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OPTIMAL SANCTIONS WHEN INDIVIDUALS
ARE IMPERFECTLY INFORMED ABOUT
THE PROBABILITY OF APPREHENSION

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**Optimal Sanctions When Individuals Are Imperfectly Informed
About the Probability of Apprehension**

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Abstract

This paper considers optimal enforcement when individuals may be imperfectly informed about the probability of apprehension. When individuals are perfectly informed, optimal sanctions are maximal because, as Gary Becker (1968) suggested, society can economize on enforcement resources by reducing the probability of apprehension while increasing sanctions. But when individuals imperfectly observe the probability of apprehension, it may be optimal to apply lower sanctions while expending more enforcement resources.

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1. Introduction

Gary Becker's (1968) classic paper introduced the point that enforcement effort and sanctions are substitutes in enforcement. A lower level of enforcement effort can be offset by increasing sanctions, which economizes on enforcement costs. Even activities involving little harm should receive the maximal sanction; the probability of apprehension for such acts may be reduced in order to avoid overdeterrence. Subsequent papers have explored and qualified Becker's insight concerning optimal enforcement policy.¹

An implicit assumption in Becker's analysis of optimal sanctions, which has been carried over to subsequent investigations of optimal enforcement policy, is that individuals accurately observe the enforcement probability set by the government.² While there presumably is a positive relationship between actual and perceived levels of enforcement, it is implausible that individuals' probability estimates are generally accurate, particularly when the probability is extremely low. Will most individuals know that the probability of being ticketed for double-parking is 2.74% while that for speeding is 0.89%? Indeed, survey evidence on individuals' perceptions indicates that estimates vary widely.³

¹ The literature has offered various explanations for why nonmaximal sanctions may be desirable: risk-aversion [Kaplow (1991); Polinsky and Shavell (1979)]; nonmonetary sanctions [Kaplow (1990b); Polinsky and Shavell (1984)]; avoidance costs [Malik (1990)]; marginal deterrence [Shavell (1991a)]; general enforcement [Mookherjee and Png (1990); Shavell (1991b)]; individuals' differences in wealth, resulting in differences in the maximum feasible fine [Polinsky and Shavell (1991)]; individuals' imperfect information about whether acts are subject to sanctions [Kaplow (1990a)]; and differences in the actual probability of apprehension [Bebchuk and Kaplow (1991)].

² Sah (1991) independently draws attention to the possibility (and discusses in greater depth the plausibility of the assumption) that individuals' estimates of the probability of apprehension may differ. His paper, however, focuses on positive analysis: he develops a rich model of how different estimates may come about and evolve over time and draws insightful implications for how crime rates may be affected. In contrast, we explore the normative question -- which he explicitly reserves -- of how different probability estimates affect optimal enforcement policy. The two papers are thus complementary.

³ Sah (1991) discusses some of the findings. Work reporting or employing such evidence includes Henshel and Silverman (1975), Montmarquette and Nerlove (1985), Parker and Grasmick (1979), and Richards and Tittle (1982).

This paper reconsiders the problem of optimal sanctions when actors' information about the probability of apprehension is imperfect. We do not assume that actors err systematically in one direction or the other. Rather, we assume that individuals observe the probability of apprehension with some noise; consequently, some individuals' estimates are too high and others' too low, with the average being unbiased.

Our primary result is that, when individuals are imperfectly informed in this manner, it may not be optimal to set the sanction at the highest feasible level. The reason is that a given error in observing the probability of apprehension affects the expected sanction in an amount that depends on the level of the sanction. To illustrate, suppose an act causes a harm of 10. The maximum possible sanction is 500, so optimal deterrence could be achieved with a probability of 2%. Alternatively, one could employ a sanction of 100 and a probability of 10%. Suppose, however, that half the individuals overestimate the probability by one percentage point and the other half underestimate it by the same amount. For the first regime, half face an expected sanction of 15 ($3\% \times 500$) and half face an expected sanction of 5 ($1\% \times 500$); for the alternative regime, half face 11 ($11\% \times 100$) and half 9 ($9\% \times 100$). Clearly, under the former regime, there will be greater overdeterrence for the individuals who overestimate the probability and greater underdeterrence for those who underestimate it. If the resulting loss in welfare exceeds the cost of raising the actual probability from 2% to 10%, the latter regime would be superior.

In the next section we present our model and analysis, after which we offer brief concluding remarks concerning the empirical importance of the phenomenon we address.

2. Model

A. Framework for Analysis

Risk-neutral individuals choose whether to commit an act that benefits the actor by b , which is assumed to be distributed uniformly in the interval $[0,1]$. Acts impose a social cost, h . We assume $h < 1$, so that some acts may be socially beneficial.

The government chooses a probability of enforcement, p , and a sanction, s , so as to maximize the sum of individuals' benefits minus the harm caused by their acts and enforcement costs, $x(p)$. We assume $x' > 0$ and $x'' > 0$. Moreover, the maximum feasible sanction is \bar{s} , which can be understood as the maximum wealth of individuals. We assume further that the sanction is costless to impose, as in the familiar Becker framework.

We depart from the familiar model by assuming that actors are imperfectly informed about p . Specifically, individuals observe p with an error: they observe either $p+e(p)$ or $p-e(p)$, each with 50% probability. The government cannot observe each actor's estimate.

We analyze first the baseline case in which $e(p) = 0$ for all p , and then the case in which $e(p) > 0$.

B. Perfect Information

When individuals accurately observe the probability of apprehension, all obviously observe the same p . The government's problem is to choose p and s so as to maximize

$$(1) \int_{ps}^1 (b-h)db - x(p),$$

subject to the constraint that $s \leq \bar{s}$. The Becker result is immediate.

Proposition 1: When individuals accurately observe the probability of apprehension, the optimal sanction is maximal.

Proof: As one increases s and reduces x so as to keep ps constant, the first term in (1) is unaffected and the magnitude of the second term falls, so the optimum is where the constraint is binding. *Q.E.D.*

C. Imperfect Information

When $e(p) > 0$, individual probability estimates differ. The government's problem is to choose p and s so as to maximize

$$(2) \quad \frac{1}{(p+e(p))s} \int (b-h)db + \frac{1}{(p-e(p))s} \int (b-h)db - x(p),$$

subject to the constraint that $s \leq \bar{s}$.

Proposition 2: When individuals observe the probability of apprehension subject to an error, the optimal sanction may be less than the maximal one.

Proof: Begin with $s = \bar{s}$ and let \bar{p} denote the optimal probability of apprehension given \bar{s} . Consider the effect on welfare of raising p and reducing s such that ps remains fixed. That is, take the derivative of (2) with respect to p , with $ds/dp = -s/p$:

$$(3) \quad \frac{1}{2}(h - (p+e)s)(se' - es/p) + \frac{1}{2}(h - (p-e)s)(es/p - se') - x' \\ = (e/p - e')es^2 - x',$$

where e' denotes de/dp . If one assumes that, at (\bar{p}, \bar{s}) , $d(e/p)/dp < 0$ -- that is, that the percentage error falls as p rises -- the first term is positive, reflecting an improvement in welfare from an improvement in behavior. It can readily be demonstrated that for enforcement technologies for which x' is not too large, the entire expression will be positive.⁴ *Q.E.D.*

⁴ The demonstration is complicated only by the fact that the first term must be evaluated at \bar{p} , which itself depends on the technology $x(p)$. Examples can most easily be constructed using a less direct technique that avoids this interdependence. Set $\bar{s} = \bar{s}$ and choose probability \bar{p} that maximizes the sum of the first two terms in (2) -- i.e., $\bar{p} = h/\bar{s} - ee'$. (It is apparent that $\bar{p} > \bar{p}$.) Then consider $\hat{s} = \bar{s}/2$ and $\hat{p} = 2\bar{p}$. Assume that $d(e/p)/dp < 0$ for $p \in [\bar{p}, \hat{p}]$, so that behavior is better at the lower sanction and higher probability. Compute the degree to which welfare is greater as a result of this behavioral effect. Finally, assume that $x(\hat{p})$ is less than this benefit, which completes the example.

The intuition behind Proposition 2 is as follows. Beginning at the maximum feasible sanction (and the correspondingly optimal probability), a reduction in the sanction accompanied by an increase in the probability that keeps the actual expected sanction constant will improve behavior whenever the relative size of the error (the ratio of the error to the actual probability) falls as the probability rises. Behavior improves because the perceived expected sanctions -- $(p+e)s$ for some and $(p-e)s$ for others -- move closer together. As a result, those who overestimate the probability are overdeterred less and those who underestimate the probability are underdeterred less.⁵ So long as it is not too costly to raise the probability somewhat, the optimum will involve a sanction that is not maximal.

3. Conclusion

We have examined the problem of optimal enforcement when individuals are imperfectly informed about the probability of apprehension. When individuals observe this probability with some random error, it may be optimal to employ less than the maximum feasible sanction with a greater probability of apprehension. While raising the probability is costly, it may improve behavior. Behavior improves if the error is a lower fraction of the actual probability as this probability increases, because less of a divergence in perceived expected sanctions will result. If behavior improves sufficiently, the higher enforcement cost will be warranted.

The importance of this phenomenon depends on the relationship between individuals' errors in estimating the probability of apprehension and the actual probability. The example we offered in the introduction suggests the plausibility of the assumption that errors are a greater fraction of the

⁵ Depending on $x(p)$, it may be that the optimal probability at \bar{s} involves both groups being underdeterred. It is nonetheless true that behavior improves: the social benefit from reducing underdeterrence of those underdeterred significantly exceeds the loss from increasing underdeterrence of those underdeterred modestly, as the harm caused by both groups' acts is the same but the benefit for the marginal individual in the former group is less than that for the latter.

probability when probabilities are very low than when they are higher. To guide enforcement policy, empirical research on this point would be useful. For example, one might attempt to infer probability perceptions from behavior, which could be accomplished in an experimental setting, or survey individuals concerning their perceptions. Work in cognitive psychology concerning probability perceptions might also illuminate the issue. Finally, one could examine analytically how individuals' probability estimates based on given priors and limited sets of observations differ when the probability generating the observations differs.⁶ Such research might reveal that relative errors are rather large for probabilities of apprehension that otherwise would be optimal with maximal sanctions, while much lower for the probabilities and sanctions actually observed.

⁶ Sah's (1991) model assumes individuals' estimates are determined by their limited observations. For his purposes, only the crime rate and not the magnitude of individuals' errors is relevant, so he does not explore how such errors may be affected by the actual probability of apprehension.

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