TAKEOVER BIDS BELOW THE EXPECTED VALUE OF MINORITY SHARES

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I. INTRODUCTION AND SUMMARY

In an already classic paper, Grossman and Hart (1980) developed a model of takeover bids and stimulated much theoretical work on the subject (see, e.g., Harris and Raviv (1986), P'ng (1985), Scharfstein (1987), Shleifer and Vishny (1986), and the survey by Dreyfus (1986)). Focussing their analysis on bids whose outcome can be predicted in advance with certainty, Grossman-Hart established the proposition, which subsequent work accepted, that successful bids must be made at or above the expected value of minority shares. This proposition led to Grossman and Hart's insightful observation that a free-rider problem exists, and it was used by them and subsequently by others to analyze the operation of takeovers. This paper shows that this important proposition does not always hold once we drop the assumption that the only successful bids are those whose success could have been predicted with certainty. It is shown that certain bids below the expected value of minority shares may succeed with a certain positive probability, that such bids may be profitable and may be used by bidders, and that these possibilities have implications for the nature of the free-rider problem and for the operation of takeovers.

Suppose that the value of a target's assets is V_o per share under the present management, and that under the control of a given "raider" the value of these assets would increase to V_t per share. Suppose also that the expected value of minority shares in the event of a takeover, V_m, exceeds V_o, because "dilution" of the value of minority shares would either be impossible or at least possible only to a limited extent. Grossman and Hart, assuming that the only successful bids are those whose success can be predicted with certainty, showed that a takeover bid by the raider at any price below V_m cannot succeed. A success with certainty, they pointed out, cannot be a rational expectations outcome of a bid below V_m: for supposing that a takeover is going to take place, each atomistic shareholder will prefer to hold out and end up with a minority share worth V_m rather than have his share acquired for the lower bid price. Thus, minority shareholders' ability to free ride on the raider's improvement (at least in part) would rule out a takeover at any price below V_m, even if this offered price exceeds the target's independent value V_o.

An important implication that Grossman-Hart drew from their conclusion that only bids at or above V_m can succeed concerns the special case in which no dilution is possible (i.e., $V_m=V_t$). In this case, Grossman-Hart suggested, the raider would not be able to make any gain on shares acquired through a successful takeover bid. Therefore, since there are costs to making a bid, the raider would not bid, and the absence of dilution would thus prevent a value-increasing takeover from taking place.

The significance of the Grossman-Hart conclusions concerning the outcome of bids below V_m has been well recognized. The free-rider corollary -- that without dilution a raider would be unable to profit on shares acquired through a bid -- motivated, and served as a useful starting point for, an examination of what explains the making of takeover bids. Grossman and Hart (1980) suggested that bids are made because legal rules and charter provisions do enable acquirers to dilute the value of minority shares; and Shleifer and Vishny (1986) demonstrated that a raider that holds a stake in the target prior to bidding will gain from the appreciation of its initial stake (even if it makes no gain on shares acquired through its bid). Furthermore, the Grossman-Hart proposition has been an important premise of subsequent models of the case in which some dilution is possible (which has been generally regarded, following Grossman and Hart, to be the common case): that bids below V_m cannot succeed has been generally assumed by analysts modelling the operation and outcome of bids.

This paper drops the Grossman-Hart assumption that the only successful bids are those whose success could have been predicted with certainty, and it then reconsiders the prospects, profitability, and use of bids below V_m. It is shown that any bid below V_m but above V_o may succeed if the bid is unconditional -- that is, a bid committing the bidder to purchase tendered shares even if the bid fails. The intuition behind this result can be stated briefly as follows. In the case of an unconditional bid below V_m but above V_o, although certain success of the bid is not a rational equilibrium outcome, neither is a certain failure. Non-tendering is not an equilibrium strategy for the target's shareholders because, if other shareholders are going to hold out and the bid is going to fail, each atomistic shareholder will prefer to tender and have his share acquired for a price exceeding the target's independent value V_o. Such an unconditional bid has a unique symmetric equilibrium, which I identify and characterize, in which shareholders use mixed strategies and the bid may consequently either succeed or fail.

Furthermore, I show that unconditional bids below V_m but above Vo may be profitable and consequently may be used by bidders. Although such a bid would produce a loss if the bid fails and the bidder must purchase shares at a price exceeding V_o, the expected payoff from the bid, exclusive of the transaction costs of making the bid, is shown to be always positive. To examine when such bids will be used, I analyze when their expected payoff will exceed both the costs of making the bid and the expected payoff from bidding at or above V_m. In particular, I show that, in the absence of dilution, a raider that can increase the value of the target's assets may bid even if it holds no initial stake in the target; and that, in the presence of some dilution, a bidder may elect to bid below V_m, even though such a bid would not succeed with certainty as would a bid at V_m.

Section II analyzes the case in which no dilution is possible and Section III analyzes the case in which some dilution is possible. Section IV then makes concluding remarks concerning the implications of the paper's analysis.

II. BIDDING WHEN NO DILUTION IS POSSIBLE

A. Assumptions

The value of the target's assets under existing management is V_o per share. If another agent -- which I shall call "the raider" -- gains control over the target, the per share value of its assets will be $V_t>V_o$. $I=V_t-V_o$ is the per share improvement that a takeover by the raider can produce.

In the event of a takeover, the bidder will be unable to dilute the value of minority shares. Thus, minority shares will have a per share value of $V_m=V_t$.

The target has N shares, where N is large. The raider will have control if and only if it owns at least kN shares, where $0<k<1$ indicates the fraction of shares necessary for control. The raider is assumed, for simplicity of exposition, to hold no initial stake in the target. (The analysis can be adjusted to apply to the case in which the raider owns initially some non-controlling fraction s ($s<k$) of the target's shares.)

Each share of the target is held by one shareholder. Because N is large, I shall assume, like Grossman and Hart, that the atomistic shareholders ignore the possibility that their decision will determine the outcome of a bid. Dropping this assumption would make the analysis more complicated but would not change its conclusions.

It is assumed, as in Grossman-Hart, that, if the raider does bid and its bid fails, the per share value of the target's shares will be the "status quo" value of V_o. That is, should the bid fail, investors will not expect the failing bidder (or another bidder) to make a subsequent bid; If the raider's failing bid requires it to purchase some shares, the raider is expected to (or even has committed itself to) sell those shares on the market or hold them as a non-controlling block.

In the event that the raider elects to bid, B will denote the bid price. For any bid B, b will denote the value of $(B-V_o)/I$; that is, b is the fraction of the takeover improvement that is offered by the raider as a premium.

T will denote the number of shares attracted by a bid. We shall consider two kinds of bids: conditional, where the raider is willing to buy shares only if $T \geq kN$, and unconditional, where no such condition is attached.

The total cost to the raider of making a bid is C. The raider will of course elect to bid only if the expected profit on shares purchased through the bid exceeds C.

It is easy to see that, although a bid with $B \geq V_t$ would succeed, it would not provide the raider with any gain on shares purchased through the bid. Thus, we should consider whether the bidder can make a gain by bidding below V_t. In particular, since it is clear that any bid below V_o would attract no shares, we should focus on bids that are below V_t but not below V_o.

B. Conditional Bid Below Vt Cannot Succeed

It is worth starting the analysis with conditional bids, the case in which the proposition that bids below V_t cannot succeed does always hold.

Consider a bid with $V_o < B < V_t$ that is conditional on $T \geq kN$. In this situation, the dominant strategy of each atomistic shareholder will be to tender. In the event that the bid fails, the shareholder's tender decision will not matter; whether he tenders or holds out, his share will not be acquired. In the event that the bid succeeds, non-tendering (and thus ending up with V_t) is clearly preferable to tendering (and having one's share acquired for $B<V_t$). Thus, we have the following proposition.

Proposition 1: Any bid $V_o<B<V_t$ ($0<b<1$) which is conditional on the bidder's gaining control ($T\geq kN$) will fail.

C. Unconditional Bids below Vt May Succeed

Let us now consider the outcome of a bid $V_o<B<V_t$ that is unconditional. In this situation, there is no equilibrium in pure strategies.

Non-tendering is no longer an equilibrium strategy (as it was in the case of a conditional bid). For, assuming that other shareholders are going to hold out, a shareholder will not be indifferent between tendering and holding out (as he was in the case of a conditional bid): he will prefer to tender and have his share acquired for B rather than remain with a share in the independent target worth V_o.

As to tendering, it is not an equilibrium strategy for the same reasons as in the case of a conditional bid. Assuming that others are going to tender and a takeover is going to take place, a shareholder will prefer to hold out and end up with a minority share worth V_t rather than tender and have his share acquired for the lower bid price of B.

It remains then to consider the possibility of an equilibrium in mixed strategies. Focussing on the possibility of a symmetric equilibrium, let us consider the shareholder strategy of tendering with probability t and holding out with probability 1-t, where $0<t<1$. If all the shareholders follow this strategy, the likelihood of a takeover will be

$$(1) \qquad Pr(t) = 1 - F(N,t,kN-1) = \sum_{j=kN}^{N} \frac{N!}{j!\,(N-j)!} t^j (1-t)^{N-j} \quad ,$$

where $F(x,y,z)$ is the binomial distribution function, indicating the probability that out of x independent trials, each with probability y of success, no more than z will be successful.

Since N is large, the binomial distribution can be well approximated using the normal distribution (see Feller, Ch. 7). In particular, $Pr(t)$ can be well approximated by

$$(2) \qquad Pr(t) = 1 - \Phi[q(t)],$$

where Φ is the standard normal distribution function and

$$(3) \qquad q(t) = \frac{kN - tN}{\sqrt{Nt(1-t)}} \quad .$$

Clearly, for t to define an equilibrium strategy, a given shareholder must be indifferent between tendering and holding out, given the probability $Pr(t)$ that the bid will succeed. In comparison to non-tendering, tendering produces a gain of $(B-V_o)$ in the event that the bid fails and a loss of (V_t-B) in the event that the bid succeeds. That is, t must satisfy

$$(4) \qquad (B-V_o)*[1-Pr(t)] - (V_t-B)*[Pr(t)] = 0,$$

or, rearranging terms

$$(5) \qquad Pr(t) = 1 - \Phi\Big[q(t)\Big] = \frac{B - V_0}{V_t - V_0} = b.$$

It can be easily seen that $Pr(t)$ goes to zero as t goes to zero, that $Pr(t)$ goes to 1 as t goes to 1, and that $Pr(t)$ increases monotonically in the interval (0,1). Thus, for any $V_o<B<V_t$, there is exactly one value of t, let us denote it by τ, which satisfies (5). We can thus state the following proposition.

<u>Proposition 2</u>: For any unconditional bid $V_o<B<V_t$, there is a unique symmetric equilibrium, in which shareholders tender with probability $\tau(B)$ satisfying

$$(6) \qquad 1 - \Phi\left[\frac{kN - \tau N}{\sqrt{N\tau(1-\tau)}}\right] = \frac{B - V_0}{V_t - V_0} \quad ,$$

and in which the bid consequently succeeds with probability

$$(7) \qquad Pr(\tau) = b = (B-V_o)/(V_t-V_o).$$

<u>Remarks</u>: (i) <u>An increase in the Bid Price B</u> will raise both the likelihood of a takeover $Pr(\tau)$ and the equilibrium probability of tendering τ. $Pr(\tau)$ will increase because it will always equal b, the fraction of the improvement that the raider's premium is offering. τ will increase to produce the necessary increase in $Pr(\tau)$. Both $Pr(\tau)$ and τ will go to 1 as B goes to V_t, and both will go to 0 when B goes to V_o.

(ii) <u>An increase in the control fraction k</u> will not affect the value of $Pr(\tau)$ but it will raise τ. Since $Pr(\tau)$ is always equal to b, it is independent of the value of other parameters. The value of τ will increase, however, to maintain this necessary equality between $Pr(\tau)$ and b; as k increases, the τ which produces a given value of $Pr(\tau)$ increases as well.

D. <u>The Expected Payoff from Bidding below Vt</u>

When a bidder makes an unconditional bid $V_o<B<V_t$, there are two possible outcome. The bid may succeed in attracting kN shares or more, in which case the bidder will gain (V_t-B) for each tendered share. Or the bid may fail, in which case the bidder will lose $(B-V_0)$ for each tendered share. Thus, the bidder's expected payoff (not counting the costs of making the bid) will be

$$(8) \qquad W(B) = -(B - V_0) * \sum_{j=1}^{kN-1} f(N,\tau,j)j \; + \; (V - B_t) * \sum_{j=kN}^{N} f(N,\tau,j)j \quad ,$$

where f(x,y,z) is the binomial density function, indicating the likelihood that out of N independent experiments, each with probability of success y, there will be exactly z successes. Again, using the normal approximation, we can get

$$(9)\quad W(B) = -\ (B-V_0)\int_{-\infty}^{q(\tau)} \psi(x)[x\sqrt{N\tau(1-\tau)} + N\tau]dx\ +$$
$$+\ (V_t - B)\int_{q(\tau)}^{\infty} \psi(x)[x\sqrt{N\tau(1-\tau)} + N\tau]dx\quad ,$$

where $\psi(.)$ is the standard normal density function.

<u>Proposition</u> <u>3</u>: The expected payoff to the bidder (not counting the costs of making the bid) from making any unconditional bid $0<B<V_t$ is positive.

<u>Remark</u>: The result in Proposition 3 can be intuitively explained as follows. If the bidder expected to purchase the same number of shares whether the bid succeeds or fails, then his expected payoff, per share to be purchased, would be $Pr(\tau)(V_t-B) - [1-Pr(\tau)](B-V_0)$. But the fact that τ is an equilibrium strategy (i.e., that shareholders are indifferent between tendering and holding out) implies that this payoff would be zero. Thus, because the bidder can expect to purchase more shares in the event that the bid succeeds ($T\geq K$) that in the event that the bid fails ($T<K$), it follows that the bidder faces a positive expected payoff.

<u>Proof</u>: Rewriting (9) we get

$$(10)\quad W(B) = -\ (B-V_0)\int_{-\infty}^{\infty} \psi(x)[x\sqrt{N\tau(1-\tau)} + N\tau]dx\ +$$
$$+\ (V_t - V_0)\int_{q(\tau)}^{\infty} \psi(x)x\sqrt{N\tau(1-\tau)}\ dx\quad +$$
$$+\ (V_t - V_0)\int_{q(\tau)}^{\infty} \psi(x)N\tau dx\quad .$$

The first expression on the right-hand side of (10) can be shown to equal $-(B-V_o)*N\tau$. The third expression on the right-hand side can be shown to equal $(V_t-V_o)*Pr(\tau)N\tau$. Thus, by the condition that τ is the equilibrium probability of tendering and thus satisfies $Pr(\tau)=(B-V_o)/(V_t-V_o)$ (see (7)), the first and third expressions add up to zero. Thus, the value of W(B) is equal to the value of the second expression. Doing the integration in that expression we get

$$(11) \qquad W(B) = \sqrt{N\tau(1-\tau)/2\pi}\; e^{-\frac{1}{2}[q(t)]^2}\; I \quad ,$$

and the right-hand side of (11) is clearly positive.

E. <u>The Optimal Bid</u>

Supposing that the raider is going to make an unconditional bid $V_o<B<V_t$, its optimal bid is given by the solution to the problem

$$(12) \qquad \max_B \; W(B) \; .$$

Solving this maximization problem provides the following proposition, which is proven in the Appendix.

<u>Proposition 4</u>: The optimal bid is one offering a premium equal to half of the potential improvement ($B = (V_t+V_o)/2$). In the face of this bid, each shareholder will tender with probability equal to the control fraction (τ=k), and the bid will succeed with likelihood 1/2.

F. <u>The Incentive to Bid</u>

We can now determine when the raider will elect to bid (even though there is no dilution). If the raider does bid, it will offer, by proposition 4, a price of $(V_t+V_o)/2$. Substituting this value for B in (11) and making the calculation shows that the raider's expected payoff in such a case will be

$$W\left[\frac{V_t + V_o}{2}\right] = \sqrt{Nk(1-k)/2\pi} * I \ . \qquad (13)$$

Since the raider will bid if and only if the expected payoff given by (13) exceeds the cost C of making the bid, we have the following proposition.

<u>Proposition 5</u>: In the absence of dilution, the raider will bid for the target if and only if the ratio of the costs of making the bid to the total improvement produced by a takeover satisfies

$$\frac{C}{N*I} < \sqrt{k(1-k)/2\pi N} \ . \qquad (14)$$

For example, supposing that N=1000 and k=1/2, a bid will take place if and only C is no larger than 0.6% of the total improvement to be produced.

III. BIDDING WHEN SOME DILUTION IS POSSIBLE

Let us now consider the case in which dilution of the value of minority shares would be possible to some extent. Specifically, suppose that in the event of a takeover the raider would be able to dilute the value of each minority share by an amount D, where I>D>0. Thus, (I-D) is that part of the improvement produced by a takeover that the raider would be unable to deny minority shareholders, and on which these shareholders could thus free ride. Thus, the value of minority shares in the event of a takeover will be $V_m = V_t - D$, where $V_o < V_m < V_t$.*

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*. Because the concern of this paper is with bids below the expected value of minority shares, there is no interest in analyzing the obvious case in which D>I and hence $V_m < V_0$. In this case, a bid below V_m cannot succeed. Note, however, that in such a case, a bid below V_0 may succeed (see Bebchuk, 1985).

As before, the interest is in examining bids with $V_o<B<V_m$. In particular, we shall examine whether such bids may succeed and whether raiders will choose to make such bids.

A. The Outcome of Bids Below Vm

The analysis of the outcome of a bid below V_m can proceed in the same way as it proceeded in Section II. For the analysis of bid outcomes in section II depended only on the relationship between B, V_o, and the expected value of minority shares V_m; V_t appeared in that analysis only because, given the assumption of no dilution, V_m was equal to V_t.

A bid below V_m that is conditional on the bidder's gaining control is bound to fail, for the same reasons as those identified in Section II. Non-tendering is a dominant strategy, because (i) assuming that the bid is going to fail (and tendered shares are consequently going to be returned), a shareholder will be indifferent between tendering and non-tendering, and (ii) assuming that the bid is going to succeed, a shareholder will prefer to remain with minority shares worth V_m than to have his share purchased for a price below that value. Thus, we have

<u>Proposition</u> <u>6</u>: Any bid with $B<V_m$ which is conditional on the bidder's gaining control ($T\geq kN$) will fail.

In the case of an unconditional bid below V_m, there is no equilibrium in pure strategies, for the same reasons as those discussed in Section II. Supposing that other shareholders are going to tender, a shareholder will prefer to hold out; and supposing that other shareholders are going to hold out, a shareholder will prefer to tender. Proceeding in the same way as in Section II, we find that there is a unique symmetric equilibrium in mixed strategies which is characterized by the following proposition.

Proposition 7: For any unconditional bid $V_o<B<V_m$, there is a unique symmetric equilibrium, in which shareholders tender with probability $\tau(B)$ satisfying

$$(15) \qquad 1 - \Phi\left[\frac{kN - \tau N}{\sqrt{N\tau(1-\tau)}}\right] = \frac{B - V_0}{V_m - V_0} \quad ,$$

and in which the bid succeeds with probability

$$(16) \qquad \Pr(\tau) = \frac{B - V_0}{V_m - V_0} \quad .$$

Thus, the likelihood that an unconditional bid is going to succeed is equal to the ratio of the premium offered to that part of the improvement that minority shareholders can capture.

B. The Expected Payoff from Bidding below Vm

When a bidder makes an unconditional bid $V_o<B<V_m$, there are two possible outcomes. If the bid fails, which has a likelihood of $(V_m-B)/(V_m-V_o)$, the bidder will lose $(B-V_o)$ on each tendered share. If the bid succeeds, which has a likelihood of $(B-V_o)/(V_m-V_o)$, the bidder will make a gain of (V_t-V_o) on each tendered share and in addition a gain of D on each non-tendered share. Since $V_t-B = D + (V_m-B)$, the bidder's gain in the event of a takeover can be alternatively described as a gain of D on all of the target's shares and in addition a gain of (V_m-B) on each tendered share. Thus, the bidder's expected payoff from the bid (not counting the costs of making the bid) will be

$$(17) \qquad W(B) = -(B - V_0) \sum_{j=1}^{kN-1} f(N,\tau,j)j \; + \; (V - B_t) \sum_{j=kN}^{N} f(N,\tau,j)j \; + DN\frac{B - V_o}{V_m-V_o} \; ,$$

where, as before, f(.,.,.) is the binomial density function. Again, using the normal approximation, we can get

$$(18) \qquad W(B) = -(B-V_0)\int_{-\infty}^{q(\tau)} \psi(x)[x\sqrt{N\tau(1-\tau)} + N\tau]dx \; + $$

$$+ (V_m - B)\int_{q(t)}^{\infty} \psi(x)[x\sqrt{N\tau(1-\tau)} + N\tau]dx \quad + \quad DN\frac{B - V_o}{V_m - V_o} ,$$

where, as before, $\psi(.)$ is the standard normal density function.

Noting the similarity between the first two expressions on the right-hand side of (18) and the two expressions on the right-hand side of (9), we can follow the steps taken in proving proposition 3 to get

$$(19) \qquad W(B) = \sqrt{N\tau(1-\tau)/2\pi}\; e^{-\frac{1}{2}[q(t)]^2}(V_m - V_o) \; + \; D\frac{B - V_o}{V_m - V_o} ,$$

and since the first expression on the right-hand side of (19) is clearly positive, we have the following proposition.

<u>Proposition</u> <u>8</u>: the expected payoff to the bidder (not counting the costs of making the bid) from making any unconditional bid $0<B<V_m$ is greater than $DN\frac{B - V_0}{V_m - V_0}$.

C. <u>The Possible Use of Bids Below Vm</u>

In the case in which no dilution is possible, the only two options of the bidder that required our consideration were bidding below V_m and not bidding; for, in the absence of dilution, the bidder could not make any gain on shares acquired through a bid at or above $V_m=V_t$. Thus, the bidder's choice was determined by comparing the maximum expected payoff from bidding below V_m with the cost of making a bid. In the case in which dilution is possible, however, a bid at $V_m=V_t-D$ will provide the bidder with a gain of D*N (not counting the costs of making the bid). Thus, it is necessary to examine whether a bid below V_m will ever be more profitable than a bid at V_m. As will be seen below, the answer to this question is affirmative.

Assessing the consequences of lowering its bid from V_m, a bidder will have to consider two effects. On the one hand, while a bid at V_m is bound to succeed, the lower bid may fail; in this case, lowering the bid would produce a loss, both because the bidder would lose on shares purchased through the failing bid and because the bidder would lose the potential gain of DN that bidding at V_m could produce. On the other hand, the lower bid may succeed; in this case, lowering the bid would produce a saving of x on each share purchased through the successful bid.

We have seen that in the no-dilution case of D=0, V_m=V_t, it would be indeed optimal for the bidder to lower its bid from V_m to $(V_m+V_o)/2$. In comparison to this no-dilution case, the presence of dilution increases the cost of lowering the bid from V_m: in the presence of dilution, a bid at V_m would produce a gain of DN, and any lowering of the bid from this level would increase the likelihood of losing this potential gain. Thus, it seems reasonable to suspect that whether the optimal bid would be lower than V_m -- and, if so, by how much -- would depend on the size of D relative to I. In particular, it seems reasonable to suspect that increasing D makes it less likely that the optimal bid will be below V_m or, if the optimal bid is below V_m, that the optimal bid fall below V_m by a substantial margin. And indeed, such a relationship is suggested by the following two propositions, which are proven in the appendix.

<u>Proposition</u> <u>9</u>: Supposing that the raider is going to bid, a sufficient condition for the optimal bid to be below V_m is that D satisfy

(20) $$D < \frac{1-k}{2-k} I \quad .$$

Proposition 10: Supposing that the raider is going to bid, a sufficient condition for the optimal bid to be above $V_m - \alpha D$ is that D satisfy

$$(21) \qquad D \geq \frac{[k(1-k)/2\pi\alpha^2 N]^{1/4}}{1 + [k(1-k)/2\pi\alpha^2 N]^{1/4}} \; I \quad .$$

To illustrate, suppose for example that k=0.50 and that N=1,000. Then, by proposition 9, a sufficient condition for the optimal bid to be below V_m is that D is no greater than 0.33*I. And, by proposition 10, a sufficient condition for the optimal bid to exceed V_m-D/4 is that D is not lower than 0.15*I.

IV. CONCLUDING REMARKS

This paper has extended the Grossman-Hart analysis to allow for bids whose outcome cannot be predicted in advance with certainty. It has been shown that, once such bids are introduced, the proposition that all successful bids must be made at or above the expected value of minority shares does not always hold. Bids below the expected value of minority shares may succeed, may be profitable, and may be used.

A crucial distinction that the analysis has highlighted is the difference in consequences between conditional bids and unconditional bids. While conditional bids below V_m cannot succeed, unconditional bids may. The power of unconditional bids results from the fact that they offer shareholders the prospect of having their shares acquired even if the bid fails. Because of shareholders' desire to have their shares acquired in such a case, a failure with certainty cannot be a rational expectations outcome of an unconditional bid. Thus, what might enable such a bid to succeed is the possibility that it will fail.

Note that the paper's analysis assumed that all the target's shareholders have the same information. In this framework, the shareholders' uncertainty about the bid's outcome (which led some of them to tender) was solely rooted in their use of mixed strategies. It should be possible, however, to construct a model in which the shareholders' uncertainty about the outcome is rooted, at least in part, in differences among the shareholders in their estimates of V_m or V_t.

It should be emphasized that, although the Grossman-Hart conclusions concerning takeover bids must be refined in the way described above, the main insights offered by their analysis do remain: the importance of dilution in inducing bids, and the importance of the expected value of minority shares in determining tender offers and bid outcomes. In particular, although it was shown that bids might be made even in the absence of dilution, there should be no question that the presence of dilution is important in explaining bids and that some amount of dilution is desirable to both society and target shareholders.

APPENDIX

Proof of Proposition 4:

Differentiating the right-hand side of (11) with respect to B gives:

$$\text{(A1)} \qquad W'(B) = \sqrt{N/2\pi}\ I\ \sqrt{\tau(1-\tau)}\ e^{-1/2[q(\tau)]^2}[-q(\tau)]\ q'(\tau)\ \tau'(B)\ +$$

$$+\ \sqrt{N/2\pi}\ I\ \frac{1}{2\sqrt{\tau(1-\tau)}}\ (1-2\tau)\ e^{-1/2[q(\tau)]^2}\tau'(B)\ .$$

To get $q'(\tau)$ we differentiate both sides of (3) and get

$$\text{(A2)} \qquad q'(\tau) = -\sqrt{N}\ \frac{\tau(1-k) + k(1-\tau)}{2[\tau(1-\tau)]^{3/2}} \qquad .$$

After substituting the right-hand side of (A2) for $q'(\tau)$ in (A1) and some algebraic manipulation we get

$$\text{(A3)} \qquad W'(B) = \sqrt{N/2\pi}\ I\ (1/2)[\tau(1-\tau)]^{-3/2}\ \tau'(B)\ *$$

$$*\ \left\{(1/N)(1-2\tau)\tau(1-\tau) + (k-\tau)[\tau(1-k) + k(1-\tau)]\right\}\ .$$

The sign of the right-hand side of (A3) depends on the sign of the sum in the braces (since all the terms outside the braces are positive). Since N is large, the value of the sum in braces (largely) depends on the second expression in the braces. And the second expression is positive for $\tau<k$ and negative for $\tau>k$. Thus, the optimal B is that (approximately) which would produce a τ equal to k.

Finally, from (2) it follows that the probability of a takeover associated with $\tau=k$ is 1/2. And from (5) it follows that the B which will produce such τ and $Pr(\tau)$ is $(V_t + V_o)/2$.

Proof of Proposition 9:

Consider the consequences of lowering a bid from V_m by an arbitrarily small ϵ. On the one hand, such lowering of the bid introduces a likelihood of $\epsilon/(V_m-V_o)$ that the bid will fail. Denoting by T_{nt} the expected number of tendered shares conditional on the bid's failure, the expected loss in the event that lowering the bid leads to the bid's failure is

(A4) $$D*N + T_{nt}*(V_m-\epsilon).$$

The first expression is the loss of the potential profit that a bid at V_m would produce, and the second expression is the expected loss on the shares that would have to be purchased through the failing bid.

On the other hand, the lower bid might also succeed with probability $(V_m-V_o-\epsilon))/(V_m-V_o)$, in which case lowering the bid would produce a saving of ϵ on each share purchased through the bid. Denoting by T_t the expected number of tendered shares in the event that the bid succeeds, the gain from lowering the bid in the event that the bid succeeds is

(A5) $$T_t*\epsilon .$$

Thus, a bid at $V_m-\epsilon$ will be superior to a bid at V_m if and only if

(A6) $$(1 - \frac{\epsilon}{V_m - V_0})\, T_t\, \epsilon > (\frac{\epsilon}{V_m - V_0})\, [D*N + (V_m - v_0 - \epsilon)\, T_{nt}] \quad ,$$

or, after rearranging terms, if and only if

(A7) $$T_t - T_t \frac{\epsilon}{V_m - V_0} > \frac{D}{V_m - V_0} N + (1 - \frac{\epsilon}{V_m - V_0})\, T_{nt} \quad .$$

Note that by setting ϵ sufficiently small we can set the value of the right-hand side of (A7) as close as we wish to T_t and the value of the second expression on the right-hand side as close as we wish to T_{nt}. Note also that

T_{nt} must by definition be lower than kN. And, finally, note that by setting ϵ sufficiently small we can get the equilibrium probability of tendering τ to be as close to 1 as we wish and thus also get T_t to be as close to N as we wish. Putting all of this together, it follows that a sufficient condition for (A7) to hold is that

$$\text{(A8)} \qquad \frac{D}{V_m - V_o} < 1\text{-}k \ ,$$

or, using the fact that V_m-V_o = I-D, that

$$\text{(A9)} \qquad D < \frac{1 - k}{2 - k} I \ .$$

__Proof__ __of__ __Proposition__ __10__:

If the bidder bids below V_m-αD, its expected payoff will be as given by (19). Using the analysis of proposition 4, the maximum value of the first expression on the right-hand side of (9) is $[Nk(1\text{-}k)/2\pi]^{1/2}*I$ (a maximum that is obtained at $B=(V_m+V_o)/2)$. And the second expression on the right-hand side of (19) must be smaller than $DN[\alpha D/(V_m\text{-}V_0)]$. Thus, the expected payoff to the bidder from any bid below V_m-αD cannot exceed

$$\text{(A10)} \qquad \sqrt{Nk(1\text{-}k)/2\pi}\ (V_m\text{-}V_o) \ + \ DN \frac{V_m - \alpha D - V0}{V_m - V_o} \ .$$

In contrast, if the bidder bids at V_m, its gain will be DN. Thus, a sufficient condition for the optimal bid to exceed V_m-αD is that DN exceed the right-hand side of (A10) and thus that

$$\text{(A11)} \qquad DN \frac{\alpha D}{Vm - V0} \geq \sqrt{Nk(1\text{-}k)/2\pi}\ (V_m - V0) \ .$$

And, after substituting I-D for V_m-V_o and rearranging terms we get

$$\text{(21)} \qquad D \geq \frac{[k(1\text{-}k)/2\pi\alpha^2 N]^{1/4}}{1 + [k(1\text{-}k)/2\pi\alpha^2 N]^{1/4}} I \ .$$

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