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DECISION RULES IN JUDICIAL PANELS: CHOOSING AMONG A CONTINUUM OF OUTCOMES

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Abstract

This paper develops an economic-Social Choice model of decision making in collegial courts or juries. It focuses on "non-dichotomous" adjudication, involving selection of a single alternative from a continuum, such as the case of setting damage awards or imprisonment terms. The simple majority method, the ordinary device for resolution of disagreements, is insufficient in this setting, since a majority winner may not exist. After considering supplementary aggregation rules this paper focuses on the mean and the median. They are evaluated in two stages: first, according to their statistical competence in advancing some axiomatically defined objectives; and second, by examining the incentives created for each judicial panel member to behave insincerely -- namely, to announce a judgement deviating from the one that would have been issued were he the only member of the panel. This analysis provides an explanation for the high frequency of unanimity in collegial courts.

I. Introduction

A typical model of decision-making in courts examines dichotomous choice problems involving binary alternative sets only. For any adjudicated issue, the ordinary approach is to divide it into sub-issues, for which the court has to determine one out of two possible outcomes: "Yes" or "No"; "Reject" or "Accept"; "0" or "1"; and so forth (Tullock and Good, 1984; Kornhauser and Sager, 1986; Kornhauser, 1990).

However, the choice problems that courts of law face are not restricted to the dichotomous ones. Situations in which a choice has to be made from a *continuum* of alternatives are important and abundant. Consider the judicial determinations of compensation awards (in civil trials) or imprisonment terms (in criminal proceedings). While these decisions could be reduced to sequences of dichotomous choices, it appears natural to characterize them as involving one, non-dichotomous stage in which the alternatives set is continuous.

This paper examines non-dichotomous choice problems faced by judicial panels such as collegial courts or juries.[1] In these settings, a new analytical problem is presented, since the majority method ordinarily used to resolve disagreements in dichotomous settings is no longer sufficient. Other "decision rules" may be required to supplement the majority method. This paper investigates particular aggregation methods that could be employed in non-dichotomous adjudication to achieve a deterministic outcome. At the center of the paper's normative analysis are two methods, the *mean* and the *median*, which, as I will later attempt to demonstrate, are superior to other possible candidates.

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[1] In the American legal system, collegial courts are typically appellate courts, whose role of reviewing lower courts' decisions can be represented as choosing from a binary set. Thus, the analysis of non-dichotomous choice is more relevant to damage awards and imprisonment judgments by juries. However, even appellate courts may be required occasionally to resolve non-dichotomous choice problems (e.g., where the court below neglected to do so). Furthermore, the analysis is applicable for adjudication systems that operate without juries, but instead use collegial courts to determine issues.

The main insights of the paper concern the patterns of *strategic behavior* among judges in collegial courts. In particular, I will examine the type of manipulative conduct that arises from each of the applicable "decision rules". An interesting result from this analysis is a general explanation for the high frequency of unanimity in collegial courts. This explanation differs from those found in the previous literature concerning jury decision-making (Klevorick and Rothchild, 1979; Schwartz and Schwartz, 1991). To set the stage for this discussion, which will be carried in section IV, I will present in section II two alternative models of "disagreements" in courts. The difference between these models stems from the varying jurisprudential interpretations of the source of these disagreements. Section III studies the quality of the various decision rules, absent any manipulative motives or insincere voting on behalf of the panel members. This section establishes the ordinary conditions under which the mean would be superior. In section IV, these results are reconsidered in light of the possibility of strategic behavior, demonstrating the manipulability of the various decision rules.

The analysis is carried out in the framework of Social Choice Theory. The previous literature applying this theory to legal contexts has neglected the non-dichotomous situations, ignoring any explicit form of strategic manipulations among judges (Spitzer, 1979; Easterbrook, 1982; Kornhauser, 1990). Although I use a formal model in my analysis, the paper is addressed primarily to legal theorists. I therefore highlight its intuitive implications, at the risk of appearing redundant in the eyes of the trained economist.

II. The Framework of the Analysis

1. General Framework. The decision making body in any given case is a $\boldsymbol{k}$-size panel of judges, drawn randomly from a population that contains $\boldsymbol{n}$ judges ($n \geq k$). There is an exogenous alternatives set $\boldsymbol{X}$ which is continuous. Each judge has a preference ordering over X, and is required to announce a unique choice ("judgment"). The individual opinions are then aggregated by a "decision rule". Several potential rules will be defined and discussed.

*2. **Alternatives***. The set of alternatives is an interval of $\mathbb{R}$:

$$X = [\alpha, \beta], \qquad \alpha, \beta \in \mathbb{R} . \tag{1}$$

This is a particular situation of non-dichotomous choice, where the alternatives set contains an infinite amount of elements, and each element may be represented by a numerical value. This formulation captures the situations where the judgement is presented in a quantitative form, such as the award of damages or the imposition of fines or imprisonment terms.[2] The bounds of X, α and β $(\alpha<\beta)$ are set exogenously, typically by a legislator.

*3. **Judges.*** A set of judges $N = \{1,2,...,n\}$ is the "population" of decision makers. Judges operate in *random* k-size panels. $K = \{1,2,...,k\}$ is a typical random panel, $K \subseteq N$, where $k>1$ is odd.

Each panel judge forms her opinion, denoted by D_i , $i \in K$. Disagreements between judges may be attributed to either of two different conceptions of the decision-making process, differing in the set of assumptions that characterize the nature of adjudication. The first views the process as a search for the single, correct, outcome, whereby disagreements are attributed to errors in uncovering this "truth". The second interpretation regards differences in opinions as the result of conflicting values or objectives, and declines to appraise the correctness of each initial standpoint.

(i) Different Estimates. There is a unique optimal decision denoted by "y". y is the solution to

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[2] A restriction implied by this characterization of X is *convexity*. The set is not limited to a sequence of discrete elements, but contains *one* continuous interval. While at first glance this may seem improper in judicial contexts (where measurements can only be divided into basic discrete units), it could be justified either as a convenient technical approximation, or in light of the possibility of mixed strategies.

a well defined optimization problem, which society and the court as its agent face. For example, the utilitarian goal discussed in Shavell (1987) may dictate a specific tort damages award as an optimal deterrent. Thus, y is an ex-ante characteristic of the decision problem in the sense that it is independent of the actual judgements that are drawn in a particular panel, after observing the data. According to this approach, all judges supposedly agree as to the existence of a common objective, and share the unique objective, but the process of revealing y in any particular case is complex enough to cause estimation errors. Formally:

$$D_i = y + e_i \ , \tag{2}$$

where e_i is the error term disturbing the accuracy of each judge's observation of y. e_i is a random variable. Hence, D_i is also a random variable. Several assumptions about its distribution will be made below.

(ii) Different objectives. The second conception regards disagreements as resulting from conflicting objectives. Judges do not share a common objective and may hold different values, each prescribing a different judgement, and each judgement is "correct", given the particular objective that underlies it. The optimal outcome is no longer deducible by some substantive omnipotent reasoning. Instead, it is defined as an ex-post parameter of a given judicial population. The n-tuple of hypothetical opinions[3] can be aggregated in any method that is considered representative and, in particular, by the mean or median measures of the center of the distribution.[4] Each judge's individual objective is modelled as a bias term, relative to the "ex-post y":

[3] They are hypothetical because judges outside the panel do not issue an actual opinion. They are included in the ex-post characterization according to their hypothetical positions.

[4] The majority method cannot be considered as an appropriate aggregation mechanism, since it is not deterministic: a majority winner may not exist.

$$D_i = y + \theta_i \ , \tag{3}$$

where θ_i is the bias term of judge i, $i \in K$. The bias is different from an error term. It is a parameter, rather than a variable. In a way, it is deliberately chosen by each judge.

The distinction between the two sources of disagreement is critical to the results I will present. It is well founded in the social choice literature. While aggregation of different objectives is the typical social choice problem, the literature has distinguished it from the problem of aggregating different estimates. The latter applies in cases where although the objective is shared by the decision makers, the decision makers vary in their skills to reveal the optimal strategy.[5]

Judges preferences over the alternatives set may be defined as follows: in the different estimates setting, each judge's top choice is, ex definitio, y. Although the judge may err in uncovering it and announce a different opinion, y is individually preferred to D_i. In the different objectives model, each judge may be characterized as having a preference ordering over X, which is a linear order.[6] Hence D_i is preferred to any other outcome, in particular to y. Let U_i represent the preferences of judge i, thus:

$$\forall x \in X, \quad \forall i \in K, \quad U_i(D_i) > U_i(x) \ . \tag{4}$$

3. The decision rule. A collegial court must issue a deterministic outcome, to resolve the dispute between the parties. The methods by which the individual judgements are aggregated are "collective choice rules" (CCR) (Sen, 1970). The majority method is the ordinary CCR that

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[5] See Sen (1977), Nitzan and Paroush (1985) for discussion of this distinction.

[6] A complete binary relation that is transitive and anti-symmetric.

applies to dichotomous cases. Some of the majority method's "legal" attributes have already been studied (Easterbrook, 1982; Tullock and Good, 1984; Kornhauser, 1990). However, the majority method's significant attribute, pair-wise "decisiveness", is absent in the non-dichotomous setting. While a majority may still form in particular cases, there may arise situations in which more than two distinct opinions are issued, and none receives the necessary majority support. A supplementary deterministic rule is required, and the following alternative CCRs are considered below:

(i) Mean. A CCR that assigns to the set of individual choices their arithmetic average:[7]

$$Mean = \overline{D} = \frac{1}{k}\sum_{i=1}^{k} D_i \quad . \tag{5}$$

(ii) Order statistics. Since the individual choices have numerical values, they can be arranged in order of increasing magnitudes. If $D_1, D_2, \ldots, D_k$ are the k-tuple of panel opinions, let $D_{(1)} \leq D_{(2)} \leq \ldots \leq D_{(k)}$ stand for the same opinions arranged in an increasing order, which will be defined as the k order statistics corresponding to the given panel. Any order statistic may be a CCR, yet I will focus on one in particular -- the median (the ½(k+1)-th order statistic), and later comment also on the extreme order statistics.

(iii) Other alternative CCRs, include the "dictatorial rule", where a specific (named in advance) judge is assigned, in case of no-majority, the absolute prerogative of choice, and the "degenerate rule", where a default alternative is the outcome whenever courts fail to reach majority (or

[7] A more general rule is the "weighted average". The "mean" is an individual case with a uniform weight.

unanimity).[8] The "plurality count" rule, i.e., the "mode", cannot be an adequate supplementary CCR, since it is not deterministic.

While the "jurisdiction" of these rules may be universal, given that they could be applicable in every non-dichotomous situation, including where a majority exists, it would be more accurate to consider them as supplementary rules in the context of adjudication. When a majority exists, the majority decision prevails, and when there is no majority -- any one of the above CCRs becomes relevant. The reason for the supremacy of the majority method is as follows: in binary contests, the applicable rule is the majority method. If a majority winner exists, the content of the *minority* group -- whether it is homogeneous or heterogeneous -- should not matter. However, the significance of this qualification is relatively small, since in the model presented, the likelihood of a "sincere" majority (which is not a product of coordination) is negligible.

III. Sincere Judges

This section investigates the hypothetical quality of the aggregation rules if each judge announces her sincere opinion, i.e., the judgment she would have issued were she the sole judge on the panel. This isolated analysis is important in demonstrating not only the outcome under those CCRs that indeed induce sincere voting (thus these qualities will not be merely hypothetical), but also the actual effects of insincerity, once the sincerity assumption is relaxed. Separate discussions are conducted for each of the two disagreement models.

1. *Different Estimates*

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[8] A particular form of the degenerate rule is where the default outcome is "0", i.e., no affirmative judgement issued.

The quality of a CCR will be determined by its accuracy in estimating y. Each judge is an estimator of y, whose precision is a product of her error distribution. Hence, the accuracy of a CCR is dependent on the distribution of its error, which in turn is a function of the individual errors. The measure of estimation accuracy is the expected square deviation of the aggregate estimator from y. Denote by $\boldsymbol{D}$ the aggregate outcome that a CCR yields. Then, the objective function in selecting a rule is:

$$\min\ [E(D-y)^2] = Var(D) + [y-E(D)]^2 . \tag{6}$$

Note, that the accuracy of an aggregate estimator is determined by two additive elements: the square of its statistical bias and its variance.[9]

In order to evaluate the CCRs, I will have to consider several alternative assumptions concerning the distributions of e_j, $j \in N$.

Assumption 1 All judges have the same error distribution:

$$\forall j \in N, \quad Var(e_j) = \sigma^2 . \tag{7}$$

Note, that the assumption $E(e_j) = 0$ is redundant, since the different estimates framework implies, *ex-definitio*, no bias. Any systematic error indicates a bias, and such a situation would be captured by the alternative model of different objectives.

(i) Mean. The mean is a CCR that has the following properties:

$$E(\overline{D}) = E(\frac{1}{k}\sum_{i=1}^{k} D_i) = \frac{1}{k}[ky+\sum_{i=1}^{k} E(e_i)] = y , \tag{8}$$

and if D_i are independent (uncorrelated):

[9] The premise underlying this model is that a statistically efficient estimation is a desirable quality for adjudication. Lack of bias implies optimal deterrence in a world of risk-neutrality, while a smaller variance indicates less discrimination and arbitrariness. Among other benefits from lack of bias and a smaller variance is a higher rate of settlements (Cooter and Rubinfeld 1989, at 1075-1082) and Coherence (Kornhauser and Sager 1986).

$$Var(\bar{D}) = Var(\frac{1}{k}\sum_{i=1}^{k} D_i) = \frac{1}{k^2} Var(\sum_{i=1}^{k} D_i) = \\ = \frac{1}{k}\sigma^2 . \tag{9}$$

Intuitively, the mean is a very accurate estimator. It is unbiased, and simply by increasing the size of the panel, its variance is reduced and its statistical efficiency is enhanced. This virtue -- quality of judgement increasing in size of panel -- was noted in the literature only with respect to the majority method (Kornhauser and Sager, 1986; Tullock and Good, 1984). The critical condition is independence of judges' opinions, which may not be fully satisfied in practice; yet for any degree of a less than perfect positive correlation, the mean reduces the dispersion of outcomes.

(ii) Median. Two claims will be stated here -- the first concerning the bias of the median and the second concerning its variance.

Claim 1 *The median is unbiased if for all $j \in N$, e_j has a symmetric distribution.*

The proof is straightforward. The probability that the median will yield an outcome that has a *positive* deviation from y greater than a distance d ($\forall$ $d>0$), is equal to the probability that the median will yield an outcome that has a *negative* deviation of the same absolute magnitude. This equality is derived from the symmetry of all e_j, and the equal likelihood of each population judge to be on any given panel (the randomness of the panel as a sample of the population of judges). This equality is sufficient for the median to have an expectation of y, and thus no bias.

Claim 2 *For every $k \geq 3$, the median's variance is greater than the mean's.*

The median's variance is given by the following formula:

$$Var(D_{(med)}) = \frac{1}{\lambda} Var(\bar{D}) \quad , \tag{10}$$

where λ is distributed so that for every $k \geq 3$, its values are less than 1, hence the median's variance is greater than the mean's.[10]

The intuition behind this result is the following: in a given panel, values that are close to y can be either the mean or median of "many" samples. However, it is clear that values that are more distant from y can be the median more often than the mean, since the median is not sensitive to the intensity of the "dispute", i.e., whether non-median judges are lightly or strongly opposed. If the median has a higher probability of reaching extreme values, its variance is greater.

(iii) Both the mean and the median are better estimators than the other CCRs. For example, a dictatorial rule is indeed unbiased (since every judge's estimate is unbiased), but displays higher variance; further, an extreme order statistic or any degenerate rule are necessarily biased.

Assumption 2 judges have different error distributions:

$$\forall j \neq j', \quad Var(e_j) \neq Var(e_{j'}) \tag{11}$$

Here, neither the mean nor the median are the Minimum Variance Unbiased Estimator. Both are unbiased. In general, the mean may not necessarily have a smaller variance relative

[10] This formula, as well as λ's distribution, are presented in Kendall & Stuart, 1977, Sec. 10.10-10.12; 14.4-14.8; 17.12. These are the values of λ for several sample sizes:

k	1	2	3	4	5	6	7	9	11	13 ...... ∞
λ	1	1	.74	.84	.70	.78	.68	.67	.66	.66 .637

to the median.[11] Intuitively, consider the effect that highly "volatile" judges have on the median and the mean. The mean gives equal weight to every judge's opinion; thus its variance is increased by the presence of highly volatile judges. (I.e., there are more potential panels from which extreme opinions may be issued.) The median, on the other hand, is less responsive to high volatility, because the extreme opinions are less likely to be panel medians and be influential over the aggregate outcome. Thus, the median rule is more suitable for a judicial populations that includes highly volatile judges, while the mean performs better when the population is more restrained.

In sum, within the different estimates model, if judges are relatively consistent and are not influenced strongly by arbitrary factors, then the mean is the more accurate estimator.

2. Different Objectives

Each judge has an individual objective, possibly other than y, which is modelled as an additive bias term θ_i. Judges do not err in observing their personal objective. To determine how close the aggregate judgment is to the "optimal" outcome y, additional information about the nature of y is required. If y is conceived as an ex-ante characteristic of the problem, such that it is determined notwithstanding the actual opinions of the population, then a specific objective

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[11] To demonstrate this ambiguity, compare the mean and the median to the Maximum Likelihood Estimator (MLE). Consider a case where each judge's opinions are distributed normally with parameters (y,σ^2_i), where σ^2_i is judge i's variance. In this case, the MLE is given by:

$$y_{ML} = \frac{\sum_{i=1}^{k} \frac{D_i}{\sigma_i^2}}{\sum_{i=1}^{k} \frac{1}{\sigma_i^2}} \tag{12}$$

The mean CCR assigns equal weights to judges' opinions, thus deviating from the optimal rule which assigns higher weights to low variance judges. As the diversity of the variances in the population grows, so does this deviation of the mean, suggesting that the median may be superior.

is in fact regarded as superior and correct. That would undermine the principal property of the rival conception of disagreements, which abstracts from such value judgments. Therefore, the only other approach to define y is ex post: to pinpoint a certain value that -- given the actual opinions of the population -- is considered representative (or stable). This can be done implicitly by proposing several assumptions about the distribution of the population's θ_j. Without defining y explicitly, it is still possible to establish the proximity of D to y.

Hence, consider the following two suppositions:

$$\textbf{\textit{S1}}:\ \bar{\theta} = \frac{1}{n}\sum_{j=1}^{n} \theta_j = 0$$

$$\textbf{\textit{S2}}:\ \forall j \in N,\ \theta_{(j)} + \theta_{(n-j+1)} = 0$$

S1 states that the sum of biases (or the average bias) in the population is 0. It implies that y is the average opinion of the population. *S2* is even stronger: it demands not only 0 average bias, but also a symmetric population, where every positively biased judge is countered by an opposite judge with a negative bias of the same absolute magnitude. Note that *S1* implies *S2*, but not vice versa. It is now possible to examine the statistical properties of the mean and the median CCRs.

(i) Mean.

Claim 3. *The mean is an unbiased estimation of y if and only if S1 holds.*

The zero-expected-value of θ_j is a necessary and sufficient condition for the mean to be unbiased. The proof is simple: if every judge may appear on a panel with the same frequency, the mean's bias is a simple average of θ_j, $j \in N$:

$$E(\bar{D}) = \frac{1}{n}\sum_{J=1}^{n} D_j = y+\frac{1}{n}\sum_{j=1}^{n} \theta_j = y+\bar{\theta} \ , \qquad (13)$$

and only when *S1* holds, the mean's expectation is y. This result is straightforward since *S1* is synonymous to defining y to be the average bias in the population. Then, the mean of a random sample is an unbiased estimator of the population's mean, which is assumed to be y.

(ii) Median.

Claim 4. *The median is an unbiased estimator if and only if S2 holds.*

If judges are symmetrically distributed around y, the median's expected value is y. This is true because for all $j \in N$, $D_{(j)}$ and $D_{(n-j+1)}$ can be the medians in an identical number of different panels. For k=3 for example, they can be the medians in exactly $(j-1)(n-j)$ panels. If the value of y is such that the judges are not symmetrically distributed around it, the median of a random sample will generally be biased.[12]

The significance of *Claims 3* and *4* lies in the stronger requirement that necessary to guarantee the median's unbiased quality, relative to that of the mean. If only *S1* (and *not S2*) holds, the mean is superior; the converse cannot occur.

(iii) The other CCRs are strictly inferior to the mean or the median. The dictatorial rule is biased, according to the specific prejudice of the dictator. The extreme order statistics are necessarily biased if the median is not, and even if the median is also biased, such statistics are still most likely biased.

[12] If the distribution of the population is symmetric around a value other than y, then this value is the expectation of the median. If the distribution is asymmetric, then unless y is defined as the population's median, the median of a random sample will be biased.

Under a compound model, where the sincere judgment of each panel member includes both a bias parameter and an error term, the results may change. Until now, the mean has displayed consistently superior statistical qualities, relative to the median. Within the compound framework, the mean would still be more likely to be unbiased; yet circumstances may emerge where the median's variance would be smaller.[13] However, under ordinary assumptions regarding the judiciary, i.e., no extreme biases and low volatility in judges' conduct, the mean is strictly more accurate than the median and the rest of the CCRs. I will now turn to re-examine this result under a strategic environment.

IV. Strategic Behavior

1. Manipulability introduced

Announcing the sincere opinion may not be the optimal strategy for a judge. Once she realizes that the court's aggregated judgement might deviate from her own, she may find strategies other than sincerity to be superior, if the manipulated outcome is more favorable to her than the sincere one.

Two main factors affect the motivation to behave strategically:[14] first, the state of knowledge -- namely, the accuracy of the prediction concerning other judges' opinions, and second, the manipulability of the CCRs. Two alternative assumptions will be discussed regarding the knowledge of each other's opinions: there is either complete and accurate

[13] In general, this outcome is likely when the population is extreme: either there are many extremely biased judges in both directions, or there is a positive correlation between the absolute bias and the error variance among judges. The mean is more sensitive to extreme values, since it reacts to any extremity, while the median is affected only by the extremity of judges who are the potential panel medians.

[14] In this paper I ignore other types of obstacles to manipulations, such as the ethics of the judiciary, which demands honesty and sincerity.

knowledge, or -- in addition -- there is common knowledge. Complete knowledge, where each judge recognizes the components of $\{D_i\}_{i \in K}$, can be gained if judges privately inquire about their counterparts' opinions, or if information passes through clerks. Common knowledge, where every judge knows not only the above, but also that the others have complete knowledge, and knows that all the others know that everyone has complete knowledge, and so on, may exist if judges share their true opinions openly, or circulate initial judgments among the panel.[15]

The other condition for strategic behavior is the manipulability of the CCRs. As will be demonstrated below, under some conditions, one CCR (the mean) might induce manipulations, while another (the median) could be strategy-proof. Under different circumstances, the opposite may hold. The main body of this section studies the manipulability characteristic of the rules.

It is necessary to distinguish between two types of strategic interactions: "non-cooperative" and "cooperative". The non-cooperative type refers to the kind of isolated, "self-interested" behavior on part of the actors, where no explicit coordination takes place. The cooperative type, by contrast, attempts to account also for the open modes of coordination and collusion that judges adopt through explicit communication, as well as the patterns of coalition-formations among sub-sets of players.[16] A CCR may be resistant to one type of strategic behavior yet subject to another.

The analysis is again conducted separately for the two models of disagreement. The conclusions of the first part, analyzing the different estimates case, strongly favor the mean, which would be, under some conditions, strategy proof. In the different objectives analysis the conclusions reverse, and the median is superior.

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[15] Situations of incomplete information are less likely to arise in courts, because a deliberation process usually precedes the judgment.

[16] For discussion of the distinction between these two branches of Game Theory, see Schelling (1980), Shubik (1984).

2. Different Estimates

(a) General. The first thing to note, which distinguishes this case from the different objectives setting, is that for each judge, D_i may not be her most preferred outcome, even though it is her best estimate of y. When judges are aware of the conditions under which they operate, in particular of the existence of errors in their judgements, they recognize that their estimates of y are not perfect. For all i, $U_i(y) \geq U_i(D_i)$.

Even though the divergence of D_i from y is recognized, the ambiguity about the value of y makes manipulations futile in promoting the individual's most desired outcome. However, each judge may identify *estimation procedures* which are more accurate than her own estimate, and therefore, each judge will be willing to concede, a priori, to that estimate's product. Hence, if the court's CCR produces estimates of y that are statistically more efficient than the individual estimate, it would be desirable for the rational judge to "surrender" to this estimate, rather than promote her own D_i. In light of this qualification, it is possible to make the following claim:

Claim 5 *In a model where disagreements are due to estimate errors, non-cooperative manipulations are individually desirable if and only if the individual's estimate is statistically more efficient than the CCR's.*

Proof. The positive claim, that strategic manipulations can occur when the individual is a better estimator than the CCR is obvious. By manipulating the outcome to promote her estimate, the judge reduces the expected error, even if she does not eliminate it. The second part of the claim, that manipulations will not occur otherwise, is more subtle. If the CCR produces better estimates than each individual judge, an equilibrium where each judge is sincere exists. It is the only equilibrium. Any non-cooperative deviation from sincerity is not profitable: if it aims in promoting D_i, it reduces the estimate's efficiency; and it cannot aim at advancing y since y is unknown.

Given this result, the following analysis examines the patterns of cooperative strategic behavior that could arise under the above assumptions.

(b) Opting-out of inefficient rules. The strategic patterns that follow from each CCR resemble the type of interactions that are induced by default rules in contract law. It is well established that inefficient rules of law (rules that do not maximize the total amount of the parties' welfare) are subject to the process of "opting-out", in which the parties privately select an alternative, more efficient arrangement (Ayres and Gertner, 1989; Bebchuk and Shavell, 1990). The same pattern arises in our judicial environment. If the CCR is not the statistically most efficient one, the judges can jointly agree to opt out from it, and informally implement a preferred rule, by unanimously supporting its outcome. If, for example, the CCR is the median rule, and the conditions are such that the mean produces a more accurate estimate of y, the entire panel could coordinate its judgments so that everyone announces the same judgment -- the mean of the sincere opinions. Even the median judge, who forgoes the opportunity to make his estimate the social choice, would conform to the cooperative strategy, since he too finds the mean superior (ex-ante) to the median.

The conditions under which such rational coordination emerges depend on the knowledge and the rationality of the panel.[17] Even if reality departs from these "Coasean" conditions, one can easily imagine judges acting in the manner described above. Instead of submitting their disagreements to the jurisdiction of the CCR, they form a coalition, either of a majority (to save transactions costs) or of the entire panel, that supports an outcome other

[17] The term "rationality" in this context suggests both knowledge of the parameters of the environment and conformity to the motivational characteristics that are discussed.

than the individual judgments. Indeed, the likelihood of such coordination is perhaps the reason for the apparent disregard, by many legal systems, of formal CCRs.

It is no longer a puzzle why courts demonstrate unanimity so frequently, where the a priori probability of such an outcome is negligible. Where individual opinions are formed independently, unanimous consent is necessarily a result of insincere behavior, of coordinated manipulations. The frequency of consent is the product of the individuals' acquiescence to voluntary aggregation processes, in order to avoid resorting to the less efficient formal CCRs. This explanation for unanimity is distinct from the type of explanation presented by Klevorick and Rothchild (1979). In contrast to their focus on the deliberation process that takes place in the jury panel, attributing unanimity to the "persuasive" power of the majority, the current explanation emphasizes the motivation of the decision makers to perform *collectively* in an optimal manner. Both explanations concentrate on types of dependencies among individuals' judgments; they, however, differ in the dynamic process that leads to unanimity. Klevorick and Rothchild view it as a stochastic process, while this paper offers a deterministic model founded on the strategic interactions among the decision makers.

Furthermore, the analysis enables us to predict the type of agreements that judges would arrive at. If judges are rational, they would be aware of the results that were discussed in section II above and realize the conditions for the mean's optimality. Hence, they would agree to support the mean as the unanimous choice. Further, the mean of the sincere opinions is the likely "compromise" even for judges who are less informed and sophisticated, as long as they share the notion that all have equal skills.

The above discussion can be summed up as follows:

Claim 6 *In the different estimates model, there is only one strategy-proof CCR -- the one that is the most accurate estimator of y. Based on section II, the mean is most likely to be the only* *non-manipulable rule, and if its statistical efficiency attributes are common knowledge, any other CCR will induce cooperative strategies that implement the mean outcome, ad hoc.*

3. Different Objectives

The critical feature that distinguishes the following analysis from the above, concerns the motivation of each judge. Here, D_i is judge i's most preferred outcome. D_i includes her subjective bias term θ_i, and she therefore wishes the collective judgment to be as close as possible to D_i. In particular, for all i, $U_i(D_i) \geq U_i(y)$. Judges will engage in non-cooperative manipulations in order to bring the aggregated judgment closer to their individual D_i. The discussion below examines the manipulability of the CCRs.

(i) Median.

Claim 7 *For a single-peaked preferences profile, the median is non-manipulable.*

Remark. Single-peakedness is a restriction on the domain of the preferences orderings that an individual judge may have. U_i is single-peaked with respect to the ordering relation > ("greater than") if:

$$\forall a,b,c \in X,\ [U_i(a) > U_i(b) \text{ and } a>b>c] \Rightarrow U_i(b) > U_i(c)\,. \qquad \textbf{(14)}$$

Equivalently, single-peakedness implies a unique local maximum. For every judge and for every two alternatives that are either both higher or both lower than D_i, the one closer to D_i is preferred.[18]

The proposition states that if all judges have single-peaked preferences, their optimal behavior is sincerity. This assumption appears to be reasonable since it only requires that

[18] Moulin (1983) at p.70; Mueller (1989) at p. 66.

judges agree on the fact that there is a unique optimal solution and that the further the decision lies from the optimum, the worse it gets. They can disagree as to the value of the optimum. However, multi-peaks are also plausible. Attitudes such as "punish severely or do not punish at all, to avoid ambivalence" can be a source of multi-peakedness.

Proof. See Appendix.

This strategy-proof result indicates that the vector $(D_1,..D_k)$ is the unique Nash equilibrium vector of strategies where D_i is the *dominant strategy* for any judge. Even if others act manipulatively, the optimal response of an individual judge is sincerity. Sincere behavior is therefore the only stable state of behavior for the entire court. The result is closely related to the median's familiar property (also dependant on single-peakedness) of being the Condorcet winner, i.e., the median's ability to defeat any other alternative in binary majority contests.[19] To override the median by a cooperative majority is impossible, since the majority (Condorcet) winner is in fact the median.

(ii) Mean. Unlike the median, the mean is sensitive to the intensity of the preferences: it responds to a shift in any judge's opinion. This property accounts for the mean's manipulability. The pattern of manipulations is the following: for every vector of the panel's preferences $(D_1,..,D_k)$, any judge $i \in K$ assigns an element of X, which promotes her own objective optimally. The judge operates according to a manipulation function $m_i : X^k \to X$, of the form:

$$\frac{1}{k}\left[\sum_{t\in K\setminus\{i\}} D_t + m_i(D_1,\ldots,D_k)\right] = D_i \tag{15}$$

[19] See Mueller (1989) at p. 66.

which yields:

$$m_i(D_1, \ldots, D_k) = kD_i - \sum_{t \in K \setminus \{i\}} D_t \ . \tag{16}$$

Hence, the only case where judge i will not issue an insincere opinion is when $m_i(D_1,..,D_k)=D_i$, which occurs when D_i is already the mean of the panel. Furthermore, manipulations are always optimal for some subset of judges, unless $D_i=D^*$ for all i is the case (sincere unanimity).

This form of manipulative behavior assumes complete knowledge and unconstrained values for m_i. I will now examine how the results changes under different assumptions.

Constrained manipulation function. For some judges, optimal manipulations cannot be exercised, since X is bounded. The general form of the manipulation function, accounting for the bounds, is:

$$m_i^* = \begin{cases} \max(\alpha, m_i) & \text{if } m_i < \beta \\ \min(\beta, m_i) & \text{if } m_i > \alpha \end{cases} \tag{17}$$

Thus, a single-peaked preference judge whose optimal strategy is infeasible has to settle for a bound of X.

Knowledge and manipulation equilibrium. The analysis of the manipulations induced by the mean has focused on the isolated motives of each individual. The problem is that the assigned strategies are not Nash equilibrium strategies. Each judge's function represents her best response to the true opinions of the others, not to their actual (manipulated) votes. One way for an equilibrium to evolve requires judges to account for the manipulations of each other. This situation would emerge if judges have common knowledge of the true preferences of the panel and of rationality. Then, each judge would realize that other judges will also manipulate, following the same patterns she does, and that others also account for her manipulation, as well as for her account of their manipulations, and so on.

In equilibrium, each judge would have to consider not only the true preferences of the other judges on the panel, but also their manipulative motivations, in order to offset their direct effect, as well as their offsetting steps against her own manipulation, and so on. Clearly, this motivation drives the equilibrium judgements of some judges to the bounds of X. Consider the following clarifying example:

$X=[0,\infty)$, $\quad k=3$

$(D_1, D_2, D_3) = (50, 60, 100)$

The "true" mean is 70. Under complete but not common knowledge, the constrained manipulations are:

$m_1^*=\max(0,-10)=0, \quad m_2^*=30, \quad m_3^*=190$

Which is not an equilibrium: the mean is 73⅓, and both judges 2 and 3 can deviate and gain. The only Nash equilibrium is where judges 1 and 2 declare "0" and judge 3 announces "300". The mean is 100, optimal to judge 3, while judges 1 and 2 cannot deviate to reduce it, since they are bounded by $\alpha=0$.

Notice that this process has negative implications on the estimation accuracy of the mean. First, the unbiased quality discussed above can be frustrated if one of X's bounds is on average more "active" than the other. The bias would tend to the opposite direction of the active bound. Second, the variance of the mean would increase. Observe (from (16)) that the absolute magnitude of the manipulation rises with the distance of D_i from the true mean. This results in a polarization of judgments. In many cases they do not cancel each other; thus the outcomes would be more dispersed.

Another manipulation feature that is related to the mean CCR is the cooperative type.

Whenever the mean has a different value than the median, there would be a subset of judges that includes the median judge, that could coordinate a simple majority coalition to endorse an outcome other than the mean. In the above example, judges 1 and 2 would be better-off by agreeing on any alternative in the interval [50,60] and providing the majority to ensure its prevalence. This follows from the fact that the majority rule has supremacy, and that if a majority winner exists, it is the median.

(iii) The last comment above applies to the other decision rules as well. Consider the degenerate rule for example, where the default outcome (in the absence of a majority winner) is "0". This method would most likely implement the median. Since the default outcome is undesirable to most judges, it would induce a subset of the panel members to agree upon a different outcome and to ensure at least a simple majority to support it. While many outcomes may override the default in this manner, only the median would itself be resistant to overriding coalitions. In fact, a "good" degenerate rule may not necessarily be a "consensus" default, pretending to mimic some reasonable compromise. Instead, it may only act as a "penalty default"[20] that stimulates the voluntarily coordination of judgements and formation of collusive majorities to override it. Similar arguments may apply to the dictatorial rule or to the extreme order statistics.

V. Conclusion

Non-dichotomous choice problems arise frequently enough in judicial contexts to merit separate treatment. The insufficiency of the majority decision method in these cases is the

[20] The term "penalty default" is used here in precisely the same sense as that specified in Ayres and Gertner (1989). While they focused on general contractual default rules, the context here is specific to "contracts" among judges in a collegial court.

driving force of the above analysis, comparing other complementary methods for resolving disagreements. The results depend strongly on the assumptions made about the source of disagreements in courts. If disagreements are attributed to varying skills or estimates (in revealing a single correct judgement), the superior method is the mean rule. Not only does it estimate the desired outcome most proficiently, it is also the only non-manipulable mechanism. The result changes significantly under the alternative model, where disagreements are conceived to be founded on value differences. Here, it is harder to point an "efficient" estimator, mainly because of the skepticism concerning the existence of an optimal outcome. Nevertheless, the median appears to be the superior rule, since it is less prone (sometimes proof) to manipulations, compared to the mean.

Even if the normative conclusions seem somewhat ambivalent, this analysis captures clearly at least one positive feature of adjudication: the overwhelming frequency of unanimity or the existence of a majority in non-dichotomous cases. Unanimity cannot be attributed to chance, since the alternatives set is too large. The strategic patterns discussed above provide an understanding of the emergence of such conformity. They also suggest an itinerary for further theoretical research that would focus on time-dimensional interactions. Relevant extensions include investigating the conditions and effects of judicial "logrolling" (vote-trading) and its dependence on the formal aggregation procedures.

Appendix

Proof of Claim 7. Denote the median of the panel by $med(D_1,...,D_k)$. The panel's judges can be partitioned into two sets: those who prefer an outcome equal to or greater than the median and those that prefer an outcome less than the median. Denote these sets by K_H and K_L, respectively:

$$K_H = \{i \in K: D_i \geq med(D_1, \ldots, D_k)\} \tag{1A}$$

$$K_L = \{i \in K: D_i < med(D_1, \ldots, D_k)\}$$

Suppose the median is manipulable. Without any loss of generality, suppose there is a judge h in K_H for whom announcing something other than D_h results in a strictly better outcome. Namely,

$$\exists\ h \in K_H,\ \exists\ z \neq D_h,\ s.t.\ U_h[med(z, D_{-h})] > U_h[med(D_h, D_{-h})] \tag{2A}$$

where z is the manipulated judgment of judge h, and D_{-h} are the judgements of the rest of the panel: $D_{-h} = (D_1,..,D_{h-1},D_{h+1},..,D_k)$.

Distinguish two possible cases: either z is less than the non-manipulated median, or it is equal to or greater than it.

(1) $z < med(D_h, D_{-h})$

Since the median is a monotonic, non-decreasing function in $(\bullet, D_{-h})$,

$$med(z, D_{-h}) \leq med(D_h, D_{-h}) \tag{3A}$$

By assumption, $h \in K_H$, thus $med(D_h, D_{-h}) \leq D_h$, and since $U_h(D_h) \geq U_h(med(D_h, D_{-h}))$, single-peakedness of U_h implies:

which contradicts the assumption made in expression (2A). Hence case (1) is not possible.

$$U_h[med(D_h, D_{-h})] \geq U_h[med(z, D_{-h})] \tag{4A}$$

(2) $z \geq med(D_h, D_{-h})$

$h \in K_H$, so $D_h \geq med(D_h, D_{-h})$. Both z and D_h have values equal to or greater than the median so that the substitution of one with another would not affect the median's value: $med(D_h, D_{-h}) = med(z, D_{-h})$, which yields,

$$U_h[med(D_h, D_{-h})] = U_h[med(z, D_{-h})] \tag{5A}$$

which again contradicts the assumption that h can deviate from D_h to some z and strictly gain. Case (2) is thus impossible.

If both case (1) and (2) are impossible, than there is no value to which the judge can deviate and which will bring about an outcome more favorable to her. The median is non-manipulable.

Q.E.D.

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