

PLAYING WITHOUT A RULEBOOK:
OPTIMAL SANCTIONS
UNDER IMPERFECT INFORMATION

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ABSTRACT

Individuals may commit sanctionable acts while being imperfectly informed about the government's enforcement policy. By engaging in behavior repeatedly, and getting caught occasionally, individuals learn some of the information over time. In this setting, Becker's argument for imposing maximal sanctions along with a low probability of apprehension may not hold. Raising the probability of apprehension increases the number of occasions in which individuals get caught, giving them more opportunities to learn. The acquired information improves behavior, a benefit that may exceed the added enforcement cost.

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I. INTRODUCTION

Gary Becker's classic argument demonstrated that society can save enforcement costs without sacrificing deterrence. By raising sanctions and reducing the probability of detection, the level of deterrence can remain fixed, while the cost of enforcement is reduced. This insight suggests that optimal sanctions be maximal, a feature that is rarely observed in reality. The divergence between Becker's normative analysis and actual enforcement policies has generated further explorations of the problem.

This paper reconsiders the optimal enforcement policy in situations in which individuals may be imperfectly informed about the enforcement efforts devoted by the government. They may not know the probability of apprehension, the magnitude of the sanction, or both. Further, they may not know the social harm that their acts impose, nor the expected sanction.¹ They engage in behavior more than once, and thus -- by getting caught occasionally -- have an opportunity to learn some information over time.²

In this setting, raising the enforcement effort above minimal and reducing fines from maximal may be desirable. A "learning effect" of two elements is generated. First, individuals are apprehended more often, and thus learn the magnitude of fines faster. Second, individuals may learn the probability of apprehension faster (the variance of the individual's estimate of the probability of apprehension may decrease as experience is gained). This effect produces a social benefit because, over time, it allows individuals to

¹ Empirical studies verify the assumption of imperfect information. Studies in the area of tax compliance suggest that agents form beliefs about the probability of detection which are imperfect, thus may comply when they are better-off evading and vice versa. See Alm et al. [1992], Cowell [1990], Klepper and Nagin [1989].

² The repeated play assumption fits many of the major enforcement areas considered in the literature, e.g., tax reporting, traffic regulation.

make better decisions about whether to engage in behavior. The benefit may or may not exceed the extra enforcement cost it requires.

The intuition underlying this result can be captured with the following simple example. Suppose a driver arrives in a new town in which the speeding rules dictate a lower speed limit than that with which he is familiar. He is mistakenly led to believe that for a particular high speed, the expected sanction is 0. Every time his benefit from speeding is positive, he will do so. He will perform the violation repeatedly, until he acquires new information and updates his belief. Such new information is gained when he is detected and sanctioned. In this setting, a higher probability of apprehension (e.g., more patrol cars) will, in expectation, reduce the time that it takes the driver to update his beliefs. At the expense of higher enforcement cost, the driver learns the actual expected sanction faster, and speeds illegally only if his benefit exceeds the expected sanction. The enforcement agency can buy superior compliance by investing more in "educating" the public, and one way of doing so is through a higher apprehension rate.³

The idea that individuals learn about the penalty schedule through their own experience has been studied recently by Sah [1991].⁴ His paper investigates evolutionary aspects of crime, such as the variance in the public's perceptions about the probability of detection and how experience shapes these perceptions. It does not, however, consider the normative issue, of designing optimal sanctions in light of the learning dynamics.

The argument in this paper joins a large body of literature that addresses and

³ The enforcement agency has other means to inform the public about its policy. One method that come to mind is advertising the policy. In the Concluding Remarks Section of this paper it is argued that the advertising instrument has a credibility shortcoming, and in order to be effective it needs at least to be supplemented by greater enforcement efforts.

⁴ For empirical study of how individuals' perceptions change over time, see Piliavin et al. [1986].

qualifies Becker's insight. Some of this literature resorts to information-related arguments.⁵ The most relevant to this paper is the argument of Bebchuk and Kaplow [1992]. They analyze a model in which individuals are imperfectly informed about the probability of detection. In this case, the individuals' perceptions of the expected sanction are distorted. They argue that the magnitude of (and the social loss from) this distortion depends on the enforcement parameters. By raising the probability of detection and lowering the magnitude of the sanction, the absolute size of the error declines. Thus, sub-maximal sanctions are desirable because they alleviate the inefficiency that arises from individuals' misperceptions.

This paper can be viewed as an extension of Bebchuk and Kaplow's argument. Instead of assuming a particular structure of error in the public's observation of the probability of apprehension, I suggest that, at least in part, the error is determined endogenously. In particular, the individuals can reduce their errors through their own experience. Higher probability of apprehension simply generates more experience from which to learn. Thus, while Bebchuk and Kaplow's argument applies only to particular error structures,⁶ the argument in this paper applies to every type of erroneous belief (including beliefs about the magnitude of the sanction), as long as the situation involves repeated behavior. In doing so, the paper departs from the familiar static model of law enforcement, and uses a dynamic model.⁷

⁵ Kaplow [1990]; Bebchuk and Kaplow [1992]; Bebchuk and Kaplow [1993].

⁶ Bebchuk and Kaplow's argument requires that the magnitude of error in individuals' estimate be independent of the probability of apprehension, or that its relative size falls as the probability of apprehension increases.

⁷ A multi-period model of law enforcement has been analyzed before. Rubinstein (1979) and Polinsky and Rubinfeld [1991] use multi-period models to show that escalating the sanctions to repeat offenders can be optimal. Landsberger and Meilijson [1982] and Greenberg [1984] demonstrate that a policy of flexible probabilities of detection can be superior to a policy of uniform probability over time.

The next section presents an example. Section III presents the model, and proves the proposition that optimal sanctions may be less than maximal. Section IV extends the model, and Section V offers concluding remarks.

II. A HEURISTIC EXAMPLE

This section uses a numerical example to illustrate the learning benefit from raising the probability of apprehension. Consider a situation in which risk-neutral individuals can engage in an act twice, sequentially. The private per-period benefit from the act is random, uniformly distributed over $[0,100]$. The social harm from the act is 50.

The enforcement agency, in an attempt to impose an expected sanction of 50, has to choose between one of the following two policies: (i) maintain a low, 1% probability of apprehension, along with a high sanction of 5000, or (ii) maintain a high, 100% probability of apprehension, along with a low sanction of 50. Assume that the cost of policy (i) is 1 per period, and the cost of policy (ii) is 200 per period.

At the first period, the individuals are uninformed about the actual enforcement policy that applies. Assume that half of the population believes the expected sanction to be 100, while the other half believes it to be 0. If an individual gets caught, he learns the actual enforcement policy and the expected sanction perfectly. If he does not get caught, he learns nothing and proceeds with the same initial beliefs to the next period.

Becker's (1968) result, stating the superiority of policy (i), with its low probability of apprehension and high sanction, applies only in an environment of perfect information. If individuals correctly anticipate the expected sanction, then it is indeed socially optimal to implement policy (i). However, when individuals have imperfect information, policy (ii) has the superior quality of informing individuals about the actual expected sanction and leading them to improve their behavior in the second period. Observe that, under

either policy, total social welfare is

$$\frac{1}{2} \int_0^{100} (b - 50) db + \frac{1}{2} p \int_{50}^{100} (b - 50) db + \frac{1}{2} (1-p) \int_0^{100} (b - 50) db - 2x(p) \quad (1)$$

where b is the benefit to the actor, p is the probability of apprehension and $x(p)$ is the enforcement cost. The first term in (1) is the net welfare from first-period acts. Only individuals who believe the expected sanction to be 0 engage in the act. The second term is net welfare from acts by informed individuals at the second period. From those who committed the act at the first period, a fraction p was apprehended and learned the actual policy. They will engage in the act at the second period only if their benefit exceeds the true expected sanction. The third term denotes social welfare at the second period from acts by individuals who were not apprehended at the first period, and did not learn any new information. The last term is the enforcement cost in the two periods.

Now compare the two policies. Total welfare under policy (i), with its low probability of apprehension, is 4.25, which consists of the benefit from superior decisions by informed individuals of 6.25, minus twice the enforcement cost of 1. Under policy (ii), with its expensive high probability of apprehension, total welfare is 225, which consists of a benefit of 625, minus twice the enforcement cost of 200. Hence, policy (ii) generates higher social welfare. It costs more to maintain the high probability of apprehension, but the high probability also guarantees that more individuals will become informed and improve their behavior.

III. A MODEL OF STRATEGIC COMPLIANCE

The example above demonstrated that raising the probability of apprehension generates a learning benefit. However, while the individuals did in fact learn, they did not

"expect to learn" when choosing whether to engage in the act at the first period. They only took into account the immediate benefit and the expected sanction from the act. In a rational behavior model, individuals take into account the additional element that by engaging in the act they also increase their chance to learn new and more precise information about the enforcement policy. An individual may act at the first period even if his benefit is less than what he believes to be the expected sanction. He may sacrifice some immediate welfare for the opportunity to learn and make better decisions in the future. This strategic motive for acting will be included in the enforcement model below.

A. Framework of Analysis

There are two periods, $t = 1, 2$. A population of risk-neutral individuals with a discount factor of 1 have to decide in each period whether to commit an act. $b \in [0, 1]$ is the random benefit an individual enjoys from his act. b is assumed to be distributed uniformly. Once it is determined, b is the same across periods. The social harm from the act is h . It is assumed that $0 < h < 1$, so that the act may be desirable or undesirable.

The enforcement agency chooses a probability of apprehension, p , and a sanction, s . It cannot vary its policy between periods. Enforcement costs are $x(p)$. It is assumed that $x' > 0$, $x'' < 0$, i.e., diminishing marginal returns to enforcement effort. The maximal feasible sanction is \bar{s} .

At $t=1$, individuals are uninformed about the enforcement parameters, p or s . They have prior beliefs about the expected sanction. For simplicity, it is assumed that beliefs have the following structure. All individuals believe that the expected sanction is either 0 or 1. Denote by e the belief about the likelihood that the expected sanction is 1. $1-e$ is the likelihood that the expected sanction is 0. Individuals vary with respect to their prior

beliefs, e . Assume that e is distributed uniformly on $[0,1]$. Finally, the individuals' belief about the probability of apprehension is denoted by π .⁸ The following learning procedure is assumed. If an individual gets caught at $t=1$, he learns the enforcement parameters perfectly, and his beliefs about the expected sanction at $t=2$ become ps . If an individual does not get caught at $t=1$, he learns nothing and holds the same distribution of beliefs at $t=2$.⁹

B. Perfect Information

The government chooses p, s so as to maximize

$$2 \int_{ps}^1 (b - h) db - 2x(p) \quad (2)$$

subject to the constraint that $s \leq \bar{s}$.

The solution to this problem yields the familiar Becker [1968] result, that optimal enforcement policy involves maximal sanctions, \bar{s} , and a corresponding optimal probability of apprehension, \underline{p} .

C. Imperfect Information

If the individual does not know the expected sanction perfectly, he will engage in the act if one of the following two conditions hold: (i) his benefit exceeds his belief about

⁸ This specification allows π to be random and vary across individuals. Since π affects the belief about the expected sanction, and this belief is random, it is permissible that either π , the belief about the magnitude of the sanction, or both, be random. In the analysis below, π can be regarded either as the fixed (deterministic) belief, or as the mean of the distribution of beliefs.

⁹ This assumption embodies the notion that more information is learned from apprehension than from escaping apprehension. It has two justifications. First, only after being apprehended the individual learns the magnitude of the sanction. Second, when the probability of apprehension is small (less than 0.5), the variance of the estimate of the probability declines with the number of apprehensions. Later, the analysis will be extended to include more general learning processes, such as Bayesian updating.

the expected sanction, or (ii) his benefit is less than his belief about the expected sanction, but he also believes that the probability of apprehension is sufficiently high. In this case, by engaging in the act he is likely to learn new information and may discover that he initially exaggerated the expected sanction. If the expected value of this added information at $t=2$ exceeds the expected loss at $t=1$, the individual will engage in the act.

Formally, note that when $b \geq e$ (e is the mean of the distribution of the individual's beliefs about the expected sanction), the individual will always engage in the act. He expects a non-negative payoff at $t=1$, and is guaranteed a non-negative expected payoff at $t=2$. For if he is not caught at $t=1$, his $t=2$ expected payoff remains $b-e$, a non-negative value, while if he gets caught he learns the actual expected sanction and takes the act only if b exceeds it.

If $b < e$, the individual will engage in the act if and only if

$$(b - e) + \pi (1 - e) b \geq 0. \quad (3)$$

The first term in (3), $b-e$, is the (negative) expected payoff from committing the act at $t=1$. With probability $1-\pi$, the individual does not get caught and does not learn any new information. In this case, he will not engage in the act at $t=2$ and his $t=2$ payoff will be 0. However, with probability π , the individual gets caught and revises his beliefs. He expects that with probability $1-e$ he will discover that the expected sanction is 0, in which case he will engage in the act at $t=2$ and expect a payoff of b . This is denoted by the second term. If the combined payoff is greater than 0, it pays to engage in the act at $t=1$, rather than not engage and have a certain payoff of 0.

Simplifying expression (3), we find that when $b < e$, the individual will engage in the act if and only if

$$b \geq \frac{e}{1 + \pi(1-e)} \equiv b^*(e, \pi) . \quad (4)$$

Notice, that for all $\pi \in [0,1]$ and for all $e \in [0,1]$, $b^*(e, \pi) \leq e$, which implies that, in general, the individual will engage in the act whenever $b \geq b^*(e, \pi)$.

Given this behavior by the individual, the government chooses p, s so as to maximize

$$\int_0^1 \int_{b^*(e, \pi)}^1 (b - h) \, db \, de + p \int_{ps}^1 (b - h) \, db + (1-p) \int_0^1 \int_e^1 (b - h) \, db \, de - 2x(p) \quad (5)$$

subject to $s \leq \bar{s}$.

The first term in (5) denotes social welfare at $t=1$, in which the individual acts only if his benefits exceeds $b^*(e, \pi)$, averaged over all possible beliefs, e . The second term denotes social welfare at $t=2$, if the individual was apprehended at $t=1$ and learned the expected sanction perfectly. The third term is social welfare at $t=2$, if the individual was not apprehended at $t=1$, and did not learn any new information. At this stage there is no longer a strategic motive for acting, and thus an individual will engage in the act if and only if b is greater than the mean of his distribution of beliefs about the expected sanction, e . It is again averaged over all possible values of e . The last term is the enforcement cost in the two periods.

Proposition. *When individuals are imperfectly informed about the enforcement policy and learn from their own experience, the optimal sanction may be less than the maximal one.*

Proof. Begin with $s = \bar{s}$ and $p = \underline{p}$ and consider the effect of raising p and reducing s such that ps remains constant. All the integral terms in expression (5) remain unchanged, and

the only effects on social welfare are the weights given to the second and third terms, and the enforcement cost. Recall that the second term in expression (5) denotes welfare under perfect information, while the third term denotes welfare under imperfect information. Clearly, welfare is higher under perfect information. Precisely, it is higher by

$$I = \int_{ps}^1 \int_{ps}^e (b - h) db de - \int_0^{ps} \int_e^{ps} (b - h) db de . \quad (6)$$

The first term in expression (6) is the social welfare foregone due to the fact that the individual may overestimate the expected sanction and refrain from acting even though his benefit exceeds the social harm. The second term in expression (6) is the social loss from the fact that the individual may underestimate the expected sanction and choose to act even though his benefit is less than the social harm.

This informational premium, I , is obtained at $t=2$ with probability p . Thus, raising p above \underline{p} is desirable as long as the added enforcement cost in the two periods, $2x'$, is not too large:

$$I \geq 2x' . \quad (7)$$

Q.E.D.

Remark. Rational Expectations. In this model, the individual's perception (or the mean of the distribution of perceptions) about the probability of apprehension, π , was assumed to be independent of the actual probability, p . However, if expectations are rational, π equals p and the individual commits the act whenever $b > b^*(e, p)$. Notice that $\partial b^*(e, p) / \partial p < 0$, which implies that an increase in p leads more individuals to engage in the act. The learning effect is magnified: the greater is p , the more inclined are individuals to take a loss at $t=1$, as they expect to be apprehended, acquire superior information, and increase

their expected payoff at $t=2$. Hence, in a rational expectations setting, the argument for a higher p is strengthened.

IV. EXTENSIONS

The model is specific in several respects, and can be further generalized.

1. *More than Two Periods.* The learning effect identified above applies in a model with $n > 2$ periods. Moreover, if condition (7) holds, i.e., if the benefit from learning exceeds the extra enforcement costs in a two period model, then this same result necessarily holds with more than two periods. The reason is that learning in early periods generates a long-term benefit in the remaining periods. To see the precise effect, suppose there are $n = 3$ periods. Enforcement cost increases by 50% ($3x(p)$ instead of $2x(p)$), but the learning benefit increases by more than 100%. Learning can now take place either at $t=1$ or at $t=2$. If an individual is detected at $t=1$, the social benefit from his learning carries over to two, instead of one, periods. Thus, the benefit that arises from detection at $t=1$ is $2I$. An additional social benefit can accrue if an individual is detected at $t=2$. This benefit applies only to those individuals that were not already detected at $t=1$. Thus, the benefit from learning at $t=2$ is $(1-p)I$. The total learning benefit over time from maintaining a probability of p is therefore $2I + (1-p)I = (3-p)I$. Hence, if condition (7) holds, then $(3-p)I > 3x'$.¹⁰

¹⁰ Similarly, in an n -period model, the total learning benefit is $(n-1)I + (1-p)(n-2)I + (1-p)^2(n-3)I + \dots + (1-p)^{n-2}I$, which can be simplified and written as

$$\left[\frac{n}{p} + \frac{1}{p^2} ((1-p)^n - 1) \right] I .$$

The extra cost involved in raising p is $nx'(p)$. As n increases, both the learning benefit and the enforcement cost increase, but the benefit increases faster. Thus, the more periods there are, the more likely is the higher enforcement cost to be warranted.

2. Different Learning Processes. This model incorporates the assumption that if an individual is detected, he learns all the information perfectly, and if he is not detected, he learns nothing. This is an extreme assumption. It may be true that from one apprehension the individual succeeds to learn perfectly the magnitude of the sanction, but it is less plausible that he can deduce the precise probability of apprehension, and thus the precise expected sanction. Further, it is plausible that when an individual is not apprehended, he adjusts his estimate of the expected sanction downwards. However, even if we apply a more appropriate learning process, such as a Bayesian updating scheme, the spirit of the above argument holds. The learning benefit is not as large as identified above, but exists all the same, and may be sufficiently large to justify the extra cost.

3. Different Erroneous Beliefs. The assumption in the model was that prior beliefs have a particular structure: the expected sanction is believed to be either 0 or 1, and the likelihood of each varies with a uniform distribution across individuals. The same result holds for different specifications of errors. For example, if individuals observe the actual expected sanction with noise, or, alternatively, if individuals start at $t=1$ with fictitious beliefs, in both cases it is still true that greater experience brings the beliefs closer to the actual expected sanction.

V. CONCLUDING REMARKS

1. Advertizing. Raising the probability of apprehension is costly. Are there cheaper ways to educate the public about enforcement policies? One way to publicize enforcement measures is to advertize them: the government advertises its tax auditing efforts, the police department advertises its highway speed control measures, etc. Yet, advertizing methods, cheaper as they may be, have an inherent credibility problem. If individuals

believe the government's statements about its enforcement policy, in particular about the probability of apprehension, the government has an incentive to cheat. By proclaiming a higher probability than it actually maintains, the government can achieve any desired deterrence level, with less cost. Unless its reputation prevents the government from spreading false information, this information dissemination mechanism may be flawed. To convince the public that its advertized policy is actually implemented, the government has to pursue it and let the individuals verify its content through their own experience. But if experience is the key to accurately informing the public, it was already established above that a higher probability of detection steps up the pace in which experience is acquired. Hence, even with advertizing, a higher probability of detection is necessary to credibly inform the public.

2. High Publicity to High Sanctions. The model assumes that individuals learn only from their own apprehension experience. It is plausible to assume that individuals learn from the experience of others as well.¹¹ This external effect alone does not change the result that more detections are desirable as they generate more rapid learning. Yet, it is usually true that high sanctions amplify this external effect, as they have a louder "echo" relative to low sanctions, through the greater (ex-post) publicity that they receive. They are better remembered, and thus are more likely to be brought to the individual's mind when contemplating the risk of apprehension.¹² Using a high-sanction-low-probability policy can still guarantee a fast pace of information dissemination. However, even if high

¹¹ Sah [1991] discusses a model in which individuals share information with acquaintances.

¹² Tversky and Kahneman [1973] identify this psychological mechanism as the "availability heuristic" for judging frequency and probability. In general, availability is employed when a person estimates a probability by the ease with which similar instances or associations can be brought to mind.

sanctions do indeed receive greater publicity, they may still be undesirable. First, high publicity may cause individuals to systematically overestimate the probability of detection, and excessive deterrence will ensue.¹³ Second, high publicity may generate more erratic knowledge. The mean of the individuals' estimates of the probability of detection would increase, but so would the variance of their estimates. Thus, maximal sanctions may not manifest the same desirable learning effect as lower sanctions coupled with higher probability of detection.

3. Buying Information. Legal advice that is acquired prior to the commission of the act can inform the individual about the enforcement policy. To some extent, the purchase of information can substitute experience. However, even if legal advice is less costly to society than administering higher probabilities of apprehension, it may still be that many acts are committed without the opportunity to obtain advice. Furthermore, the quality of legal advice depends on the experience of lawyers, who in turn learn faster the greater is the apprehension rate.

4. Actual Use of High Apprehension Rates. An ordinary feature of actual enforcement policies is the use of less than maximal sanctions, coupled with higher than minimal apprehension rates. Apparently, there is one particular area in which this feature arises from information dissemination concerns. Occasionally, enforcement agencies announce their intention to combat particular offenses by concentrating greater enforcement efforts for a fixed period of time. For example, traffic police may announce an increase in patrols

¹³ Consider, for example, the way the movie "Jaws" distorts the public's perception of the risk from shark attack, or how news reports about airplane crashes distort the perception of the risk of crash. Similarly, high publicity to large sanctions can distort the perceived frequency of apprehension events. See Slovic et al. [1982], Tversky and Kahneman [1973].

at a particular highway for a given period, or tax authorities may raise their inspection of a specific activity for a limited time. Usually, these short-term policies apply in areas in which individuals engage in behavior repeatedly, and the policies are justified by the desire to inform the public. Although these campaigns cannot relay to the public the true probability of apprehension (the one that is maintained during normal times), they do inform the public about the sanctionability of the acts and about the magnitude of sanctions.

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