consider price caps.

The important exceptions are DellaVigna and Malmendier (2004), Heidhues and Koszegi (2010), Bar-Gill and Bubb (2012), Agarwal et al (2013) and Armstrong and Vickers (2012). DellaVigna and Malmendier (2004) and Heidhues and Koszegi (2010) demonstrate the potential welfare benefits of price regulation for a specific type of misperception, naiveté about time preferences, which is related to the utility misperception studied here. Bar-Gill and Bubb (2012) and Agarwal et al (2013) focus on a different misperception – price underestimation. The model in Armstrong and Vickers (2012) appears to cover both utility and price misperception, but in a way that masks the differences between the two types of misperception. The current paper advances the literature by analyzing and comparing, in a unified framework, the positive and normative implications of different types (and directions) of misperception. The analysis covers misperceptions that have not been studied before and yields several novel results.

Roadmap. The framework of analysis is developed in Section 2. The main results are derived in Sections 3-5. Section 3 characterizes (second-best) optimal prices, given misperception. Section 4 derives equilibrium prices in a competitive market and proves that optimally-calibrated price caps increase social welfare. Section 5 considers the monopoly case. Section 6 (briefly) discusses two extensions: indirect forms of price regulation, beyond price-caps; and quality floors in markets where product quality (rather than price) is multi-dimensional. Proofs are relegated to an appendix.

2. Framework of Analysis

A. Basic Setup

Assume a two-dimensional product (X, Y). The consumer chooses how much to consume on each dimension, i.e., the consumer chooses consumption levels (x, y). For simplicity, assume that X is a binary dimension, i.e., $x \in \{0,1\}$, with $x = 1$ representing a decision to purchase the product and $x = 0$ representing a decision not to purchase the product. If the consumer decided to purchase the product, she must then decide how intensely to use the product on the Y dimension, where $y \in R^+$. (The model can be extended to accommodate continuous decisions on both dimensions, e.g., when a consumer decides how much to borrow, on a credit card, in an introductory period (x) and how much to borrow in the post-introductory period (y).)

The assumption is that X and Y are two dimensions of a single product. Or, equivalently, that X and Y are effectively bundled, such that a consumer who purchases X from one seller will not purchase Y from another seller. Moreover, it is assumed that all sellers are offering both X and Y, and that no seller can offer just X (or just Y). The idea is that X and Y are very difficult to separate or, alternatively, that there are substantial efficiencies from bundling them together (or that bundling is very profitable for behavioral reasons – see Bar-Gill, 2006).

The seller's cost of providing the product is separable, with an independent per-unit cost for each dimension of the product. There is a fixed cost, c_x, of serving any consumer who chooses to purchase the product, and a per-unit cost, c_y, for each unit of use on dimension Y. The seller's total cost, for a consumer who decided to purchase the product, is: $C(c_x, c_y) = c_x + yc_y$.

The (gross) value of the product to the consumer is: $v + u(y)$, where v is a base-value that is distributed among consumers according to the CDF $F(v)$, and $u(y)$ is a use value that varies with use levels on the Y dimension but in a manner common to all consumers. I assume that $u'(y) > 0$ and $u''(y) < 0$.

B. The Seller's Decisions

The seller sets a two-dimensional price, which is comprised of a per-unit price for each dimension of the product. The per-unit prices are: p_x and p_y. The price p_x will be referred to as the base price; the price p_y will be referred to as the per-use price. The total price is: $P(p_x, p_y) = p_x + yp_y$. The seller's profit per-product purchased is:

$$\pi(p_x, p_y) = P(p_x, p_y) - C(c_x, c_y) = (p_x - c_x) + y(p_y)(p_y - c_y)$$

Note that $\pi(p_x, p_y)$ is increasing in p_x. And I assume that $\pi(p_x, p_y)$ is also increasing in p_y, namely that: $\frac{\partial \pi(p_x, p_y)}{\partial p_y} = y(p_y) + \frac{dy(p_y)}{dp_y}(p_y - c_y) > 0$, which follows immediately from:

Assumption 1: Profits on the use dimension $y(p_y)(p_y - c_y)$ are monotonically increasing in the per-use price, p_y, in the relevant range.

The seller's total profit function is:

$$\Pi(p_x, p_y) = \pi(p_x, p_y) \cdot D(p_x, p_y)$$

where $D(p_x, p_y)$ represents the demand for the seller's product, i.e., the number of consumers who purchase the product. The demand function is derived below.

The prices that the seller sets, and the profit that the seller makes, depend, among other things, on market structure. I will consider two different assumptions about the structure of the market: perfect competition and monopoly. In a perfectly competitive market, prices will be set to maximize the (net) value of the product, as perceived by consumers, subject to a zero-profit constraint: $\Pi(p_x, p_y) = 0$. In a monopolistic market, prices will be set to maximize $\Pi(p_x, p_y)$.

C. The Consumer's Decisions

The consumer makes two decisions: (1) whether to purchase the product, and (2) how intensely to use a product that is purchased. I begin by describing the use decision. The prior purchase decision is (potentially) influenced by consumer misperception. I, therefore, present the different types of misperception, before turning to the purchase decision itself.

1. Use Decision

A consumer who decides to purchase the product will choose a use level, y, that solves: $\max_y \langle u(y) - yp_y \rangle$. The First-Order Condition (FOC) is: $u'(y) = p_y$, which implicitly defines the optimal use level as a function of the per-unit price, p_y: $y = y(p_y)$.

Let $\eta_{y,p_y} \equiv \frac{dy(p_y)/y(p_y)}{dp_y/p_y}$ denote the elasticity of use levels with respect to the per-use price.

2. Consumer Misperception

I consider two different types of misperception: utility misperception, where the consumer believes that her use value will be $\hat{u}(y) = \delta u(y)$, with $\delta \in [0, \infty)$; and price misperception, where the consumer believes that the per-use price will be $\hat{p}_y = \delta p_y$, with $\delta \in [0, \infty)$. I study the two types of misperception separately. Therefore, I can use the same parameter, δ, for both types of misperception. The benchmark case, where the consumer does not suffer from any misperception, is captured by $\delta = 1$. Underestimation is captured by $\delta < 1$, and overestimation is captured by $\delta > 1$.

Both types of misperception apply only ex ante. Ex post, when the actual use decision is made, the consumer learns her true use value, $u(y)$, and the actual per-use price, p_y, and sets the use level, y, accordingly (as described in subsection 1 above). (Compare: naïve hyperbolic discounters in DellaVigna and Malmendier (2004) and Heidhues and Koszegi (2010).) But, ex ante, when making the purchase decision, the consumer thinks that she will choose a different use level: With utility misperception, the consumer thinks that she will choose a use level, y, that solves $\max_y \langle \delta u(y) - y p_y \rangle$. With price misperception, the consumer thinks that she will choose a use level, y, that solves $\max_y \langle u(y) - y \delta p_y \rangle$. The FOCs – $\delta u'(y) = p_y$ with utility misperception and $u'(y) = \delta p_y$ with price misperception – implicitly define the anticipated use level as a function of the per-unit price, p_y, and the misperception parameter, δ: $\hat{y} = \hat{y}(p_y; \delta)$.

3. Purchase Decision

The decision whether to purchase the product depends on the (net) value of the product, as perceived by the consumer. The (net) value of the product to a consumer is: $V(v, p_x, p_y) = v + u(y) - (p_x + yp_y)$. This (net) value might be misperceived by the consumer. Specifically, the use dimension – the per-use price, the use level and the use value – are subject to (possible) misperception. The perceived (net) value of the product is: $\hat{V}(v, p_x, p_y; \delta) = v + \hat{u}(\hat{y}) - (p_x + \hat{y}\hat{p}_y)$. This formulation captures the two types of misperception defined in subsection 2 above: With utility misperception, we have $\hat{u}(\hat{y}) = \delta u(\hat{y})$, $\hat{p}_y = p_y$ and $\hat{y} = \hat{y}(p_y; \delta)$; with price misperception, we have $\hat{u}(\hat{y}) = u(\hat{y})$, $\hat{p}_y = \delta p_y$ and $\hat{y} = \hat{y}(p_y; \delta)$.

The consumer will purchase the product iff the perceived (net) value is positive, i.e., iff $\hat{V}(v, p_x, p_y; \delta) > 0$. There exists a threshold value, $\tilde{v}(p_x, p_y; \delta) = (p_x + \hat{y}\hat{p}_y) - \hat{u}(\hat{y})$, such that only consumers with $v > \tilde{v}(p_x, p_y; \delta)$ will purchase the product. Assuming a unit mass of consumers, the demand for the product is: $D(p_x, p_y; \delta) = 1 - F\left(\tilde{v}(p_x, p_y; \delta)\right)$. The perceived overall consumer surplus is: $\hat{S}(p_x, p_y; \delta) = \int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} \hat{V}(v, p_x, p_y; \delta) f(v) dv$, whereas the actual overall consumer surplus is: $S(p_x, p_y; \delta) = \int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} V(v, p_x, p_y) f(v) dv$.

D. Social Welfare

1. The Social Welfare Function

Total social welfare is the sum of utilities enjoyed by consumers who choose to make a purchase, i.e., consumers with $v > \tilde{v}(p_x, p_y; \delta)$, minus the cost – to the seller (or sellers) – of serving these consumers. The social welfare function is:

$$W(p_x, p_y; \delta) = \int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} \left[v + u\left(y(p_y)\right) - \left(c_x + y(p_y)c_y\right)\right] f(v)dv$$

2. The First-Best Optimum

A consumer who decides to purchase the product should choose a use level, y, that solves: $\max_y \langle u(y) - yc_y \rangle$. The FOC, $u'(y^*) = c_y$, implicitly defines the optimal use level as: $y^* = y(c_y)$. A consumer should choose to purchase the product *iff* $v > \tilde{v}^* = \tilde{v}(c_x, c_y) = \left(c_x + y(c_y) \cdot c_y\right) - u\left(y(c_y)\right)$. The product should be purchased by the following number of consumers: $D^*(c_x, c_y) = 1 - F\left(\tilde{v}(c_x, c_y)\right)$. Therefore, social welfare at the first-best optimum is:

$$W^* = W(c_x, c_y; 1) = \int_{\tilde{v}(c_x, c_y)}^{\infty} \left[v + u\left(y(c_y)\right) - \left(c_x + y(c_y)c_y\right)\right] f(v)dv$$

E. The Law

I study the effects of a rule that restricts the permissible magnitude of either p_y or p_x. Specifically, I consider a price cap, $\bar{p}_y$, that adds a "legal constraint" $p_y \leq \bar{p}_y$ to the seller's optimization problem; and a price cap, $\bar{p}_x$, that adds a "legal constraint" $p_x \leq \bar{p}_x$ to the seller's optimization problem. The question is under what conditions will such a rule help consumers and increase social welfare.

F. Applicability of the Framework

This framework can be used to study important consumer markets. In the credit cards market, a consumer makes a Dimension X decision whether to get a credit card and then a Dimension Y decision how intensely to use the card. The base price, p_x, would be the annual fee charged by the card issuer. Dimension Y could capture different use dimensions. It could be the amount borrowed, and then the per-use price, p_y, would be the interest rate charged by the issuer. It could be the propensity to use the card's late payment feature, and then the per-use price would be the late fee. Or it could be the dollar amount of transactions made outside the U.S. using foreign currency, with the currency conversion fee as the per-use price. It is easy to imagine both utility and price misperception with respect to these possible use dimensions. And one of the associated per-use prices, the late fee, has recently been capped in the U.S.

In the cell phones market, a consumer makes a Dimension X decision whether to get a new smartphone and a Dimension Y decision how intensely to use the smartphone. The base price, p_x, would be the up front cost of the phone or the fixed monthly fee. Dimension Y could, once again, capture different use dimensions: number of minutes talked, messages sent, data used – each with its associated per-use price. Dimension Y could also capture the extent of the consumer's roaming activity, with the associated roaming fees. Again, it is not difficult to imagine both utility and price misperception regarding these use dimensions. And the EU caps roaming fees.

The analysis proceeds as follows. I begin, in Section 3, by deriving the second-best optimum that can be attained given a certain level of misperception (δ) – both for utility misperception and for price misperception. Then I derive the equilibrium outcomes and welfare levels and compare them to the second-best benchmark. The analysis depends on market structure. The perfect competition case is considered in Section 4. The monopoly case is considered in Section 5.

3. The Second-Best Optimum

The benchmark for the welfare analysis is the second-best optimum. The second-best optimal prices are those that maximize social welfare subject to a zero-profit constraint, given a certain level of misperception (δ):

$$(1) \quad \left(p_x^*(\delta), p_y^*(\delta)\right) = argmax_{p_x,p_y} W\left(p_x, p_y; \delta\right) \text{ s.t. } \Pi\left(p_x, p_y; \delta\right) = 0$$

An alternative definition would replace the zero profit constraint with a participation constraint, $\Pi\left(p_x, p_y; \delta\right) \geq 0$. But our goal is to study the positive and normative effects of imposing price caps, starting with a competitive market where sellers earn zero profits. It is, therefore, useful to focus on the maximum welfare that can be obtained in a zero-profit environment. The solution to Program (1) depends on the type and direction of the misperception, as detailed below.

Consider utility misperception. With utility underestimation demand is too low. The most efficient way to increase demand is by setting $p_y^*(\delta) > c_y$ and $p_x^*(\delta) < c_x$. Utility underestimation results in an underestimation of the use level, which in turn results in an underestimation of the importance of the per-use price. Therefore, we can increase demand by shifting pricing towards the per-use price and away from the accurately perceived base price. This reduces the distortion in purchase levels caused by the misperception. Optimal pricing balances this benefit from reducing distortions in purchase levels against the cost of distorted use levels (caused by the deviation from $p_y = c_y$). With utility overestimation we have the opposite problem – demand is too high. But, again, the most efficient way to increase demand is by setting $p_y^*(\delta) > c_y$ and $p_x^*(\delta) < c_x$. Utility overestimation results in an overestimation of the use level, which in turn results in an overestimation of the importance of the per-use price. Therefore, we can decrease demand by shifting pricing towards the per-use price and away from the accurately perceived base price. This reduces the distortion in purchase levels caused by the misperception. Here too, optimal pricing balances this benefit from reducing distortions in purchase levels against the cost of distorted use levels.

Next consider price misperception. With price underestimation demand is too high. The (second-best) optimal response is to shift pricing away from the underestimated dimension, namely, to reduce the per-use price below cost, such that $p_y^*(\delta) < c_y$, and to increase the base price above cost, such that $p_x^*(\delta) > c_x$. With price overestimation demand is too low. Yet the (second-best) optimal response is the same – to shift pricing away from the misperceived dimension. In both cases, optimal pricing balances the benefit from reducing distortions in purchase levels against the cost of distorted use levels.

These results are summarized in the following lemmas.

Lemma 1 (Second-Best Optimum, Utility Misperception): With both utility underestimation and utility overestimation, the second-best per-use price, p_y, satisfies: $p_y^*(\delta) > c_y$, and the second-best base price, p_x, satisfies: $p_x^*(\delta) < c_x$.

Before summarizing the results for price misperception, I introduce the following assumption:

Assumption 2: With price misperception, $\delta\hat{y}(p_y;\delta)$ is monotonically increasing in δ for all p_y.

Note that, with price misperception, a higher δ reduces the perceived use-level $\hat{y}(p_y;\delta)$ and so without Assumption 2 the effect of δ on $\delta\hat{y}(p_y;\delta)$ would be ambiguous (and the analysis more complicated). Assumption 2 guarantees that a shift in pricing away from the misperceived p_y and towards the accurately perceived p_x reduces demand when the per-use price is underestimated and increases demand when the per-use price is overestimated.

Lemma 2 (Second-Best Optimum, Price Misperception): With both price underestimation and price overestimation, the second-best per-use price, p_y, satisfies: $p_y^*(\delta) < c_y$, and the second-best base price, p_x, satisfies: $p_x^*(\delta) > c_x$.

4. Competition

A. Equilibrium Prices

In a competitive market, sellers set prices to maximize the perceived consumer surplus or, equivalently, to maximize demand, subject to a break-even constraint. Formally, the seller solves the following maximization problem:

$$\max_{p_x, p_y} \hat{S}(p_x, p_y; \delta) \text{ s.t. } \Pi(p_x, p_y) = 0$$

Sellers in a competitive market care about maximizing the perceived value to their customers, not about maximizing actual consumer surplus (or social welfare). This is the source of the inefficiency.

In particular, with both utility and price underestimation, sellers will raise the per-use price and reduce the base price, in order to increase demand for their product. With price underestimation, shifting price from the accurately perceived dimension to the underestimated dimension increases demand. With utility underestimation, consumers underestimate the use level and thus the importance of the per-use price. Again, raising the per-use price and reducing the base price increases demand.

Now compare these equilibrium prices to the second-best optimal prices (from Section 3): With price underestimation, the second-best optimal response to price underestimation is to *reduce* the per-use price and *raise* the base price. The equilibrium prices move in the wrong direction. With utility underestimation, equilibrium prices move in the same direction as the second-best prices: the per-use price is above cost and the base-price is below cost. But equilibrium prices overshoot – the per-use price is too high (above the second-best level) and the base-price is too low (below the second-best level). The second-best optimum balances the benefit of increased demand with the cost of distorted use-levels. Sellers, on the other hand, care only about increasing demand.

Related distortions occur with overestimation of utility and of price. Sellers will reduce the per-use price and raise the base price, in order to increase demand for their product. The second-best response to utility overestimation is the opposite: raise the per-use price and reduce the base price. With price overestimation, equilibrium prices move in the same direction as the second-best prices, but they overshoot – the per-use price is too low and the base-price is too high.

These results are summarized in the following lemmas.

Lemma 3 (Competitive Equilibrium, Utility Misperception): In a competitive market –

(a) With utility underestimation ($\delta < 1$), the per-use price, p_y, satisfies: $p_y^C(\delta) > p_y^*(\delta) > c_y$, and the base price, p_x, satisfies: $p_x^C(\delta) < p_x^*(\delta) < c_x$.

(b) With utility overestimation ($\delta > 1$), the per-use price, p_y, satisfies: $p_y^C(\delta) < c_y < p_y^*(\delta)$, and the base price, p_x, satisfies: $p_x^C(\delta) > c_x > p_x^*(\delta)$.

Lemma 4 (Competitive Equilibrium, Price Misperception): In a competitive market –

(a) With price underestimation ($\delta < 1$), the per-use price, p_y, satisfies: $p_y^C(\delta) > c_y > p_y^*(\delta)$, and the base price, p_x, satisfies: $p_x^C(\delta) < c_x < p_x^*(\delta)$.

(b) With price overestimation ($\delta > 1$), the per-use price, p_y, satisfies: $p_y^C(\delta) < p_y^*(\delta) < c_y$, and the base price, p_x, satisfies: $p_x^C(\delta) > p_x^*(\delta) > c_x$.

B. Price Caps

We can now study the effects of imposing a price cap $\bar{p}_y$ or $\bar{p}_x$, namely of adding a legal constraint $p_y \leq \bar{p}_y$ or $p_x \leq \bar{p}_x$. We first note the standard result that, without misperception, a price cap can only reduce social welfare. In our model, it is the misperception that generates the welfare costs and opens the door for potentially welfare-enhancing regulation. In particular, with underestimation of utility or price, i.e., when $\delta < 1$, the per-use price, p_y, will be excessively high without legal intervention (see Lemma 3 and Lemma 4), and so a price cap, $\bar{p}_y$, can increase social welfare. With overestimation of utility or price, i.e., when $\delta > 1$, the base price, p_x, will be excessively high without legal intervention (see Lemma 3 and Lemma 4), and so a price cap, $\bar{p}_x$, can increase social welfare.

These results are summarized in the following propositions.

Proposition 1 (Competition, Utility Misperception): In a competitive market –

(a) With utility underestimation ($\boldsymbol{\delta} < 1$), a mild, though still binding, price cap, $\bar{\boldsymbol{p}}_{\boldsymbol{y}}$, satisfying $\boldsymbol{p}_{\boldsymbol{y}}^{\boldsymbol{C}}(\boldsymbol{\delta}) > \bar{\boldsymbol{p}}_{\boldsymbol{y}} \geq \boldsymbol{p}_{\boldsymbol{y}}^{*}(\boldsymbol{\delta}) \geq \boldsymbol{c}_{\boldsymbol{y}}$, reduces the per-use price, $\boldsymbol{p}_{\boldsymbol{y}}$, and increases the base price, $\boldsymbol{p}_{\boldsymbol{x}}$, raising social welfare.

(b) With utility overestimation ($\boldsymbol{\delta} > 1$), a mild, though still binding, price cap, $\bar{\boldsymbol{p}}_{\boldsymbol{x}}$, satisfying $\boldsymbol{p}_{\boldsymbol{x}}^{\boldsymbol{C}}(\boldsymbol{\delta}) > \bar{\boldsymbol{p}}_{\boldsymbol{x}} \geq \boldsymbol{p}_{\boldsymbol{x}}^{*}(\boldsymbol{\delta})$, reduces the base price, $\boldsymbol{p}_{\boldsymbol{x}}$, and increases the per-use price, $\boldsymbol{p}_{\boldsymbol{y}}$, raising social welfare.

Proposition 2 (Competition, Price Misperception): In a competitive market –

(a) With price underestimation ($\boldsymbol{\delta} < 1$), a mild, though still binding, price cap, $\bar{\boldsymbol{p}}_{\boldsymbol{y}}$, satisfying $\boldsymbol{p}_{\boldsymbol{y}}^{\boldsymbol{C}}(\boldsymbol{\delta}) > \bar{\boldsymbol{p}}_{\boldsymbol{y}} \geq \boldsymbol{p}_{\boldsymbol{y}}^{*}(\boldsymbol{\delta})$, reduces the per-use price, $\boldsymbol{p}_{\boldsymbol{y}}$, and increases the base price, $\boldsymbol{p}_{\boldsymbol{x}}$, raising social welfare.

(b) With price overestimation ($\boldsymbol{\delta} > 1$), a mild, though still binding, price cap, $\bar{\boldsymbol{p}}_{\boldsymbol{x}}$, satisfying $\boldsymbol{p}_{\boldsymbol{x}}^{\boldsymbol{C}}(\boldsymbol{\delta}) > \bar{\boldsymbol{p}}_{\boldsymbol{x}} \geq \boldsymbol{p}_{\boldsymbol{x}}^{*}(\boldsymbol{\delta}) > \boldsymbol{c}_{\boldsymbol{x}}$, reduces the base price, $\boldsymbol{p}_{\boldsymbol{x}}$, and increases the per-use price, $\boldsymbol{p}_{\boldsymbol{y}}$, raising social welfare.

C. Comparison: The Object (and Direction) of Misperception

To facilitate a comparison between the welfare and policy implications of different types (and directions) of misperception, we collect the results from Lemmas 3 and 4 in the following Table.

	Utility Misperception	Price Misperception
Underestimation	$p_y^C(\delta) > p_y^*(\delta) > c_y$ $p_x^C(\delta) < p_x^*(\delta) < c_x$	$p_y^C(\delta) > c_y > p_y^*(\delta)$ $p_x^C(\delta) < c_x < p_x^*(\delta)$
Overestimation	$p_y^C(\delta) < c_y < p_y^*(\delta)$ $p_x^C(\delta) > c_x > p_x^*(\delta)$	$p_y^C(\delta) < p_y^*(\delta) < c_y$ $p_x^C(\delta) > p_x^*(\delta) > c_x$

Table 1: Price Distortions for Different Types (and Directions) of Misperception

With utility misperception, underestimation results in high per-use prices, whereas overestimation results in high base prices. Accordingly, different price caps will be relevant in the two cases – $\bar{p}_y$ for underestimation and $\bar{p}_x$ for overestimation. Also, equilibrium prices and second-best prices move in the same direction with underestimation, but in opposite directions with overestimation. This comparison may prove useful to a policymaker who is considering whether to impose a price cap. Specifically, with overestimation, if the policymaker has good information about the per-unit cost, she can use this information to increase social welfare by setting a price cap, $\bar{p}_x$, that is equal to (or greater than) the per-unit cost, c_x. With underestimation, a price cap, $\bar{p}_y$, equal to (or greater than) the per-unit cost, c_y, might reduce social welfare.

With price misperception, underestimation results in high per-use prices, and a potential benefit from a cap $\bar{p}_y$, whereas overestimation results in high base prices, and a potential benefit from a cap $\bar{p}_x$, as with utility misperception. But now equilibrium prices and second-best prices move in the same direction with overestimation, and in opposite directions with underestimation. This implies that good information about the per-unit cost could help the policymaker set price caps, if the underlying problem is underestimation, but not if the underlying problem is overestimation.

The preceding analysis highlights the importance, for policymakers, of identifying the precise nature – type and direction – of the underlying misperception. The equilibrium response to misperception depends on the direction of the misperception (under- vs. overestimation) but not on the type of misperception (utility vs. price misperception), whereas second-best prices depend on the type of misperception but not on the direction of misperception. Since the benefit from a price cap, and the optimal magnitude of the cap, depend on both the second-best prices and the equilibrium prices, a separate analysis of each one of the four type-direction combinations is necessary.

5. Monopoly

A. Equilibrium Prices

We next consider a monopolistic market. A monopolistic seller sets prices, p_x and p_y, to maximize its profits: $\Pi(p_x, p_y) = \pi(p_x, p_y) \cdot \left[1 - F\left(\tilde{v}(p_x, p_y; \delta)\right)\right]$. Under the (standard) assumption that consumers do not suffer from any misperception, the monopolist would set an efficient per-use price, $p_y^M = c_y$, to maximize total surplus (a different p_y distorts use-level decisions and reduces total surplus) and use the base price to extract monopolistic rents. The base price would be set above the competitive level, which leads to an inefficiently small number of products purchased. The result is a welfare loss – the monopoly deadweight loss.

Our focus, however, is on the implications – both descriptive and normative – of consumer misperception. Solving the monopolist's problem, we find that the per-use price, p_y, is identical to the per-use price in a competitive market: $p_y^M = p_y^C$ (for both utility and price misperception). This result follows from the separability of the X and Y dimensions in this model. The base-price, however, is, higher in a monopolistic market as compared to a competitive market: $p_x^M > p_x^C$. These results are summarized in the following lemma.

Lemma 5 (Equilibrium Prices – Monopoly vs. Competition):

(a) The per-use price, p_y, is independent of market structure: $p_y^M(\delta) = p_y^C(\delta)$.

(b) The base price, p_x, is higher in a monopolistic market than in a competitive market: $p_x^M(\delta) > p_x^C(\delta)$.

What are the welfare implications of the higher base price that the monopolist sets? Since $p_y^M(\delta) = p_y^C(\delta)$, we have:

$$W\left(p_y^M = p_y^C, p_x^C\right) - W\left(p_y^M, p_x^M\right) =$$

$$= \int_{\tilde{v}(p_y^M, p_x^C; \delta)}^{\tilde{v}(p_y^M, p_x^M; \delta)} \left[\left(v + u\left(y\left(p_y^M\right)\right) - c_x - y\left(p_y^M\right)c_y\right)\right] f(v)dv$$

With utility underestimation and price overestimation, this difference is positive – monopoly pricing reduces welfare. The misperception inefficiently reduces demand and the higher base price set by the monopolist pushes demand further down.[1] With utility overestimation and price underestimation, the difference can be either positive or negative. The higher base price reduces demand. The reduced demand avoids purchases that generate a social loss, but it might also deter purchases that generate a social gain. Accordingly, with utility overestimation and price underestimation the net welfare effect of monopoly pricing is indeterminate. In all four cases, the higher base price hurts the infra-marginal consumers who would purchase the product anyway. This distributional effect reduces consumer surplus.

[1] To confirm that the lost purchases would have generated a social gain, namely, that $v + u\left(y\left(p_y^M\right)\right) - c_x - y\left(p_y^M\right)c_y \geq 0$, note that: (i) $v + u\left(y\left(p_y^M\right)\right) - c_x - y\left(p_y^M\right)c_y \geq v + u\left(y\left(p_y^M\right)\right) - p_x^M - y\left(p_y^M\right)p_y^M$, since $p_x^M + y\left(p_y^M\right)p_y^M \geq c_x + y\left(p_y^M\right)c_y$; (ii) actual value exceeds perceived value with utility underestimation: $v + u\left(y\left(p_y^M\right)\right) - p_x^M - y\left(p_y^M\right)p_y^M \geq v + \delta u\left(\hat{y}\left(p_y^M; \delta\right)\right) - p_x^M - \hat{y}\left(p_y^M; \delta\right)p_y^M$; and (iii) perceived value is non-negative: $v + \delta u\left(\hat{y}\left(p_y^M; \delta\right)\right) - p_x^M - \hat{y}\left(p_y^M; \delta\right)p_y^M \geq 0$ (otherwise the consumer would not purchase).

B. Price Caps

We can now study the effects of imposing a price cap. Begin with the no misperception case and consider a cap $\bar{p}_x$ on the high monopoly base price. In a standard monopoly model, with a single price dimension, a price cap can reduce the monopoly deadweight loss and thus increase welfare. In our two-dimensional price model, a cap on the base price can similarly reduce the monopoly deadweight loss. But the cap will also result in a use-level distortion, as the monopolist increases p_y in response to the cap on p_x. The overall welfare effect of the price cap is, therefore, ambiguous.

Our focus, however, is on the effect of a price cap, given consumer misperception. Starting with a cap $\bar{p}_x$ on the base price, we have seen that with utility underestimation and price overestimation the higher monopoly base price reduces welfare. Therefore, a price cap $\bar{p}_x$ is desirable. With utility overestimation and price underestimation, the higher monopoly base price either increases or decreases welfare, as compared to the competition case. Accordingly, a price cap $\bar{p}_x$ that pushes the base price down can be harmful.

Next, consider a cap $\bar{p}_y$ on the per-use price. With underestimation – both utility underestimation and price underestimation – the pre-cap per-use price is excessively high *and* independent of market structure. It is, therefore, instructive to compare the effect of a similar price cap $\bar{p}_y$ in a monopolistic vs. competitive market. I begin by asking how a cap on the per-use price affects the base price and, specifically, how this effect differs between Monopoly and Competition. A decrease in the per-use price (because of the cap) reduces the seller's revenue from the use dimension. In a competitive market, sellers will have to increase the base price to compensate for this shortfall in revenues. The same is not true in a monopolistic market: With a mild price cap, the monopolist may decide *not* to increase p_x in response to the reduction in p_y. Before increasing a price, the monopolist considers the detrimental effect of the price increase on demand for its products. For the per-use price, p_y, this detrimental effect is moderated by the misperception (the consumer underestimates the per-use price itself or underestimates the use-level and thus the total use-based price). No such moderation exists for the base price, p_x. Therefore, the monopolist may decide to absorb the reduction in profit imposed by the price cap, or some of it, rather than to try and compensate by increasing the base price.

I can now state the following result.

Lemma 6 (Underestimation, Effects of a Cap on the Unregulated Price): With both utility underestimation and price underestimation, the increase in the base price, p_x, as a result of a price cap, $\bar{p}_y$, will be smaller in a monopolistic market, as compared to a competitive market. The difference is increasing in the magnitude of the misperception.

With utility underestimation, demand is inefficiently low and the higher monopoly base price reduces demand even further, deterring purchases that would have generated a social gain. A cap on the per-use price increases welfare, as in the Competition case; more so when the monopolist is expected to respond with only a minor increase in the base price (see Lemma 6). With price underestimation, demand is inefficiently high. The higher monopoly base price efficiently deters negative-value purchases, but might also inefficiently deter positive-value purchases. If (pre-cap) the marginal purchase generates a social loss, then a price cap that reduces demand can increase welfare, as in the Competition case. However, if the marginal purchase generates a social gain, then a price cap might reduce welfare.

The analysis in the monopoly case produces more limited policy implications. Still, I can state the following results.

Proposition 3 (Monopoly): In a monopolistic market –

(a) With utility underestimation and price overestimation, a price cap, $\bar{p}_x$, can raise social welfare.

(b) With utility underestimation, a price cap, $\bar{p}_y$, can raise social welfare.

6. Extensions

A. Beyond Price-Caps

The analysis in this paper and the policy implications that follow from it may apply beyond price-caps, to other, indirect forms of price regulation. Policymakers can, and do, restrict prices in other ways. For example, the CARD Act restricts sellers' ability to reprice credit card debt. Lawmakers reduce prices by changing defaults and demanding that consumers explicitly opt-into the targeted service (as with credit card overlimit fees and overdraft protection). Finally, policymakers can influence pricing by mandating conspicuous disclosure of a specific price dimension (e.g., large font, Bold face terms in the standardized credit card disclosure, the Schumer Box) or including a certain price dimension in an influential aggregate disclosure (e.g., specifying what fees are included in the "finance charge" definition, which underlies the APR disclosure). If these disclosure strategies succeed in focusing competition on the targeted price dimension, the result would be downward pressure on the regulated price dimension.

Like price caps, these alternative price-control policies often target one price dimension within a multi-dimensional pricing structure. While each policy has unique features and merits further study, the analysis in this paper should be informative – in terms of both market outcomes and social welfare.

B. Multi-Dimensional Quality and Quality Floors

This paper focused on multi-dimensional pricing and examined the implications of capping a single price in such a multi-dimensional pricing scheme. A similar analysis applies to multi-dimensional quality. For many consumer products and services, quality is measured on multiple dimensions. Consider the cellphone market. Relevant quality dimensions include the functionality of the phone itself (the handset), the scope and duration of the warranty, the reliability of the cellular service (reception, dropped calls, etc.), the accessibility and professionalism of the provider's customer service department, the degree of protection afforded to the customer's personal data, the efficacy and fairness of the contractually-specified dispute resolution mechanism, etc. The level of transparency (or disclosure) about any of these features is yet another quality dimension.

And, as with price, lawmakers often target a single quality dimension for regulation. Rather than capping certain price dimensions, lawmakers set minimal acceptable levels, or floors, for certain quality dimensions. Sellers' ability to disclaim implied warranties is restricted by law. The Food and Drug Administration (FDA) regulates certain dimensions of pharmaceutical products. The Consumer Product Safety Commission (CPSF) specifies minimum safety requirements for certain dimensions of certain consumer products. The unconscionability doctrine is used by courts to regulate dispute resolution mechanisms. Consumer protection law imposes minimum disclosure requirements, bans certain contractual terms, mandates cancellation or withdrawal rights (in certain cases), and so on.

Like price, quality is subject to consumer misperception. Consumers might overestimate a certain quality dimension. For example, a cellphone subscriber might overestimate the coverage provided by the carrier's network. Consumers might also misperceive the utility associated with a certain quality dimension. For example, the cellphone subscriber might underestimate the likelihood of traveling to other parts of the country and, therefore, underestimate the utility from broad cellular coverage. Quality misperception corresponds to price misperception. And utility misperception affects price and quality in a similar way. Accordingly, the positive and normative implications of quality floors, as a function of the underlying misperception, can be studied using a framework similar to the one developed in this paper.

References

Agarwal, Sumit, Chomsisengphet, Souphala, Mahoney, Neale and Stroebel, Johannes (2013), Regulating Consumer Financial Products: Evidence from Credit Cards, NBER Working Paper 19484.

Armstrong, Mark and Vickers, John (2012), Consumer Protection and Contingent Charges, Journal of Economic Literature, 50, 477-93.

Bar-Gill, Oren (2012), Seduction by Contract: Law, Economics and Psychology in Consumer Markets (Oxford University Press).

Bar-Gill, Oren (2006), Bundling and Consumer Misperception, University of Chicago Law Review, 73, 33-61.

Bar-Gill, Oren and Bubb, Ryan (2012), Credit Card Pricing: The Card Act and Beyond, Cornell Law Review, 97, 967-1018.

DellaVigna, Stefano and Malmendier (2004), Ulrike, Contract Design and Self-Control: Theory and Evidence, Quarterly Journal of Economics, 119, 353-402.

Farrell, Joseph (2008), Some Welfare Analytics of Aftermarkets, working paper.

Farrell, Joseph and Klemperer, Paul (2007), Coordination and Lock-In: Competition with Switching Costs and Network Effects, in Armstrong, Mark and Porter, Rob (eds.), Handbook of Industrial Organization, Vol. 3 (2007).

Gabaix, Xavier and Laibson, David (2006), Shrouded Attributes, Consumer Myopia and Information Suppression in Competitive Markets, Quarterly Journal of Economics, 121, 505-540.

Heidhues, Paul, and Koszegi, Botond (2010), Exploiting Naivete about Self-Control in the Credit Market, American Economic Review, 100, 2279-2303.

Heidhues, Paul, Koszegi, Botond and Murooka, Takeshi (2012), Inferior Products and Profitable Deception, Working Paper.

Spiegler, Ran (2011), Bounded Rationality and Industrial Organization (Oxford University Press).

Appendix

Proof of Lemma 1

Consider the partial derivatives of the social welfare function with respect to the base price and to the per-use price:

$$\frac{\partial W}{\partial p_x} = -\frac{\partial \tilde{v}(p_x, p_y; \delta)}{\partial p_x}\left[\tilde{v}(p_x, p_y; \delta) + u\left(y(p_y)\right) - \left(c_x + y(p_y)c_y\right)\right] f\left(\tilde{v}(p_x, p_y; \delta)\right)$$

$$\frac{\partial W}{\partial p_y} = -\frac{\partial \tilde{v}(p_x, p_y; \delta)}{\partial p_y}\left[\tilde{v}(p_x, p_y; \delta) + u\left(y(p_y)\right) - \left(c_x + y(p_y)c_y\right)\right] f\left(\tilde{v}(p_x, p_y; \delta)\right)$$
$$+ \int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} \left[\left(u'\left(y(p_y)\right) - c_y\right)\frac{dy(p_y)}{dp_y}\right] f(v)dv$$

Increasing either price increases $\tilde{v}(p_x, p_y; \delta)$ and reduces demand. Whether it is beneficial or harmful to reduce demand depends on the social value of the marginal purchases, $w \equiv \tilde{v}(p_x, p_y; \delta) + u\left(y(p_y)\right) - \left(c_x + y(p_y)c_y\right)$. When increasing the per-use price, we also have an infra-marginal effect: $\int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} \left[\left(u'\left(y(p_y)\right) - c_y\right)\frac{dy(p_y)}{dp_y}\right] f(v)dv$. Any deviation from $p_y = c_y$ distorts use levels and reduces social welfare.

We substitute $p_x = c_x - y(p_y) \cdot (p_y - c_y)$ (from the zero-profit constraint, $\Pi(p_x, p_y) = 0$) into $W(p_x, p_y; \delta)$ and solve for the optimal per-use price. Consider the derivative:

$$\frac{dW}{dp_y} = -\frac{d\tilde{v}(p_x, p_y; \delta)}{dp_y}\left[\tilde{v}(p_x, p_y; \delta) + u\left(y(p_y)\right) - \left(c_x + y(p_y)c_y\right)\right] f\left(\tilde{v}(p_x, p_y; \delta)\right)$$
$$+ \int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} \left[\left(u'\left(y(p_y)\right) - c_y\right)\frac{dy(p_y)}{dp_y}\right] f(v)dv$$

In particular, we evaluate the sign of $\frac{dW}{dp_y}$ at $p_y = c_y$. The second expression in $\frac{dW}{dp_y}$, capturing the effect on infra-marginal consumers, is zero at $p_y = c_y$, since $u'\left(y(c_y)\right) - c_y = 0$. Therefore, at $p_y = c_y$, the sign of $\frac{dW}{dp_y}$ is determined by the sign of the first expression, which captures the effect on the marginal consumers. This effect, in turn, is comprised of two components: the demand component and the value component.

The effect on demand for the product is given by:

$$\frac{d\tilde{v}(p_x, p_y; \delta)}{dp_y} = \frac{dp_x}{dp_y} - \left[\delta u'\left(\hat{y}(p_y, \delta)\right) - p_y\right] \frac{d\hat{y}(p_y, \delta)}{dp_y} + \hat{y}(p_y, \delta) =$$

$$= \frac{dp_x}{dp_y} + \hat{y}(p_y, \delta) = -\frac{dy(p_y)}{dp_y}(p_y - c_y) - \left(y(p_y) - \hat{y}(p_y, \delta)\right)$$

(The zero-profit constraint, $\Pi(p_x, p_y; \delta) = 0$, implies: $p_x = p_x(p_y) = c_x - y(p_y) \cdot (p_y - c_y)$. And so $\frac{dp_x}{dp_y} = -\frac{dy(p_y)}{dp_y}(p_y - c_y) - y(p_y)$.) At $p_y = c_y$, this derivative is: $\frac{d\tilde{v}(p_x, p_y; \delta)}{dp_y} = -\left(y(p_y) - \hat{y}(p_y, \delta)\right)$, which is negative for $\delta < 1$ and positive for $\delta > 1$.

The value component, $\tilde{v}(p_x, p_y; \delta) + u\left(y(p_y)\right) - \left(p_x + y(p_y)p_y\right)$, can be written as:

$$u\left(y(p_y)\right) - y(p_y)p_y - \left[\delta u\left(\hat{y}(p_y, \delta)\right) - \hat{y}(p_y, \delta)p_y\right]$$

(after substituting $\tilde{v}(p_x, p_y; \delta) = p_x + \hat{y}(p_y, \delta)p_y - \delta u\left(\hat{y}(p_y, \delta)\right)$). The value component is positive for $\delta < 1$ and negative for $\delta > 1$. This means that the marginal consumer gains from the purchase with underestimation and loses from the purchase with overestimation.

Combining the demand component and the value component: With underestimation, the marginal consumer gains from a purchase and so we want to increase demand. This is accomplished by increasing p_y. At $p_y = c_y$, $\frac{d\tilde{v}(p_x,p_y;\delta)}{dp_y} < 0$ and, since the value component is positive, $\frac{dW}{dp_y} > 0$. Therefore, $p_y^*(\delta) > c_y$. With overestimation, the marginal consumer loses from a purchase and so we want to decrease demand. This is accomplished by increasing p_y. At $p_y = c_y$, $\frac{d\tilde{v}(p_x,p_y;\delta)}{dp_y} > 0$ and, since the value component is negative, $\frac{dW}{dp_y} > 0$. Therefore, $p_y^*(\delta) > c_y$.

The optimal base price can be derived from the zero-profit condition, $\Pi(p_x, p_y) = 0$: $p_x = c_x - y(p_y) \cdot (p_y - c_y)$. Since $p_y^*(\delta) > c_y$, the zero-profit condition implies: $p_x^*(\delta) < c_x$.

QED

Proof of Lemma 2

The proof parallels the proof of Lemma 1, with the following differences:

The effect on demand for the product is given by:

$$\frac{d\tilde{v}(p_x,p_y;\delta)}{dp_y} = \frac{dp_x}{dp_y} - \left[u'\left(\hat{y}(p_y,\delta)\right) - \delta p_y\right]\frac{d\hat{y}(p_y,\delta)}{dp_y} + \delta\hat{y}(p_y,\delta) =$$

$$= \frac{dp_x}{dp_y} + \hat{y}(p_y,\delta) = -\frac{dy(p_y)}{dp_y}(p_y - c_y) - \left(y(p_y) - \delta\hat{y}(p_y,\delta)\right)$$

At $p_y = c_y$, this derivative is: $\frac{d\tilde{v}(p_x,p_y;\delta)}{dp_y} = -\left(y(p_y) - \delta\hat{y}(p_y,\delta)\right)$, which is negative for $\delta < 1$ and positive for $\delta > 1$ (Assumption 2).

The value component, $\tilde{v}(p_x,p_y;\delta) + u\left(y(p_y)\right) - (c_x + y(p_y)c_y)$, can be reduced to:

$$u\left(y(p_y)\right) - y(p_y)p_y - \left[u\left(\hat{y}(p_y,\delta)\right) - \hat{y}(p_y,\delta)\delta p_y\right]$$

In contrast to Lemma 1, the value component is negative for $\delta < 1$ and positive for $\delta > 1$. This means that the marginal consumer loses from the purchase with underestimation and gains from the purchase with overestimation.

Combining the demand component and the value component: With underestimation, the marginal consumer loses from a purchase and so we want to decrease demand. This is accomplished by decreasing p_y. At $p_y = c_y$, $\frac{d\tilde{v}(p_x, p_y; \delta)}{dp_y} < 0$ and, since the value component is negative, $\frac{dW}{dp_y} < 0$. Therefore, $p_y^*(\delta) < c_y$. With overestimation, the marginal consumer gains from a purchase and so we want to increase demand. This is accomplished by decreasing p_y. At $p_y = c_y$, $\frac{d\tilde{v}(p_x, p_y; \delta)}{dp_y} > 0$ and, since the value component is positive, $\frac{dW}{dp_y} < 0$. Therefore, $p_y^*(\delta) < c_y$.

Since $p_y^*(\delta) < c_y$, the zero-profit condition implies $p_x^*(\delta) > c_x$.

QED

Proof of Lemma 3

We substitute $p_x = c_x - y(p_y) \cdot (p_y - c_y)$ (from the zero-profit constraint, $\Pi(p_x, p_y) = 0$) into $\hat{S}(p_x, p_y; \delta)$ and solve for the optimal per-use price. Consider the derivative:

$$\frac{d\hat{S}}{dp_y} = -\frac{d\tilde{v}(p_x, p_y; \delta)}{dp_y} \cdot \hat{V}\left(\tilde{v}(p_x, p_y; \delta), p_x, p_y; \delta\right) f\left(\tilde{v}(p_x, p_y; \delta)\right)$$

$$+ \int_{\tilde{v}(p_x, p_y; \delta)}^{\infty} \left[\frac{d\hat{V}(v, p_x, p_y; \delta)}{dp_y}\right] f(v) dv$$

Since $\hat{V}\left(\tilde{v}(p_x, p_y; \delta), p_x, p_y; \delta\right) = 0$ (by definition), the marginal effect is zero: For the marginal consumer, the perceived value from purchasing the product is zero. We are thus left with the infra-marginal effect:

$$\frac{d\hat{S}}{dp_y} = \int_{\tilde{v}(p_x,p_y;\delta)}^{\infty} \left[\frac{d\hat{V}(v, p_x, p_y; \delta)}{dp_y}\right] f(v)dv$$

Using the Envelope Theorem, the derivative $\frac{d\hat{S}}{dp_y}$ can be rewritten as:

$$\frac{d\hat{S}}{dp_y} = -\left[\hat{y}(p_y; \delta) + \frac{dp_x}{dp_y}\right]\left[1 - F\left(\tilde{v}(p_x, p_y; \delta)\right)\right]$$

The FOC w.r.t. p_y is: $\hat{y}(p_y; \delta) + \frac{dp_x}{dp_y} = 0$. Using the zero-profit constraint to find $\frac{dp_x}{dp_y}$, the FOC becomes:

$$\text{(A1)}\quad p_y = c_y + \left[\hat{y}(p_y; \delta) - y(p_y)\right] / \frac{dy(p_y)}{dp_y}$$

We begin by establishing the relationship between the equilibrium prices, p_y^C and p_x^C, and costs, c_y and c_x. First, we show that $p_y^C > c_y$ for $\delta < 1$ and that $p_y^C < c_y$ for $\delta > 1$. We show that the expression $\left[\hat{y}(p_y; \delta) - y(p_y)\right] / \frac{dy(p_y)}{dp_y}$ is positive for $\delta < 1$ and negative for $\delta > 1$. Taking the derivative of the FOC that determines the use level, $u'(y) = p_y$, w.r.t. p_y, we obtain: $\frac{dy(p_y)}{dp_y} = \frac{1}{u''} < 0$. It remains to show that $\hat{y}(p_y; \delta) - y(p_y) < 0$ for $\delta < 1$ and that $\hat{y}(p_y; \delta) - y(p_y) > 0$ for $\delta > 1$. To see this note that $\hat{y}(p_y; \delta = 1) = y(p_y)$ and that $\frac{d\hat{y}(p_y;\delta)}{d\delta} > 0$. This last inequality follows if we take the derivative of the FOC that determines the perceived use level, $\delta u'(\hat{y}) = p_y$, with respect to δ: $\frac{d\hat{y}(p_y;\delta)}{d\delta} = -\frac{p_y}{\delta^2 u''} > 0$. The results for the base price, that $p_x^C < c_x$ for $\delta < 1$ and that $p_x^C > c_x$ for

$\delta > 1$, follow when the preceding results for the per-use price are plugged into the zero-profit constraint.

Next, we consider the comparison with optimal prices. For $\delta > 1$, we saw that $p_y^C < c_y$ and we know (from Lemma 1) that $p_y^*(\delta) > c_y$. Similarly, we have: $p_x^C > c_x > p_y^*(\delta)$. For $\delta < 1$, recall that in the competitive equilibrium, $\frac{dp_x}{dp_y} + \hat{y}(p_y;\delta) = 0$ (see above). Plugging $\frac{d\tilde{v}(p_x,p_y;\delta)}{dp_y} = \frac{dp_x}{dp_y} + \hat{y}(p_y;\delta) = 0$ into $\frac{dW}{dp_y}$ (from the proof of Lemma 1), we get:

$$\left.\frac{dW}{dp_y}\right|_{p_y=p_y^C} = \int_{\tilde{v}(p_x,p_y;\delta)}^{\infty}\left[\left(u'\left(y(p_y)\right) - c_y\right)\frac{dy(p_y)}{dp_y}\right] f(v)dv$$

which is negative for all $p_y > c_y$. This implies that the equilibrium price is too high for $\delta < 1$ (when $p_y > c_y$): $p_y^C > p_y^*(\delta)$. Moving on to the base price: Does a higher p_y (as compared to the second-best optimal $p_y^*(\delta)$) result in a lower or higher p_x (as compared to the second-best optimal $p_x^*(\delta)$)? The base price, p_x, is a function of p_y, as defined by the zero profit condition, $\Pi(p_x, p_y) = 0:\ p_x = p_x(p_y) = c_x - y(p_y)\cdot(p_y - c_y)$. We take the derivative w.r.t. p_y:

$$\frac{dp_x}{dp_y} = -\frac{d[y(p_y)(p_y - c_y)]}{dp_y} = -\left[y(p_y) + \frac{dy(p_y)}{dp_y}(p_y - c_y)\right]$$

$$= -\left[1 + \eta_{y,p_y}\frac{p_y - c_y}{p_y}\right] y(p_y)$$

From Assumption 1, we know that this derivative is negative. Therefore, an increase in p_y results in a decrease in p_x: $p_x^C < p_x^*(\delta)$

QED

Proof of Lemma 4

The proof of Lemma 4 is similar to the proof of Lemma 3 and is, therefore, omitted.

Proof of Proposition 1

The results stated in Proposition 1 follow from Lemma 3. A detailed proof is omitted.

Proof of Proposition 2

The results stated in Proposition 2 follow from Lemma 4. A detailed proof is omitted.

Proof of Lemma 5

The proof focuses on utility misperception. A similar analysis applies to price misperception. We solve the monopolist's profit-maximization problem. Taken together, the two FOCs, $\frac{\partial \Pi(p_x,p_y)}{\partial p_x} = 0$ and $\frac{\partial \Pi(p_x,p_y)}{\partial p_y} = 0$, imply:

$$\left[1 + \eta_{y,p_y} \cdot \left(\frac{p_y - c_y}{p_y}\right)\right] \cdot y(p_y) = \hat{y}(p_y, \delta)$$

This equation is equivalent to Equation (A1), which defines the equilibrium per-use price in a competitive market (see proof of Lemma 3). We thus have: $p_y^M = p_y^C$. Turning to the base price, p_x, the FOC, $\frac{\partial \Pi(p_x,p_y)}{\partial p_x} = 0$, implies:

$$p_x = c_x - y(p_y)(p_y - c_y) + \frac{1 - F\left(\tilde{v}(p_x,p_y)\right)}{f\left(\tilde{v}(p_x,p_y)\right)},$$

as compared to $p_x = c_x - y(p_y)(p_y - c_y)$ in a competitive market. Therefore, $p_x^M > p_x^C$.

QED

Proof f Lemma 6

In a competitive market, $p_x(p_y) = c_x - y(p_y)(p_y - c_y)$. To examine the effects of a price cap that reduces p_y, we consider the derivative:

$$\frac{dp_x}{dp_y} = -\frac{d[y(p_y)(p_y - c_y)]}{dp_y} = -y(p_y)\left[1 + \eta_{y,p_y}\frac{p_y - c_y}{p_y}\right]$$

which is negative (see Assumption 1). In a monopolistic market, $p_x(p_y) = c_x - y(p_y)(p_y - c_y) + A\left(\tilde{v}(p_x(p_y), p_y)\right)$, where $A\left(\tilde{v}(p_x, p_y)\right) \equiv \frac{1-F\left(\tilde{v}(p_x,p_y)\right)}{f\left(\tilde{v}(p_x,p_y)\right)}$. To examine the effects of a price cap that reduces p_y, we consider the derivative:

$$\frac{dp_x}{dp_y} = -y(p_y)\left[1 + \eta_{y,p_y}\frac{p_y - c_y}{p_y}\right] + \frac{dA}{d\tilde{v}}\left[\frac{\partial \tilde{v}}{\partial p_y} + \frac{\partial \tilde{v}}{\partial p_x}\frac{dp_x}{dp_y}\right]$$

Or:

$$\frac{dp_x}{dp_y} = \frac{1}{1 - \frac{dA}{d\tilde{v}}}\left\{-y(p_y)\left[1 + \eta_{y,p_y}\frac{p_y - c_y}{p_y}\right] + \frac{dA}{d\tilde{v}} \cdot \frac{\partial \tilde{v}}{\partial p_y}\right\}$$

Comparing $\frac{dp_x}{dp_y}$ in a competitive market and $\frac{dp_x}{dp_y}$ in a monopolistic market, we find that the effect in a monopolistic market is smaller as long as:

$$\text{(A2)} \quad y(p_y)\left[1 + \eta_{y,p_y}\frac{p_y - c_y}{p_y}\right] \geq \frac{\partial \tilde{v}}{\partial p_y}$$

where $\frac{\partial \tilde{v}}{\partial p_y} = \hat{y}(p_y, \delta)$ with utility underestimation and $\frac{\partial \tilde{v}}{\partial p_y} = \delta\hat{y}(p_y, \delta)$ with price underestimation. I show that Inequality (A2) holds for all $\delta \leq 1$. At $\delta = 1$, (A2) holds with equality, since $\hat{y}(p_y, \delta = 1) = y(p_y)$ (and also $\delta\hat{y}(p_y, \delta = 1) = y(p_y)$) and $p_y = c_y$ in the absence of misperception. And, since $\frac{\partial \hat{y}(p_y,\delta)}{\partial \delta} > 0$ for utility misperception and $\frac{\partial \delta\hat{y}(p_y,\delta)}{\partial \delta} > 0$ for price misperception (see Assumption 2), it follows that Inequality

(A2) holds for all $\delta \leq 1$. It also follows that the difference between $\frac{dp_x}{dp_y}$ in a competitive market and $\frac{dp_x}{dp_y}$ in a monopolistic market is increasing in the magnitude of the misperception.

QED

Proof of Proposition 3

The results stated in Proposition 3 follow from the analysis in Section 5. A detailed proof is omitted.