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BARGAINING AND THE DIVISION OF VALUE IN CORPORATE REORGANIZATION

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Abstract

This paper develops a sequential bargaining model of the negotiations in corporate reorganizations under Chapter 11. We identify the expected outcome of the bargaining process and examine the effects of the legal rules that shape the bargaining. We determine how much value equityholders and debtholders receive under the Chapter 11 process, and compare the value obtained by each class with the "contractual right" of that class. We identify and analyze three reasons that the equityholders can expect to obtain some value even when the debtholders are not paid in full. Finally, we show how the features of the reorganization process and of the company filing under Chapter 11 affect the division of value, and in this way we provide several testable predictions.

I. INTRODUCTION

Reorganization is one of the two alternatives open to an insolvent corporation under the Bankruptcy Code. A bankrupt corporation may file either for liquidation under Chapter 7 or for reorganization under Chapter 11 of the Bankruptcy Code.[1] Upon a filing for liquidation, a court immediately appoints a trustee to sell the firm's assets, either piecemeal or as a going concern, to outside buyers. The proceeds from this sale are divided among those who have rights against the corporation, with the division made according to the ranking of these rights by legal priority.

The firm may instead go into Chapter 11, under which the firm can be "reorganized." In reorganization there is no sale to third parties. Rather, there is a "hypothetical sale" of the firm to the existing "participants" -- all those who hold claims or rights against the insolvent company. These participants surrender their claims and rights in exchange for claims and rights against the new corporation. For example, a bankrupt company may emerge from reorganization with all its debt canceled and with the former debtholders holding some or all of the equity of the reorganized company.

If the court supervising a reorganization could observe the value of the reorganized firm, it would allocate that value among the participants according to the legal priority of their claims. For example, consider a company that has equityholders and that owes debtholders $200, and suppose that the court observes that the value of the reorganized company will be $150; in this case, the court would order that all of the reorganized company's securities be given to the debtholders, and none be given to the equityholders. Below we will use the term "contractual right" to mean that which a class would receive if the bankruptcy court could observe the firm's value and distributed it among the classes according to the initial contracts, i.e., strictly in the order of the legal priority of their claims.

Because a court cannot determine accurately an objective figure for this value,

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[1]A firm may commence a voluntary bankruptcy case by filing a petition with a bankruptcy court under section 301 of the Bankruptcy Code. For a general discussion of both bankruptcy proceedings, see White (1984, 1989). For a review of the law on business reorganizations, see Trost (1979).

however, the law leaves the division of the reorganized company's value to a process of bargaining among the classes of participants. Under section 1129(a)(8) of the Bankruptcy Code, each class of equityholders and debtholders whose interests are impaired must vote to approve a reorganization plan, which would include a division of value. The approval by a class requires (as specified by section 1126 of the Bankruptcy Code) a certain majority of the class members to vote in favor of the plan. As is generally believed by participants in reorganization and as will be shown in this paper, the outcome of this bargaining process often diverges from the contractual rights of the classes.[2]

Section II of this paper develops a sequential bargaining model of the negotiations in corporate reorganizations under Chapter 11. We identify the expected outcome of the bargaining process and examine the effects of the legal rules that shape the bargaining. We determine how much value each class will receive and what factors give each class an advantage or a disadvantage in the bargaining process. We also compare the share of the reorganized firm's value that each class obtains under the existing legal regime with the contractual right of that class.[3]

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[2]Note that bargaining in Chapter 11 is very different from debt renegotiations prior to Chapter 11 filing, in which each debtholder who does not agree individually to surrender some claims must be paid in full. As we will discuss in Section II.F in greater detail, this difference creates possible "holdout" problems outside Chapter 11 and makes prior debt renegotiation much more difficult.

[3]In spite of the growing interest by economists in corporate reorganization (see, e.g., Cutler and Summers (1988) and Franks and Torous (1989)), little work has been done thus far to model bargaining under Chapter 11. Brown (1989) developed a model of this bargaining, but his model does not include certain important features of the process which we seek to incorporate. First, whereas time plays no role in the reorganization game analyzed by Brown, we analyze the implications of the critical fact that the reorganization game may be played over a significant period -- that, upon filing for Chapter 11, a company may be kept in Chapter 11, protected from creditors, for some time. As our model shows, the bargaining outcome is very much shaped by the possibility of such a delay, because the value available for division may well change with time due to the realization of uncertainty and the company's incurring financial distress costs. Moreover, whereas in Brown's model no class can make many offers, we develop, consistent with our assumption about the Chapter 11 period, a full sequential bargaining model in which each class can make many offers over the Chapter 11 period.

In surveying past work, note should be made of the related literature about debt renegotiation (see, e.g., Bergman and Callen (1990), Hart and Moore (1989), Giammarino (1989), and Webb (1987)). (Of these papers, the closest to ours in modelling approach is Bergman and Callen, which also uses a sequential bargaining model, though not one in which the firm value may fluctuate during the bargaining period due to the realization of uncertainty.) Debt renegotiation prior to formal bankruptcy proceedings, however, differs from that within such proceedings. (Indeed, pre-bankruptcy bargaining is always shaped by the parties' expectations about what will happen should the company file for Chapter

The critical features of the situation that we model are as follows. Once an insolvent company files for reorganization under Chapter 11, an "automatic stay" prevents debtholders from seizing the company's assets as long as the company is in Chapter 11. Of course, the company will not remain in Chapter 11 indefinitely; if there is no agreement on a reorganization plan, eventually the supervising court would convert the bankruptcy proceedings to a Chapter 7 sale. For the debtholders to obtain any value before such a conversion, however, the two classes, equityholders and debtholders, must agree on a division of value (i.e., on a reorganization plan).

If the parties do not agree immediately, and the company remains in Chapter 11 for some time, then the value available for division may well change, for two reasons. First, the company would incur some "financial distress costs," to be discussed later, which would reduce its value. Second, uncertainty will be realized: random shocks may increase or decrease the firm's value. Furthermore, if the two classes ultimately do not reach agreement, and the assets are sold under Chapter 7, such a sale might sometimes involve a loss of value. These potential consequences of the parties' failure to reach agreement are important elements of our model, because they provide the background against which the parties would decide which offers to make or accept.

Practitioners and observers of corporate reorganization, e.g., Franks and Torous (1989), White (1989, p. 147) and Trost (1973, p. 550), generally believe that Chapter 11 often enables equityholders to obtain a share of the value of the reorganized company even when that value is less than sufficient to cover debtholders' claims. It is observed that debtholders will agree to a plan that gives the equityholders a share of this value even when the debtholders are not paid in full. Our model confirms this common perception. More

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11 or 7, and the above literature therefore makes various assumptions about the outcome of such a filing -- assumptions that can be justified ultimately only by constructing a model of the bargaining within bankruptcy proceedings.) Our focus, of course, is on bargaining within Chapter 11 with special emphasis on how the legal rules of Chapter 11 shape this process.

Finally, after developing our model, we learned about an independent effort to develop a sequential bargaining model of Chapter 11 by Baird and Picker (1990). While they share our approach, their model differs substantially from ours; in particular, in their model the company's value is assumed not to change during the Chapter 11 period.

important, it identifies and analyzes three sources of the equityholders' ability to obtain value and the parameters that determine how much value they will obtain.

As will be shown, Chapter 11 gives equityholders the ability to obtain value, even though the company is insolvent. The fact that their consent is necessary for a division of value enables equityholders to obtain value for three reasons. The first two reasons arise from the ability of the equityholders to delay the adoption of a reorganization plan from the beginning of the Chapter 11 period to a later date. First, if equityholders delay agreement, there may be a favorable resolution of uncertainty that would cause the value of the firm to exceed the value of its debt. Thus, the equityholders have an "option value," and to forego it they must be compensated by the agreed distribution of value. Second, if the equityholders withhold their consent and thereby delay agreement, the company will be expected to incur "financial distress costs" that will erode the value that debtholders can expect to receive. Thus, the equityholders' consent to a division of value can save the firm "financial distress costs," and therefore they can obtain a share of these savings. The fraction of these savings that they obtain depends upon the rules that govern the Chapter 11 bargaining. In particular, the equityholders in effect have an exclusive power to propose reorganization plans for a period of time, which increases the fraction of the savings of "financial distress costs" that the equityholders can capture.

The third reason that the equityholders can obtain value (even when the value of the reorganized firm is insufficient to pay the debt in full) arises when the parties expect a Chapter 7 sale to entail a loss of value. In this case, the equityholders' consent to a division of value is necessary to avoid this loss. Therefore, they can obtain a fraction of the value gained by avoiding a Chapter 7 sale.

The model in Section II reveals how much each of the elements described above contributes to the value that equityholders will receive. Section II also presents comparative statics analysis. In particular, the amount that equityholders will receive tends to increase in (1) the volatility of the value of the company's assets, (2) the extent to which reorganization imposes "financial distress costs," (3) the length of the reorganization period,

(4) the length of the period during which the equityholders have the exclusive right to make offers, (5) the extent to which liquidation imposes a loss in value, and (6) the extent to which the value of the company's assets covers the company's debts. As will be discussed, these results provide several testable implications of the model.

Finally, Section III offers concluding remarks on some issues for further research. While this paper focuses on the positive analysis of the bargaining process under Chapter 11, this analysis is a prerequisite for a normative analysis of the efficiency costs of Chapter 11. Accordingly, Section III discusses the implications of the model for the welfare effects of Chapter 11.

II. THE MODEL

A. Framework of Analysis

We consider a company with one class of equity and one class of debt that files for bankruptcy under Chapter 11. Let V be the value of the company's assets at the time of filing.[4] Upon filing for bankruptcy, however, the company owes an amount D to debtholders, where D>V. Thus, the company is insolvent.

Faced with this insolvency, the company files for Chapter 11 at time t = 0. The firm is to emerge from the reorganization with an all-equity structure or some other capital structure that avoids the "financial distress costs" to be described below. Under Chapter 11, the corporate reorganization process ends when the various classes of investors accept a plan including a division of value. Reorganization is not open-ended, however: if the parties fail to agree on a reorganization plan, the supervising court would eventually convert the bankruptcy proceedings to a Chapter 7 liquidation (usually following a petition by creditors). For concreteness, we assume that if the parties fail to reach an agreement within a period T, the company will be liquidated. The firm remains in Chapter 11 reorganization until time

[4]More specifically, V is the value that the company would have if its capital structure were such that the firm is not expected to incur the "financial distress costs" to be described below.

t = T unless the parties reach agreement before then.

For an agreement on the distribution of value, then, one class must propose a reorganization plan, and then both classes must accept it. The rules governing the solicitation of acceptances, however, require some minimum time to pass between offer and response. In particular, section 1125(b) of the Bankruptcy Code requires that after the filing of a Chapter 11 case, acceptances may not be solicited unless the plan is transmitted to the accepting class with a written disclosure statement, which the supervising court must approve as containing adequate information after notice and a hearing. Furthermore, Bankruptcy Rule 3017(a) requires the court to hold this hearing on at least 25-days notice to the parties in interest. These notice and hearing requirements introduce a delay -- no offer can be accepted immediately. In this respect, bargaining in the reorganization context differs from that in other contexts, in which each round of bargaining -- an offer and the response to it -- can take a very short period of time.

Let Δt be the length of one round of bargaining, i.e., the time required until a proposed reorganization plan can be accepted or rejected under Chapter 11. Let $n = T/\Delta t$, assumed for simplicity to be an integer greater than 1, be the number of bargaining rounds during the reorganization period. Let V_i denote the value of the company if the parties adopt a reorganization plan at the end of round i, where $i = 1, ..., n$.[5]

The firm's value V_i evolves over time. That is, if the parties approve a reorganization plan in any round after round 1, the value divided might differ from V_1. Specifically, if delay were to occur, the value may change for the following two reasons. One reason is that the firm will bear "financial distress costs" -- efficiency costs that the firm must incur while it is in Chapter 11 and that it would not incur if it had a new capital structure. There are several

[5]Once the firm is reorganized and its shares are distributed, its new shareholders would obviously have the power to select the company's management. We do not analyze the possibility that either the equityholders or the debtholders have any special management skills necessary for running the company. Ignoring this possibility appears quite appropriate for publicly traded firms which are often run by professional managers. Baird and Picker (1990), who focus on the reorganization of closely held firms with a small group of shareholders-managers, consider the interesting implications of the presence of shareholders-managers with some special management skills.

sources of financial distress costs. First, Chapter 11 bankruptcy involves significant administrative costs; indeed, the mere fees in a Chapter 11 reorganization of a publicly traded company are often in the order of tens of millions of dollars (see, e.g., Sontag (1990)). Second, potential business partners may be reluctant to deal with the company or demand especially favorable terms, for those doing business with a firm in financial distress may incur greater information costs, monitoring costs, enforcement costs, and collection costs. Third, financial distress might lead to inefficient management decisions, especially with respect to the choice of projects and investments (see, e.g., Myers (1977), Hart and Moore (1990)). We assume that as a result of these financial distress costs, the company loses an amount of its value continuously over time at the rate of α per period Δt, i.e., per round of bargaining, as long as the company is in Chapter 11.[6]

The second reason for V_i to change over time is that uncertainty will be realized. Random shocks may increase or decrease the value of the company's assets. We assume that during each round of bargaining, unpredictable events would cause this value to be either higher or lower than expected by the amount θ, and that either state of the world occurs with equal probability. The parameter θ, then, represents the volatility of the firm's value. For concreteness, we assume that at the beginning of each round, information is revealed that causes the value of the firm to either rise or fall instantaneously by the amount θ.

Let the value realized after revelation of information at the start of round 1 be V and V_i be the value at the end of round i. Thus, $V_1 = V-\alpha$, and $V_i = V_{i-1}+\delta_i$ for i = 2, ..., n, where $\delta_i = -\alpha-\theta$ with probability ½, and $\delta_i = -\alpha+\theta$ with probability ½. The value of δ_i is realized at the start of round i, before the i-th offer is made. We assume $0<\alpha<\theta$, so that the company's

[6]Note that the financial distress costs in our model result from the firm's mere presence in Chapter 11 and would be eliminated only if a reorganization agreement were reached (creating a sound financial structure). We abstract from the possibility that the shareholders might be able to cause some deterioration beyond the one resulting from the firm's being in Chapter 11 and, seeking to get concessions from the debtholders, might threaten to do so. Bergman and Callen (1990) present an interesting model of threats by shareholders of solvent firms to deliberately cause some deterioration in the situation of the firm. As they discuss, a critical question with respect to such threats is that of credibility. The credibility issue does not arise with respect to the financial distress costs in our model, because these costs would be incurred, whether or not the shareholders wish to have them, as long as the firm is in Chapter 11.

value will increase with probability ½ despite the financial distress costs. Nevertheless, the expected value of δ is $-\alpha$. We also assume that $(n-1)(\alpha+\theta) < V_1$, so that the company's value cannot disappear completely during the reorganization period.[7]

We also assume the following timing of events. In each round, once the uncertainty is realized, one party proposes a reorganization plan, i.e., a division of the value that will exist at the end of the round. At the end of the round, after the delay created by the rules governing the solicitation of acceptances, the value has deteriorated by the amount α. At this point, the two classes decide whether to accept or reject the plan. The class proposing the plan, naturally, votes to accept the proposal. If the other class also accepts the proposal, the plan is confirmed, and the firm immediately emerges from Chapter 11 with its value divided according to the plan. If the other class instead rejects the proposal, then the parties go to the next round of bargaining.[8]

As in any sequential bargaining model, it is necessary to specify a procedure that determines which party makes the offer in each round. In the case of Chapter 11 bargaining, the Bankruptcy Code determines some of this bargaining protocol. Section 1121(b) of the Bankruptcy Code establishes a period during which only the debtor (i.e., the incumbent management) has the right to propose reorganization plans. The incumbent management generally will have at least six months in which to produce a reorganization plan and obtain the necessary acceptances, and courts may grant extensions of this period (see Trost (1979, p. 1325)). During the first several months of reorganization, then, usually the parties can adopt only a plan proposed by management.

Although the insolvent corporation and its management have fiduciary duties to

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[7]For simplicity, we can assume that the interest rate is zero, so that there is no discounting. Alternatively, we can allow for a positive interest rate and interpret all variables representing values, such as V, V_1, and D as present discounted values.

[8]The above description of the sequence of events assumes implicitly that each class of claimants acts as a single agent. Given our assumptions, there is no conflict of interest among the class members, and they will all agree on whether a plan should be accepted or rejected. Furthermore, there is no advantage to casting a dissenting vote, because under Chapter 11 each class binds all of its members by majority vote.

creditors as well as shareholders, most observers expect management to favor the interests of shareholders. As Normandin (1989, pp. 56-58) notes, management not only is elected by the shareholders but also often holds a substantial interest in the corporation's stock. Consistent with this observation, this paper assumes that with respect to proposals to divide the value of the reorganized firm between equityholders and debtholders, management acts on behalf of the equityholders.[9]

To represent the period during which the management has the exclusive right to propose plans, we assume that the equityholders make at least the first offer and possibly the first several offers. That is, they have the exclusive right to make offers for the first e rounds, where e is an integer such that $0<e<n$.[10] Over the remaining n-e rounds, both classes may make offers. (This possibility is indeed contemplated by Chapter 11, and firms often remain in Chapter 11 even after the exclusive period runs out.) To capture the possibility of either party making the offer in any given round, we assume that at the beginning of each of these n-e rounds, the identity of the class making the offer is determined randomly, with the probability of each equal to ½.[11]

If the parties were to fail to reach agreement in round n, we assume that the court would convert the proceedings to a Chapter 7 liquidation. In that event, the assets of the company would be sold. Let the value obtained through such a sale be L, which would then be divided according to the legal priority of the parties' claims. That is, equityholders get an amount $VE_L = \max(0, L-D)$, and debtholders get the complementary share, $VD_L = L - \max(0,$

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[9]It is not our assumption that management acts in the equityholders' interests in all respects. As Hart and Moore (1990, p. 4) argue, the interests of the equityholders and of management may well diverge with respect to certain investment decisions. With respect to management's proposals to divide the value of the reorganized firm between equityholders and debtholders, however, we believe it reasonable to assume that management will prefer the interests of equityholders.

[10]We can easily extend our analysis to the case in which e = n. See footnotes 13, 15, and 16.

[11]Having each party make the offer with probability ½ in each round is one of the two conventional alternatives in modelling a situation in which two parties can make offers. The second alternative is to assume that the two parties alternate in making offers. In a model with a finite number of rounds (such as ours), however, the first alternative ensures that the division of value is not sensitive to whether the number of rounds, n, is even or odd. We prefer the first method because it renders it unnecessary to make such arbitrary assumptions regarding n.

L-D) = min(L, D). At this point, we will assume that $L = V_n$; that is, we assume that the sale procedure occurs immediately and does not involve any loss of value. In Section II.E, however, we will drop the assumption that $L = V_n$ and extend the analysis to cover the case in which $L<V_n$.

We also assume that all parties are risk neutral. Thus, each class seeks to maximize the expected value of its share. Finally, we assume that the structure of the bargaining game described above, including the values of the parameters, V_1, D, L, α, θ, T, e, and n, is common knowledge to the participants.

B. The Division of Value

Under the assumption of common knowledge, the parties will reach an agreement at the end of round 1. As in the sequential bargaining game analyzed by Rubinstein (1982), to obtain agreement, the party making the proposal in round i would offer the other at least what the other could obtain in expected value in round i+1, where i = 1, ..., n-1. Each party would offer just this amount to the other in order to maximize its own share. It cannot expect to do better by asking for any larger amount, which would delay agreement and allow the other party to make the next offer.

The party whose turn it would be in round i, denoted by X, offers to take the amount VX_i of the total value V_i. By the argument above:

$$VX_i = V_i - E_i[V_{i+1} - VX_{i+1}], \tag{1}$$

where VX_{i+1} denotes the amount X would obtain under the plan proposed in the next round, and E_i denotes the expected value conditional on information available in round i, i.e., conditional on the realized value of V_i. By our assumptions about δ, we also know that the expected "efficiency gain" from an agreement in round i rather than round i+1 is:

$$V_i - E_i[V_{i+1}] = -E_i[\delta_{i+1}] = \alpha. \tag{2}$$

Using (2) to substitute into (1):

$$VX_i = \alpha + E_i[VX_{i+1}]. \quad (3)$$

Thus, when the party making the offer, X, holds the other to its expected payoff next round, X thereby takes for itself the expected "efficiency gain," α, plus its own expected payoff next round.

Under liquidation at the end of round n, equityholders would receive VE_L, and debtholders would receive VD_L, where $VE_L + VD_L = L = V_n$. In round n, the party making the offer cannot gain by proposing any other division, because the other party will reject any proposal offering it a smaller share. Therefore, $VE_n = VE_L$ and $VD_n = VD_L$. The party making the nth offer cannot capture any "efficiency gain" from avoiding financial distress costs, because we have assumed here that liquidation occurs costlessly and immediately upon rejection of the offer.

We solve for the unique subgame perfect equilibrium by backwards induction. In round n-1, the party making the offer would hold the other to its expected round n payoff and take for itself the expected "efficiency gain" (from agreement in round n-1 rather than round n), α, plus its own expected round n payoff. Similarly, in round n-2, the offering party would hold the other party to its expected round n-1 payoff (which is $\frac{1}{2}\alpha$ plus its expected round n payoff, if it would make the offer in round n-1 with probability $\frac{1}{2}$) and take the balance. That balance amounts to the expected "efficiency gain" (from agreement in round n-2 rather than round n-1), α, plus its own expected round n-1 payoff. Its expected round n-1 payoff, in turn, is also $\frac{1}{2}\alpha$ plus its expected round n payoff, if it would make the offer in round n-1 with probability $\frac{1}{2}$.[12]

In sum, in each round, each party receives its expected round n payoff plus α times the number of offers it would expect to make from the current round to round n-1. In round 1, therefore, the equityholders receive their expected round n payoff plus α times the number

[12]All expectations are taken conditional on information available at the time of the current offer, i.e., in round n-2. Recall that by the law of iterated expectations, the expectation of an expectation conditional on more information is simply the unconditional expectation. In the example above, $E_{n-2}[E_{n-1}[VX_n]] = E_{n-2}[VX_n]$.

of offers they would expect to make before round n. Because each party would expect to make ½(n-1-e) offers from round e+1 to round n-1, and the equityholders in addition would make all e offers from round 1 to round e, the equityholders receive:

$$VE_1 = \alpha e + \tfrac{1}{2}\alpha(n-1-e) + E_1[VE_n], \tag{4}$$

and the debtholders receive:

$$VD_1 = \tfrac{1}{2}\alpha(n-1-e) + E_1[VD_n], \tag{5}$$

at the end of round 1. Moreover, because we assume liquidation is costless, (4) equals:

$$VE_1 = \tfrac{1}{2}\alpha(e+n-1) + E_1[\max(0, V_n-D)], \tag{6}$$

and (5) equals:

$$VD_1 = \tfrac{1}{2}\alpha(n-1-e) + E_1[\min(D, V_n)]. \tag{7}$$

The second term in (6) is the expected value of what equityholders would receive if there is no agreement through round n and there is instead liquidation. In the alternative, we may express this expected value as:

$$E_1[\max(0, V_n-D)] = \Pr(V_n>D)E_1[V_n-D\,|\,V_n>D], \tag{8}$$

where $\Pr(V_n>D)$ is the probability of $V_n>D$. We can also express this expected value as a function of our basic parameters, making use of the particular probability distribution of V_n:

Lemma 1: The expected values of what the parties would receive from a costless liquidation if they failed to reach agreement by time t = T are as follows:

$$E_1[\max(0,V_n-D)] = \sum_{k=0}^{n-1} (\tfrac{1}{2})^{n-1}\binom{n-1}{k}\max[0,V_1-D-(\alpha+\theta)(n-1)+2\theta k] \tag{9}$$

for the equityholders, and:

$$E_1[\min(D,V_n)] = \sum_{k=0}^{n-1} (\tfrac{1}{2})^{n-1}\binom{n-1}{k}\max[D,V_1-(\alpha+\theta)(n-1)+2\theta k] \tag{10}$$

for the debtholders.

Proof: Each round is a Bernoulli trial in which δ may take on one of two values with equal probability, so n-1 such trials from round 2 to round n leads to 2^{n-1} equally likely sequences of δ_i. Let k represent the number of times δ takes on the high value, $\theta-\alpha$, from the end of round 1 to the end of round n, so that n-1-k is the number of times it takes on the low value, $-\theta-\alpha$. The number of distinct sequences with the same k is given by the binomial coefficient. Thus, k is a random variable, taking values from 0 to n-1, that follows the binomial distribution:

$$b(k,n-1) = (\tfrac{1}{2})^{n-1}\binom{n-1}{k} = (\tfrac{1}{2})^{n-1}\frac{(n-1)!}{k!(n-1-k)!}\ . \tag{11}$$

Given any set of values for the parameters known in round 1, including V_1, there are n possible values for V_n, because there are n possible values for k:

$$V_n(k) = V_1-\alpha(n-1)+\theta k-\theta(n-1-k) = V_1-(\alpha+\theta)(n-1)+2\theta k. \tag{12}$$

Therefore, the probability of each $V_n(k)$ is given by the binomial formula in (11). Together (11) and (12) yield the above lemma. ■

Using (9) and (10) to substitute into (6) and (7), respectively, we can conclude with the following proposition:[13]

Proposition 1: The two classes of claimants will adopt a reorganization plan in round 1, with the equityholders obtaining:

[13]The above discussion assumes that e < n. As long as we assume that liquidation is costless, the division of value when e = n is simply the same as that implied by this proposition when e = n-1. That is, if a Chapter 7 sale involves no loss of value, then it does not matter which class makes the offer in round n.

$$VE_1 = \tfrac{1}{2}\alpha(e+n-1)+\sum_{k=0}^{n-1}(\tfrac{1}{2})^{n-1}\binom{n-1}{k}\max[0,V_1-D-(\alpha+\theta)(n-1)+2\theta k], \quad (13)$$

and the debtholders obtaining the complementary share:

$$VD_1 = \tfrac{1}{2}\alpha(n-1-e)+\sum_{k=0}^{n-1}(\tfrac{1}{2})^{n-1}\binom{n-1}{k}\min[D,V_1-(\alpha+\theta)(n-1)+2\theta k]. \quad (14)$$

C. The Sources of the Equityholders' Power

Although debtholders' claims exceed the value of the firm in round 1, equityholders receive a round 1 share greater than 0. We can isolate two distinct sources of the equityholders' power to obtain more than its contractual right. The equityholders' share, VE_1 as expressed in (13), is composed of two terms. Each term corresponds to a different reason for equityholders to get a positive share even though $D>V_1$. (In Section II.E, when we introduce the possibility that conversion to Chapter 7 liquidation may cause the firm to lose value, there will be a third source of the equityholders' power to obtain value.)

1. The Financial Distress Costs Created by Delay

The total "efficiency gains" from reaching agreement in round 1 rather than leaving the firm in Chapter 11 for n rounds is $\alpha(n-1)$, because such agreement thereby avoids the deterioration in firm value that would occur during the next n-1 rounds. Because the consent of the equityholders is necessary to avoid incurring these financial distress costs, the equityholders can obtain part of these savings in exchange for that consent. The first term in (13) is the equityholders' share of these savings and represents the gain to equityholders from the presence of financial distress costs. If we let α go to 0 in the limit, then the savings from avoiding delay also go to 0, and the equityholders would receive only the second term, $E_1[VE_n]$.

The first term in (13), $\frac{1}{2}\alpha(e+n-1)$, may be expressed as the sum of two elements, as in (4). First, during the first e rounds, when the equityholders have the exclusive power to make offers, they expect to capture all the savings from agreement in round 1 rather than

in round e+1. This surplus amounts to αe. Second, during the last n-e rounds, when both the equityholders and the debtholders can make offers, each class expects to capture half of the savings from agreement in round e+1 rather than in round n. Therefore, the equityholders can obtain half of this surplus, i.e., $\frac{1}{2}\alpha(n-1-e)$, in round 1.

2. The "Option Value" Created by Delay

The second term in (13) represents the gain to equityholders from the possibility of V_n greater than D. The equityholders could deny their consent to any plan and delay a division of value until round n. Given the volatility of V_i, there is some chance that at the end of round n, V_n will exceed D. Although the expected value of V_n is less than V_1, the actual value of V_n might exceed V_1 and even D. The Chapter 11 process, by giving the equityholders the ability to insist on delay, gives equity the option to receive the difference V_n-D. To give up this option, equityholders must get compensation in return.

Specifically, equityholders receive the expected value of VE_n, i.e., the full value of the equityholders' "option." This "option value" is positive if and only if $Pr(V_n>D) > 0$. Furthermore, $Pr(V_n>D) > 0$ if and only if $V_n>D$ when all uncertainty is resolved favorably, i.e., $\delta = \theta-\alpha$ in each round, so that k = n-1 in (12). Therefore, the following two conditions are equivalent:

$$Pr(V_n>D) > 0 \Leftrightarrow (n-1)(\theta-\alpha) > D-V_1. \tag{15}$$

D. Comparative Statics

The equityholders' share of V_1 consists of two terms that depend on the following parameters: the initial undercoverage of debt, $D-V_1$; the volatility of the firm's value from period to period, θ; the financial distress costs per unit of time Δt, α; the length of the reorganization period measured in rounds of bargaining, n; and the length of the period during which the equityholders control the agenda, also measured in rounds of bargaining, e. We will now examine how each of these parameters affects the equityholders' share, VE_1.

1. The Initial Undercoverage of Debt

By the initial undercoverage of debt, we mean the extent to which the debt exceeds the value of the firm's assets at the end of round 1, $D\text{-}V_1$.

Proposition 2: The share of V_1 obtained by equityholders, VE_1, is nonincreasing in $D\text{-}V_1$, and thus in D. Specifically, if $D\text{-}V_1 < (n\text{-}1)(\theta\text{-}\alpha)$, then increasing $D\text{-}V_1$ will cause VE_1 to decrease. If $D\text{-}V_1 \geq (n\text{-}1)(\theta\text{-}\alpha)$, then further increasing $D\text{-}V_1$ will have no effect on VE_1.

Remark: The intuition underlying Proposition 2 is as follows. A rise in $D\text{-}V_1$ affects only the second term in (13). In particular, it shifts down the entire probability distribution of V_n-D in the expression for the equityholders' "option value" in (13). That "option value" arises from the possibility of V_i rising from V_1 to above D due to positive shocks. Because a higher $D\text{-}V_1$ makes this event less likely, it reduces this "option value." Thus, as shown in the proof, a rise in $D\text{-}V_1$ cannot make equityholders better off, and if $D\text{-}V_1$ is small enough to imply $Pr(V_n>D) > 0$, then such a rise must cause their "option value" (and their share of V_1) to decrease.

Proof: See the Appendix.

2. The Volatility of Firm Value

The volatility of the firm's value over time, represented by θ, depends upon the nature of the firm's business and thus may vary greatly from firm to firm.

Proposition 3: The share of V_1 obtained by equityholders, VE_1, is nondecreasing in θ. Specifically, if $\theta < \alpha + (D\text{-}V_1)/(n\text{-}1)$, then increasing θ will have no effect on VE_1. If $\theta \geq \alpha + (D\text{-}V_1)/(n\text{-}1)$, then increasing θ will cause VE_1 to increase.

Remark: The intuition underlying Proposition 3 is as follows. The volatility of the firm's value affects only the second term in (13). As shown in the proof, the greater that volatility, θ, the greater the likelihood that V_n will exceed D, and the greater will be the equityholders'

"option value." Thus, a rise in θ must leave equityholders at least as well off as before, and if θ is large enough to imply $Pr(V_n \geq D) > 0$, then their "option value" (and their share of V_1) must increase in θ.

Proof: See the Appendix.

3. The Financial Distress Costs

The financial distress costs per unit of time that the firm remains in Chapter 11, α, is likely to vary greatly among firms. For example, a company that owns a number of buildings leased under long-term leases would incur small financial distress costs (because few transactions would have to take place during the Chapter 11 period) relative to a company that deals frequently with many business partners and makes frequent investment decisions.

Proposition 4: The share of V_1 obtained by equityholders, VE_1, is increasing in α.

Remark: The intuition underlying Proposition 4 is as follows. The size of the financial distress costs, α, will affect both terms in (13). Thus, a rise in α has two effects. On the one hand, it increases the financial distress costs that can be saved if the equityholders consent to a plan, thus improving their bargaining position. On the other hand, it decreases their "option value" because it makes it less likely that V_i can climb beyond D. If α increases to the point that $Pr(V_n > D) = 0$, then the equityholders' share is affected by α only through the first effect and therefore a rise in α makes equityholders unambiguously better off. Furthermore, as shown in the proof, the first effect is always greater than the second, so that the equityholders' share increases in α.

Proof: See the Appendix.

4. Length of the Reorganization Period

The length of the reorganization period, represented in our model by T (or equivalently

by the number of rounds n), depends on how long the bankruptcy court will wait, in the face of failure to work out a reorganization plan, before converting from Chapter 11 to Chapter 7. Let us consider increases in T by multiples of Δt, which would cause n to rise by an integer, so as to maintain the structure of the model which assumes that n is an integer.

Proposition 5: The share of V_1 obtained by equityholders, VE_1, is nondecreasing in n. Furthermore, VE_1 is increasing in n if either $(D-V_1) + \alpha(n-1) \geq \theta$, as is likely, or n is odd.

Remark: The intuition underlying Proposition 5 is as follows. A rise in T affects both terms in (13) through n. First, it raises the total financial distress costs that the firm would incur if equityholders never gave their consent to a plan. This effect strengthens their bargaining power. Second, once $Pr(V_n>D) > 0$, an increase in n also would have an effect on their "option value." As shown in the proof, some increases will cause an upward change in the "option value" (because high values for V_n become more likely) whereas others will cause a downward change (because the expected value of V_n falls). We show in the proof, however, that even in the case of a downward change, the overall effect on the equityholders' share is nonnegative. That is, in those cases in which a rise in n causes equity's "option value" to fall, the positive effect upon equity's share of the "efficiency gains" would be at least as great as the absolute value of the negative effect. Furthermore, the proof shows that under conditions that ensure $Pr(V_n>D) < ½$, which is likely to hold for plausible parameter values, the positive effect must strictly dominate the negative effect.

Proof: See the Appendix.

5. The Equityholders' Initial Agenda Control

As already noted, the equityholders have control over the agenda for the first six months of bargaining, and the courts often extend this period. Our model captures this feature of Chapter 11 by assuming that all offers in the first e rounds, $0<e<n$, must be made by the equityholders. The size of e depends on the courts' willingness to extend the initial

six-month period.

Proposition 6: The share of V_1 obtained by the equityholders, VE_1, is increasing in e. Specifically, any unit increase in e causes VE_1 to increase by $\frac{1}{2}\alpha$.

Remark: The intuition underlying Proposition 6 is as follows. Control over the reorganization agenda is valuable. During each round in which the equityholders have this control, they would capture all of the financial distress costs, α. In any round in which both sides can make offers, however, the equityholders expect to capture only $\frac{1}{2}\alpha$. The incremental gain to the equityholders from any unit increase in e is therefore $\frac{1}{2}\alpha$. The greater are the financial distress costs, the greater is the value of this unit increase (the derivative of the equityholders' gain with respect to α is $\frac{1}{2}$). Furthermore, a higher e implies that a rise in α benefits equityholders even more than it would otherwise, because only they gain bargaining power from the deterioration of firm value from round 1 to round e+1. The derivative of the "efficiency gains" component in (13) with respect to α is $\frac{1}{2}(e+n-1)$, and each unit increase in e raises this derivative by $\frac{1}{2}$.

Proof: The proof is clear from the above remark.

E. Loss from Chapter 7 Liquidation

Until now we have assumed that a sale of the firm's assets under Chapter 7 would not impose any efficiency loss. That is, we assumed that after n rounds a Chapter 7 sale would produce a value of $L = V_n$. This assumption would be reasonable if one adopted the view of Baird (1986). Under that view, there is no reason to expect that a sale under Chapter 7, properly administered, would produce a value significantly below the firm's going-concern value. If the firm has greater value as a going concern, then the assets would be sold as a going concern.

Baird's view of Chapter 7, however, is by no means universally accepted. Many scholars and players in corporate insolvency believe, correctly or incorrectly, that Chapter 7

would involve some efficiency loss -- that is, it might not bring in the full value V_n. For this reason, we now introduce the possibility that a Chapter 7 sale at t = T would produce a value $L<V_n$.[14] Specifically, let $L = V_n-\lambda$, where $0 \le \lambda < V_1-(n-1)(\theta+\alpha)$, so that $0<L\le V_n$ in all states of the world, i.e., for any k. All other assumptions are as before. As the following proposition indicates, the loss in value from liquidation introduces a third source of the equityholders' bargaining power.

Proposition 7: If the two classes of claimants expect a Chapter 7 sale to involve a loss of value λ, then they will adopt a reorganization plan in round 1, with the equityholders obtaining:

$$VE_1 = \tfrac{1}{2}\alpha(e+n-1)+\tfrac{1}{2}\lambda+\sum_{k=0}^{n-1}(\tfrac{1}{2})^{n-1}\binom{n-1}{k}\max[0,V_1-D-\lambda-(\alpha+\theta)(n-1)+2\theta k]. \qquad (16)$$

Proof: By the same reasoning used in Section II.B -- backward induction from round n to round 1 -- the equityholders would obtain (4), i.e., $VE_1 = \tfrac{1}{2}\alpha(e+n-1) + E_1[VE_n]$. If $L<V_n$, however, then VE_n will not equal VE_L. Consider the reasoning used to derive (3) with respect to the financial distress costs, but substitute λ for α, round n for round i, and liquidation for round i+1. One can show that the party making the offer in round n would hold its opponent to its payoff from liquidation and thereby obtain the surplus from avoiding liquidation, λ, plus its own payoff from liquidation. In liquidation, the equityholders would receive $VE_L = \max(0, V_n-\lambda-D)$, and the debtholders would receive the complementary share $VD_L = L-VE_L = V_n-\lambda-\max(0, V_n-\lambda-D) = \min(V_n-\lambda, D)$.

Each party would make the offer in round n (and thus capture the surplus λ) with probability ½. In this case, $E_1[VE_n] = \tfrac{1}{2}\lambda + E_1[VE_L]$, and substituting this expression in (4) yields:

$$VE_1 = \tfrac{1}{2}\alpha(e+n-1) + \tfrac{1}{2}\lambda + E_1[VE_L]. \qquad (17)$$

[14]Furthermore, to the extent that liquidation does not occur immediately upon rejection of the last offer in round n, the firm would incur some financial distress costs between the end of round n and liquidation. In the alternative, then, the possibility of $L < V_n$ could represent the deterioration in the firm's value between the end of round n and liquidation.

Consider the reasoning used to prove Lemma 1, but applied to L rather than to V_n. An analogous lemma implies that (17) equals (16).[15] ■

Let us now examine the change that is introduced by the loss λ from Chapter 7. As the following proposition indicates, the equityholders' share of V_1 tends to be larger as this loss is larger.

Proposition 8: The share of V_1 obtained by equityholders, VE_1, is nondecreasing in λ. Thus, the equityholders are at least as well off when $\lambda>0$ as they are when $\lambda = 0$. Furthermore, VE_1 is increasing in λ if and only if either $(D-V_1) + \alpha(n-1) + \lambda \geq \theta$, as is likely, or n is odd.[16]

Remark: The intuition underlying Proposition 8 is as follows. The expectation of the liquidation loss λ would strengthen the equityholders' bargaining power and thereby increase their share VE_1. Without their consent to a reorganization plan, the firm would not only suffer financial distress costs for n rounds, but would also suffer the losses from a Chapter 7 sale for less than the firm's full value. Thus, the equityholders' consent can save the firm the liquidation loss, λ, and therefore they can expect to capture some part of these savings in exchange for their consent. At the same time, in our model, such a liquidation loss would reduce the equityholders' "option value," $E_1[VE_L]$. As shown in the proof, however, the positive effect is at least as great as the negative effect on this "option value." Furthermore, the proof shows that under conditions that ensure $Pr(V_n>D) < ½$, which is likely to hold for plausible parameter values, the positive effect must strictly dominate the negative effect.

Proof: See the Appendix.

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[15]If e = n rather than e<n, then only the equityholders could make the offer in round n and capture λ. In this case, $E_1[VE_n] = \lambda + E_1[VE_L]$. This value plus the $\alpha(n-1)$ they would capture from round 1 to round n-1 yields:

$$VE_1 = \alpha(n-1) + \lambda + E_1[VE_L]. \quad (18)$$

In either case, the equityholders would have an additional source of bargaining power represented by the new second term.

[16]Furthermore, if e = n rather than e<n, then VE_1 is always increasing in λ. See the Appendix for the proof.

F. Renegotiation Prior to Chapter 11 Filing

Our model has assumed that the bargaining game begins upon the firm's filing under Chapter 11. One might want to consider, however, whether the parties would anticipate the agreement expected under Chapter 11 by renegotiation of their claims prior to filing under Chapter 11. At first glance, it seems that this would be the case. To the extent that some financial distress costs are incurred prior to the filing, as is likely as the company approaches insolvency, the parties would have much to gain from such an earlier renegotiation. By this reasoning, if the parties know that once there is filing for Chapter 11 the firm's value will be divided in a certain way, then there will be no such filing because they will agree to such a division beforehand. Brown (1989), for example, assumes in his model that such renegotiation will always take place.

Various factors, however, suggest that such renegotiation may not succeed outside of Chapter 11, because there may be a "free rider" or "holdout" problem. Consequently, renegotiation would instead take place, as in our model, only after the filing, and in accordance with the rules governing Chapter 11 reorganization. Outside of Chapter 11, each debtholders' claim cannot be waived unless the debtholder individually consents. Thus, each debtholder may hold out, preferring that other debtholders make the necessary concessions. In contrast, under Chapter 11 a provision for majority rule solves this collective action problem: the specified majority in any given class can vote to make concessions on behalf of each of the members of the class, without the unanimous consent of its members, and can thereby impose the plan on its members who dissent.

Furthermore, in one important instance, the legal rules prevent the establishment of contractual mechanisms to solve this collective action problem outside of Chapter 11. The Trust Indenture Act prohibits contractual provisions that would enable the majority of bondholders to concede any part of the principal on the bonds. As Roe (1987) notes, the unanimous approval requirements of that law may generate holdout problems within a class and thereby inhibit a troubled firm's ability to avert bankruptcy proceedings.

III. CONCLUDING REMARKS

This paper has sought to develop a sequential bargaining model of the Chapter 11 negotiation process, to show the effects of the legal rules that govern this process, and to determine the shares of the firm's value that equityholders and debtholders may expect to receive under a reorganization plan. The model has confirmed the widely held belief that equityholders generally can obtain value under Chapter 11 even if the value of the firm's assets is less than its debt. The model has identified and analyzed three possible sources of the equityholders' power -- if they were to delay or prevent agreement, (i) the firm would incur financial distress costs, (ii) the volatility of its assets' value may create some probability that the firm becomes solvent, and (iii) a Chapter 7 sale may entail a loss in value.

Our analysis has identified precisely how various parameters, reflecting both features of the company and of the legal regime as enforced by the courts, shape the Chapter 11 division of value. Our results regarding the effects of the legal regime enable us to predict how the division of value would change if the legal rules or judicial attitudes were to change. Our results regarding the effects of the company's features provide testable implications of the model. In particular, one could test the hypotheses that equityholders tend to capture a larger fraction of the value of the reorganized company when (i) the value of the company's assets is volatile (as measured, say, by the past volatility of the company's total stock and bond value), (ii) the nature of the company's business is such that financial distress costs are likely to be relatively high, or (iii) the total value of the reorganized company is a relatively large fraction of the outstanding debt.

This paper has focused on a positive analysis of the formal reorganization process under the bankruptcy laws. This description of the distribution of value, however, has not addressed an important issue: the efficiency effects of Chapter 11. One alleged benefit of Chapter 11 that it avoids Chapter 7 liquidation, which would waste value. This possibility, which we introduced in Section II.E, is the subject of debate as noted. See Baird (1986). Whatever the size of this benefit of Chapter 11, however, a full assessment of Chapter 11 requires an understanding of its costs. It is worth discussing briefly these efficiency costs

and in particular how our positive model does some important preparatory work for a complete understanding of these costs. Chapter 11 entails both financial distress costs after filing and lost value due to suboptimal management before filing. Both ex post and ex ante costs are important in evaluating proposed reforms in the law of corporate reorganization.

A. Ex Post Welfare Costs

In the model, because it was assumed that there was no asymmetric information, the resolution of the bargaining occurred in round 1. Note that because the rounds are not of negligible duration, the company incurred some financial distress costs: the firm lost α in value during round 1. Thus, even in this model with perfect information, the parties bear some limited efficiency costs.

Suppose one were to introduce asymmetric information about the firm's value V. For example, suppose that the equityholders (or more precisely, the management of the debtor who might be acting in their interests) have some private information not held by the debtholders. In this case, the sequential bargaining literature suggests that reaching an agreement may well take some time (as indeed often happens in reality[17]) and that the parties will incur significant efficiency costs. See, e.g., Grossman and Perry (1986), Fudenberg and Tirole (1983), and Crampton (1984).

The structure imposed upon the bargaining by Chapter 11 is crucial to this result. Gul and Sonnenschein (1988) point out that asymmetric information between bargaining parties cannot by itself explain delay in reaching agreement, because as the length of each round is allowed to go to 0, so does the total delay. This objection does not apply to the reorganization context, however, because the legal rules governing Chapter 11 bargaining imply that, unlike the case in other contexts, there is a limit to how short the bargaining rounds can be.

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[17]White (1984, pp. 35, 37) found the average length of the reorganization period, from bankruptcy filing to confirmation of a plan, to be 17 months. See also Franks and Torous (1989, p. 753).

B. Ex Ante Welfare Costs

As the analysis has shown, Chapter 11 enables equityholders to obtain some value even if the firm is insolvent, that is, if the value of the firm is not enough to pay the debtholders' claims. Because the parties anticipate this outcome, Chapter 11 has ex ante effects. One ex ante effect is an initial interest rate on the debt that is higher than it would be otherwise. Chapter 11, then, does not actually create a net expected transfer from debtholders to equityholders, because the possibility of an ex post transfer is reflected in the interest rate chosen ex ante. Furthermore, these effects do not imply any efficiency consequences.

Chapter 11 has another ex ante effect, however. The equityholders anticipate obtaining some value in the event of insolvency, and this expectation may affect the way in which the management runs the firm ex ante. As will be shown in another paper, this expectation leads to inefficient ex ante management decisions, which should be regarded as a significant efficiency cost.[18]

[18]The above discussion suggests that Chapter 11 produces significant efficiency costs. For a proposal for changing Chapter 11 to eliminate these costs, see Bebchuk (1988). This proposal suggests a method for distributing the value of the reorganized firm. The distribution would be fully consistent with all the participants' contractual rights and would occur without delay. As a result, it would eliminate both the ex post and the ex ante efficiency costs of Chapter 11.

APPENDIX

In this appendix we derive our comparative statics results. To analyze the effect of the parameters on VE_1, it will prove useful to make some preliminary observations. Consider the general model in Section II.E, in which $\lambda \geq 0$ and $L \leq V_n$. (To apply the following reasoning to the model in Section II.D, which addresses a special case of the more general model, let $\lambda = 0$, $L = V_n$, and $VE_L = VE_n$.)

First, we derive a lemma pertaining to the "option value" component of VE_1. Note that VE_L is a continuous function of L-D. In particular, as the parameters change, VE_L will equal (and therefore increase and decrease one-for-one with) L-D if L>D, will remain constant at 0 if L<D, and will increase one-for-one (but not decrease) with L-D if L = D. Therefore, we can state the following lemma:

Lemma A1: For any parameter x that does not affect the probability distribution of the random variable k, i.e., for D-V_1, θ, and α, the following holds for increases in x:[19]

$$\frac{\partial E_1[VE_L]}{\partial x} = E_1\left[\frac{\partial VE_L}{\partial x}\right] = \tag{A.1}$$
$$Pr(L = D)\max\left[0, \frac{\partial (L-D)}{\partial x}\Big|_{L=D}\right] + Pr(L>D)E_1\left[\frac{\partial (L-D)}{\partial x}\Big| L>D\right].$$

Second, we will find it useful to place an upper limit on Pr(L>D). Given the symmetric probability distribution of k, if n is odd, then the median value for k is $E_1[k] = \frac{1}{2}(n-1)$. In that case, the median value for L is similarly $E_1[L] = V_1 - \alpha(n-1) - \lambda$, which must be less than D. Therefore, if n is odd, then $Pr(L>D) < \frac{1}{2}$.

If n is instead even, then there is no unique median value for k. In this case, $Pr(L>D) < \frac{1}{2}$ if and only if $L \leq D$ for $k = \frac{1}{2}n$. As one can see from (12), if $k = \frac{1}{2}n$, then $V_n = V_1 - \alpha(n-1) + \theta$.

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[19]The notation for the partial derivative of VE_L (or its expected value) in this lemma and in the proofs below will represent the limit of $\Delta VE_L/\Delta x$ as we approach x from the right side, in order to describe the effects of an increase in x. To describe decreases in x, we would approach x from the left side and would replace "max" in (A.1) with "min." These two limits are not equal (i.e., although $VE_L(x)$ is continuous, it is not differentiable) at values of x where Pr(L>D) changes, because this probability changes only in discrete amounts.

Therefore, if $(D-V_1) + \alpha(n-1) + \lambda \geq \theta$, then $Pr(L>D) < \frac{1}{2}$. Furthermore, $Pr(L>D) > \frac{1}{2}$ would require that L>D for $k = \frac{1}{2}(n-2)$. As one can see from (12), however, if $k = \frac{1}{2}(n-2)$, then $V_n = V_1-\alpha(n-1)-\theta$, which must be less than D. Therefore, $Pr(L>D) > \frac{1}{2}$ is impossible. We summarize these results in the following lemma:

Lemma A2: $Pr(L>D) \leq \frac{1}{2}$, and $Pr(L>D) < \frac{1}{2}$ if and only if either $(D-V_1) + \alpha(n-1) + \lambda \geq \theta$ or n is odd.

Using these two lemmas, we can now provide proofs of the propositions stating our comparative statics results.

Proof of Proposition 2: Note that the derivative of V_n-D with respect to $D-V_1$ is -1 in any state of the world, i.e., for any k. Therefore, recalling that $VE_L = VE_n$ and $L = V_n$, and using Lemma A1:

$$\frac{\partial E_1[VE_n]}{\partial(D-V_1)} = -Pr(V_n>D). \tag{A.2}$$

Thus, increases in $D-V_1$ will decrease the equityholders' "option value" until $D-V_1 \geq (n-1)(\theta-\alpha)$. At that point, by (15), the probability of $V_n>D$ (and the "option value") reaches 0, and by (A.2), increases in $D-V_1$ then will have no further effect. ■

Proof of Proposition 3: First, note that the partial derivative of V_n in (12) with respect to θ is $2k-(n-1)$, which is greater or less than 0 as k is greater or less than $\frac{1}{2}(n-1)$. That is, V_n increases in θ in those states of the world in which k is larger than $E_1[k] = \frac{1}{2}(n-1)$, which by (12) are those states in which V_n is larger than $E_1[V_n] = V_1-\alpha(n-1)$. Because V_n can affect $E_1[VE_n]$ only in those states in which V_n exceeds or equals D, which in turn exceeds $V_1-\alpha(n-1)$ (indeed, exceeds V_1), θ can affect $E_1[VE_n]$ only in those states in which V_n is increasing in θ. In the relevant states, therefore, $2k-(n-1) > 0$, and a rise in θ can only increase V_n-D in the expression for the equityholders' "option value" in (13). Recalling that $VE_L = VE_n$ and $L = V_n$, and using Lemma A1, then:

$$\frac{\partial E_1[VE_n]}{\partial \theta} = Pr(V_n \geq D)E_1[2k-(n-1) \mid V_n \geq D]. \quad \text{(A.3)}$$

Furthermore, by reasoning analogous to that used to derive (15), the following two conditions are equivalent:

$$\Pr(V_n \geq D) > 0 \Leftrightarrow (n-1)(\theta-\alpha) \geq D-V_1. \quad \text{(A.4)}$$

A rise in θ, by (A.3) and (A.4), will have no effect on the expected value of VE_n until $\theta \geq \alpha + (D-V_1)/(n-1)$; further increases will raise equityholders' "option value." ■

Proof of Proposition 4: First, note that the equityholders' "option value" is nonincreasing in α. A rise in α shifts the entire probability distribution of V_n downward and so reduces V_n-D in the expression for the equityholders' "option value" in (13). In all states of the world, i.e., for any k, the derivative of V_n-D with respect to α will be -(n-1). Recalling that $VE_L = VE_n$ and $L = V_n$, and using Lemma A1, then:

$$\frac{\partial E_1[VE_n]}{\partial \alpha} = -(n-1)Pr(V_n>D). \quad \text{(A.5)}$$

Thus, α will have no effect if $\Pr(V_n>D) = 0$ already. That is, by (15), an increase in α will reduce the "option value" until $\alpha \geq \theta + (V_1-D)/(n-1)$. Then the "option value" will be 0, and further increases in α will have no effect on it.

It will be useful to place an upper limit on the absolute value of the negative effect of α on the "option value." By Lemma A2, $\Pr(V_n>D)$ can be at most ½. Therefore, (A.5) implies that the derivative of $E_1[VE_n]$ with respect to α can range from 0 to -½(n-1).

The full effect of α on the equityholders' share, unlike the effect of θ or of $D-V_1$, will also depend on an effect on the "efficiency gains" component, i.e., the first term in (13). An increase in α has an unambiguously positive effect on this component of the equityholders' share: the derivative of ½α(e+n-1) with respect to α is ½(e+n-1). This effect will always be greater than the absolute value of the negative effect on the "option value" component in (13), because e>0. ■

Proof of Proposition 5: By (15), the "option value" component of the equityholders' share will remain at 0 until $n > 1 + (D-V_1)/(\theta-\alpha)$. Once n rises above this level, their expected round n payoff rises above 0. Thus, at that point the "option value" increases in n.

The "option value," however, will not be monotonically nondecreasing in n. Instead, it may decrease in particular cases. Note that $E_1[V_n] = V_1-\alpha(n-1)$, so that each unit increase in n causes the expected value of V_n to fall by α. Indeed, an increase from n = N to n = N+1 will reduce the expected value of V_n, conditional on any particular V_N, by the amount: $V_N - E_N[V_{N+1}] = \alpha$. Therefore, it is easy to construct cases in which the expected value of V_n, conditional on $V_n>D$, will fall with an increase from n = N to n = N+1. The equityholders' expected round n payoff will then fall, provided that $Pr(V_n>D)$ does not rise.[20]

It will prove useful to place a ceiling on the absolute value of the negative effect that n may have on the equityholders' "option value." A unit rise in n cannot cause the expected value of V_n, conditional on $V_n>D$, to fall by more than α. Thus, a unit rise in n can reduce $E_1[VE_n]$ by at most $\alpha Pr(V_n>D)$, because it can only reduce VE_n in those states of the world in which $V_n>D$. Recall also that Lemma A2, with $\lambda = 0$ and $L = V_n$, implies that $Pr(V_n>D)$ can never exceed ½.

To consider the effect of n upon the "efficiency gains" component, note that the derivative of $\frac{1}{2}\alpha(e+n-1)$ with respect to n is $\frac{1}{2}\alpha$, which is unambiguously positive. This positive effect upon the "efficiency gains" component of the equityholders' share must be at least as great as the absolute value of the negative effect upon the "option value" component. Thus, a unit increase in n cannot cause VE_1 to decrease. Furthermore, by Lemma A2, if

[20]As n increases, new probability mass will rise above D at regular intervals: first, the V_n associated with k = n-1 will exceed D, then that with k = n-2, and then that with k = n-3, and so forth. As one can see in (12), $V_n(k)$ falls by 2θ for each drop in k, and the V_n corresponding to each n-1-k will rise by $\theta-\alpha$ for each unit rise in n. Therefore, n must rise by $2\theta/(\theta-\alpha)$ to bring the next possible V_n above D. During these intervals, probability mass will be falling below D because each unit increase in n reduces the expected value of the V_n just above D; in particular, half of the probability mass at this marginal V_n falls below D. Once the marginal V_n (which by definition can be no more than $D+2\theta$) is at least $D+\theta-\alpha$ (so that the half of the probability mass that falls will at least offset the half that rises), but still no more than $D+\theta+\alpha$ (so that no new probability mass will rise above D), $E_1[VE_n]$ falls unambiguously with a unit increase in n.

either $(D-V_1) + \alpha(n-1) \geq \theta$ or n is odd, then $Pr(V_n>D) < \frac{1}{2}$, and a unit increase in n must cause VE_1 to increase. ■

Proof of Proposition 8: The loss from liquidation, λ, affects VE_1 through both the second and the third terms in both (17) and (18) in footnote 13. Note that the derivative of L-D with respect to λ is -1 in any state of the world, i.e., for any k. Therefore, using Lemma A1:

$$\frac{\partial E_1[VE_L]}{\partial \lambda} = -Pr(L>D). \tag{A.6}$$

By (A.6), the derivative of the third term, $E_1[\max(0, L-D)]$, with respect to λ, is -Pr(L>D). The derivative of the second term with respect to λ, however, equals ½ if e<n (and equals 1 if e = n). The positive effect on the second term would be greater than or equal to the absolute value of the negative effect on the third term, because by Lemma A2, $Pr(L>D) \leq \frac{1}{2}$. Therefore, VE_1 is nondecreasing in λ. Moreover, if either $Pr(L>D) < \frac{1}{2}$ or e = n, then VE_1 is increasing in λ. Lemma A2, which states that if either $(D-V_1) + \alpha(n-1) + \lambda \geq \theta$ or n is odd, then $Pr(L>D) < \frac{1}{2}$, completes the proof. ■

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