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PARTIAL OWNERSHIP AS A STRATEGIC VARIABLE TO FACILITATE TACIT COLLUSION

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Abstract

This paper investigates the role of partial ownership of a competitor as a strategic device to facilitate tacit collusion. The paper discusses partial ownership as a decision variable in the hands of the firms, and not as an exogenous parameter, as in previous literature. Once partial ownership is acknowledged as a decision variable, it is unambiguously shown that it can be used by oligopolists to facilitate tacit collusion. This result will be shown to hold in a Bertrand model with and without cost asymmetries. It will also be demonstrated that this result, in essence, carries over to the Cournot framework as well.

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1. Introduction

This paper shows how passive partial ownership of a competitor, in an oligopolistic framework, can be used as a strategic tool to facilitate repeated interaction (noncooperative) tacit collusion. Previous literature deals almost exclusively with the effects of partial ownership on static Cournot equilibrium (Reynolds and Snapp (1985); Farrel and Shapiro (1990); Reitman (1994)). Malueg (1992) examines the question if an increase in cross ownership levels in a Cournot repeated game facilitates tacit collusion. He concludes that such an increase in cross ownership may in fact make tacit collusion more difficult, depending on the form of the demand function. Unlike Malueg, who treats cross ownership as an exogenous parameter,[1] this paper examines partial ownership as a decision variable. It

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[1] The same assumption that partial ownership is exogenous characterizes the rest of the literature on cross ownership, that deals, as stated earlier, with static (one period) Cournot equilibrium.

will be shown that once we acknowledge partial ownership as a strategic choice variable in the hands of oligopolists, partial ownership, if indeed chosen by the firms, unambiguously facilitates tacit collusion.

In each of the models to be discussed, the strategic nature of partial ownership will be captured through a two stage game. At the first stage, the firms simultaneously choose partial ownership levels in one another (anticipating the impact of cross ownership on behavior in the second stage). The second stage consists of an infinitely repeated price game.

The paper focuses on the anti-competitive effects of partial ownership. To the extent that there are social efficiencies to partial ownership, or factors motivating partial ownership other than the strategic behavior discussed in the paper, they should be weighed against (or aside from) these strategic motivations for partial ownership.

The results of the economic analysis present straightforward policy implications. As will be shown in an accompanying legal policy paper, under existing U.S. antitrust law partial acquisitions of minority interests in competitors are generally exempt from antitrust liability, as long as courts (or the antitrust agencies) are assured that such interests are absolutely passive (i.e., that the partial owner will possess no rights to elect the board, to influence the firm's actions or to have access to sensitive information). This stems from the courts' and agencies' interpretation of an exemption for stock purchases "solely for the purpose of investment" included in Section 7 of the Clayton Act. In contrast, this paper shows that even totally passive partial ownership of a competitor in an oligopolistic framework may be used as a strategic device to facilitate tacit collusion. Hence such partial acquisitions possess clear anti-competitive effects. Accordingly, the results obtained in this paper present a strong case for reform in the antitrust treatment of partial passive acquisitions.

Section 2 of the paper examines cross ownership as a decision variable in a Bertrand duopoly with symmetric firms. Section 3 analyzes the case of cost asymmetries. Section 4 briefly explains why the qualitative results of the paper are valid in the Cournot framework with symmetric firms as well.

2. A Bertrand model with symmetric firms

The impact of partial ownership on tacit collusion is easily shown when firms compete in prices. While cross ownership between competitors has been dealt with in quantity setting models (Reynolds & Snapp (1986); Farrel & Shapiro (1990); Malueg (1992); Reitman (1994)), prices may be viewed as a more natural strategic variable than quantities (or capacities) where collusive price schemes are concerned. Therefore, it is useful to examine the role of cross ownership in a price-setting framework. In the current section, firms will be assumed to be identical. Price competition between firms with different marginal costs will be discussed in section 3.

2.1 The Model

Consider a basic Bertrand duopoly model with firms producing homogenous products. Accordingly, there are two firms, denoted firm 1 and firm 2.[2] The number of firms in the industry is assumed to be fixed.[3] The firms are identical and have the same marginal cost of production c. Market demand is q=D(p). The demand each firm faces is:

$$D_i(p_i,p_j)=\begin{cases} D(p_i) & \text{if } p_i < p_j \\ \frac{1}{2}D(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

[2] The analysis in this section could be easily extended to an industry with n firms and blockaded or deterred entry.
[3] In other words, (large scale) entry is assumed to be blockaded, or effectively deterred by the incumbents. Entry of price taking fringe firms will not affect the results.

where $i,j=1,2$. Profit is assumed to rise monotonically in price below the monopoly price (the price which maximizes industry profit). Each firm is capable of supplying total demand. The two firms choose prices independently and simultaneously. The well known Bertrand (1883) result is that this static price game yields a unique Nash equilibrium in which both firms charge the competitive price, i.e., $p_1=p_2=c$.

Let us now assume that firm 1 owns a fraction x of firm 2 and that firm 2 owns a fraction y of firm 1, where $0 \leq x,y < 1/2$.[4] These ownership fractions represent passive ownership rights of the competitor. Thus an ownership fraction of x in a competitor entitles its holder to a fraction x of the competitor's profit flow.[5] This ownership fraction, however, does not entitle its holder to any influence over the activities of the competitor.

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[4] The assumption that $x,y<1/2$ means that firms seek only a minority interest in their competitor. This paper focuses on passive partial ownership. It would therefore be unrealistic for the model to include majority ownership of a firm, which normally involves active influence in this firm's activities.

[5] It is assumed here that there is no separation between ownership and control within the firm. The (active) owner of the firm is also the manager, and maximizes her own profits. The same *qualitative* results would hold if we assumed that the manager maximizes the total profits generated by the firm (including the profits that would be transferred to the competitor as a passive investor in the firm). This point will be pursued further in note 8.

2.2 Static equilibrium

As stated in proposition 1 below, cross ownership in this model does not change static equilibrium.

Proposition 1. In the unique static Nash equilibrium (with cross ownership levels x and y) both firms charge the competitive price, i.e. $p_1^*=p_2^*=c$, as long as $x+y<1$.

Proof. See Appendix A.

2.3 Dynamic Equilibrium

Let us now examine the case in which the two firms repeat the above-mentioned static game for an infinite number of periods.

As in the basic supergame model, let us assume each of the firms pursues the following trigger strategy: Each firm charges the industry monopoly price p^m (which maximizes $D(p)(p-c)$) in period 0. Additionally, each firm charges p^m in any period t if in every period preceding t neither firm has deviated from p^m. If in a preceding period there was a deviation from p^m, each firm charges the marginal cost c (reverting to the static, one period equilibrium) forever. In the case without cross ownership it has been shown[6] that p^m is sustainable if and only if $\delta \geq 1/2$, where δ is the prevailing industry discount factor.[7]

It will be shown below that in this model cross ownership can enable p^m to be sustainable not only for $\delta \geq 1/2$, but for every δ, however small it may be. In other words, cross ownership can be used to lower the "critical discount factor" (denoted below by δ^c) from 1/2 to a positive number arbitrarily close to zero. Proposition 2 below states the conditions for sustainability of tacit collusion in the cross ownership case.

Proposition 2. Under cross ownership p^m is sustainable if and only if:

$$\delta \geq \max\left\{\frac{1}{2} - \frac{x}{2(1-y)}, \frac{1}{2} - \frac{y}{2(1-x)}\right\} \equiv \delta^c$$

Proof. Denote π^m as the industry monopoly profit (the maximum of D(p)(p-c)). Firm 1 would not find it profitable to deviate from the monopoly price if and only if:

6 Friedman (1971).

7 That is, $\delta \equiv e^{-rt}$, where r is the instantaneous rate of interest and t is the real time between periods.

$$(1-y)\pi^m - (1-y)\frac{\pi^m}{2} - x\frac{\pi^m}{2} \le \frac{\delta}{1-\delta}\left[(1-y)\frac{\pi^m}{2} + x\frac{\pi^m}{2}\right] \qquad (1)$$

The left hand side of (2) constitutes firm 1's short term gain from deviation and the right hand side is equal to firm 1's long term loss from deviation.[8] This yields firm 1's incentive compatibility (no undercutting) constraint (denoted by ICC_1):

$$ICC_1 \quad \delta \ge \frac{1}{2} - \frac{x}{2(1-y)} \qquad (2)$$

Similarly, due to symmetry, firm 2's incentive compatibility constraint is:

[8] If we were to assume that the managers of firm 1 maximize total firm profits, including the profits that will be transferred to firm 2 as a partial owner, the terms 1-y on both sides of (2) should be omitted. It can be shown that in such a case p^m can be sustained if and only if: $\delta \ge \max\left\{\frac{1}{2}-\frac{x}{2}, \frac{1}{2}-\frac{y}{2}\right\}$. As noted earlier, this does not change the qualitative result of proposition 2 that cross ownership facilitates tacit collusion and lowers the critical discount factor (δ^c), or that δ^c is monotonically decreasing in x and y, as stated below. Such an assumption would, however, decrease the magnitude of such facilitation. For example, we can see that for x,y<1/2 δ^c can only be lowered down to 1/4 (unlike in the case discussed in the text, where managers maximize only the controller's profits, and δ^c can be lowered down to zero).

$$\text{ICC}_2 \quad \delta \geq \frac{1}{2} - \frac{y}{2(1-x)} \tag{3}$$

The binding incentive constraint belongs to the firm with the largest lower bound on δ. Accordingly, p^m would be sustainable if and only if:

$$\delta \geq \max\left\{\frac{1}{2} - \frac{x}{2(1-y)}, \frac{1}{2} - \frac{y}{2(1-x)}\right\} \equiv \delta^c \tag{4}$$

Q.E.D.

The intuition behind the incentive constraints (2) and (3) is straightforward. There are two factors in cross ownership that make cheating on the collusive price less profitable. First, partial ownership of the competitor (represented in (2) by x and in (3) by y) increases the partial owner's profits at the collusive state. Second, the competitor's partial ownership of the decision making firm (represented in (2) by y and in (3) by x) makes cheating by it less profitable.[9]

[9] One might claim that a firm wishing to price cut might sell its ownership rights to a third party before price cutting and thus circumvent the above-mentioned incentive constraint. In such a case, so the argument goes, cross ownership will not serve as a credible commitment to reduce the profitability of price cutting, and will not facilitate tacit collusion. It is impossible, however, to make such a sale without signaling the intentions to price cut to the other firm, which will retaliate immediately, preventing the short term gain that could be made by price cutting. This would certainly be true in closely held corporations, or partnerships, where sales of ownership rights to third

There are a few notable conclusions to be drawn from proposition 2: First, in order to facilitate tacit collusion, partial ownership needs to be bilateral (though it need not be symmetric). It can easily be seen from (4) that if either x or y equals zero, the critical discount factor becomes 1/2, which brings us back to the ordinary supergame result with no cross ownership.[10] Second, if both x and y are close enough to 1/2, it can be seen from (4) that δ^c approaches zero. In such a case, tacit collusion will always be sustainable, for any positive discount factor prevailing in the industry. Thus, it is enough if each firm owns nearly half of the other firm for maximum collusion facilitation.

Finally, it can easily be seen from (4) that $\delta^c(x,y)$ monotonically decreases as x and y rise. The higher the cross ownership levels, the lower the critical discount factor, and thus the more tacit collusion is facilitated.

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parties cannot be kept secret, and moreover are often limited contractually. This point is also true, however, in the case of publicly traded corporations. It is hard to imagine that one of the firms can sell all (or most) of its shares in the open market without signaling the market and the issuing firm that a price cut is contemplated.

[10] The requirement that cross ownership be bilateral need not be present when firms are not completely identical. See section 3.2.

2.4 A two stage game--partial ownership as a strategic variable

Partial ownership is a decision variable on the part of the firms involved. As shown in the preceding analysis, cross ownership may be used to facilitate tacit collusion. Therefore, it would be natural and correct to treat partial acquisition as an action used strategically by oligopolists with the aim of facilitating tacit collusion. The strategic nature of cross ownership will be captured in the following two stage game. At the first stage, firms 1 and 2 simultaneously choose x and y respectively. The second stage consists of an infinitely repeated Bertrand price game as the one illustrated in section 2.3 above.

It will further be assumed that there is a cost $c_p(x)$ to partial acquisition of a competitor, where x is the level of partial ownership to be acquired and $\frac{\partial c_p(x)}{\partial x} \geq 0 \; \forall x$.[11] This assumption is useful when we consider partial ownership as a decision variable. It acknowledges that the decision to partially acquire a competitor may involve, in the real world, some kind of trade-off between the benefits from partial ownership in facilitating tacit collusion and the

[11] This cost could be explained by transaction costs, imperfect financial and capital markets causing non-positive returns on the investment, legal constraints, etc. It is assumed, for simplicity of exposition, that this cost function is identical for the two firms, though this assumption is not critical to the analysis.

costs of gaining partial ownership. In addition denote the price that the acquiring firm pays for its partial ownership right as T(x). In order to focus on the collusion-facilitating motivation for partial acquisition it is assumed that T(x) exactly equals the expected profit flow that the partial ownership brings. Thus, in the collusive state, $T(x)=\frac{1}{1-\delta}\frac{\pi^m}{2}x$.[12]

Partial ownership of a firm may either require or not require its consent. Clearly, if the firms are closely held, partial ownership in each of the firms would require the firm's consent.[13] In such a case, the first stage game is cooperative. On the other hand, if both firms are publicly traded corporations in which partial ownership could be achieved through purchases of shares in the open market, such partial ownership could be obtained unilaterally,[14]

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[12] In other words, it is assumed that capital markets function perfectly. This assumption simplifies the analysis to come. If we were to allow positive returns on such an investment (i.e., $T(x)<\frac{1}{1-\delta}\frac{\pi^m}{2}x$), such gains (in the noncooperative case to be discussed below) would constitute an additional motivation for partial acquisition beside collusion facilitation. The model could easily be modified, however, to capture the case of negative returns on the investment. In such a case, the cost function $c_p(x)$ would include such negative returns.

[13] This would also be the case in a publicly traded corporation if its controller possesses a sufficiently large block.

[14] A natural extension of this analysis would allow for delegation problems within the publicly traded corporation. Such an extension is, however, beyond the scope of this paper.

without the consent of the firm that was partially acquired. In such a case, the first stage game would be non-cooperative.[15] Let us examine these two cases separately.

2.4.1. Partial ownership of a firm requires its consent

The maximization problem facing the firms (acting cooperatively) is:

$$\left.\begin{array}{l} \max_{x,y} \dfrac{1}{1-\delta}\pi^{m} - c_p(x) - c_p(y) \\ s.t. \\ 1.\ \delta \geq \max\left[\dfrac{1}{2} - \dfrac{x}{2(1-y)}, \dfrac{1}{2} - \dfrac{y}{2(1-x)}\right] \\ 2.\ x, y < \dfrac{1}{2} \end{array}\right\} \quad if\ c_p(x^{*}) + c_p(y^{*}) < \frac{1}{1-\delta}\pi^{m}$$[16]

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[15] In practice there might be an intermediate case, in which one of the firms is publicly traded and the other is closely held. Under perfect information, however, this case is identical to the case where both firms are closely held. Consider the contract settling partial ownership of the closely held firm. It will factor in the anticipated unilateral acquisition of the publicly traded corporation by the closely held firm. Furthermore, since such unilateral action is perfectly observable and verifiable, this contract can even specify the level of partial ownership of the publicly traded corporation that will be pursued "unilaterally" by the closely held firm.

[16] The condition $c_p(x^{*}) + c_p(y^{*}) < \frac{1}{1-\delta}\pi^{m}$, means that the solutions to the maximization problem x^{*} and y^{*} will be chosen only if the costs of such partial acquisition are not prohibitively large (a similar notation of x^{*}, y^{*} will be used in the following sections as well). If this condition is not met, the firms will refrain from using cross ownership to enable tacit collusion and prefer their (zero profit) static equilibrium.

(5)

and

$$x = y = o\} \quad if \ c_p(x^*) + c_p(y^*) \geq \frac{1}{1-\delta}\pi^m$$

The first constraint in (5) is the incentive (no undercutting) constraint required to sustain tacit collusion.[17] This constraint follows from proposition 2. The second constraint is due to the underlying assumption of this paper that partial ownership of a competitor is passive and thus firms seek only a minority interest in one another.[18] The first term in the objective function ($\frac{\pi^m}{1-\delta}$) is independent of the choice variables x and y. This is not surprising since the firms are maximizing joint profits. Therefore, the internal split of industry profits corresponding to x and y is irrelevant to the firm's joint

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17 It is assumed here that lowering δ^c even below δ involves additional costs and no benefits (since $\delta^c=\delta$ is enough to sustain tacit collusion). Following the work of Bernheim and Whinston (1990), however, if firms 1 and 2 have parent or sister firms that meet in other markets, considerations of multi-market contact may change this assumption. The firms may then choose more cross ownership than is needed to sustain tacit collusion in their own market in order to help relax the incentive (no undercutting) constraints in other markets.

18 If there is an additional upper bound on x and y due to antitrust scrutiny, this would add an additional constraint according to which $x,y \leq A_0$, where $A_0<1/2$ is the maximum partial acquisition level that would not be deterred due to fear of antitrust intervention. The level of A_0, however, according to existing U.S. antitrust law, is quite close (if not equal) to 1/2 when partial ownership is assured to be completely passive.

decision. Accordingly, the maximization problem boils down to satisfying the constraints with minimum costs of partial acquisition. The firms will not elect more cross ownership than is needed to sustain tacit collusion.

2.4.2 Partial ownership of a firm does not require its consent

Here the first stage game is, as already mentioned, noncooperative. Accordingly, assume that in this first stage game each of the two firms simultaneously (and noncooperatively) chooses its partial ownership level in the other firm. The maximization problem facing firm 1 (an identical analysis would follow, due to symmetry, for firm 2) is:

$$\left.\begin{array}{l} \max_x \dfrac{1}{1-\delta}\dfrac{\pi^m}{2} - c_p(x) \\ s.t \\ 1.\ \delta \geq \max\left[\dfrac{1}{2} - \dfrac{x}{2(1-y)}, \dfrac{1}{2} - \dfrac{y}{2(1-x)}\right] \\ 2.\ x, y < \dfrac{1}{2} \end{array}\right\} \quad if\ \frac{1}{1-\delta}\frac{\pi^m}{2} > c_p(x^*)$$[19]

[19] The objective function is derived as follows: Firm 1 earns $\frac{1}{1-\delta}\frac{\pi^m}{2}(1-y+x)+\frac{1}{1-\delta}\frac{\pi^m}{2}y-\frac{1}{1-\delta}\frac{\pi^m}{2}x-c_p(x)$. The second term is the consideration firm 1 gets from firm 2 for a portion y of firm 1's shares. The third term is the consideration firm

(6)

and

$$x = y = 0\} \quad if \frac{1}{1-\delta}\frac{\pi^m}{2} \le c_p(x^*)$$

The first term in firm 1's collusive objective function is independent of x and y. Therefore, as in the cooperative case, the maximization problem boils down to satisfaction of the constraints with minimum costs of partial acquisition, as long as the costs of doing so are not prohibitive. Limiting ourselves to pure strategy equilibria, the resulting equilibria are pairs x^* and y^* that satisfy:

$$\left.\begin{array}{l} \delta = \max\left[\frac{1}{2} - \frac{x^*}{2(1-y^*)}, \frac{1}{2} - \frac{y^*}{2(1-x^*)}\right] \\ s.t. \\ x^*, y^* < 1/2 \end{array}\right\} \quad if \frac{1}{1-\delta}\frac{\pi^m}{2} > \max\left[c_p(x^*), c_p(y^*)\right]$$

(7)

and

$$x = y = 0\} \quad if \frac{1}{1-\delta}\frac{\pi^m}{2} \le \max\left[c_p(x^*), c_p(y^*)\right].$$ [20] [21]

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1 pays to firm 2 for a portion x of firm 2's shares (see note 12).

[20] To see that these are equilibria, suppose that firm 2 chooses y^* as in (7). Denote x^* as the level of x which exactly satisfies the incentive constraint in (7). Firm 1 would not want to choose x so that $x<x^*$ because such an x would fail to satisfy the incentive constraint thereby making both firms earn zero profits. Firm 1 would find it

We can see that both in the cooperative and noncooperative cases, the firms do not choose more cross ownership than is needed to sustain tacit collusion. At one extreme, when $\delta \geq 1/2$ (i.e., where p^m is sustainable even without cross ownership), the incentive constraints in (5) and (6) are never binding, even for x=y=0. Here cost minimization would impose zero cross ownership. At the other extreme, we can see that the firms can (and will) use cross ownership to enable tacit collusion even for the lowest possible industry discount factors (i.e., δ close to zero), as long as the costs of obtaining such cross ownership aren't prohibitive.

From an industry point of view, the costs of cross ownership (if chosen) are worth their while for the firms since cross ownership enables monopoly profits. From the social point of view, however, the costs of cross ownership are a social waste (assuming all cross ownership achieves is tacit collusion over the monopoly price, which is in itself socially harmful). Therefore, the costs of obtaining cross ownership are, under these assumptions, an additional factor

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profitable to increase x up to x* unless the costs of doing so are prohibitive, in which case firm 1 would set x=0. Note that firm 1 wouldn't want to choose x so that $x>x^*$, because $x=x^*$ would be enough to sustain p^m and any larger x would just impose additional costs. Due to symmetry, an analogous analysis follows for firm 2.

21 x=y=0 (regardless of the cost of partial acquisition) is also a Nash equilibrium. This Nash equilibrium, however, is Pareto inferior to the equilibria described in (7).

working against partial acquisition from the social welfare perspective. On top of this social waste lies the obvious deadweight loss involved in the monopoly pricing that cross ownership enables.

Despite the similarities between the cooperative and noncooperative outcomes, there are some differences between them. First, as shown in Appendix B, costs of partial acquisition are prohibitive more often (i.e., for a lower cost function $c_p(x)$) in the noncooperative case than in the cooperative case. Intuitively, partial ownership in the noncooperative case constitutes a "public good". The firm partially acquiring a larger portion of its competitor (incurring larger costs) contributes more to enabling tacit collusion, but still receives only half the fruits of this contribution (namely, $\frac{1}{1-\delta}\frac{\pi^m}{2}$). Therefore, it is possible that costs are prohibitive in the noncooperative game while they would not be prohibitive had the game been cooperative. In such a case, the noncooperative game would involve underinvestment in cross ownership from an industry point of view. Second, the firms in the cooperative game choose the least costly combination of cross ownership levels that enable tacit collusion. In the noncooperative case, on the other hand, this combination is not necessarily the least costly one rather it is determined by the Nash game's outcome.

3. Bertrand model with cost asymmetries

The case of firms with different marginal costs raises interest in the context of partial acquisition because it introduces the possibility that even unilateral partial acquisition will facilitate tacit collusion. We shall focus here on unilateral partial acquisition of the high cost firm by the low cost firm. This case is of particular interest because in practice it seems that most instances of partial ownership are those in which a large oligopolist partially owns a smaller oligopolist. Since we can interpret a large market share, in appropriate cases, as stemming from a cost advantage, it is appealing to investigate the motivation a low cost firm has for partially acquiring a high cost firm. Moreover, the low cost firm is generally more "trigger happy" than the high cost firm in the context of tacit collusion.[22] Therefore, it is worthwhile focusing on partial acquisition by the low cost firm as a commitment by it to be less aggressive. This would keep the formal analysis tractable without considerable loss of explanatory value.

[22] This stems from the low cost firm's ability to earn positive profits even without tacit collusion as well as from the fact that the low cost firm has a lower monopoly price (see section 3.2).

It will be shown, in the analysis that follows, that partial ownership of the high cost firm by the low cost firm unambiguously facilitates tacit collusion. Farrel and Shapiro (1990) (examining a static Cournot model with cost asymmetries) find that the low cost (larger) firm will not find it profitable to acquire (as a passive investor) part of the high cost (smaller) firm. With regard to the prevailing phenomenon of large firms partially acquiring smaller firms, they conclude that "such purchases will be profitable only if [the larger firm] gains control over [the smaller firm's] actions." The analysis presented here suggests that even totally passive partial ownership (with no rights of control) of the higher cost (smaller) firm by the lower cost (larger) firm may be beneficial to both parties when repeated interaction and tacit collusion are introduced. Such partial acquisition will be unilaterally profitable to the low cost firm since it will serve as a credible commitment on its part to make deviations by it less profitable. This will prevent situations in which the high cost firm is induced to price cut solely out of fear that the low cost firm will price cut itself.

3.1 Static Equilibrium

Let us modify the duopoly model of section 2.1 by assuming that firm 1 and 2's constant unit costs are c_1 and c_2 respectively. Assume that firm 1 has a cost advantage over firm 2 (i.e., $c_1<c_2$). The firms' profit functions ($D(p)(p-c_i)$, $i=1,2$) are assumed to be concave for all p and c and are denoted by ϕ_i. Denote $p^m(c_i)$ as the monopoly price of firm i ($i=1,2$), that is, the price that maximizes ϕ_i. The low cost firm owns a portion x ($0\leq x<1/2$) of the high cost firm's profit flow. Let us first consider the static Bertrand-Nash equilibrium:

Proposition 3. The static Bertrand-Nash equilibrium with partial ownership of x by the low cost firm of the high cost firm is identical to the equilibrium without such partial ownership, i.e., firm 1 serves the whole market charging slightly below c_2 (if $c_2\leq p^m(c_1)$), and charging $p^m(c_1)$ (if $c_2>p^m(c_1)$).

Proof. See Appendix C.

The case in which $c_2>p^m(c_1)$ is of no interest in the current context since it is easily shown that in such a case firm 1 will always prefer static equilibrium upon collusion. Therefore, we shall concentrate on the case in which $c_2 \leq p^m(c_1)$.

3.2 Dynamic Equilibrium

Since firms with different marginal costs have different monopoly prices there is no obvious "focal price" that firms can collude upon. The firms must employ some sort of mechanism to overcome their conflict of interest regarding the collusive price. It is assumed that side payments are not possible (say, due to fear of antitrust intervention)[23] . The mechanism assumed to be employed is a "market sharing" mechanism, according to which firm 2 is guaranteed a profit of π^{2*} in $[0, \pi^{2m}]$, (where π^{2m} is the monopoly profit of firm 2) and both firms charge the same price. Accordingly, the firms choose market shares s_1 and s_2 (such that $s_1+s_2=1$).[24] Recall that we have assumed the

[23] In an accompanying paper I am working on, it will be shown that partial ownership may constitute a mechanism in itself to overcome the conflict of interest stemming from cost asymmetries. In the current paper, however, the focus is upon partial ownership as facilitating the sustainablity of the well known market sharing mechanism.

[24] The model and proofs presented in this section follow the analysis of the case with no partial ownership in Tirole

price firms pay for partial ownership equals the expected profit stemming from partial ownership (firm 1 thus pays $T(x)=xs_1\phi_2$ for partial ownership of firm 2). Therefore, partial ownership does not affect firm 2's guaranteed profit (π^{2*}) and the firms' resulting market shares in the market sharing scheme. The only thing partial ownership may change is the ability to sustain the market sharing scheme in repeated interaction, as will be shown below.

Denote p^* as the price stemming from an efficient market allocation among the two firms (where firm 2 was guaranteed a profit of π^{2*}, the corresponding market shares are s_i^* and the collusive profits generated by firm i's plant are ϕ_i^* i=1,2).[25] Assume that the two firms follow a trigger strategy similar to the one employed in section 2.3. Each firm charges p^* and produces $s_i^*D(p^*)$ (i=1,2) as long as both firms behaved in such a manner previously. If a firm has deviated in the past, both firms revert to the static Bertrand Nash equilibrium forever (in which firm 1 serves the whole market charging slightly below c_2).

In the case without partial ownership, it can be shown that tacit collusion is sustainable if and only if the two

(1988) 242, 271. See also Schmalensee (1987) 354 and Bishop (1960) 948.

[25] An efficient market allocation is one that maximizes firm 1's profit subject to the constraint imposed by firm 2's guaranteed profit. See Tirole (1988) 242, 271.

following conditions are met: 1) $s_1^* \phi_1^* > k$, where k is firm 1's profit in the static (one period) Bertrand equilibrium ($k \equiv D(c_2)(c_2 - c_1)$). This is the "preliminary condition" for tacit collusion to be sustainable;[26] and 2) $\delta \geq \frac{\pi^{1m}}{\pi^{1m} + \phi_1^* - k} \equiv \delta^c$, where π^{1m} is firm 1's monopoly profit ($\pi^{1m} \equiv \max_p \phi_1(p)$). This is "the condition concerning δ".[27] The preliminary condition and the condition concerning δ, acting together, define the set of pairs (k,δ) under which tacit collusion is sustainable.

Now let us consider the case in which firm 1 owns a portion x of firm 2. As stated in the following proposition, such partial ownership facilitates tacit collusion in the sense that it makes tacit collusion sustainable under a larger range of cost advantages and discount factors.

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26 The preliminary condition guarantees that firm 1's loss in deviating from the collusive price is positive. This condition may not be met when firm 1's cost advantage is so great that it would rather stick to the static Bertrand result (earning k) than collude, regardless of the industry's discount factor δ (see Tirole (1988) 272-273).

27 The condition concerning δ guarantees that there is a range in which a market sharing scheme is maintained while both firms' incentive (no undercutting) constraints are satisfied *(id.)*. Both the preliminary condition and the condition concerning δ offer some insight regarding the point made earlier about the low cost firm being more "trigger happy". Both these conditions depend in a direct way only on firm 1's characteristics, namely, its monopoly profit π^{1m}, its collusive profit ϕ_1^*, and its static equilibrium profit k.

Proposition 4. Under a market sharing scheme, if firm 1 partially owns a portion x of firm 2, tacit collusion is sustainable for a larger set of (k,δ) pairs.

Proof. If firm 1 owns x of firm 2, firm 1's incentive constraint is:

$$\pi^{1m} - s_1^*\phi_1^* - x(1-s_1^*)\phi_2^* \leq \frac{\delta}{1-\delta}\left[s_1^*\phi_1^* + x(1-s_1^*)\phi_2^* - k\right] \qquad (8)$$

The left hand side of (8) constitutes firm 1's short term gain from deviation. Its most profitable deviation is cutting the price from p^* to its monopoly price. The right hand side constitutes firm 1's long term loss from deviation. It would lose the future flow of its collusive profits and instead earn its static Bertrand profit forever.

First, firm 1's loss from deviation must be positive in order to satisfy the preliminary condition for sustainability. This condition is met for $s_1^*\phi_1^* + x(1-s_1^*)\phi_2^* > k$. Thus, the preliminary condition is more easily met (i.e., is met for larger levels of k) in the partial ownership case than in the case with no partial ownership (which corresponds to x=0). Since k correlates with firm 1's cost advantage ($k \equiv D(c_2)(c_2 - c_1)$), it follows that partial ownership enables tacit collusion to be sustainable under higher cost advantages.[28]

Now let us derive the critical discount factor for the partial ownership case. Given δ, (8) yields the following bottom threshold on s_1^*, denoted as $\underline{s}_1$:

$$\text{ICC}_1 \quad s_1^* \geq \frac{\pi^{1m} - \delta(\pi^{1m} - k) - \phi_2^* x}{\phi_1^* - \phi_2^* x} \equiv \underline{s}_1 \tag{9}$$

Firm 2's most profitable deviation is slightly undercutting p^* and serving the whole market. Thus firm 2's incentive constraint can be shown to be:

$$s_1^* \phi_2^* (1-x) \leq \frac{\delta}{1-\delta}\left(1-s_1^*\right)\phi_2^*(1-x) \tag{10}$$

Hence partial ownership of firm 2 does not influence its incentive constraint. Given δ, this yields an upper bound for s_1^*:

28 Note that even with partial ownership, however, the preliminary condition may not be met for large enough levels of k. For $k \geq s_1^* \phi_1^* + \frac{1}{2}(1-s_1^*)\phi_2^*$, even x=1/2 will not suffice to sustain tacit collusion.

ICC$_2$ $s_1^* \leq \delta$ (11)

Both incentive constraints will be met if and only if:

$$\underline{s}_1 \leq s_1^* \leq \delta \tag{12}$$

The condition for such a range for s_1^* to exist (and for tacit collusion to be sustainable) is thus $\delta > \underline{s}_1$, which, using (9), yields:

$$\delta \geq \frac{\pi^{1m} - \phi_2^* x}{\pi^{1m} + \phi_1^* - k - \phi_2^* x} \equiv \hat{\delta}^c \tag{13}$$

It is easily shown that $\delta^c > \hat{\delta}^c$ (i.e., partial ownership lowers the critical discount factor) if and only if $\phi_1^* > k$. If $k \geq \phi_1^*$, it is easily seen that the preliminary condition will never be met, either with or without partial ownership, regardless of the level of δ. Accordingly, whenever the level of δ matters at all, it is always true that $\phi_1^* > k$ and $\delta^c > \hat{\delta}^c$. Thus, partial ownership enables tacit collusion for a larger set of (k, δ) pairs. *Q.E.D.*

The latter part of the above-mentioned proof implies that the degree by which $\delta^c > \hat{\delta}^c$ is greater for a greater margin between ϕ_1^* and k. This means that the smaller is firm 1's cost advantage (which implies a smaller k) and the larger are the profits produced by firm 1's plant under tacit collusion (ϕ_1^*), the more tacit collusion will be facilitated by partial ownership. The fact that a higher ϕ_1^* yields higher effectiveness of partial ownership as a collusion facilitator has an important implication. It can be easily shown that ϕ_1^* is higher for lower levels of π^{2*} (firm 2's guaranteed profit).[29] Therefore, the firms are able to make partial ownership a more effective tool in facilitating tacit collusion by lowering firm 2's guaranteed profit.

In the symmetric case, we observed that the critical discount factor decreased monotonically with partial ownership levels. The following proposition reveals that this result carries over to the case with cost asymmetries discussed in the current section. This property exists both with regard to the preliminary condition for tacit collusion to be sustainable and the condition concerning δ.

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[29] See Tirole (1988) 271.

Proposition 5. (A) The preliminary condition $s_1^*\phi_1^* + x(1-s_1^*)\phi_2^* > k$ is more easily met for higher levels of x. (B) Whenever $\phi_1^*>k$ (i.e., whenever δ matters) $\hat{\delta}^c(x)$ decreases monotonically with x.

Proof. Part (A) is easily seen from the fact that $(1-s_1^*)\phi_2^* > 0$.

To prove part (B) of the proposition, it can be shown that the derivative of $\hat{\delta}^c(x)$ with respect to x is negative when $\phi_1^*>k$. As mentioned above, if $k\geq\phi_1^*$, the preliminary condition will not be met either with or without partial ownership, regardless of the level of δ. Accordingly, in all relevant cases (i.e., cases in which δ matters at all) $\hat{\delta}^c(x)$ monotonically decreases with x. *Q.E.D.*

One property that does not carry over from the symmetric case to the case with cost asymmetries is the ability to lower the critical discount factor all the way to zero. Taking account of the constraint that x<1/2, it is easily seen that the lowest possible level of $\hat{\delta}^c(x)$ (denoted $\min(\hat{\delta}^c)$ is given by:

$$\lim \hat{\delta}^{c}_{x \to \frac{1}{2}} = \frac{\pi^{1m} - 1/2\phi_2^{*}}{\pi^{1m} + \phi_1^{*} - k - 1/2\phi_2^{*}} \equiv \min(\hat{\delta}^{c}) \qquad (14)$$

3.3 A two stage game--partial acquisition by firm 1 as a decision variable

Let us now examine partial acquisition of firm 2 by firm 1 as a decision variable. At the first stage of the game firm 1's partial ownership level of firm 2 is determined. The next stage consists of an infinitely repeated price game.

Since the cooperative and noncooperative cases have similar results, they will be discussed jointly. In the cooperative case, the firms' joint (collusive) objective function is:

$$\pi_{1+2} \equiv s_1^{*}\phi_1^{*} + (1 - s_1^{*})\phi_2^{*} - c_p(x) \qquad (15)$$

In the noncooperative case, firm 1's collusive objective function is:

$$\pi_1 \equiv s_1^{*}\phi_1^{*} - c_p(x)$$ [30] (16)

Both the cooperative and noncooperative collusive objective functions are decreasing in x. The firms will thus choose only the smallest level of x that still enables tacit collusion (subject to the constraint that x<1/2). This will occur unless the costs of partial acquisition for firm 1 are "prohibitive" (as formalized below), in which case the firms will choose x=0. Similarly, x=0 will be chosen if x is not needed, or unable, to sustain tacit collusion, since it would then entail costs and no benefits.

Consider first the case in which x is not needed to satisfy the preliminary condition, but is needed, and able, to satisfy the condition concerning δ.[31] Here the choice of x, both in the cooperative and noncooperative cases, will be the following:[32]

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[30] Note that costs are prohibitive more often (i.e., for lower cost functions $c_p(x)$) in the noncooperative case. This stems from the fact that $\pi_1<\pi_{1+2}$ (see expressions (15),(16)). The intuition is analogous to the symmetric case, as discussed in section 2.4.

[31] Formally this is the case in which $s_1^{*}\phi_1^{*}>k$ but $\min(\delta^{c\prime}(x))\leq\delta<\delta^{c}$.

[32] The condition for costs to be prohibitive in the cooperative case is noted first while the parallel condition for the noncooperative case is given to the right in brackets.

$$x = \begin{cases} x^*: \ \delta = \hat{\delta}^c(x^*) \ \ if \ \pi_{1+2}(x^*) - k > 0 \ \ [\pi_1(x^*) - k > 0] \\ 0 \ \ if \ \ \pi_{1+2}(x^*) - k \leq 0 \ \ [\pi_1(x^*) - k \leq 0] \end{cases} \qquad (17)$$

Consider now the case in which x is not needed to satisfy the condition concerning δ but is needed (and able) to satisfy the preliminary condition.[33] In such a case, the choice of x will be as follows:

$$x = \begin{cases} x^*: \ k = s_1^*\phi_1^* + x^*(1-s_1^*)\phi_2^* - \varepsilon \ \ if \ \pi_{1+2}(x^*) - k > 0 \ \ [\pi_1(x^*) - k > 0] \\ 0 \ \ if \ \ \pi_{1+2}(x^*) - k \leq 0 \ \ [\pi_1(x^*) - k \leq 0] \end{cases} \qquad (18)$$

Where ε is positive and arbitrarily small.

Next let us consider the case in which x is needed and able to satisfy both the preliminary condition and the condition concerning δ. Here, again, the firms will choose the smallest x that enables tacit collusion. This would be the partial ownership level that satisfies the more strict condition. Denote x_1^* as the partial ownership level satisfying $k = s_1^*\phi_1^* + x^*(1-s_1^*)\phi_2^* - \varepsilon$, and x_2^* as the partial ownership level satisfying $\delta = \hat{\delta}^c(x_2^*)$. From the monotonicity result of proposition 5, $\max(x_1^*, x_2^*)$ would solve the more strict condition. Accordingly, the firms will choose:

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33 Here $\delta \geq \delta^c$ but $s_1^*\phi_1^* \leq k < s_1^*\phi_1^* + \frac{1}{2}(1-s_1^*)\phi_2^*$.

$$x = \begin{cases} Max(x_1^*; x_2^*) \equiv x^* & if\ \pi_{1+2}(x^*) - k > 0 \quad [\pi_1(x^*) - k > 0] \\ 0 \quad if & \pi_{1+2}(x^*) - k \le 0 \quad [\pi_1(x^*) - k \le 0] \end{cases} \tag{19}$$

Finally, in the case where partial ownership is not needed,[34] or unable,[35] to sustain tacit collusion, as stated, the firms will choose x=0.

To sum up the results illustrated above we can see that both in the cooperative and noncooperative games, if partial ownership is chosen it is to enable tacit collusion. At the first stage game firm 1 acquires part of firm 2 in order to commit itself not to deviate in the repeated price game to be played at the next stage. This will induce firm 2 to cooperate itself.

3.4 Welfare analysis

From a welfare perspective, in the cost asymmetries case, partial ownership may involve both deadweight loss and inefficiency in production. These inefficiencies result when partial ownership is used to enable tacit collusion. The deadweight loss is obvious and stems from enabling tacit collusion over a higher price than the static equilibrium

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[34] This would happen when $\delta \geq \delta^c$ and $s_1^* \phi_1^* > k$.

[35] Here $\min(\delta^{c\prime}(x)) > \delta$ or $k \ge s_1^* \phi_1^* + \frac{1}{2}(1 - s_1^*)\phi_2^*$.

price. The inefficiency in production stems from the worsened allocation of production that partial ownership involves when it enables tacit collusion: Without partial ownership, there would be no tacit collusion and the static one period equilibrium would prevail with the low cost firm serving the whole market. With partial ownership, the market sharing collusive mechanism is sustainable, and thus both the high cost and low cost firms share the market. To these welfare losses can be added the loss involved in the costs of partial acquisition itself.

4. Validity of the results in a Cournot framework

We shall now briefly inquire to what extent the results of section 2 (for symmetric firms) carry over to the Cournot context. The basic difference between the Cournot and Bertrand cases in our context is that in the Cournot case, unlike the Bertrand case, cross ownership changes static (one period) equilibrium. In particular, symmetric cross ownership, in the Cournot framework, was shown to raise static equilibrium price (and reduce equilibrium output) thereby raising the firms' profits in the one period (noncollusive) equilibrium.[36] Since punishments for deviations in the supergame model constitute reversions to the one period Cournot-Nash equilibrium, cross ownership has the effect of softening punishments in the Cournot case. As

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36 See Reynolds and Snapp (1985); Farrel and Shapiro (1990).

a result, the critical discount factor (δ^c) in the Cournot case is not necessarily a monotonically decreasing function of the cross ownership levels (x and y) (see Malueg (1992). This is the reason for Malueg's result that an increase in cross ownership may hinder (rather than facilitate) tacit collusion).

Nevertheless, the basic result obtained in this paper that cross ownership may be used as a strategic variable to facilitate tacit collusion carries over to the Cournot case as well. This can be understood once we acknowledge that partial ownership is a decision variable in the hands of the firms and not exogenously given. It may thus be used by them to facilitate tacit collusion as long as there exist cross ownership levels that facilitate tacit collusion (i.e., which lower δ^c). For this to be true $\delta^c(x,y)$ need not be monotonically decreasing in x and y. It is enough if $\delta^c(x,y)$ is not monotonically increasing. In other words, for cross ownership to be a possible strategic variable facilitating tacit collusion, it is enough to show that there is a range (however small it may be) in which the detrimental effect of partial ownership on the profitability of deviation dominates its softening effect on punishments.[37]

[37] As can be shown (for the case of symmetric cross ownership levels), this will occur as long as there exists a (symmetric) partial ownership level v (or a range of such partial ownership levels) in which: $\frac{\pi^n{}_v}{-\pi^d{}_v} < \frac{\pi^m - \pi^n(v)}{\pi^d(v) - \pi^m}$ where

Accordingly, even without extending the formal analysis of this paper to the Cournot case, the following should become clear: First, unlike the symmetric Bertrand result, cross ownership in the Cournot case will not always be able to lower δ^c to δ. When cross ownership is unable to sustain tacit collusion, in the cooperative case, it will still be chosen for a more profitable (and less competitive) static equilibrium. In the noncooperative case, on the other hand, if cross ownership is unable to sustain tacit collusion, it will not be chosen unilaterally by any of the firms. This is because with quantities as strategic substitutes, partial ownership of a competitor makes the competitor more aggressive and the acquirer less aggressive, which is not unilaterally profitable in static equilibrium. On the other hand, where cross ownership is needed, and able to sustain tacit collusion (i.e., δ^c can be brought down to δ), the firms, either cooperatively or noncooperatively (in Pareto optimal equilibria), will choose cross ownership, but no more than is needed to sustain tacit collusion. This would be true unless costs of such cross ownership are prohibitive, in which case x=y=0 will be chosen.

(following Malueg's (1992) notation) π^m denotes each firm's collusive profits; $\pi^n(v)$ denotes its profits in the one period Cournot outcome; $\pi^d(v)$ is a firm's short term gain from deviating; and the v subscript denotes partial derivatives with respect to v.

5. Conclusion

It was shown that oligopolists can use partial ownership as a strategic device to facilitate tacit collusion. This tool proves to be the strongest in the homogenous product price-setting context, where firms can (and will) always enable tacit collusion by choosing the appropriate cross ownership levels, unless the costs of such partial acquisitions are prohibitive. The same basic result carries over to the cost asymmetry case. It was shown that partial ownership by the low cost firm of the high cost firm may be used to facilitate tacit collusion under a market sharing scheme. Here (unlike the symmetric case) partial ownership is not always able to facilitate tacit collusion. Finally, in the Cournot framework, qualitatively similar results can be obtained.

Appendix A

Proof of proposition 1:

Suppose the equilibrium prices of the two firms, p_1^* and p_2^* satisfy $p_1^*>p_2^*>c$.[38] In such a case all of the demand would be supplied by firm 2. Firm 1's profit would be $xD(p_2^*)(p_2^*-c)$. If firm 1 price cuts by a small but positive ε, it will then supply total demand and earn almost $(1-y)D(p_2^*)(p_2^*-c)$. Therefore, firm 1 will price cut if and only if $x+y<1$.[39]

[38] This proof is similar to the one in Tirole (1988) 210 for the case without cross ownership.
[39] Due to the underlying assumption that $x,y<1/2$, this condition is met in all cases.

Suppose now that $p_1^*=p_2^*>c$. In such a case firm 1 makes a profit of $\frac{1}{2}(1-y+x)D(p_1^*)(p_1^*-c)$. If firm 1 price cuts by ε it gets almost $(1-y)D(p_1^*)(p_1^*-c)$. Thus it will price cut if and only if x+y<1, which is true by assumption (see note 39).

It was thus far proven that either firm 1 or firm 2 must charge the competitive price c (as long as x+y<1). Let us now show that they will both charge c. Suppose $p_1^*>p_2^*=c$. In such a case, firm 2, which serves the whole market, makes a profit of $(1-x)D(p_2^*)(p_2^*-c)=0$. But in such a case, firm 2 could raise its price above the competitive price slightly by ε and still serve total demand, making a higher profit of $(1-x)D(p_2^*+\varepsilon)(p_2^*+\varepsilon-c)$, and therefore this too cannot be an equilibrium. Accordingly, $p_1^*=p_2^*=c$ (as long as x+y<1). *Q.E.D.*

Appendix B

Comparison between the cooperative and non cooperative Bertrand symmetric cases as to how often costs are "prohibitive" (section 2.4):

In the cooperative case, costs of acquiring cross ownership levels x,y will not be "prohibitive" if and only if:

$$c_p(x)+c_p(y)<\frac{1}{1-\delta}\pi^m \tag{20}$$

In the noncooperative case costs are not prohibitive if and only if:

$$\max\left[c_p(x),c_p(y)\right]<\frac{1}{1-\delta}\frac{\pi^m}{2} \text{ [40]} \tag{21}$$

Since $2\max\left[c_p(x),c_p(y)\right]\geq c_p(x)+c_p(y)$, it is clear that if (21) is satisfied, then (20) is satisfied as well (i.e., whenever costs are not prohibitive in the noncooperative case, they are also not prohibitive in the cooperative case. Conversely, where $c_p(x)+c_p(y)<\frac{1}{1-\delta}\pi^m<2\max\left[c_p(x),c_p(y)\right]$, costs will be prohibitive in the noncooperative case while they would not be prohibitive in the cooperative case. Therefore, costs are prohibitive more often in the noncooperative case than they are in the cooperative case.

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40 In this proof it is assumed that the same pair (x,y) prevails both in the cooperative and noncooperative equilibrium. This is, of course, a simplification, since in the noncooperative case joint costs of partial acquisition are minimized whereas they are not necessarily minimized in the noncooperative case. This point only strengthens the conclusion obtained in the proof that costs are prohibitive more often in the noncooperative case.

Appendix C

Proof of proposition 3:

Denote p_i^* as the price charged by firm i (i=1,2) and $\phi_i(p_i^*)$ as the profit generated by firm i's plant. Assume $p_1^*>p_2^*>c_2$.[41] In such a case, firm 2 serves the whole market and firm 1's profit is $x\phi_2(p_2^*)$. If firm 1 price cuts p_2^* by a small ε (ε>0), it captures the whole market and makes a profit of almost, $\phi_1(p_2^*)$, which is larger than $x\phi_2(p_2^*)$ for small enough ε. Thus $p_1^*>p_2^*>c_2$ is not an equilibrium.

Furthermore, assuming firm 2 does not play a dominated strategy, we know that $p_2 \geq c_2$.

Assume now that $p_1^*=p_2^*>c_2$. In such a case, firm 1 earns $(1/2)\phi_1(p_1^*)+(1/2)x\phi_2(p_1^*)$. If firm 1 deviates by a small and positive ε it will earn almost $\phi_1(p_1^*)$, which is larger for a small enough ε. Thus $p_1^*=p_2^*>c_2$ too is not an equilibrium.

Finally, assume that $p_1^*=p_2^*=c_2$. Here the firms share the market and firm 1 earns $\frac{1}{2}D(c_2)(c_2-c_1)$. If firm 1 charges ε below c_2, however, it will serve the whole market and earn almost $D(c_2)(c_2-c_1)$, which is clearly larger.

Therefore, if $p^m(c_1)>c_2$ the equilibrium consists of firm 1 charging slightly below c_2 and serving the whole market. The same reasoning implies that if $p^m(c_1)<c_2$ firm 1 will charge $p^m(c_1)$, since, by definition of the monopoly price, it brings firm 1 profits exceeding $D(c_2)(c_2-c_1)$. This result is identical to the case with no partial ownership. *Q.E.D.*

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[41] This proof resembles the proof in Tirole (1988) 211, 234 for the case without partial ownership.

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