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# THE WELFARE EFFECTS OF METERING TIES

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## Abstract

Critics of current tying doctrine argue that metering ties can increase consumer welfare and total welfare without increasing output and that they generally increase both welfare measures. Contrary to those claims, we prove that metering ties always lower consumer welfare and total welfare unless they increase capital good output. We further show that under market conditions we argue are realistic (which include a lognormal distribution of usage rates that are independently distributed from per-usage valuations), metering ties *always* harm consumer welfare, even when output increases. Under those market conditions, we show that whether metering ties raise or lower total welfare depends on the dispersion of desired usage, the cost structure, and the dissipation of profits caused by metering. For realistic cost values, metering ties will reduce total welfare if the dispersion in desired usage of the metered good is below 0.62. (For comparison, 0.74 is the dispersion of income in the U.S.) If 5% of metering profits are dissipated, metering will reduce total welfare for all cost levels unless the dispersion in desired usage exceeds 150% of the dispersion of income in the U.S.



JEL Codes: C72, K21, L12, L40, L41, L42.

Keywords: antitrust, bundling, consumer welfare, metering, price discrimination, quasi per se rule, ties, tying, requirements ties.


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# THE WELFARE EFFECTS OF METERING TIES

## I. INTRODUCTION

A firm with market power may meter usage of its product by requiring users to purchase at elevated prices a tied product that is needed to use its product. The classic case is tying a capital product, like printers, to a consumable used with the capital product, like ink. Because buyers who use the tying product more often tend to value it more highly, metering ties can achieve a form of price discrimination when market power exists. However, the relationship of usage to value is typically imperfect and thus so is the resulting price discrimination.

Metering ties have been central to many Supreme Court cases on tying.[1] While there is little controversy about ties that foreclose a substantial share of the tied market, critics have focused on the fact that current doctrine allows condemnation of ties without proof of a substantial foreclosure share.[2] Thus, we here address the direct effect of a tie without any substantial foreclosure.

We focus on metering ties because the economics of metering ties has been comparatively more controversial and less well developed in rigorous models. Focusing on metering ties also tackles what has so far seemed the form of tie that offers the best case for the critique that the current quasi per se rule is overbroad.[3]

Hovenkamp and Hovenkamp, (2010: 925–928) conclude that the "great majority" of metering ties benefit both consumer welfare and total welfare, and indeed "only harm consumer welfare in the most flagrant situations." and "consumers likely benefit most of the time." Lambert, (2011: 934, 939) concludes that metering ties "typically enhance total welfare" and "most instances of metering enhance consumer welfare." According to Wright (2006, 349),

[1] *See* Illinois Tool Works v. Independent Ink, 547 U.S. 28, 32 (2006) (ink used with printers); Int'l Salt Co., Inc. v. United States, 332 U.S. 392 (1947) (salt used with salt-injecting machines); Int'l Bus. Machs. Corp. v. United States, 298 U.S. 131 (1936) (cards used with tabulating machines).

[2] See Carlton and Heyer (2008), Carlton and Waldman (2012), Crane (2012), Hovenkamp and Hovenkamp (2010), Lambert (2011), and Semeraro (2010). Bowman (1957) provided an early critique of applying antitrust to metering ties.

[3] That quasi per se rule is really a form of rule of reason review in that it condemns ties only when the plaintiff proves tying market power and the defendant fails to prove any offsetting procompetitive justification. *See* Elhauge (2009: 420, 425); Areeda and Hovenkamp (2011: 372–82), and Elhauge (2011: 368-69); Elhauge (2016: 466).

"Second-degree price discrimination in the metering context is therefore an approximate form of perfect price discrimination."

For these reasons, they argue that metering ties should be per se legal absent proof of a substantial foreclosure share. Contrary to this prior literature, our economic model suggests a presumption that metering lowers consumer welfare and rejects any general presumption that metering improves total welfare.

We begin with some general results regarding the welfare effects of second-degree price discrimination. Although in our view consumer welfare is and should be the legal standard (Elhauge 2009, 2011), throughout the following analysis we also address total welfare effects because some argue that the legal standard is or should be total welfare.

When we turn to consider the welfare effects of metering ties, we put aside the possibility that the tie might be designed to achieve some productive efficiency, such as risk-sharing, lowering product costs or improving quality. Initially, we also assume there is no dissipation of the profits reaped from metering ties. To the extent that profit dissipation occurs, total welfare will be reduced, and we show how our results extend to these cases.[4]

We divide our analysis into four parts. Following this introduction, in Part II we offer a general proof that when a firm has market power over a capital good that is used with a consumable for which there is a competitive market, then profitable metering ties—and indeed any form of price discrimination—lowers consumer welfare and total welfare unless it increases output of the capital product. This is directly analogous to the result previously known about third-degree price discrimination. This paper thus extends that result to all metering ties and second-degree price discrimination whenever a firm has market power over a capital product but there is no market power on the consumable. The implication of this theorem is not just hypothetical: we provide specific conditions under which profitable metering ties fail to increase output and thus lower both consumer welfare and total welfare.

Although expanding capital good output is a *necessary* condition to show that a metering tie increases consumer welfare or total welfare, it is not a *sufficient* condition. A metering tie that

[4] Ties might dissipate profits because of the costs of implementing or enforcing the tie or because the profits increase ex post managerial inefficiency or ex ante expenditures to obtain the market power to tie; see Elhauge (2016: 486–89, 510–14). Wright and Ginsburg (2013: 2412 n.44) argue the other side, that price discrimination, by increasing profits, increase dynamic efficiency such as the incentive to innovate.

leaves output unchanged will reallocate some output from higher-value buyers to lower-value buyers, which standing alone is welfare reducing. Metering ties can enhance welfare only when they increase overall output *and* the size of that output increase offsets the welfare harm from reallocating some output from higher-value buyers to lower-value buyers. Whether the net welfare effects of metering ties are harmful will turn both on the distribution of buyer types and on the welfare standard one uses.

Kwoka (1984) provides an example with two customer types that shows second-degree price discrimination can lower output and total welfare, while Hovenkamp and Hovenkamp (2010) provide a two-buyer example where metering raises total welfare.[5] An important feature of the Hovenkamp and Hovenkamp model is that there is perfect correlation between the usage rate the buyer desires and buyer's average value per usage. In other words, they assume there are no buyers who want a large number of consumable units but have a low value per unit. This assumption favors metering because those are precisely the customers who are served under single pricing but excluded from the market under metering. Indeed, we can show that in our model with such perfect correlation, metering always raises total welfare but lowers consumer welfare.

This leads us to Part III where we develop a more realistic two-dimensional model in which the number of consumable units desired and the value per unit vary separately. In order to determine an intuition for welfare effects, we adopt the plausible assumption that there is a lognormal distribution of buyer usage rates. Empirically, the lognormal distribution provides a good fit for both the income distribution and the distribution of firm size.[6] Thus, to the extent that desired usage is proportional to either income or firm size, our assumption of a lognormal distribution is appropriate.[7] We also assume: (1) for each buyer, the value per usage is constant up to the desired usage level; and (2) across buyers, the value per usage is uniformly distributed

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[5] The Hovenkamp and Hovenkamp example has the problem that the hypothesized metering is not profit maximizing. In our paper, we restrict attention to metering prices that are profit-maximizing, and thus market relevant.

[6] This observation goes back Gibrat (1931). For more recent confirmation, see Stanley *et. al.* (1995). The fit works well, except at the very upper tails of firm size.

[7] For large firms, it seems likely that they are using multiple capital products to produce their output, and thus their usage rate of *each* individual capital product will not grow linearly with firm size. For that reason, we break down a firm into its different establishments. We also note that for firms at the upper tail, vendor prices are often negotiated on an individual basis, and thus fall outside our model.

and independent of how many units are desired. We discuss and justify these assumptions at length in section IIB.

Under these assumptions, Lemma 2 and Theorem 4 demonstrate that metering ties are always profitable and *always* reduce consumer welfare, *even* in those cases when they increase tying product output. Thus, there is no need to inquire into tying product output: consumer welfare falls. This is true for all cost levels and usage dispersion levels, though the result does depend on the lognormal distribution and our other modeling assumptions described just above.

The results for total welfare depend on the cost and the dispersion level. With zero capital good costs, metering is able to bring all levels of buyer usage into the market. Even so, metering ties will lower total welfare unless they increase output of the tying product by more than thirty-seven percent. That level of output expansion arises only when the standard deviation of the natural log of buyer usage rates is greater than 0.78. To put this in perspective, 0.78 is slightly greater than the estimated 0.74 standard deviation of the lognormal income dispersion in the United States, but below the 1.06 estimated standard deviation of the lognormal distribution of establishment size (based on employees) in the United States.[8]

When capital costs are positive, metering will reduce total welfare over a smaller range of parameters. For reasonable cost levels, Theorem 5 shoes that metering ties still lower total welfare unless the dispersion in buyer usage rates exceeds 0.62.[9] This is to be expected. While metering creates inefficiencies, its central advantage comes from being able to serve customers with a wide range of usage rates.

If metering leads to even a small dissipation of profits, metering will lower total welfare over a much wider range of parameters. For example, with a 5% dissipation in profits, total

[8] See Pinkovskiy and Xavier Sala-i-Martin (2009) for income distribution. The lognormal parameter for firm establishment size was calculated by the authors using 2013 U.S. Census data available at http://www.census.gov/ces/dataproducts/bds/data_estab.html. Note that establishment size is different than firm size in that each plant of a firm is recorded as a different establishment. To the extent that firms are buying separate capital goods at each establishment, establishment size is a more appropriate measure than firm size. Even the dispersion of establishment size can overstate the dispersion of consumable usage per capital good to the extent that a single establishment uses multiple capital goods (like an office building with a hundred printers, each of which is tied to ink cartridges).

[9] Capital good costs can be measured relative to the desired usage of the median potential buyer. As discussed following Theorem 5, we think a cost ratio of 20% or less is reasonable. In that case, metering must increase sales of the tying product by thirty percent to increase total welfare, which arises when the dispersion of the lognormal exceeds 0.62.

welfare is lower under metering for *all* costs, provided that the dispersion in desired usage of the metered good is below 150% of the dispersion of income or the dispersion of establishment size (based on employees) in the United States.

Finally, Part IV concludes with a discussion of the policy implications of our results.

## II. GENERAL RESULTS

Perfect (or first-degree) price discrimination charges each buyer a price for the tying product that precisely equals its valuation of that product. This clearly reduces consumer welfare (by taking all consumer surplus), but increases total welfare by selling the product to anyone who values it at or above its marginal cost. In the real world, price discrimination is inevitably imperfect, and falls into two categories. Second-degree price discrimination offers a menu of product options and prices that causes buyers to self-select in ways that imperfectly correlate with their willingness to pay. Third-degree price discrimination charges different categories of buyers different prices that roughly correlate to the value that buyers in each category put on the product.

Varian (1985, 1989) proves that third-degree price discrimination reduces total welfare unless it increases output. With linear demands, third-degree price discrimination leaves output unchanged and thus reduces total welfare unless there is a category of buyers who would buy none of the product at a single profit-maximizing price but would at the discriminatory price; see Robinson (1969) and Schmalensee (1981).

The status of second-degree price discrimination is less clear. The literature does not provide any general proof of whether second-degree price discrimination can increase total welfare without any output increase. Some scholars argue that the proposition that imperfect price discrimination cannot increase total welfare unless it increases output does not apply to second-degree price discrimination. According to former FTC commissioner Joshua Wright (2006, 348):

> This conventional welfare analysis is of limited utility. … [A]ftermarket metering arrangements, like those involved in *Independent Ink*, do not involve third-degree price discrimination.

According to Hovenkamp and Hovenkamp (2010: 928–929) [10]

> Third-degree price discrimination that does not increase output necessarily decreases welfare. This is not true of second-degree price discrimination.... Equally important is the fact that third-degree price discrimination transfers output from higher-value to lower-value customers, thus reducing welfare even if output remains constant. Second-degree price discrimination does not have this effect.

The truth of this statement depends on what output is being measured. Hovenkamp and Hovenkamp focus on *consumable* output. It is straightforward to provide an example where consumable output falls and total welfare rises under second-degree price discrimination.[11] Later, following Theorem 2, we provide an example where consumable output rises and welfare falls. Together these examples show that there is no general relationship between consumable output and welfare.

We conclude that *capital* good output is the appropriate measure of output as it is the good over which the firm has a monopoly. As we demonstrate in Theorem 1, the welfare result for third-degree price discrimination has a direct analogy for second-degree price discrimination in the case when there is a capital good whose usage requires a complementary consumable good, like using ink with a printer. In such markets, *second-degree price discrimination reduces total welfare unless it increases capital good output*. Furthermore, Theorem 3 shows that with linear demands, second-degree price discrimination will not increase capital good output unless there is a category of buyers who would buy none of the product at the single profit-maximizing price but would with price discrimination. Here, too, the welfare results for second-degree price

[10] See also Lambert (2011: 938) "second-degree price discrimination in the form of metering, unlike a third-degree price discrimination scheme, need not increase total output in order to enhance welfare."

[11] Take the case where there are 13 potential buyers, divided into three types, A, B, and C:

| | Number of Customers | Copies desired | Value per copy | Willingness to pay |
|---|---|---|---|---|
| A | 1 | 10 | \$1.00 | \$10 |
| B | 1 | 100 | \$0.10 | \$10 |
| C | 11 | 1 | \$1.50 | \$1.50 |

If we assume that all costs are zero, then the profit-maximizing single price is \$10 for the printer, the firm earns \$20, and total welfare is also \$20. Under linear metering, the printer is given away for free, the profit-maximizing usage charge is \$1 per copy, the number of copies made falls from 110 to 21, but profits rise to \$21, consumer welfare rises from 0 to \$5.50, and welfare rises to \$26.50.

discrimination parallel those for third-degree price discrimination when one uses the analogous measure of output.

Because the connection between welfare and output turns so much on which output one is considering, we try to be extra clear about which product is which. For example, the printer is the tying product, and the toner cartridge is the tied product. That said, the use of “tied” and “tying” product can lead to confusion. We think it is simpler terminology to think of the firm as having market power over a capital good, such as a printer or razor. The tied good is then the consumable product, such as the toner cartridge or the blade. Thus in our discussion below we adopt this convention. A firm has market power over a capital good, and there is no power over the consumable used with it.

To test whether price discrimination necessarily decreases total welfare, we look at the output of capital goods, not the complementary consumables. Metering may reduce the number of toner cartridges sold while increasing total welfare. But metering cannot increase total welfare without increasing the number of printers sold.

To test whether metering necessarily increases welfare when it increases consumable output, there is analogous result (Theorem 2) that runs in the opposite direction, but is less general. If metering increases consumable sales, then it increases total welfare in the case where the capital good is costless and provided for free. The result is not as general because metering may lead to an inefficiently large number of capital good sales, and thus the proof requires that the capital good be costless and given away. Moreover, if this test is passed, then sales of the capital good must also have increased.

### *A. Any Price Discrimination (Including Metering Ties) Must Increase Capital Good Output to Increase Welfare*

We assume a market with a capital good whose usage requires a complementary consumable good. Neither the capital good (printer) nor the consumable good (ink) provide any utility on its own. The price-discriminating monopolist can monitor and charge for usage directly or can lock the customer into the purchase of its consumables either by technology or by contract.[12] While

[12] In the *Independent Ink* case, the manufacturer of the printer (Trident) contractually required its customers to use its ink exclusively. There was no technological incompatibility. The price of its ink was

the buyer typically purchases the consumable good after making the capital good purchase, buyers correctly anticipate their future use of the consumables and its price, and they take this information into account when deciding whether to purchase the capital good.

The seller has market power only over the capital good. There is a competitive market for the consumable good, which is produced and sold at its marginal cost of $c_1$. For simplicity, we assume the capital good can be produced at a constant marginal cost of $c_0$.

Without any metering tie or price discrimination, the seller charges a single price $p_s$ for its capital good, and its customers are free to purchase the consumable good from the competitive market at price $c_1$. Thus, with single pricing, a buyer who uses $n$ consumable units pays a total of $p_s + c_1 n$. A buyer of type $\alpha$ will purchase the capital good if there is some $n$ for which the value $V_\alpha(n) \geq p_s + c_1 n$, where $V_\alpha(n)$ is the overall value a buyer of type $\alpha$ gets from using the capital good with $n$ consumable units. We assume buyers are only interested in one unit of the capital good.

With a metering tie, the seller requires the buyers to purchase all of the consumables from it as a condition of buying the capital good. Thus, with a metering tie, the seller charges a price of $p_m$ to buy the capital good upfront and then a metering price of $m > c_1$ for each unit of the consumable good, so that a buyer who uses $n$ units of the consumable pays a total of $p_m + mn$. A buyer of type $\alpha$ will thus purchase the capital good if there is some $n$ for which $V_\alpha(n) \geq p_m + mn$.[13]

Under metering, the price of the consumables is typically constant per unit, but it need not be. More generally, the seller might employ non-linear pricing for the consumables. To use a non-constant metering tie, a seller will have to observe directly how many consumable units each buyer is buying and to prevent buyers from reselling consumables to each other. In that case, $m$ becomes the average price paid per consumable unit by a buyer who uses $n$ consumable units, which we will call $m(n)$ because it is a function of $n$. Taking into account the cost of the capital good, a buyer who uses $n$ units of the consumable good pays a total of $p_m + m(n)n$. The results

some 2.5 to 4 times more expensive than the compatible ink sold by Independent Ink; see Nalebuff, Ayres, and Sullivan (2005).

[13] We follow convention and assume that an indifferent buyer will make a purchase.

that follow apply to any form of second-degree price discrimination done via linear or non-linear pricing for the consumable good.[14]

Our results also apply when the metering or price discrimination is done *without* the use of a forced tie. For example, if the firm could attach a meter to the capital good and directly observe how often each buyer used it, then sellers could directly charge per usage without a need to impose a metering tie that restrains purchases of the tied consumable. In such a case, the buyer would purchase the consumable (toner) in the competitive market at price $c_1$ and then pay an additional per-unit toll of $m(n) - c_1$ to the firm. Equivalently, the firm's price of the capital good could vary based on usage of the consumable.

Because such direct observation is often not feasible, metering ties can be used to facilitate price discrimination that cannot otherwise be accomplished. Assessing the effects of direct price discrimination is important because the welfare effects are the same whether the metering is done directly or via a tied sale. Indeed, our results might suggest legal condemnation should also apply in the situations when direct price discrimination is used but fails to expand output of the capital good.[15] However, there are several reasons why the law might treat direct price discrimination differently than metering ties.[16]

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[14] In our model, there is only one good being consumed, which limits the scope for second-degree price discrimination. For example, the firm cannot engage in bundled pricing across multiple goods.

[15] Under U.S. law, unless such direct price discrimination involves a condition that restrains the buyer from purchasing a product from the seller's rival, it would typically be legal absent below-cost prices or a distorting effect on downstream competition among buyers; see Brooke Grp. v. Brown & Williamson Tobacco, 509 U.S. 209 (1993). European competition law differs on this score because it condemns direct price discrimination by a dominant firm that increases the exploitation of market power in a way that harms consumer welfare, even if no price is below cost and downstream competition is undistorted; see TFEU Article 102, Case 27/76, United Brands v. EC, [1978] E.C.R. 207, though enforcement of this rule is currently limited.

[16] We offer five reasons: (i) A metering tie requires an agreement or condition that restrains the buyer from purchasing the consumable from rival sellers, and thus falls within Sherman Act §1 and Clayton Act §3; see Elhauge (2016: 496-501, 505) and Jefferson Parish Hosp. v. Hyde, 466 U.S. 2, 14-15 (1984) (stressing that one of the anticompetitive effects of tying agreements that make them illegal is that they "can increase the social costs of market power by facilitating price discrimination"). In contrast, a firm needs no such restraining agreement or condition to price discriminate with direct metering. (ii) Metering ties tying can inflict ancillary inefficiencies by foreclosing sales by rival consumable suppliers that have lower costs or offer benefits through quality or variety. (iii) Direct metering is often unfeasible and thus less often problematic. (iv) Even when direct metering is feasible, the metered price makes it clear to the buyer the extent of the markup. In contrast, with tied metering, the markup is often shrouded. For example, most consumers do not know much extra they are paying per page for using an HP toner cartridge rather than generic ink. By preserving the opportunity to employ direct metering, we preserve

We focus on situations where metering ties or other forms of price discrimination are profitable, by which we mean they strictly increase seller profits compared to charging a single profit-maximizing price. If they did not increase profits, then a seller would have no incentive to engage such conduct. Thus, our results apply to all the cases where a seller would want to engage in metering ties or any other price discrimination.

**Theorem 1: If a seller has market power over a capital good and there is no market power over a consumable used with it, then any profitable price discrimination (including a metering tie) cannot increase total welfare unless it increases total sales of the *capital* good.**

*Proof:* We first observe that for all potential buyers—those who purchase the capital good and those who do not—the total welfare from that buyer is maximized when the consumable is sold at marginal cost. This is because conditional on buying the capital good, the buyer will then purchase the amount of the consumable that maximizes $V_\alpha(n) - c_1 n$. Here, buyer utility and total welfare coincide. Thus those who buy the capital good choose the efficient amount of consumables and maximize total welfare from their purchase. Even those who do not purchase are maximizing their potential contribution to total welfare. They may not purchase because the capital good price is too high, but had they purchased they would have contributed as much as possible to total welfare.

With single pricing, the price for the capital good is $p_s$, so all customers who bought the capital good must have valued it at $p_s$ or greater, and thus each contributes $p_s - c_0$ or more to total welfare.[17] All customers who did not purchase the capital good with single pricing must have valued it at less than $p_s$, and thus each necessarily would have added something less than $p_s - c_0$ to total welfare if they had bought. By the argument above, they could not have contributed any more under any other pricing scheme.

potential benefits to buyers such as risk sharing, while reducing the opportunity for the firm to exercise its market power and extract consumer surplus. (v) Regulating pricing involves greater issues of legal administrability because, unlike tying agreements, setting prices is an unavoidable market activity. Thus, just as the law prohibits agreements that facilitate oligopolistic coordination, even though it does not directly prohibit oligopolistic pricing itself, so too the law may reasonably choose to prohibit tying agreements that facilitate price discrimination, even though it does not directly prohibit price discrimination itself; see Elhauge (2016: 491-92).

[17] Note that $p_s - c_0 \geq 0$ as the seller is maximizing profits and would lose money at any price below $c_0$.

If price discrimination leaves capital good output unchanged but changes the set of customers who buy, it must replace one or more of the original customers with an equal number of new customers. That replacement necessarily strictly lowers total welfare because the lost original customers contributed more to total welfare than the new customers can add. If price discrimination reduces output of the capital good, then it must further reduce total welfare because it also loses one or more of the original customers without replacing them. Therefore, price discrimination cannot raise total welfare without also raising capital good output. □

From Theorem 1, we know that total welfare cannot increase unless capital good sales also rise. Because total welfare is the sum of profits and consumer surplus, if price discrimination increases profits without also increasing total welfare, it must reduce consumer surplus. This leads to Corollary 1:

*Corollary 1: Any profitable price discrimination always reduces consumer welfare unless it increases total sales of the capital good.*

Theorem 1 leaves open two borderline case questions: (i) what happens to total welfare where capital good sales are constant, and (ii) when does total welfare strictly fall? We know from the proof of Theorem 1 that total welfare will fall if any buyers under the single price rule are replaced by new buyers under price discrimination. Even if price discrimination leaves unchanged both capital good output and the customers who buy, if it raises the consumable price in a way that reduces usage by customers who valued that usage above the consumable cost, then it necessarily reduces total welfare. Only in the special case where price discrimination leaves unchanged capital good output, customer identity, and the usage of any consumable valued above cost, does it leaves total welfare unchanged.

*Corollary 2: Unless it increases total sales of the capital good, profitable price discrimination strictly reduces total welfare other than in the special case where the price discrimination leaves total welfare unchanged if it both (i) does not change any of the customers who buy and (ii) does not reduce consumable usage by those who valued that usage above the consumable cost.*

*Remarks.* There are several points to note about these results. First, Theorem 1 is designed to be the analog of results proven in earlier literature (Robinson (1969), Varian (1985), Schmalensee (1981) for third-degree price discrimination. Indeed, the structure of the proof builds on a textbook example in Tirole (1998) and is almost a direct translation of the third-degree price discrimination proof in Schwartz (1990). Theorem 1 (and the corollaries) extend those prior results to *any* price discrimination—including metering ties and second-degree price discrimination—that is used when a seller has market power over a capital good but there is no power over the consumable used with it.[18]

To be sure, a single-pricing firm with market power over a capital good is inefficient in that it prices too high and thereby cuts off demand from customers for whom it would be efficient to purchase the good. But a single-pricing seller is inefficient in a maximally efficient way—the customers excluded from the market are those who have the lowest value from the good. The single-pricing firm sells the consumable in the most efficient manner, so as to extract the greatest possible price upfront. *A price-discriminating seller with market power cannot beat the efficiency of a single-pricing seller for the buyers who were served in the market*. Its only hope for improving efficiency is to expand the set of buyers who are served.

Second, the results do not show that price discrimination can never increase consumer welfare or total welfare. Theorem 1 shows only that in this situation it cannot do so *unless* it increases output of the *capital* good. The change in capital good output provides a useful test to determine if a welfare increase is possible. In section II.B below, we present a model specification in which profit-maximizing price discrimination via metering ties fails to increase output; therefore, by Corollary 1, it must strictly lower consumer welfare and we will show that Corollary 2 applies so that it also strictly lowers total welfare in this case.

The result in Theorem 1 does not apply to consumable output. As shown earlier, consumable output can fall while total welfare rises (see fn. 11). It is also possible for consumable output to rise and total welfare to fall, as we demonstrate below in Remark 3. In the following paragraphs, we show that in some limited circumstances a rise in consumable output always leads to a total welfare gain.

[18] Our results do not extend to situations where the seller also has market power over the consumable or when the tied and tying goods are not used together and have separate demand.

*A More Limited Converse Result for Consumable Output.* We derive a more limited converse result based on the observation that the consumables sold under metering are allocated efficiently among the buyers in the market. That is, a tying firm that charges a price $m > c_1$ inefficiently limits consumable sales with a price above cost, but efficiently allocates consumables to those who value them the most (among those who have purchased the capital good). It might seem that this observation produces a general converse result: metering ties increase total welfare whenever they increase output of the consumable good. But this is not true. The reason is that metering is not efficient in terms of its allocation of the capital good. Too many consumers who have a high per-usage value and low usage rate will be brought into the market,[19] while other consumer with high usage rate and low per-usage value will be excluded. But this is only an issue if the capital good is costly. If the capital good can be produced at zero cost and the metering tie provides the capital good for free, then there is no potential inefficient allocation of the capital good as everyone can be given the good. All that matters is the allocation of the consumable good, which metering does efficiently. With costless and free capital goods, we establish the converse result with the addition of the standard assumption of declining marginal utility.

**Theorem 2: If (i) the capital good is costless ($c_0$ = 0), (ii) the price with a profitable metering tie is 0 for the capital good, and (iii) the marginal value of the consumable is weakly decreasing for each buyer type, then a metering tie will increase total welfare if it increases total sales of the *consumable* good.**

*Proof:* The proof of this result follows the same argument as in Theorem 1, except from the opposite direction. With a metering tie, the price for the consumable is $m$, and $m > c_1$ as otherwise the metering would not be profitable. All consumable units purchased must be valued at or above $m$ (given declining marginal utility), and each consumable sale contributes $m - c_1$ or more to total welfare. Because the cost of the capital good is zero, the only customers who do not take the capital good are those who value even the first unit of the consumable at less than $m$, and

[19] Because of the high profits on toner cartridges, laser printers are almost free, especially as they come with a starter cartridge. The low price of laser printers has resulted in three of them being scattered around each of the authors' households; absent metering, the printer would be more expensive and each would have only one printer used three times as much.

hence, each would have added less than $m - c_1$ per consumable unit to total welfare had they purchased anything. Thus metering leads to the maximally efficient allocation of consumables.

If a metering tie increases the total consumables sold it must increase total welfare because any reallocation of consumable output from original to new customers cannot decrease total welfare and any additional consumable output adds to total welfare. □

*Remark 1.* Theorem 2 does not apply to consumer welfare.

*Remark 2.* In the case where the capital good has zero cost and is provided for free with the metering tie, total welfare increases with tying if outputs of both the capital good and consumable rise, and decreases if both outputs fall. When capital good output rises while consumable output falls, the directions of output changes do not alone suffice to determine the direction of welfare change. In contrast, a decrease in the capital good output combined with an increase in consumable output would create a paradox because it would indicate an increase in total welfare under Theorem 2, but a decrease under Theorem 1 and its corollaries; thus, if the capital good is costless and given away, a metering tie cannot both decrease capital good output and increase consumable output.

*Remark 3:* In the case where the capital good has positive marginal cost, consumable sales can increase while total welfare falls, so a stronger result than Theorem 2 is not possible. An example shows how this is possible. There are 30 potential buyers in the market, of types A, B, C, and D described below:

| | Number of Customers | Copies desired | Value per copy | Willingness to pay |
|---|---|---|---|---|
| A | 1 | 100 | $0.50 | $50 |
| B | 2 | 50 | $0.50 | $25 |
| C | 1 | 100 | $0.25 | $25 |
| D | 26 | 4 | $0.50 | $2 |

The cost of the consumable is $0, and the cost of the capital good is $1.9. Thus, the profit-maximizing single price of the capital good is $25. Buyer types A, B, C will purchase. Total profits are 100 – 4*1.9 = $92.4. Total welfare is $117.4 (because buyer A gets $25 of consumer surplus and buyers B and C get none) and 300 consumables are sold.

With a metering tie, the profit-maximizing prices are \$0 for the capital good plus \$0.50 for the consumable. In that case, buyer types A, B, and D all purchase. Total profits are 100 – 3*1.9 + 26 * (2 – 1.9) = \$96.9, and the firm will find it profitable to adopt a metering tie. Total welfare is also \$96.9 (because consumer surplus drops to zero), and thus the metering tie lowers total welfare. Here, 304 consumables are sold.

The reason why metering raises consumable sales (from 300 to 304) yet reduces total welfare is that in order to extract more surplus from buyer A, the metering firm was willing to sacrifice the sales of 100 consumable units to buyer C, sales that were highly efficient because they required only one unit of the capital good. While those consumable sales were made up via buyer D, once one takes the capital cost into account, the new 104 units of consumables that came into the market brought a very low welfare contribution, namely 0.1/4 = \$0.025 per unit.

What Theorem 2 shows is that, just as single pricing is efficient in allocating the capital good, a metering tie is efficient in allocating the consumable goods, subject to the constraint that all buyers get access to the capital good with the metering tie. The assumption of zero cost and zero price for the capital good biases the scale in favor of metering ties because metering ties bring in more low-demand buyers and so require more capital goods. If these capital goods are costly to produce, then this move will be less advantageous to welfare.

Theorem 2 has less practical relevance than Theorem 1. In a typical metering tie, the capital good is something like a printer and the consumable is something like ink or vinyl. Thus, typically the variable cost of the capital good is high and the variable cost of the consumable is low. A metering tie cannot guarantee a more efficient allocation of consumables once capital goods have a positive cost or even if the metering seller maximizes profits by charging a positive price for the capital good.

### *B. Metering Ties Can Fail to Increase Capital Good Output and Thus Can Lower Welfare*

Although the general proof of Theorem 1 shows that metering ties must increase capital good output in order to increase welfare, that result would be less interesting if metering ties always increased capital good output. In this section, we provide a simple model in which the profit-maximizing metering ties fail to increase capital good output and thus reduce both consumer and total welfare. We continue to apply this model in Part III, where we add a specific distribution assumption regarding usage.

**Market Assumptions:** (i) As before, we assume that both the capital good and consumable can be produced at a constant marginal cost, $c_0$ for the capital good and $c_1$ for the consumable. (ii) A customer of type $(a, n)$ wants to use $n$ units of the consumable and values each unit at $a$. (iii) The value of $a$ is distributed uniformly over [0, 1], and $n$ is distributed independently from $a$ according to the non-atomic density function $g(n)$, with cumulative distribution $G(n)$.[20] (iv) Finally, we assume there are some customers for whom $n(1 - c_1) - c_0 > 0$.

Assumption (iv) was made to prevent trivial outcomes; it ensures that there are some customers in the market that can be profitably served. In other words, costs are not so high that the capital good would never be produced.

Assumptions (ii) and (iii) are both important and warrant further discussion. We assume customers have what might be called "rectangular demand." Instead of the more traditional downward-sloping demand, each buyer has a constant willingness to pay for up to $n$ units and then the valuation drops to zero. We think this is a reasonable assumption in the context of many commercial metering situations where demand for the consumable is practically exogenous. For example, in the *Independent Ink* case, the metering was done in the context of a markup on ink used to print on corrugated cardboard cartons of beer. Given the high margins at the manufacturer level, the manufacturer wants to sell as many cartons of beer as buyers will purchase. Thus, for a specific customer, the demand for "copies" (in this case printed logos) is exogenously given by temperature and other factors influencing beer consumption. If the monopolist were to raise the price of ink, it would not shrink how many units were demanded—until the point that the manufacturer decided that the price was too high to justify the printing technology. At that point, demand would fall to zero as the firm would switch over to a different printing technology, likely one that is less efficient but not controlled by a monopolist.

To put this in perspective, imagine that the older, competitively supplied, technology costs 0.1¢ per carton and the new monopoly technology is able to reduce that cost to 0.04¢. In this case, we would say that the $a$ for the buyer is 0.06¢. At prices up to 0.06¢, the cost savings

[20] While we assume the distribution of $n$ is not all concentrated on a single type, in this section we are not constraining the distribution to have any specific form such as a uniform, normal, or lognormal distribution.

justify using the newer technology. As the metered price varies from 0 to 0.06¢, it is possible that there would be a tiny demand response if the firm were to raise its price. However, we think it is realistic to assume that demand is essentially flat over this region, and then drops to zero when the metered price is so high that the total cost is no longer competitive against other technologies.[21] The key insight is that the monopolist has some cost or quality advantage over some other non-monopolized technology. The $a$ represents that advantage, and the $n$ represents the expected usage for the specific customer.

Our model of buyer preferences is two-dimensional in that preferences depend on both $a$ and $n$. In contrast, Hovenkamp and Hovenkamp (2010) assume that different types of buyers vary only in a unidimensional "intensity level." In their model, buyers can be lined up in a single dimension based on how much they value the consumable.[22] This one-dimensionality assumption has the effect of biasing the analysis because it excludes customers who have high usage rates but a low per-usage value, precisely the buyers who are inefficiently priced out of the market under tying. Indeed, in the supplement appendix to this paper, we show that for the case where $a$ and $n$ are perfectly correlated, metering always leads to an increase in total welfare and a decrease in consumer welfare.

We assume that $a$ and $n$ are independently distributed. We think independence is the most interesting case both for theory and as a practical matter. When the correlation between $a$ and $n$ is negative, there is little dispersion in total valuations. Single pricing is able to capture most of the consumer surplus, and there is little reason to engage in price discrimination. At the other extreme, when $a$ and $n$ are strongly positively correlated, this is close to the one-dimensional model, and thus metering will tend to increase total welfare (as there are few low $a$, high $n$ types

[21] This also seems to fit the fact pattern in historical antitrust cases on metering, such as Heaton-Peninsular Button-Fastener Co. v. Eureka Specialty Co., 77 Fed. 288, 1896 U.S. App. (6th Cir. 1896). Heaton had a patent on its button fastener machines used to staple buttons on "high-button" shoes and required the buyer to use its expensive staples. The machine replaced human labor and so the amount a shoe manufacturer would pay per staple ($a$) was its labor cost savings per button. Provided the metered price was below that level, the manufacturer's demand for staples ($n$) was essentially exogenous as it primarily depended on the fashion for buttons on shoes.

[22] In Hovenkamp and Hovenkamp, a high-value buyer starts with an initial value of, say, 10 and this falls to 0, while a low-value buyer starts with an initial value of 2, which declines to 0, both with the same slope. As a result, in any pricing scenario, if the low-value buyer is in the market, then the high-value buyer will be as well. This eliminates any potential for inefficient allocation across buyers.

who would be excluded under metering) but decrease consumer welfare. Independence is the interesting case in the middle.

This is similar to the situation with bundling, except with bundling we care about the sum of the two valuations, not the product $(a \times n)$. In the case with perfect negative correlation, bundling leads to perfect price discrimination (Adams and Yellen (1976)), and in the case with perfect positive correlation, bundling provides no advantage. Thus the importance of McAfee, McMillan, and Whinston (1989), which showed that bundling always increases profits when the two goods are independent (and by continuity when positive correlation is not too strong).

We also think the independent case is a good representation of reality. One might think it instead makes sense to presume that consumers with a high $n$ usually have a high $a$, so there should be positive correlation between the two. But a strong positive correlation would imply that total valuation implausibly goes up like the square, not just linearly. A law firm that makes 100 times as many copies as another likely values their copier 100 times higher— but not $100^2 = 10{,}000$ times as much, as would be the case if value of $a$ and $n$ were perfectly correlated. More generally, positive correlation means the total valuation go up more than linearly with $n$, and negative correlation means that the total valuation goes up less than linearly with $n$.

To gain perspective on which assumption is more reasonable, return to the commercial example where the metering firm has a monopoly over a printer technology. A large beer maker (high $n$) will demand more printed cartons than a craft brewery (low $n$), but that does mean that the extra value the monopoly technology provides per carton should be higher or lower for the big brewery. That depends on what other (often older) technologies are available. The craft brewery may find it easier to use an older technology as its production line moves slower or may find it harder to switch to another technology if it has less volume over which to amortize the equipment. The key insight is that while we expect the total value to increase with $n$, we have no general reason to expect the value per usage to also increase with $n$.

The case of a camera and photos illustrates this as well. Here $n$ is how many photos the buyer wants to take, and $a$ is the average value of each photo. Professional users have a high $n$ and a high $a$. Teenagers have a high $n$ and a low $a$. Absent-minded professors (who only bring their cameras to important events) have a low $n$ and a high $a$. Those with both a low $n$ and a low

$a$ are unlikely to be served in the market under either pricing structure and thus are less relevant. Here again, knowing the value of $n$ does not help predict the value of $a$.

Finally, we note that the independence between $a$ and $n$, combined with the uniform distribution of $a$ over [0, 1] leads to aggregate demands for the capital good that are linear and rotate from a choke point on the x-axis as seen in Figure 1 following Theorem 3. This demand specification is used in Malueg and Schwartz (1994) in their analysis of price discrimination across countries.

Having presented the modeling assumptions, we proceed to solve the model and provide welfare comparisons.

*Metering Tie.* We look at the case of metering with a two-part linear pricing scheme. The seller charges a price $p_m$ for the capital good plus a price $m$ per copy. As we now show, this simple pricing scheme maximizes profits in our model.

*Lemma 1: Under our Market Assumptions, a two-part linear tariff maximizes profits over all non-linear metering options.*

*Proof:* We start by allowing the seller to choose a general non-linear pricing scheme. Indeed, we go one step further and allow the monopolist to *know* each customer's desired usage rate $n$ and to set a profit-maximizing price schedule $\{p_{in}\}$ tailored to the customer group with that desired usage rate. Here, $p_{in}$ is the price for the $i^{\text{th}}$ consumable unit for the buyer who is interested in purchasing $n$ units. The customer type *(a, n)* will purchase $q$ units where $q$ solves:

$$\max_{0 \leq q \leq n} aq - \sum\nolimits_{i=1}^{q} p_{in} \tag{1}$$

The first point to note is that, among all *(a, n)* customers, there is some minimal $a$ type that is willing to make a purchase. Call that cutoff $a_n^*$ and let $q^*$ be the purchases made by $a_n^*$. All customers with $a > a_n^*$ will purchase at least $q^*$ units (as their value per usage is higher), and those with $a < a_n^*$ will not make any purchase.

The next point to note is that, if profits are being maximized, we can set $p_{in} = a_n^*$ for $i = \{1, \dots, q^*\}$. At that price, the $a_n^*$ type is just willing to purchase $q^*$ units. It is not possible to get more revenue from the sale of the first $q^*$ units and still have the $a_n^*$ type purchase $q^*$ units. Note also that this pricing will have no influence on the purchases of those with $a > a_n^*$.

It is also the case that maximal profits can be achieved by setting $p_{in} = \text{Max}[a_n^*, \frac{1+c_1}{2}]$ for all the remaining units. The profit on each of the remaining units is $(p - c_1)(1 - a_n^*\ )$ for $p < a_n^*$ and $(p - c_1)(1 - p)$ for $p \geq a_n^*$. This is maximized at $p = a_n^*$ for $a_n^* \geq \frac{1+c_1}{2}$ and at $p = \frac{1+c_1}{2}$ for $a_n^* < \frac{1+c_1}{2}$. There is no point charging less than $a_n^*$. But if $a_n^* < \frac{1+c_1}{2}$, profits are increased by selling to only a subset of active buyers at a higher price.

Finally, it follows that since profits are being maximized, $a_n^* \geq \frac{1+c_1}{2}$. If not, then the monopolist can make more money by raising the price of the first $q^*$ units up to $\frac{1+c_1}{2}$. Putting aside the capital good costs, such pricing will generate higher profits on the first $q^*$ consumable units, and it will lead any active buyers with $a < \frac{1+c_1}{2}$ not to purchase at all, which will further raise profits by saving the monopolist the capital good costs for those customers.

Thus, if the monopolist knows *n*, profits can be maximized by selecting an $a_n^*$, and setting $p_{in} = a_n^*$ for all *i*. All buyers in the market will purchase *n* units at a total price of $a_n^* n$. It is straightforward to calculate the profit-maximizing $a_n^*$:

$$\max_{a_n^*}(a_n^* n - nc_1 - c_0)(1 - a_n^*) \tag{2}$$

The solution is $a_n^* = \frac{1+c_1}{2} + \frac{c_0}{2n}$. Note that the total price paid by buyers active in the market is $a_n^* n = \frac{c_0}{2} + n\frac{1+c_1}{2}$

Up until now we have assumed that the monopolist knows *n* and can choose $\{p_{in}\}$. But note that the firm can achieve the *exact* same result without knowledge of *n*. All that is required is a two-part metering tariff. If the firm charges

$$p_m = \frac{c_0}{2} \text{ and } m = \frac{1 + c_1}{2} \tag{3}$$

then all consumers will buy either *0* or *n* units, and the indifferent type $a_n^*$ will be $\frac{1+c_1}{2} + \frac{c_0}{2n}$. □

The fact that a linear two-part metering tariff can be as profitable as any non-linear metering tariff does depend on the independence between *a* and *n* and the assumed uniform distribution of *a*. But, given those assumptions, we obtain generality in that our simple metering tariff maximizes profits over all second-degree price discrimination schemes.

With a metering tie, the profit-maximizing price for the capital good is half its marginal cost and for the consumable is the halfway point between the marginal cost and maximum valuation on the consumable. If the capital good has zero cost, then $p_m = 0$ and the profit-maximizing metering scheme can rely entirely on a metering price.

*Single Pricing.* With single pricing, the seller sells the capital good alone for a uniform price of $p$. A customer of type $(a, n)$ will purchase the capital good if and only if

$$n(a - c_1) \geq p \text{ or } a \geq \frac{p}{n} + c_1 \text{ which has probability } \left[1 - c_1 - \frac{p}{n}\right]. \tag{4}$$

Note that unless $n > p/(1 - c_1)$, the probability of a purchase by customers of type $n$ is zero. That is because the highest valuation per usage is 1, and thus the highest valuation any customer of type $n$ can put on a capital good is $n(1 - c_1)$.

The price is chosen to maximize profits where

$$Profits = \int_{p/(1-c_1)}^{\infty} (p - c_0)\left[1 - c_1 - \frac{p}{n}\right] g(n)dn. \tag{5}$$

Profits are maximized at

$$\frac{dProfits}{dp} = 0 \implies \int_{p/(1-c_1)}^{\infty} \left[1 - c_1 - \frac{2p - c_0}{n}\right] g(n)dn = 0. \tag{6}$$

The seller's profit-maximizing single price $p^*$ is thus the solution to (6)

$$p^* = \frac{c_0}{2} + \frac{\frac{1}{2}(1 - c_1)}{\int_{p^*/(1-c_1)}^{\infty} \frac{1}{n} \frac{g(n)}{1 - G(p^*/(1-c_1))} dn} \tag{7}$$

The expression for $p^*$ has some parallels to pricing under metering. In both cases, the buyer pays half the cost of the capital good, $\frac{c_0}{2}$. Under metering, each buyer pays an additional $\frac{1}{2}(1 + c_1)n$ for the consumables, but since the firm has to pay the cost, its net profit on the consumables is $\frac{1}{2}(1 - c_1)n$. With single pricing, each customer who buys the capital good pays an additional $\frac{1}{2}(1 - c_1)H$, where H is the harmonic mean of the value of $n$ among those who buy. Thus the pricing is similar, in that single-price firm's price for the capital good in effect includes the same mark up on consumable usage, but instead of being multiplied by each customer's actual usage with the tie, it is multiplied by the harmonic average usage of all the customers who buy without the tie.

*Lemma 2: Under our Market Assumptions, a metering tie strictly increases profits for any distribution of usage rates.*

*Proof*: Under metering, the seller could perfectly replicate the profits under single pricing by choosing the combination of $p_m = p^*$ and $m = c_1$. In this case, the metering firm sells (or resells) the consumable good at cost and makes all of its profits on the single upfront price. The fact that the metering firm does not pick this price combination means that it can earn strictly higher profits by choosing $p_m = c_0/2$ and $m = (1 + c_1)/2$.

As shown above in (2), profits under metering depend on $p_m$ and *m* only via $p_m + nm$. Because single pricing is a special case of (non-profit-maximizing) metering, this proposition is equally true for it. Thus, the profits with single pricing and profit-maximizing metering could coincide only if $p_m + nm$ has the same value for each, which is when:

$$p^* + c_1 n = \frac{c_0}{2} + \frac{1 + c_1}{2} n \quad \text{or} \quad n = \frac{2p^* - c_0}{1 - c_1}. \tag{8}$$

Since metering profits are strictly concave in $p_m + mn$ (over the range of *a* and *n* where demand is positive), profits must be strictly higher with profit-maximizing metering for all but the one *n* where they coincide. Thus, integrating over *n*, profits could be equal only if all those who buy have the same value of *n*. But that would contradict our Market Assumption that *g(n)* has no mass points. □

The importance of Lemma 2 is that it shows that firms with market power will want to engage in metering if given the opportunity. It is also the case that, with our Market Assumptions, metering never reduces sales of the capital good no matter what the distribution of usage rates.

*Lemma 3: Under our Market Assumptions, metering ties will either increase capital good sales or leave them unchanged for any distribution of usage rates.*

*Proof*: With a metering tie, a consumer with preferences (*a, n*) will purchase if

$$n\left(a - \frac{1 + c_1}{2}\right) \geq \frac{c_0}{2} \text{ or } a \geq \frac{1 + c_1}{2} + \frac{c_0}{2n}. \tag{9}$$

Thus demand for the capital good is

$$\frac{1}{2}\int_{c_0/(1-c_1)}^{\infty} \left[1 - c_1 - \frac{c_0}{n}\right] g(n)dn. \tag{10}$$

Under single pricing, total Capital Demand is

$$\int_{p^*/(1-c_1)}^{\infty} \left[1 - c_1 - \frac{p^*}{n}\right] g(n) \ = \ \frac{1}{2}\int_{p^*/(1-c_1)}^{\infty} \left[1 - c_1 - \frac{c_0}{n}\right] g(n)dn\,, \qquad (11)$$

where the second equality follows substitution of equation (6).

The only difference in the total capital demands with metering versus single pricing is the starting point of the integral. In the case of metering, the integral starts at $c_0/(1-c_1)$. In the case of single pricing the integral starts at $p^*/(1-c_1)$. Since profits are positive, we know that $p^* > c_0$ and $1 - c_1 - \frac{c_0}{n} > 0$ for $n > \frac{c_0}{1-c_1}$. Therefore

$$\frac{1}{2}\int_{p^*/(1-c_1)}^{\infty} \left(1 - c_1 - \frac{c_0}{n}\right) g(n)dn \ \leq \frac{1}{2}\int_{\frac{c_0}{1-c_1}}^{\infty} \left[1 - c_1 - \frac{c_0}{n}\right] g(n)dn. \qquad (12)$$

Demand for the capital good will be strictly greater under metering unless *g(n)* is zero for $c_0/(1-c_1) \ \leq n < p^*/(1-c_1)$. □

Note that in the special case where $c_0 = 0$, capital good demand is $\frac{(1-c_1)}{2}$ under metering and $\frac{(1-c_1)}{2}[1 - G(p^*)]$ under single pricing.

The next theorem uses the condition that each customer group of usage rate *n* has at least one customer who buys the capital good with single pricing. This condition means that the entire body of the distribution of *n* lies above $p^*/(1-c_1)$, that is $G(p^*/(1-c_1)) = 0$. For example, suppose $c_0 = c_1 = 0$ and the distribution of *n* were uniform over [50, 100]. Then the harmonic mean would be 50/[ln100 – ln50] = 72, and the profit-maximizing single price would be half the harmonic mean, or 36; here the condition would be satisfied because this is less than the minimum *n* of 50. Indeed, for any uniform distribution of *n* ranging from [$\underline{n}$, 100] this condition is satisfied as long as $\underline{n} \geq 28.5$.

**Theorem 3. Under our Market Assumptions, if each customer group of usage rate *n* has at least one customer type who buys the capital good with single pricing, then profit-maximizing metering ties will reduce both total welfare and consumer welfare.**

*Proof:* Given the condition that $G(p^*/(1-c_1)) = 0$, Lemma 3 shows that total capital good output with single pricing is exactly the same as with a metering tie. This is because there are no potential buyers with $\frac{c_0}{1-c_1} \leq n < \frac{p^*}{1-c_1}$.

By Lemma 2, profit with a metering tie must be strictly higher than under single pricing. Because capital good output is unchanged and profits are higher, it follows directly from Theorem 1 and its corollaries that consumer welfare must fall.

Next, we show that some buyers under metering replace customers under single pricing. This implies total welfare must strictly fall as capital good output has remained constant. Under single pricing, the indifferent buyer is

$$a = \frac{p^*}{n} + c_1$$

Under metering, the indifferent buyer is

$$a = \frac{c_0}{2n} + \frac{1 + c_1}{2}$$

The indifferent buyer can be the same for at most one value of $n$. Thus, there must be a different set of buyers for almost all values of $n$. The exception would be if demand were positive only at a single value of $n$. But that would contradict the assumption that profits are strictly positive, which requires a positive interval of demand because the distribution of $n$ has no mass points. □

*Remark 1.* In this simple case, we find that metering ties produce the same total sales of the capital good, but more of those sales go to buyers with small values of $n$ and thus the allocation is less efficient. This example also shows that there should be no general presumption that metering ties or any form of price discrimination will strictly increase sales of the capital good.

*Remark 2.* The result here is parallel to earlier results for 3rd-degree price discrimination. The condition that each customer group of usage rate $n$ buys at least some of the capital good with single pricing is similar to the assumptions used by Robinson (1969), Schmalensee (1981), and Varian (1985). It says that when this assumption holds, there is some buyer of each $n$ type that is in the market. Thus when the seller raises its price, it will lose customers from each $n$ type. If this assumption were not true, then raising price would be less costly, because once all of an $n$-type are driven from the market, there is no longer additional loss from raising price to that type.

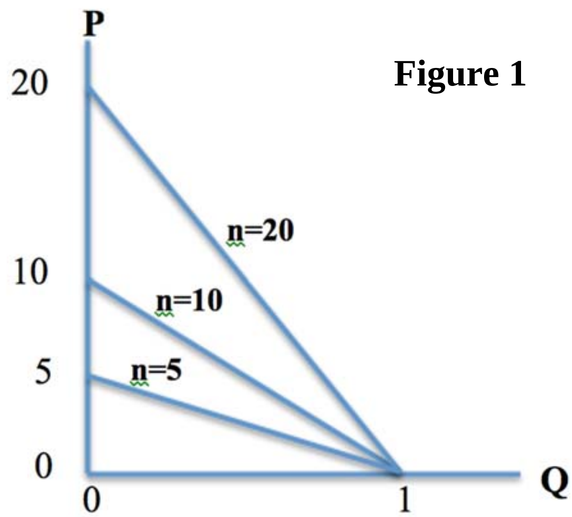
The similarity in results is not accidental. Imagine that the monopolist could engage in 3rd-degree price discrimination and charge a different capital good price to the group of buyers based on their $n$. The profit-maximizing price for group $n$ under 3rd-degree price discrimination maximizes the integrand in (5), $(p - c_0)\left[1 - c_1 - \frac{p}{n}\right]$. The solution is

$$p(n) = \frac{c_0}{2} + n\frac{1-c_1}{2}. \tag{13}$$

In addition to this amount, buyers would pay the marginal cost $c_1$ for the consumable. Added together, this is the exact same price that the *(a, n)* buyer faces under metering. Thus, the same conditions on the distribution of demand should and do lead to the same results.

Our Market Assumptions correspond to the linear demand assumption used by Robinson (1969), although instead of a series of parallel demand curves with different intercepts, our model of individual rectangular demand by buyers leads to an aggregate demand that is similar to the "rotating demand" of Malueg and Schwartz (1994); see Figure 1 below drawn for the case with $c_1 = 0$. Malueg and Schwartz (1994) look at the result of price discrimination across countries. Using our notation, the inverse demand in country *n* is given by $p(q) = n(1-q)$.

**Figure 1**




The demand looks the same in our model. The difference is that each demand line is the aggregation of capital good demand from different *a* types all with the same consumable usage *n*. Consider, for example, the demand from customers with *n=10*. When *p=6*, those with $a - c_1 \geq 0.6$ buy the capital good (and 10 units of the consumable). The *q* here measures the fraction of the *n* types that purchase the capital good.

The condition in Theorem 3 requires that that the buyer with the highest *a* type (*a=1*) and the lowest *n* type must buy with single pricing or that $n_{min} > p^*/(1-c_1)$. With linear demand from each group, metering ties will not expand output if they do not expand the customer groups that buy.

We know that, more generally, it is possible that metering can increase or decrease consumer welfare and total welfare. In the next section, we show that the effects can be signed given further distributional assumptions regarding the demand for consumables. Here, we end this section with a result that shows that both consumer and total welfare would be improved if metering could be reduced.

**Under our Market Assumptions, a small mandated reduction in the metered price of the consumable good leads to an increase in both consumer welfare and total welfare.**

The proof of this result is in the supplemental appendix. Before providing the intuition, it is useful to explain how this result differs from the typical result on monopoly power. Consider a monopolist restricted to a single price, $p_s$. If the monopolist were forced to lower its price from the monopoly level this would clearly be a win for buyers (who gain from lower prices). From the monopolist's perspective, since $p_s$ was maximizing profits, there would be no first-order loss to profits. Since we have a first-order gain to buyers and no first-order loss to the firm, a small reduction in price from the monopoly level will increase both consumer and total welfare.

In our case, the result is not as straightforward. When a monopolist is forced to lower $m$ from its profit-maximizing level, this is clearly good for buyers (who once again prefer lower prices) and has no first-order impact on profits (as $m$ was chosen to maximize profits). The complication is that as the monopolist lowers $m$, it raises the corresponding capital good price, $p_m$. Changing the capital good price also has no first-order impact on profits (as $p_m$ was chosen to maximize profits), but it hurts buyers. Thus the question: Is the reduction in the metering charge $m$ more or less offset by the increase in $p_m$? The net change in consumer surplus is just equal to the change in the amount existing buyers pay under the new pricing scheme. In an appendix to our working paper, we show that as $m$ decreases, the savings on consumables always exceeds the increased spending on the capital good (because the arithmetic mean exceeds the harmonic mean), and so consumer surplus increases. And because there is no first-order impact on profits, the change in total welfare is the same as the change in consumer surplus.

## III. LOGNORMAL BUYER USAGE DISTRIBUTION

Theorems 1 and 2 are general in that they apply to any distribution of buyer usage rates. But the total welfare result of Theorem 3 requires the Market Assumptions and that the single profit-maximizing price leaves some of each usage type in the market. This is a sufficient condition for metering ties to reduce total welfare, but it is not a necessary condition. To see what happens when a single price excludes some buyer types, we consider the results for some plausible distributions of buyer usage rates.

Each of us has, in separate prior works, considered the case with zero costs and a uniform distribution of both buyer usage rates and buyer valuation per usage; Elhauge (2009) and Nalebuff (2009). In this case, metering is always profitable and always reduces consumer surplus. The effect on total surplus depends on the ratio of the maximum usage rate $H$ to the minimum usage $L$. If $H/L$ is below 4.64 then total welfare is higher under single pricing and if $H/L$ is above 4.64 then total welfare is higher under metering.[23] This is in line with our intuition. The advantage of metering arises when there is greater dispersion in usage and some of the market is left unserved with the single price. Thus when dispersion, here H/L, is large enough, then metering leads to greater efficiency. Note that this is directly parallel to the earlier results in Malueg and Schwartz (1994), who show that 3rd-degree price discrimination always lowers consumer welfare and will increase total welfare if the dispersion of a uniformly distributed $n$ is sufficiently high.

Although the assumption of zero capital good costs and a uniform distribution of buyer usage rates in our prior work (or "choke points" in Malueg and Schwartz) simplifies the math, it is unrealistic. Most capital goods have positive costs. And, in most things in life, extreme preferences are less common than moderate preferences. There might exist some buyers who would use a printer only once at competitive ink prices, and other buyers who would use it a million times, but it would be surprising if such extreme buyers were just as likely as buyers who would use their printer more moderately. We continue to employ the collection of rotating demand curves of Malueg and Schwartz, but now the distribution of those curves is no longer uniform, and we allow for positive costs.

[23] See Nalebuff (2009: Theorem 3). In this theorem, L=1, but the results scale so that all that matters is the ratio of H to L. The theorem uses the ratio of 4.58, but that is slightly low due to earlier rounding.

Here, we assume a lognormal distribution of buyer usage rates with parameters $(\mu, \sigma)$. As mentioned earlier, the lognormal distribution offers a good fit for the distribution of income and firm size, and thus to the extent that usage ($n$) is proportional to size it is an appropriate assumption. In addition, it is mathematically tractable. It allows for a closed-form solution for consumer surplus, profits, and total welfare.

**Theorem 4: Under our Market Assumptions, metering ties always lower consumer surplus for any lognormal distribution of buyer usage rates.**

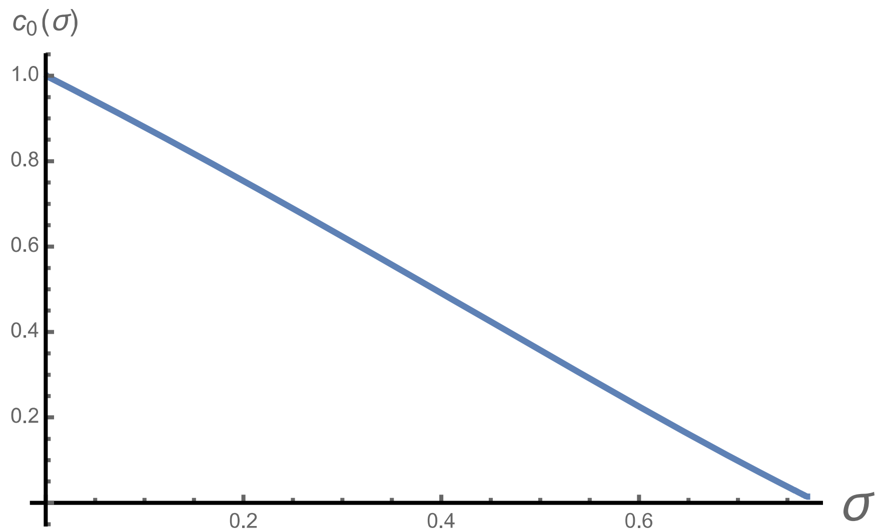
*Proof*: See Mathematical Appendix

*Remark 1.* Assuming a lognormal distribution of buyer usage rates, metering ties are always profitable (by Lemma 2) and always reduce consumer welfare. It is worth emphasizing that this result holds for *all* cost levels for the capital good and the metered good. The result does however rely on our Market Assumptions, most specifically that for each buyer, the per-usage valuation is constant up to the desired usage, and across buyers, the value per-usage is uniformly distributed over the relevant range and independent of the desired usage.

*Remark 2.* Under the consumer-welfare standard, this theorem supports condemning metering ties unless firms with market power can show that Theorem 4 does not apply. They could do so by demonstrating: (1) the tie creates productive efficiencies that are passed on to consumers sufficiently to offset the consumer welfare harm; (2) the Market Assumptions do not apply; or (3) the market exhibits a different distribution of buyer usage rates that creates an increase in consumer welfare under metering.

Turning to total welfare, the results now depend on costs and dispersion. Lemma 4 in the appendix shows that the comparison between metering and single pricing depends on the costs $c_0$ and $c_1$ only via the ratio $\frac{c_0}{(1-c_1)e^{\mu}}$, where $e^{\mu}$ is the median $n$ of all potential buyers in the lognormal. Thus, we can think of the capital good cost as being measured relative to maximum possible surplus for potential buyers with the median usage rate $(1 - c_1)e^{\mu}$. With this interpretation in mind, and without loss of generality, we normalize variables so that $\mu = 0$, $c_1 = 0$, and $c_0$ represents this cost ratio.

**Theorem 5. Under our Market Assumptions and a lognormal distribution of usage rates *n*, for $\boldsymbol{\sigma < 0.78}$, metering ties lower total welfare provided $\boldsymbol{c_0 < c_0(\sigma) \approx 1 - \frac{\sigma}{0.78}}$; see graph below. For $\boldsymbol{\sigma > 0.78}$, metering ties increase total welfare for all values of $\boldsymbol{c_0}$.**




*Proof*: See Mathematical Appendix

*Remark 1.* This result is in line with our intuition and prior results from the uniform case. Metering does well when there is great variability in the quantity demanded. Indeed, if all customers had the same per-usage valuation and differed only in the quantity demanded, then metering would achieve perfect price discrimination and maximize total welfare.

If antitrust law adopts the total welfare standard, our lognormal result supports condemning metering ties by a firm with market power unless the firm shows that the tying output expansion or usage dispersion is large, there are productive efficiencies large enough to offset the total welfare losses, or that different assumptions are more realistic to the case at hand usage (such as that usage and per-usage value are highly correlated) and that those different assumptions turn around the result.

*Remark 2.* We know from Theorem 3 that metering can only increase total welfare if there are some buyer types that are completely excluded from the market with single pricing. The results from the lognormal case in Theorem 5 show the extent to which metering needs to expand the customer set in order to offset its inefficiency. With capital good costs of zero, metering is just tied with single pricing in terms of total welfare when $\sigma = 0.78$. As shown after Lemma 3, when capital costs are zero, sales of the capital good under metering are $\frac{1-c_1}{2}$ and are $\frac{1-c_1}{2}[1 -$

$G(p^*)]$ under single pricing. Thus, with zero capital costs, metering has to expand capital good sales by a ratio of $\frac{1}{1-G(p^*)} = 1.37$ or 37% before total welfare is equalized.[24]

*Remark 3.* Positive capital good costs are the more relevant case. When we write that $c_0 < c_0^*(\sigma)$, this is based on the normalizations that $\mu = 0$ and $c_1 = 0$. Absent these normalizations, the condition would be $\frac{c_0 e^{-\mu}}{1-c_1} < c_0^*(\sigma)$. To interpret this condition, we take the case where $\sigma = 0.624$, so that $c_0^*(\sigma) \approx 0.2$. To say that $c_0 < 0.2 * (1 - c_1)e^{\mu}$ means that the cost of the capital good is less than twenty percent of the maximum possible surplus for potential buyers with the median usage rate.

Consider the case where the device is an office copier machine. Suppose the median desired usage over the five-year life of the machine is 200,000 copies and the maximum value per page (after paying toner cost) is 25¢. Thus, metering would lead to lower total welfare provided that the production cost (not price) of the office copier is below $0.2 * \$0.25 * 200{,}000 = \$10{,}000$. Under metering (and our Market Assumptions), the capital good price is half of cost. Putting this all together, if our valuation assumptions are correct and high-end office copiers are monopolized and sold under metering with a price below $\$5{,}000$, then metering will lower total welfare provided $\sigma < 0.624$.[25]

Another way to estimate a reasonable cost is to look at the payback period. The monopolist loses $\frac{c_0}{2}$ on the upfront sale, but will then make $\frac{(1-c_1)}{2}e^{\mu}$ based on the median individual's demand.[26] If we think that the lifetime of the capital good is five years or more and that the monopolist expects to recover its costs in the first year, then $\frac{(1-c_1)}{2}e^{\mu}/c_o$ will be below five, which means the cost ratio will be below 20%, and so metering will lower total welfare if $\sigma < 0.624$.

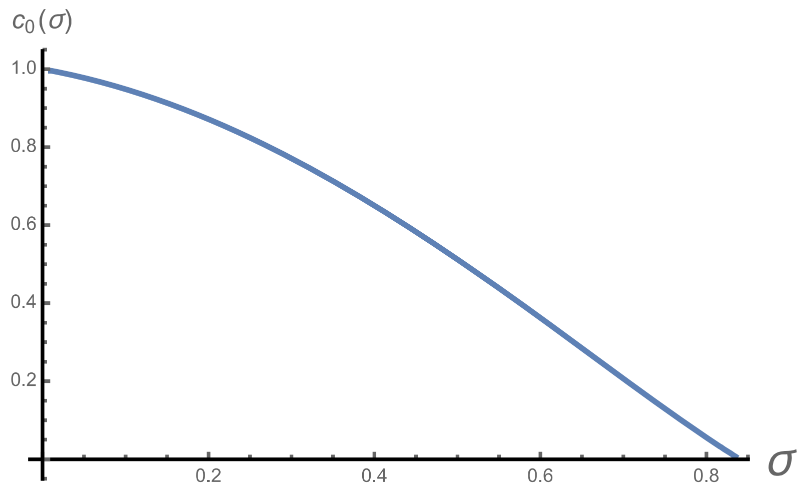
[24] At $\sigma = 0.78$, $p^*$ is 0.62, $\ln(p^*)/\sigma = -0.61$. Thus, $1 - G(p^*) = 1 - \Phi(-0.61) = 0.73$. Note that up to 27% of buyer types can be entirely excluded from the market before metering increases total welfare.

[25] At $\sigma = 0.624$, a single-price monopolist serves 29.4% of the market and a metering monopolist serves 38.2%. Metering must expand capital good sales by 30% (0.382/0.294 – 1) in order to improve total welfare.

[26] Note that this is the median potential buyer in the population. Among those who purchase the good, the median demand will be higher.

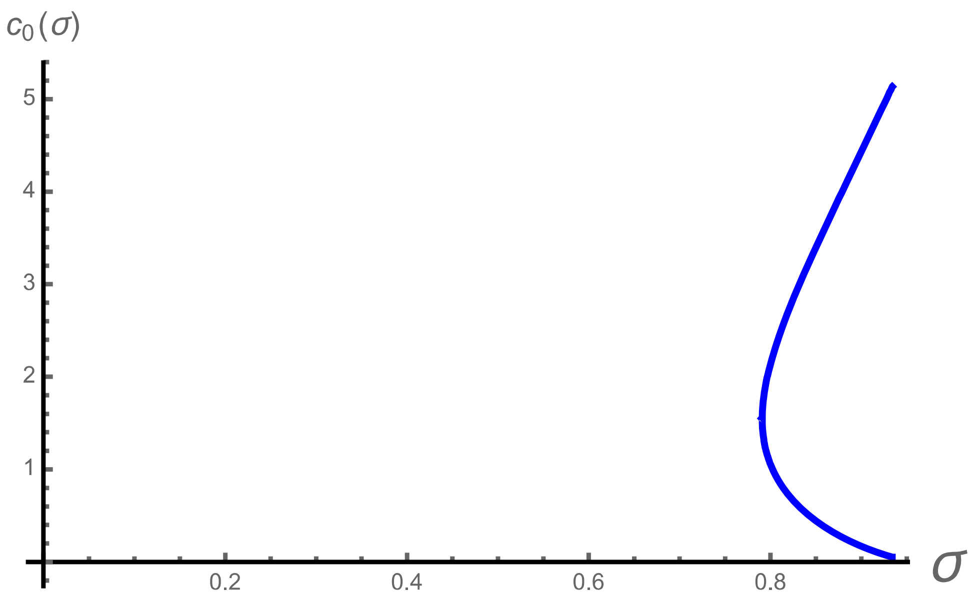
Theorem 5 assumes no profit dissipation and no productive efficiencies from tying. If there were productive efficiencies, those efficiencies might offset the total welfare harm from metering ties. Conversely, if there were profit dissipation, then metering ties could reduce total welfare even if the standard deviation were higher than 0.78. Corollary 3 in the Mathematical Appendix provides the new critical value of $c_0(\sigma)$ when dissipation is a factor.

Let $d$ represent the fraction of metering profits that are dissipated. When $d = 1\%$, the result is that the dividing line bows upwards and extends further to the right, out to $\sigma = 0.83$, as shown in the figure below.

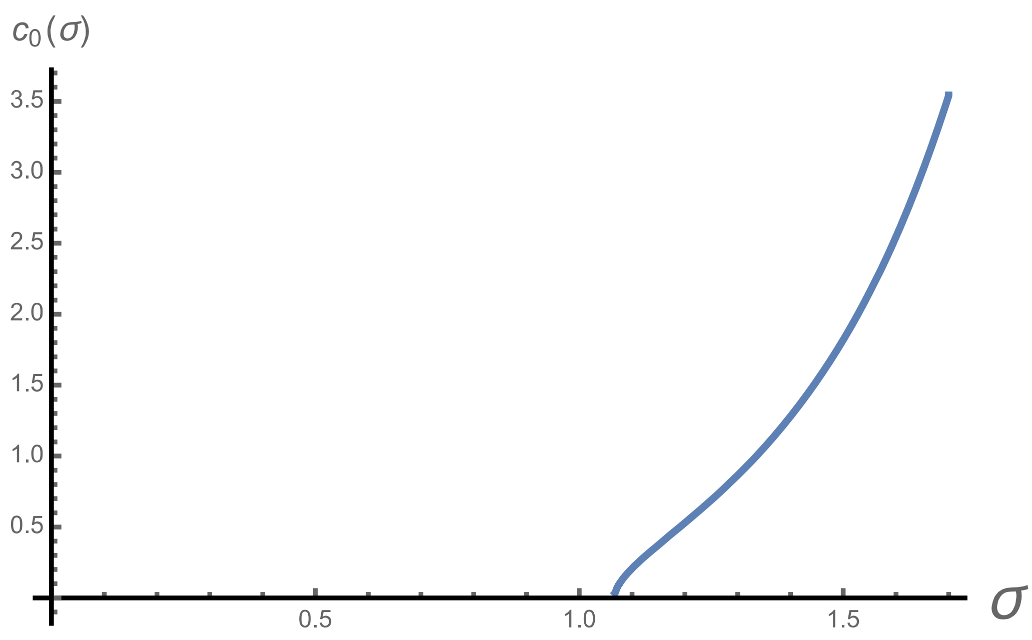



If $d = 3\%$, then metering lowers total welfare for *all* $c_0$ provided $\sigma < 0.79$. Once $\sigma > 0.79$ then metering lowers total welfare if and only if $c_0$ is (in the following figure) below the bottom curve or above the top one. For example, if $\sigma = 0.9$ then metering lowers total welfare if $c_0 < 0.17$ or if $c_0 > 4.4$. Realistically, only the bottom part of the curve is relevant at that point because costs above 4.4 will require prices so high that demand is limited to niche markets.[27]

[27] If $c_0 = 4.4$, then $p^* > 4.4$, which requires $n > 4.4$ for positive demand. Recall that $c_0 = 4.4$ is based on a normalization where the median demand is 1. This means that the capital good will be sold only to buyers whose desired usage is over *four times* the median usage, which given the lognormal distribution means $n$ must be at least $\ln(4.4) = 1.5\sigma$ above the median which is just 7% of the population.




If $d = 5\%$, then single pricing leads to higher surplus for *all* $c_0$ provided $\sigma < 1.07$. Thereafter, single pricing leads to higher surplus provided $c_0 > c_0(\sigma)$ defined by the line below. The importance of this result is not that it extends the region for $c_0 = 0$ out from 0.78 to 1.07 but that it extends the region for *all* costs as long as $\sigma < 1.07$.




Although we have framed the analysis above in terms of tying, by Lemma 1 all the same formulas and analysis would apply if one considered the case of any optimal non-linear pricing scheme. If the only good is the consumption good (so there is no capital good) then $c_0$ is naturally equal to 0. And if demand for the consumption good follows our Market Assumptions along with lognormality in $n$, then the profit-maximizing non-linear tariff will always lower consumer surplus and will reduce total welfare when $\sigma < 0.78$.

## IV. CONCLUSION

This paper questions the claim that second-degree pricing discrimination in general, and metering ties in particular, are better forms of price discrimination than the more frequently analyzed third-degree versions. We prove that when a firm has market power over a capital good that is used with a competitive consumable, then (just like third-degree price discrimination) metering ties and second-degree price discrimination lower consumer welfare and total welfare unless they increase capital good output. This proof supports, at a minimum, requiring that firms with market power show that their metering ties have the procompetitive effect of expanding capital good output.[28]

Expanding capital good output is a necessary but not sufficient condition to improve total welfare.[29] With a lognormal distribution of usage rates, reasonable costs, and our Market Assumptions, metering ties reduce total welfare unless the dispersion of the desired usage of the metered good is large. Moreover, if 3% or more of metering profits are dissipated, then metering ties will lower total welfare for all cost levels unless the dispersion of desired usage is above 0.79, which is more than the dispersion of income in the United States.

Even when metering ties increase capital good output enough to increase total welfare, it does not follow that they increase consumer welfare. To the contrary, we prove that for all cost levels and all lognormal distributions of usage intensity, metering ties harm consumer welfare. While this result is quite general, it does rely on our Market Assumptions, in particular that the value per usage is independent of the desired usage rate.

It is tempting to imagine that since perfect price discrimination improves total welfare, any move towards more price discrimination will lead in that same direction. But, while giving a firm with market power more price discrimination tools will generally raise its profits, this gain may come at the expense of consumers and result in lower overall efficiency.

[28] This requirement corresponds to the current quasi per se rule, which condemns ties by a firm with tying market power unless it can prove a procompetitive justification; see also footnote 4.
[29] Expanding consumable output is sufficient to demonstrate an increase in total welfare (but not consumer welfare) in the special case where the capital good is both costless and provided for free.

The advantage of metering is that it is well designed to handle heterogeneity in desired usage. The disadvantage of metering is that it excludes buyers with a high overall valuation who combine a high usage rate with a low value per usage. The advantage of a single price is that it is maximally efficient given the sales that occur. The disadvantage of single pricing is that the firm will inefficiently restrict supply, especially to buyers who combine a low usage rate with a high value per usage.

Antitrust law allows a firm with market power to charge an inefficiently high price. It does so in order to provide incentives to innovate and lower costs. At the same time, the law provides limits on the application of market power—patents are limited to twenty years and firms may not extend or abuse their market power. The results of this paper show that we might want to preserve a historical limit on market power: limiting the ability of a firm with market power to use tied sales for metering.

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## MATHEMATICAL APPENDIX

Before proving Theorems 4 and 5, we employ two normalizations to simplify the mathematics without loss of generality. The issue is that there are four variables, namely $c_0, c_1$ from the cost function and $\mu, \sigma$ from the lognormal. The analysis is much more manageable once we demonstrate that three of these variables only enter into the calculation as a ratio.

*Lemma 4: Under our Market Assumptions, the comparison between metering and single pricing only depends on $\mu, c_0, and\ c_1$ via $\frac{e^{-\mu}c_0}{1-c_1}$. Thus without loss of generality we normalize variables so that $\mu = 0$ and $c_1 = 0$.*

*Proof*: The first normalization ($\mu = 0$) follows once we recognize that if we double both demand and capital good costs, then profits double, along with consumer surplus and total surplus. This is true for both metering and single pricing. By doubling demand, we mean that each buyer type *(a, n)* demands *2n* rather than *n* units. If the capital good cost $c_0$ also doubles, then we can think of this doubling as equivalent to each buyer adding a clone of him or herself. The combined consumable demand of the buyer and the clone is double the prior demand. And the combined capital good cost for the pair is also double the original cost as the two of them have to buy *two* of the original capital goods. The new situation would be unchanged if the original customer and the clone could share one capital good, but that one capital good was double the original cost.

With the doubled demand and doubled capital cost in place, the profit-maximizing capital good price under metering is $\frac{2c_0}{2} = c_0$ and the consumable price remains at $\frac{1+c_1}{2}$. Profits from the "*2n*" type are now

$$\left(2n\frac{(1-c_1)}{2} + c_0 - 2c_0\right) * \left[\frac{(1-c_1)}{2} - \frac{c_0}{2n}\right] = 2 * \frac{1}{4n}(n(1-c_1) - c_0)^2 \qquad \text{(A1)}$$

which is twice the prior metering profits. Consumer surplus and total surplus also double.

The same logic about splitting the buyer in two shows that the profit-maximizing single price remains at $p^*$ when the customer and his or her clone each buy a capital good that costs $c_0$. Similarly, if we combine the two clones back into one, and the new capital cost is $2c_0$, then the profit-maximizing single price is $2p^*$ and profits, consumer surplus, and total surplus are all double.

There is nothing special about doubling here—any scale factor would lead to the same invariance. Bringing this to our model, we note that for the lognormal distribution, $e^{\mu}$ (which is the median of the distribution) acts just like a scale factor. Thus, the above argument shows that a comparison of consumer surplus or total surplus between metering and single pricing is unchanged (in terms of ratios) provided that $c_0/e^{\mu}$ remains constant.[30] Without loss of generality, we can scale demand so that $e^{\mu} = 1$ or $\mu = 0$ when comparing surplus in metering versus single pricing.

The second normalization ($c_1 = 0$) arises when we realize that our comparison of surplus in the two cases only depends on $c_0$ and $c_1$ via the ratio $\frac{c_0}{1-c_1}$. This normalization relies on the assumption that the per-usage valuations are uniformly distributed, which is part of the Market Assumptions. To see this, consider a different problem where demand is double for each buyer and $1 - c_1$ falls by half, say from 1 to ½. Since the consumable unit cost has risen from 0 to ½, the value of each unit purchased will range from 0 to ½, instead of from 0 to 1 (as all those who only valued each unit at something less than ½ are driven out of the market.) But while the per-unit value ranges from 0 to ½, the per "two-units" value ranges uniformly from 0 to 1. Thus, if we think of buyers as purchasing consumable units in *pairs* under the new regime, nothing is changed. However, we cannot double demand because we have normalized median demand to equal 1. But doubling demand is the same as dividing capital good cost in half in terms of keeping the ratio $c_0/e^{\mu}$ constant. Thus, if we halve both $c_0$ and $1 - c_1$, then once again all of the surplus ratios will remain constant: any comparison of surplus between metering and single pricing (in terms of ratios) only depends on $c_0$ and $1 - c_1$ via their ratio $\frac{c_0}{1-c_1}$. Without loss of generality, we can set $c_1 = 0$. □

In the proof of Theorems 4 and 5 we employ several properties of the lognormal.

*Lemma 5: A lognormal distribution with* $\mu = 0$ *has the following properties:*

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[30] Another way of saying this is that when comparing the ratio of surplus, either consumer or total, between metering and single pricing, $c_0$ and $e^{\mu}$ only enter the calculation in the form of a ratio, $\frac{c_0}{e^{\mu}}$.

The cumulative distribution function is

$$G(p^*) = \Phi\left(\frac{ln(p^*)}{\sigma}\right) \tag{A2}$$

where Φ() is the cumulative distribution of a standard normal distribution (mean 0 and standard deviation of 1). The conditional expectation for a lognormal distribution is:

$$E[n|n \geq p^*] = \frac{e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(p^*)}{\sigma}\right)}{1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)} \tag{A3}$$

The conditional harmonic mean for a lognormal distribution is:

$$H[n|n \geq p^*] = \frac{1}{E\left(\frac{1}{n}\middle|n \geq p^*\right)} = \frac{e^{-\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right]}{1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)} \tag{A4}$$

**Theorem 4: Under our Market Assumptions, metering ties always lower consumer surplus for any lognormal distribution of buyer usage rates.**

*Proof*: By Lemma 4, we can set $c_1 = 0$ and $\mu = 0$. Without tying, the maximum consumer surplus for buyers of type *n* is *n* – *p* and the minimum surplus is 0. Thus the average consumer surplus for type *n* buyers is $\frac{n-p}{2}$.

$$CS_s = \int_{p^*}^{\infty} \frac{n - p^*}{2}\ (1 - \frac{p^*}{n}) g(n)dn \tag{A5}$$

$$= \int_{p^*}^{\infty} \left(\frac{n}{2} - p^* + \frac{{p^*}^2}{2n}\right) g(n)dn$$

$$= \frac{1 - G(p^*)}{2}\left[E[n|n \geq p^*] - 2p^* + \frac{{p^*}^2}{H[n|n \geq p^*)}\right]$$

Substituting in the values from Lemma 5 leads to:

$$CS_s = \frac{1}{2}[e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(p^*)}{\sigma}\right) - 2p^*\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right] + p^{*2}e^{\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)\right]] \quad \text{(A6)}$$

The value of $p^*$ is defined by (7). Note that $p^*$ is an (implicit) function of $c_0$ and $\sigma$:

$$p^* = \frac{1}{2}[c_0 + H[n|n \geq p^*]] = \frac{1}{2}c_0 + \frac{1}{2}\frac{e^{-\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right]}{1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)} \quad \text{(A6)}$$

While it is possible to solve for $p^*$ as a function of $c_0$ given $\sigma$, it is trivial to solve for the inverse relationship: $c_0 = 2p^* - H[n|n \geq p^*]$. Thus it is simpler to write $CS_s$ as a function of $p^*$ and then employ $c_0(p^*)$ in the metering equation. As a consistency check, note that if $c_0 \leq 1$, $p^*$ converges to $\frac{1+c_0}{2}$ as $\sigma \to 0$ and Consumer Surplus converges to $\frac{(1-c_0)^2}{8}$.

With a metering tie, $p_m = \frac{c_0}{2}$ and $m = \frac{1}{2}$, so that

$$CS_m = \int_{c_0}^{\infty} \frac{n(1-m) - p_m}{2}\ (1 - \frac{p_m}{n} - m)g(n)dn \quad \text{(A8)}$$

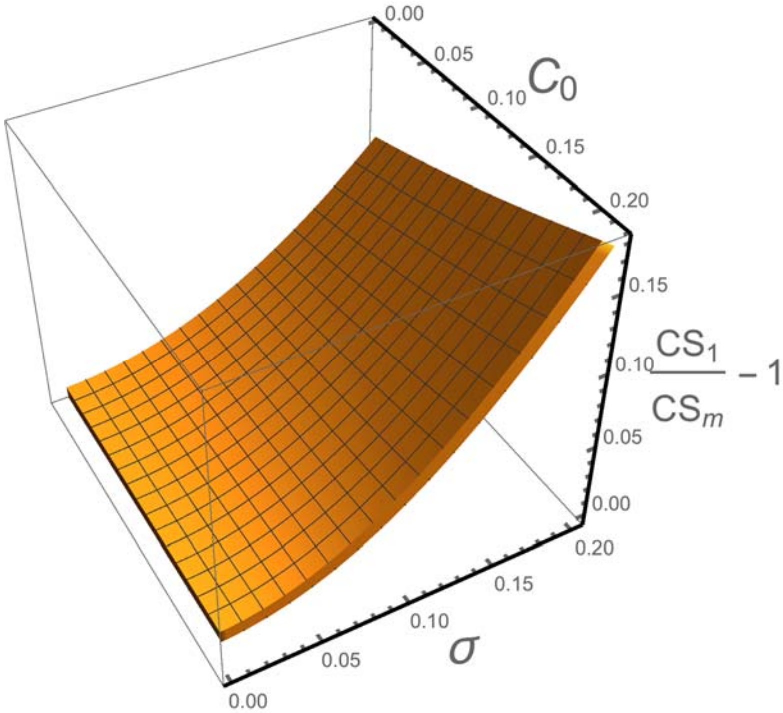
$$= \frac{1}{8}\int_{c_0}^{\infty}\left(n - 2c_0 + \frac{c_0^2}{n}\right)g(n)dn$$

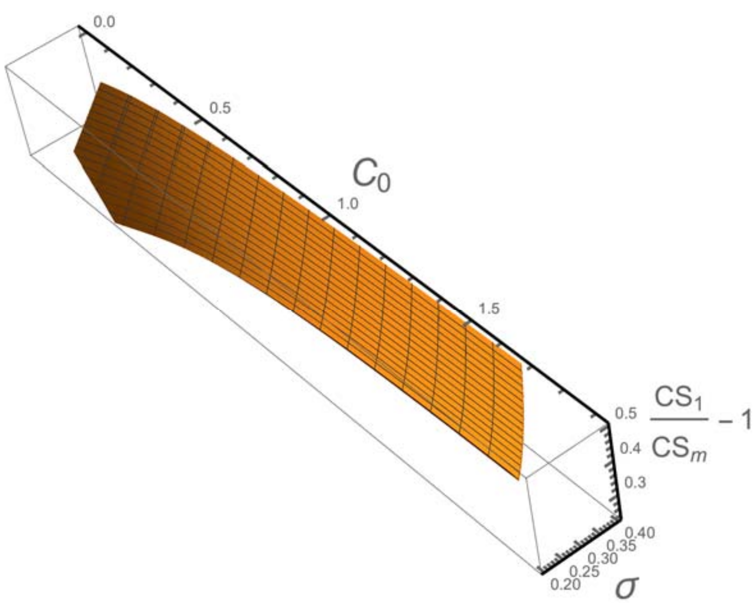
$$= \frac{1 - G(c_0)}{8}\left[E[n|n \geq c_0] - 2c_0 + \frac{c_0^2}{H[n|n \geq c_0)}\right]$$

$$CS_m = \frac{1}{8}\left[e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(c_0)}{\sigma}\right) - 2c_0\left[1 - \Phi\left(\frac{ln(c_0)}{\sigma}\right)\right] + c_0^2 e^{\frac{1}{2}\sigma^2}[1 - \Phi\left(\frac{ln(c_0)}{\sigma} + \sigma\right)]\right] \quad \text{(A9)}$$

As a consistency check, note that if $c_0 \leq 1$, Consumer Surplus under metering also converges to $\frac{(1-c_0)^2}{8}$ as $\sigma \to 0$.

While the two consumer surpluses converge as $\sigma \to 0$, as the 3D-plots below show, consumer surplus under single pricing is always above that from metering when $\sigma > 0$. In both graphs, the z-axis is $\frac{CS_s}{CS_m} - 1$, the ratio of the two consumer surplus levels minus 1. The first graph shows that this is always positive for small values of $\sigma$. The second graph shows that the function is even larger with greater values of $\sigma$ and $c_0$.




□

**Theorem 5. Under our Market Assumptions and a lognormal distribution of usage rates $n$, for $\sigma < 0.78$, metering ties lower total welfare provided $c_0 < c_0(\sigma) \approx 1 - \frac{\sigma}{0.78}$. For $\sigma > 0.78$, metering ties increase total for all values of $c_0$.**

*Proof*: With a metering tie, profits are just double consumer surplus:

$$\Pi_m = \int_{c_0}^{\infty} (\frac{n}{2} - \frac{c_0}{2})\,(\frac{1}{2} - \frac{c_0}{2n}) g(n) dn \tag{A10}$$

$$= \frac{1}{4}\int_{c_0}^{\infty} (n - 2c_0 + \frac{c_0^2}{n}) g(n) dn = 2CS_m$$

Since TS = CS + Π, it follows that Total Surplus is just triple consumer surplus:

$$TS_m = \frac{3}{8}\left[e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(c_0)}{\sigma}\right) - 2c_0\left[1 - \Phi\left(\frac{ln(c_0)}{\sigma}\right)\right] + c_0^2 e^{\frac{1}{2}\sigma^2}[1 - \Phi\left(\frac{ln(c_0)}{\sigma} + \sigma\right)]\right] \tag{A11}$$

We next turn to calculate Total Surplus under a single profit-maximizing price of $p^*$. Among those of type *n* who purchase the capital good, the minimum value they receive is $p^*$ and the maximum value is *n*, so that the average surplus created is $\frac{n+p^*}{2} - c_0$. The fraction that purchase the capital good is $1 - \frac{p^*}{n}$. Thus:

$$TS_s = \int_{p^*}^{\infty} \left(\frac{n + p^*}{2} - c_0\right)\left(1 - \frac{p^*}{n}\right) g(n) dn \tag{A12}$$

$$= \frac{1}{2}\int_{p^*}^{\infty} \left[n - 2c_0 - (p^* - 2c_0)\frac{p^*}{n}\right] g(n) dn$$

$$= \frac{1}{2}\int_{p^*}^{\infty} \left[n - 6p^* + 2H[n|n \geq p^*] + \frac{3p^{*2}}{n}\right] g(n) dn$$

where the final equality employed the substitution $c_0 = 2p^* - [n|n \geq p^*]$. Using Lemma 5:

$$TS_s = \frac{1}{2}[e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(p^*)}{\sigma}\right) - 6p^*\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right] + \frac{2e^{-\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right]^2}{1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)} + 3p^{*2}e^{\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)\right] \quad \text{(A13)}$$

Multiplying both surplus expressions by 2, total surplus is lower with a metering tie if

$$e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(p^*)}{\sigma}\right) - 6p^*\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right] + \frac{2e^{-\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right]^2}{1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)} + 3p^{*2}e^{\frac{1}{2}\sigma^2}[1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)]$$
$$> \frac{3}{4}\left[e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(c_0)}{\sigma}\right) - 2c_0\left[1 - \Phi\left(\frac{ln(c_0)}{\sigma}\right)\right] + c_0^2e^{\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(c_0)}{\sigma} + \sigma\right)\right]\right] \quad \text{(A14)}$$

For each value of $\sigma$ we can find the value of $c_0$ that leads to equality in the above relationship. This is graphed following the statement of Theorem 5 in the text. As one can see, this is very close to a straight line that starts at (0, 1) and falls to (0.78, 0). □

**Corollary 3. If metering leads to dissipation of fraction *d* of metering profits, then single pricing leads to greater total surplus provided:**

$$e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(p^*)}{\sigma}\right) - 6p^*\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right] + \frac{2e^{-\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(p^*)}{\sigma}\right)\right]^2}{1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)} + 3p^{*2}e^{\frac{1}{2}\sigma^2}[1 - \Phi\left(\frac{ln(p^*)}{\sigma} + \sigma\right)]$$
$$> \frac{3-2d}{4}\left[e^{\frac{1}{2}\sigma^2}\Phi\left(\sigma - \frac{ln(c_0)}{\sigma}\right) - 2c_0\left[1 - \Phi\left(\frac{ln(c_0)}{\sigma}\right)\right] + c_0^2e^{\frac{1}{2}\sigma^2}\left[1 - \Phi\left(\frac{ln(c_0)}{\sigma} + \sigma\right)\right]\right] \quad \text{(A15)}$$

*Proof:* Note that the only change to (A14) is $\frac{3}{4}$ is replaced by $\frac{3-2d}{4}$. Recall that Total Surplus under metering is consumer surplus plus profits, and profits are now only a $2(1-d)$ multiple of consumer surplus. Thus the multiple of consumer surplus is $1 + 2(1-d) = 3 - 2d$ rather than 3. □

The inequality in (A15) is graphed for $d = 0.01, 0.03,$ and $0.05$ in the main text.

## SUPPLEMENTAL APPENDIX

**Theorem: Under our Market Assumptions, a small mandated reduction in the metered price of the consumable good leads to an increase in both consumer welfare and total welfare.**

Proof: The monopolist chooses $m$ and $p$ jointly to maximize profits. We could equivalently say that the monopolist chooses $m$ where the upfront price is $p(m)$. To prove this theorem , we need show

$$\frac{dCS}{dm} < 0 \text{ and } \frac{dTS}{dm} < 0$$

at the $m$ that maximizes profits. If so, a small reduction in $m$ will improve consumer and total welfare. Because $m$ and $p(m)$ are chosen by the monopolist to maximize profits, it follows immediately that $\frac{d\Pi}{dm} = 0$. Thus the change in consumer surplus is the same as the change in total welfare. And the change in consumer surplus is just equal to the change in the amount existing consumers pay under the new pricing scheme. (This follows as the lost marginal consumers had no surplus and the level of consumption is fixed at $n$ or $0$.) To determine the change in payments, we begin by calculating $\frac{dp}{dm}$.

$$\text{Profits} = \int_{\frac{p}{1-m}}^{\infty} (n(m - c_1) + p - c_0)\left(1 - m - \frac{p}{n}\right) g(n)dn. \qquad \text{(S1)}$$

$$\frac{d\text{Profits}}{dp} = \int_{\frac{p}{1-m}}^{\infty} (1 + c_1 - 2m - \frac{2p - c_0}{n}) g(n)dn = 0. \qquad \text{(S2)}$$

The first-order condition in (S2) defines $p(m)$. Taking the derivative with respect to $m$ provides the equation for $\frac{dp}{dm}$. Note that at the profit-maximizing $m$,

$$p = \frac{c_0}{2}, \qquad m = \frac{1 + c_1}{2}.$$

Thus the integrand in (S2) is zero for all values of $n$ at the profit-maximizing value of $m$, so that we can ignore that effect of the change in the range of integration.

$$\int_{\frac{p}{1-m}}^{\infty}\left(-2-\frac{2}{n}\frac{dp}{dm}\right)g(n)dn\ = 0 \tag{S3}$$

$$\frac{dp}{dm} = -H\left[n\middle|n \geq \frac{p}{1-m}\right],$$

where *H[n]* is the harmonic mean of *n*, conditional on *n* being at least $\frac{p}{1-m}$.

We can now compute the change in consumers' payments (and thus change in consumer surplus). Demand can be broken into two parts:

$$\text{Capital Good Demand } = \int_{\frac{p}{1-m}}^{\infty}(1-m-\frac{p}{n})g(n)dn$$

$$= (1-G\left(\frac{p}{1-m}\right))(1-m-\frac{p}{H});$$

$$\text{Consumable Good Demand } = \int_{\frac{p}{1-m}}^{\infty}n(1-m-\frac{p}{n})g(n)dn$$

$$= \ (1-G\left(\frac{p}{1-m}\right))((1-m)E[n]-p)$$

As $m$ increases, the consumers pay $dm$ more on consumables and $\frac{dp}{dm}dm$ less on the capital good:

$$\frac{dCS}{dm} = -1 * \text{Consumable Good Demand} + H * \text{Capital Good Demand} \tag{S5}$$

$$= \left[1-G\left(\frac{p}{1-m}\right)\right]\left(-(1-m)E[n]+p+H[n](1-m-\frac{p}{H[n]})\right)$$

$$= \left[1-G\left(\frac{p}{1-m}\right)\right](1-m)(H[n]-E[n]) < 0.$$

where *H[n]* and *E[n]* are the harmonic and arithmetic means conditional on *n* being greater than $\frac{p}{1-m}$. The final inequality follows from the fact that the arithmetic mean is always greater than the harmonic mean. The inequality is strict provided that the density is not all concentrated on a single mass point, which is ruled out by out Market Assumption. Other than the no mass point, we do not require any assumption on the distribution *g(n)*. □

**Theorem: With perfect correlation between *a* and *n* it follows that metering always leads to (i) higher total welfare and (ii) lower consumer surplus.**

*Proof:* For the case with perfect correlation, we can set $n = ka$ where $a$ is (as before) uniform over [0, 1].

Single pricing: The consumer will buy if

$$n(a - c_1) \geq p$$

$$ka(a - c_1) \geq p$$

$$a^2 - c_1 a - \frac{p}{k} \geq 0$$

$$a \geq a^* = \frac{c_1}{2} + \sqrt{\frac{c_1^2}{4} + \frac{p}{k}}$$

Instead of having the firm choose $p$, It is easier to solve the problem if we think of the monopolist as choosing $a^*$ and then translating that into the corresponding price. Given $a^*$

$$p = ka^{*2} - kc_1 a^*$$

$$\text{Profits} = \left(ka^{*2} - kc_1 a^* - c_0\right)(1 - a^*)$$

Profits are maximized at

$$\frac{d\text{Profits}}{da^*} = 0 \quad => (2ka^* - kc_1)(1 - a^*) - \left(ka^{*2} - kc_1 a^* - c_0\right) = 0$$

$$(2ka^* - kc_1) - \left(2ka^{*2} - kc_1 a^*\right) - \left(ka^{*2} - kc_1 a^* - c_0\right) = 0$$

$$-3a^{*2} + 2(1 + c_1)a^* - c_1 + c_0/k = 0$$

$$a^* = \frac{(1 + c_1)}{3} + \frac{1}{3}\sqrt{(1 + c_1)^2 - 3\left(c_1 - \frac{c_0}{k}\right)} \tag{S6}$$

Note that the negative solution to the quadratic would lead to the firm making zero profits in the case of zero costs.

Metering: The consumer will buy if

$$n(a - m) \geq p_m$$

$$ka(a - m) \geq p_m$$

$$a^2 - ma - \frac{p_m}{k} \geq 0$$

$$a \geq a^* = \frac{m}{2} + \sqrt{\frac{m^2}{4} + \frac{p_m}{k}}$$

Again, this is easier to solve if we change variables and optimize with respect to *(a*, m)* and then set $p_m = ka(a - m)$.

$$\text{Profits} = \Big(ka^{*2} - ka^*m - c_0 + (m - c_1)(\tfrac{k}{2}(1 + a^*)\Big)(1 - a^*)$$

Optimizing with respect to *m* reveals that the first-order condition is

$$\frac{k}{2}(1 + a^*) - ka^* = \frac{k}{2}(1 - a^*) > 0 \text{ for a}^* < 1$$

Since the first-order condition is always positive whenever demand is positive, it follows that we want to keep raising *m* (and lowering $p_m$) until we arrive at a corner solution. We will assume that the lowest possible upfront price for the capital good is zero. Thus *m* will rise until the point where $p_m = 0$. And, at that point, $a^* = m$. Thus we set $p_m = 0$ and now find the optimal *m*:

$$\text{Profits} = \Big(-c_0 + (m - c_1)(\tfrac{k}{2}(1 + m)\Big)\ (1 - m)$$

Profits are maximized at

$$\frac{d\text{Profits}}{dm} = 0 \ \ => \ c_0 + (-2m)\frac{k}{2}(m - c_1) + \frac{k}{2}(1 - m^2) \ = 0$$

$$\frac{2c_0}{k} + (-2m)(m - c_1) + 1 - m^2 \ = 0$$

$$-3m^2 + m(2c_1) + (\frac{2c_0}{k} + 1) = 0$$

$$m^* = \frac{c_1}{3} + \frac{1}{3}\sqrt{c_1^2 + 3(1 + \frac{2c_0}{k})} \tag{S7}$$

We know that $m \geq c_1$ as otherwise profits are negative. Thus over the range of positive profits, note that the second-order condition is $-6m + 2c_1$, which is negative, and so the relevant profit function is concave.

We now can now compare the result of metering versus single pricing. First note that if the optimal metering rate is below the $a^*$ under single pricing then more consumers will be in the market. As there is no inefficiency in terms of which consumers are in the market, this means total welfare will be higher.

**Comment #1.**

Both equations are easy to solve in the case where all costs are zero. In that case,

Single pricing: $a^* = \frac{2}{3}$ and $m = \frac{1}{\sqrt{3}} < \frac{2}{3}$.

Thus total welfare is higher under metering. Turing to consumer surplus, under single pricing *p=k4/9*, and

$$CS_{single\ pricing} = k\int_{2/3}^{1}[a^2 - \frac{4}{9}]da = \frac{1}{3} - \frac{8}{81} - \frac{4}{27} = \frac{7k}{81}$$

Under metering,

$$\begin{aligned} CS_{metering} &= \int_{m}^{1}[ka(a-m)]da \\ &= \frac{k(1-m^3)}{3} - \frac{km(1-m^2)}{2} \\ &= \frac{k}{6}[\frac{1}{3\sqrt{3}} - 3/\sqrt{3} + 2] \\ &= \frac{k}{3}\left[1 - \frac{4}{3\sqrt{3}}\right] < \frac{7k}{81} \end{aligned}$$

as $81 - 36\sqrt{3} = 18.65 < 21.$ As we see below, this is not just the case for zero costs.

*Total welfare is always higher under metering.* To show this, we could do a direct comparison of the solutions for $a^*$ and *m*. Instead, it is simpler to demonstrate that the first-order condition for *m* under metering is negative at the value of *m* that is the solution to a* under single pricing. As the profit function under metering is concave where positive, that implies that the profit-maximizing value of *m* is below $a^*$.

Recall that the first-order condition for metering is

$$-3m^2 + m(2c_1) + (\frac{2c_0}{k} + 1)$$

If we substitute $m = a^*$ and use the equality below that defines $a^*$ to substitute in the value of $-3{a^*}^2$

$$-3{a^*}^2 + 2(1 + c_1)a^* - c_1 + c_0/k \ = 0$$

Then we get that the derivative of metering profit with respect to *m* at $m = a^*$ is

$$a^*(2c_1) + \left(\frac{2c_0}{k} + 1\right) - [2(1 + c_1)a^* - c_1 + c_0/k]$$

We can take out the two $a^*(2c_1)$ terms that cancel leaving us with

$$\left(\frac{2c_0}{k} + 1\right) - [2a^* - c_1 + c_0/k]$$

This simplifies to

$$\frac{c_0}{k} + 1 + c_1 - 2a^*$$

Plugging in the value of a*

$$a^* = \frac{(1 + c_1)}{3} + \frac{1}{3}\sqrt{(1 + c_1)^2 - 3(c_1 - \frac{c_0}{k})}$$

reveals

$$\frac{c_0}{k} + 1 + c_1 - 2a^* = \frac{c_0}{k} + \frac{1 + c_1}{3} - \frac{2}{3}\sqrt{(1 + c_1)^2 - 3(c_1 - \frac{c_0}{k})}$$

Thus the first-order condition will be negative if and only if

$$\frac{c_0}{k} + \frac{1 + c_1}{3} < \frac{2}{3}\sqrt{(1 + c_1)^2 - 3(c_1 - \frac{c_0}{k})}$$

$$9\left(\frac{c_0}{k}\right)^2 + (1 + c_1)^2 + 6\frac{c_0}{k}(1 + c_1) < 4\,[(1 + c_1)^2 - 3(c_1 - \frac{c_0}{k})]$$

Collecting terms we find

$$9\left(\frac{c_0}{k}\right)^2 - 6\frac{c_0}{k}(1-c_1) < \ 3(1+c_1)^2 - 12c_1$$

Dividing all terms by 3

$$3\left(\frac{c_0}{k}\right)^2 - 2\frac{c_0}{k}(1-c_1) < (1+c_1)^2 - 4c_1 = (1-c_1)^2$$

In order for any demand to exist,

$$\frac{c_0}{k} < 1 - c_1$$

Hence

$$3\left(\frac{c_0}{k}\right)^2 - 2\frac{c_0}{k}(1-c_1) < 2\left(\frac{c_0}{k}\right)^2 + (1-c_1)^2 - 2\frac{c_0}{k}(1-c_1)$$

Thus it will be sufficient to show that

$$2\left(\frac{c_0}{k}\right)^2 + (1-c_1)^2 - 2\frac{c_0}{k}(1-c_1) < (1-c_1)^2$$

or

$$2\left(\frac{c_0}{k}\right)^2 - 2\frac{c_0}{k}(1-c_1) < 0$$

$$\left(\frac{c_0}{k}\right)^2 < \frac{c_0}{k}(1-c_1)$$

or

$$\left(\frac{c_0}{k}\right) < 1 - c_1$$

But this must true in order for there to be the potential for any profitable demand to exist.

Thus we have shown that the monopolist will always serve more of the market under metering. In this one-dimensional model, this is sufficient to demonstrate that metering will increase total welfare (as consumers are ordered by *a* and thus there is no potential for inefficient allocation across consumers).

Next we show that consumer welfare is always lower under metering. Consumer welfare is

$$CS_{single} = \int_{a^*}^{1} [ka(a - c_1) - p]da$$

$$= \frac{k - ka^{*3}}{3} - \frac{c_1 k(1 - a^{*2})}{2} - k(a^{*2} - c_1 a^*)(1 - a^*)$$

$$CS_{single}/k = \frac{1 - a^{*3}}{3} - \frac{c_1(1 - a^{*2})}{2} - (a^{*2} - c_1 a^*)(1 - a^*)$$

$$= \frac{1}{3} + \frac{2}{3}a^{*3} - \frac{c_1}{2}[1 + a^{*2}] - a^{*2} + c_1 a^*$$

$$= \frac{1}{3} + \frac{2}{3}a^{*3} - \frac{c_1}{2}[1 + a^{*2} - 2a^*] - a^{*2}$$

$$= \frac{1}{3} + \frac{2}{3}a^{*3} - \frac{c_1}{2}[1 - a^*]^2 - a^{*2}$$

$$CS_{meter} = \int_{m}^{1} [ka(a - m)]da$$

$$= \frac{k(1 - m^3)}{3} - \frac{km(1 - m^2)}{2}$$

$$= \frac{k}{6}[m^3 - 3m + 2]$$

Factoring out a k from both expressions leaves us with the condition that single pricing will lead to higher surplus if and only if

$$\frac{1}{3} + \frac{2}{3}a^{*3} - \frac{c_1}{2}[1 - a^*]^2 - a^{*2} > \frac{1}{6}[m^3 - 3m + 2]$$

or

$$4a^{*3} - 3c_1[1 - a^*]^2 - 6a^{*2} > m^3 - 3m$$

From (S5) and (S6) we know both the value of $a^*$ and *m* as a function of $(c_1, \frac{c_0}{k})$. An Excel table of the function over the two-dimensional space shows that this is inequality always holds. □

The table below is consumer surplus under single pricing minus consumer surplus under metering

| | | c1 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.000000 | 0.050000 | 0.100000 | 0.150000 | 0.200000 | 0.250000 | 0.300000 | 0.350000 | 0.400000 | 0.450000 |
| co/k | 0.000000 | 0.009687 | 0.008783 | 0.007920 | 0.007094 | 0.006302 | 0.005542 | 0.004814 | 0.004121 | 0.003465 | 0.002852 |
| | 0.050000 | 0.008327 | 0.007440 | 0.006603 | 0.005811 | 0.005065 | 0.004363 | 0.003705 | 0.003093 | 0.002530 | 0.002019 |
| | 0.100000 | 0.007180 | 0.006328 | 0.005531 | 0.004787 | 0.004096 | 0.003457 | 0.002870 | 0.002335 | 0.001855 | 0.001431 |
| | 0.150000 | 0.006192 | 0.005386 | 0.004639 | 0.003951 | 0.003319 | 0.002743 | 0.002224 | 0.001762 | 0.001356 | 0.001007 |
| | 0.200000 | 0.005331 | 0.004577 | 0.003885 | 0.003254 | 0.002683 | 0.002170 | 0.001716 | 0.001320 | 0.000980 | 0.000697 |
| | 0.250000 | 0.004573 | 0.003875 | 0.003240 | 0.002667 | 0.002156 | 0.001704 | 0.001311 | 0.000975 | 0.000695 | 0.000469 |
| | 0.300000 | 0.003901 | 0.003260 | 0.002682 | 0.002168 | 0.001715 | 0.001321 | 0.000985 | 0.000705 | 0.000478 | 0.000302 |
| | 0.350000 | 0.003303 | 0.002719 | 0.002199 | 0.001741 | 0.001344 | 0.001005 | 0.000722 | 0.000493 | 0.000314 | 0.000181 |
| | 0.400000 | 0.002769 | 0.002242 | 0.001779 | 0.001376 | 0.001032 | 0.000745 | 0.000511 | 0.000328 | 0.000191 | 0.000096 |
| | 0.450000 | 0.002294 | 0.001823 | 0.001414 | 0.001064 | 0.000771 | 0.000532 | 0.000344 | 0.000202 | 0.000103 | 0.000041 |
| | 0.500000 | 0.001872 | 0.001456 | 0.001099 | 0.000799 | 0.000554 | 0.000360 | 0.000214 | 0.000110 | 0.000044 | 0.000010 |
| | 0.550000 | 0.001500 | 0.001136 | 0.000829 | 0.000578 | 0.000378 | 0.000226 | 0.000117 | 0.000048 | 0.000011 | 0.000000 |
| | 0.600000 | 0.001173 | 0.000860 | 0.000601 | 0.000395 | 0.000238 | 0.000125 | 0.000051 | 0.000012 | 0.000000 | |
| | 0.650000 | 0.000890 | 0.000625 | 0.000413 | 0.000250 | 0.000132 | 0.000055 | 0.000013 | 0.000000 | | |
| | 0.700000 | 0.000649 | 0.000430 | 0.000262 | 0.000139 | 0.000058 | 0.000013 | 0.000000 | | | |
| | 0.750000 | 0.000447 | 0.000273 | 0.000146 | 0.000061 | 0.000014 | 0.000000 | | | | |
| | 0.800000 | 0.000284 | 0.000152 | 0.000064 | 0.000015 | 0.000000 | | | | | |
| | 0.850000 | 0.000159 | 0.000067 | 0.000016 | 0.000000 | | | | | | |
| | 0.900000 | 0.000070 | 0.000079 | 0.000000 | | | | | | | |
| | 0.950000 | 0.000017 | 0.000000 | | | | | | | | |
| | 1.000000 | 0.000000 | | | | | | | | | |

| | | c1 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.500000 | 0.550000 | 0.600000 | 0.650000 | 0.700000 | 0.750000 | 0.800000 | 0.850000 | 0.900000 | 0.950000 | 1.000000 |
| co/k | 0.000000 | 0.009687 | 0.008783 | 0.007920 | 0.007094 | 0.006302 | 0.005542 | 0.004814 | 0.004121 | 0.003465 | 0.002852 | 0.002287 |
| | 0.050000 | 0.008327 | 0.007440 | 0.006603 | 0.005811 | 0.005065 | 0.004363 | 0.003705 | 0.003093 | 0.002530 | 0.002019 | |
| | 0.100000 | 0.007180 | 0.006328 | 0.005531 | 0.004787 | 0.004096 | 0.003457 | 0.002870 | 0.002335 | 0.001855 | | |
| | 0.150000 | 0.006192 | 0.005386 | 0.004639 | 0.003951 | 0.003319 | 0.002743 | 0.002224 | 0.001762 | | | |
| | 0.200000 | 0.005331 | 0.004577 | 0.003885 | 0.003254 | 0.002683 | 0.002170 | 0.001716 | | | | |
| | 0.250000 | 0.004573 | 0.003875 | 0.003240 | 0.002667 | 0.002156 | 0.001704 | | | | | |
| | 0.300000 | 0.003901 | 0.003260 | 0.002682 | 0.002168 | 0.001715 | | | | | | |
| | 0.350000 | 0.003303 | 0.002719 | 0.002199 | 0.001741 | | | | | | | |
| | 0.400000 | 0.002769 | 0.002242 | 0.001779 | | | | | | | | |
| | 0.450000 | 0.002294 | 0.001823 | | | | | | | | | |
| | 0.500000 | 0.001872 | | | | | | | | | | |