PROPERTY RIGHTS AND THE NATURE OF THE FIRM

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Abstract

The paper provides a framework for addressing the question of when transactions should be carried out within the firm and when through the market. Following Grossman and Hart (1986), we identify a firm with the assets which its owners control: ownership confers the right to decide how the assets are to be used in events unspecified in an initial contract. We study how changes in ownership affect non-owners of assets, such as employees. We show that a party 1 will put more weight on a party 2's objectives if 1 is an employee of 2 (i.e., 1 works with assets that 2 owns), than if 1 is an employee of some other party 3, or if 1 is an independent contractor providing 2 with a service. We use these ideas to give a partial characterization of an optimal control structure. Our framework is broad enough to encompass more general control structures than simple ownership: for example, partnerships, and worker and consumer cooperatives, all emerge as special cases.

Our principal findings are, first, that assets should be owned or controlled together if they are (strongly) complementary; second, that ownership should be given to agents who are indispensable even though they may not have important (uncontractible) actions; third, that concentration of ownership through integration improves the incentives of the acquiring firm's employees, but has an ambiguous effect on employees in the acquired firm; and fourth, that when an asset is crucial to several firms, it may be optimal for the firms to share control of the asset via majority rule.

1. INTRODUCTION

What is a firm? How do transactions within a firm differ from those between firms? These questions, first raised by Coase over fifty years ago, have been the subject of much discussion by economists, but satisfactory answers to them, particularly at the level of formal modeling, have still to be provided. The present paper is an attempt to fill the gap. Extending the work of Grossman and Hart (1986), it explores the idea that a firm is a collection of assets, where the owner (or owners) of these assets has the right to use the assets in uncontracted-for contingencies. The key difference between transactions within firms and those across firms is that in the former case the major assets are in the hands of one person or group, whereas in the latter case they are dispersed among many people or groups. We argue that party 1 will put more weight on party 2's objectives if 1 is an employee of 2 than if 1 is an employee of some other party 3, or if 1 is an independent contractor providing 2 with a service. We use these ideas to analyze the optimal assignment of assets to owners and the determinants of the extent of the firm.

Our analysis is consistent with and builds on the ideas developed by Coase (1937), Williamson (1975, 1985) and Klein, Crawford and Alchian (1978).[1] Coase argued in his 1937 paper that the firm is a response to transaction costs and arises in order to economize on the haggling and recontracting costs that occur between independent parties in a long-term relationship. Williamson (1975), and Klein, Crawford and Alchian refined this idea by emphasizing that firms will be important in situations where parties must make large specific investments and where, because of the impossibility of writing detailed long-term contracts, the quasi-rents from these investment cannot be divided up appropriately in advance. Integration is seen as a way of reducing the opportunistic behavior and hold-up problems that can arise in such circumstances. The idea is that, if party 2 brings party 1 into his firm, party 1 will have both less ability and less desire to hold up party 2, and as a result, ex-post inefficiency will be reduced and the incentive to invest

[1]For reviews of this literature and further references, see Joskow (1985), Holmstrom and Tirole (1988) and Hart (1988).

ex-ante will increase.[2]

The Coase-Williamson-Klein-Crawford-Alchian approach is clearer on the benefits of integration than its costs. In Grossman and Hart (1986), the approach is modified so as to provide a unified framework for understanding the costs and benefits of integration. Ownership of a physical asset is taken to provide the owner with residual rights of control over that asset, i.e. the owner has the right to do whatever he likes with the asset except to the extent that particular rights have been given away via an initial contract. Transferring ownership of an asset from party 1 to party 2 will increase 2's freedom of action to use the asset as he sees fit, and will therefore increase his share of ex-post surplus and his ex-ante incentive to invest in the relationship. At the same time, however, 1's share of ex-post surplus and incentive to invest falls. Hence concentrating ownership in 2's hands will be good to the extent that 2's investment decision is important relative to 1's, but bad if the opposite is the case. In this way, the costs and benefits of integration can be understood as two sides of the same coin.

The Grossman-Hart analysis is restrictive in several respects. First, it focuses on control over physical assets rather than human assets, and, secondly, it views the costs and benefits of integration solely in terms of the incentive effects on top management. In this paper, both these conditions are relaxed. First, we allow for the possibility that an asset is worked on by several people, some of whom (employers) have ownership rights and others of whom (employees) do not. A major part of our analysis will be concerned with how employees' incentives change as integration occurs, i.e. as asset ownership becomes more or less concentrated. Secondly, while physical assets will continue to play a role in our analysis, it will be a more limited one than in Grossman-Hart. We will suppose that the sole benefit accruing to the owner of an asset comes from his ability to exclude others from the use of that asset. That is, the owner of a machine can decide who can and who cannot work on that machine in the future.[3] We will see that control over a physical asset in this sense can lead indirectly to control over human assets. For

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[2] He is a shorthand for he/she in this paper.

[3] The notion that the boss of a firm can exclude employees from access to the firm's assets may be found in Alchian and Demsetz (1972).

example, if a group of workers requires the use of an asset to be productive -- this asset could be a machine or inventories or a list of clients or even just a location -- then the fact that the owner has the power to exclude some or all of these workers from the asset later on (i.e. he can fire them) will cause the workers to act partially in the owner's interest. That is, this view of the firm as a collection of physical assets leads to the intuitive conclusion that an employer 2 will have more "control" over party 1 if 1 is an employee than if he is an independent contractor.

Our approach will be sufficiently general to allow for the possibility of multiple ownership of assets. This means that ownership by workers (worker cooperatives) or by consumers (consumer cooperatives) will arise quite naturally as possible outcomes as well as more traditional ownership patterns. In fact one of the benefits of our analysis is that it provides an indication as to when such nonstandard arrangements are likely to be optimal.

We will use the following model to formalize these ideas. We consider a situation where agents take actions today which affect their (actual or perceived) productivity or value tomorrow. These actions might represent an investment in human capital; participation in on the job training; or a signaling activity. For example, an agent may learn how to be a good production line worker or sales manager or corporate lawyer tomorrow by being one today, or by learning certain skills today. Or, by taking a particular action today, an agent may signal his type, e.g. that he is able or that his cost of effort is low or that he's hard-working or loyal. We also suppose that it is costly for agents to write detailed long-term contracts which specify precisely current and future actions as a function of every possible eventuality; and that, as a result, the contracts that are written are incomplete and will be subject to renegotiation later on. Finally, we suppose that at least some of the actions taken today "pay off" in the future only if the agents have access to particular assets and/or can trade with particular people, i.e., some of skill or productivity acquisition is asset-specific and/or person-specific (this means that there are lock-in effects).[4]

[4]As in Becker (1964).

These assumptions have the following implications. First, the incompleteness of contracts means that the future return on an individual's current action will depend on his "marketability" or bargaining position tomorrow in ways which cannot be controlled via the original contract. Secondly, an agent's marketability or bargaining position will depend on which assets he has access to and hence will be sensitive to the allocation of asset ownership. As a result, an agent's actions will depend not only on whether he owns a particular asset, but, in the event he does not own it, on who does.

This last point can be illustrated by an example. Suppose that two agents 1 and 2 will together provide a service to agent 3 at date 1. Assume further that to do this 1 and 2 will require access to a particular machine, and that at cost 100 to himself 1 can take an action at date 0 (e.g. work hard or acquire a skill), which will reduce 2's date 1 costs by 101 and will raise 3's date 1 benefits by 102 (all costs and benefits are measured in date 1 dollars). Suppose in addition that if 3 does not take delivery of the service tomorrow, it can be sold on the spot market; moreover, while the spot price will be lower, it will fully reflect 1's past action (i.e. the spot price will be 102 dollars higher if 1 acted at date 0 than if he did not). In contrast, assume 2 is indispensable; i.e. if he refuses to supply his part of the service tomorrow, 1 and 3 will have no gainful trade. Finally, suppose that transaction costs prevent the writing of any long-term contract at date 0.

We have set things up so that it is socially efficient that 1 act at date 0. However, if 3 owns the asset, 1 will not act. The reason is that, looking ahead to date 1, 1 will recognize that in order to realize the gains from his date 0 action he needs to reach agreement with 2 and 3: 3 because 1 needs access to the machine he owns, and 2 because without 2's cooperation 1's service is useless. Assuming that the date 1 gains from trade are split three ways, this means that 1 will receive a return of 1/3 (101+102) on his date 0 action, which does not cover the initial cost of 100.

In contrast, if 2 owns the asset, 1 <u>will</u> act. The reason is that in this case 1 needs only the cooperation of 2 at date 0 to realize the gain from his investment (since the 102 dollar increase of 3's benefit can also be realized if 1 and 2 sell on the spot market). Assuming a two-way split, 1's return will now be 1/2 (101+102) > 100.

This example captures the idea that 1 is more likely to take an action which is specific to 2 -- or, to put it another way, is specifically in 2's interest -- if 2 is his boss (i.e. 2 owns the asset that he is productive with) than if 3 is his boss. (The action is specific to 2 in the sense that the cost saving which it achieves cannot be realized without 2's participation.) The same idea would lead to the conclusion that 3 rather 2 should own the asset if the cost savings of 101 could instead be earned even if 2 were replaced, but the benefit increase of 102 were specific to 3. (Another possibility is that 1 should own the asset. This may not be optimal, however, if 2 and 3 also have productivity-enhancing actions to take.)

We can modify this example to illustrate a point that we will make much of in the paper: assets that are used together should be owned together. Suppose now that 1's action is not specific to either 2 or 3, but allow 2 and 3 also to take (non-specific) actions. Suppose that the machine has two pieces, the motor and the chassis, say. Neither piece is of any use without the other. Would it ever be optimal for 2 to own the motor (for example) and 3 to own the chassis?

The answer is no. The reason is that 1 would not act at date 0 if the pieces were separately owned, since he then would have to reach agreement with both 2 and 3 at date 1 in order to gain access to the entire machine. And as we have seen, a three-way split of his marginal product (101+102) is not enough to cover his costs (100).

It would be better to give the entire machine to either 2 or 3. Consider giving 2 the chassis as well as the engine. Then clearly his incentive to act will not fall (and in general will rise). Interestingly, 3 will be unaffected by the amalgamation of ownership. For when 3 had the chassis, he had to reach agreement with 2 (who owned the engine) anyway, so it makes no difference to him at the margin that 2 will now own both pieces: 3 gets 50% of his marginal product under either ownership arrangement. The important effect is on 1. Now he will only have to reach an agreement with 2, and a two-way split of his marginal product more than covers his costs.

In the above examples, the bargaining problem at date 1 is relatively simple -- either all three agents are required to obtain the gains from trade, or a particular coalition of two agents is sufficient. In general, more complicated situations will arise where partial gains from trade can be realized by subcoalitions of the grand coalition. We shall take a cooperative rather than a noncooperative approach to this bargaining problem, adopting Shapley value as our solution concept. We suspect, however, that the main ideas of the paper will hold true under a variety of other divisions of the surplus.

The paper is organized as follows. The formal model and our general results are laid out in Sections 2 and 3. In Section 4 we apply and develop these results in some special cases. Section 5 extends our general model to situations in which the agents do not make investments *per se* at date 0; we will employ a version of Holmstrom's (1982) reputation model for part of this analysis. Section 6 contains concluding remarks. There are two Appendices: the first provides a noncooperative justification for the use of Shapley value, and the second gives a number of the proofs of the Propositions.

2. THE MODEL

We will consider a part of the economy comprising a set $\underline{S}$ of I risk neutral individuals or agents $i = 1,\ldots,I$ and a set $\underline{A}$ of N assets $(a_1,\ldots,a_n,\ldots,a_N)$. There are two periods, dates 0 and 1. All costs and benefits are measured in date 1 dollars.

At date 0, each agent i takes an action x_i. This action affects the agent's productivity or value at date 1. As explained in the Introduction, this action might represent an investment in human capital, on the job training, or participation in an activity which increases perceived rather than actual productivity in the future.

For ease of exposition we shall adopt the first interpretation: we take x_i to be a pure investment in human capital. In Section 5, however, we will show that the analysis can be extended to include some of the other interpretations. As a further simplification, we confine attention to the case where an agent chooses only how much to invest (or under the other interpretations what level of service to provide). That is, we suppose that x_i is a scalar lying in $[0, \bar{x}_i]$, where $\bar{x}_i \geq 0$. Additional issues arise if an agent can also choose what type of investment to make (or type of service to provide); we discuss these briefly in the Conclusion.

Denote the cost to agent i of action x_i by $C_i(x_i)$. We assume:

<u>Assumption A1</u> $C_i(x_i) \geq 0$ and $C_i(0) = 0$.

C_i is twice differentiable.

If $\bar{x}_i > 0$, then $C_i'(x_i) > 0$ and $C_i''(x_i) > 0$ for $x_i \in (0,\bar{x}_i)$, with $\lim_{x_i \to 0} C_i'(x_i) = 0$ and $\lim_{x_i \to \bar{x}_i} C_i'(x_i) = \infty$.

At date 1, production and trade occur. We suppose that x_i is too complicated to be specified in a date 0 contract and that transaction costs (e.g. uncertainty about the future) also prevent the inclusion of plans about future production and trade in such a contract. Hence production occurs de novo at date 1: a new contract must be written to consummate the gains from trade. Although x_i is complex, we assume that it is observable to other agents at date 1, so that this date 1 contract is negotiated under symmetric information (each agent's date 1 productivity or value is known to other agents).[5]

Let $x = (x_1, \ldots, x_I)$. Consider a coalition S of the agents who control a subset A of the assets. We shall suppose that the absence of a date 0 contract means that this coalition can use the assets in A as they like; in particular, they can exclude all agents outside S from access to them. Denote by $v(S,A|x)$ the (dollar) value that the coalition can generate through efficient trades among themselves at date 1, where we are supposing that each agent's costs and benefits are measured in money terms. Such trades may be quite complex; e.g. agents i and j may have to work with asset a_n, sell the item they produce to agent k who works on it some more and sells it to agent ℓ. The point is that we assume that such trades can be agreed to at date 1 even though they could not be planned ahead of time at date 0. For example, it may be impossible for agent j to contract with i at date 0 about the skill level that i should acquire, but it may be quite possible later on for j to hire i, knowing the skill level that i selected. Note that in a multi-period model, agents would acquire further skills at date 1 for use at later dates. In a two period context, however, we can ignore this possibility.

In coalition S, agent i's marginal return on investment is given by

$$\frac{\partial}{\partial x_i} v(S,A|x) \equiv v^i(S,A|x), \text{ say.}$$

For each i, S, A and x we assume:

<u>Assumption A2</u> $\quad v(S,A|x) \geq 0$; and $v(\phi,A|x) = 0$ where ϕ is the empty set.

$v(S,A|x)$ is twice differentiable in x.

If $\bar{x}_i > 0$, then $v^i(S,A|x) \geq 0$ for $x_i \in (0,\bar{x}_i)$.

$v(S,A|x)$ is concave in x.

<u>Assumption A3</u> $\quad v^i(S,A|x) = 0 \quad$ if $i \notin S$.

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[5]For further discussion of these assumptions, see Grossman and Hart (1986).

Assumption A4 $\quad \frac{\partial}{\partial x_j} v^i(S,A|x) \geq 0$ for all $j \neq i$.

Assumption A5 $\quad$ For all subsets $S' \subseteq S$, $A' \subseteq A$,

$$v(S,A|x) \geq v(S',A'|x) + v(S\backslash S',A\backslash A'|x).$$

Assumption A6 $\quad$ For all subsets $S' \subseteq S$, $A' \subseteq A$,

$$v^i(S,A|x) \geq v^i(S',A'|x).$$

(A3) says that an agent's marginal investment only affects the value of coalitions of which he is member. It captures the idea that an agent's investment enhances his own productivity rather than that of any asset he works with. (A4) says that investments are complementary at the margin. (A5) is a natural superadditivity assumption: a coalition could always divide if the values of the partition added up to more than the value of the whole. (A6) is stronger than the others; it says that marginal return on investment increases with the number of other agents and assets in the coalition (that is, marginal and total values are positively correlated).

Note that in writing the value of the coalition S as $v(S,A|x)$, we are allowing for the possibility that S recruits additional members for the coalition from outside the agents $\underline{S}\backslash S$; for example, S could hire from the date 1 spot market. Such new members will typically be less productive than the agents $\underline{S}\backslash S$ whom they are replacing -- they won't have acquired the relevant asset-specific or person-specific skills, or come with the appropriate assets -- and it is for this reason that we expect <u>strict</u> superadditivity to hold: $v(\underline{S},\underline{A}|x) > v(S,A|x) + v(\underline{S}\backslash S,\underline{A}\backslash A|x)$.

Superadditivity implies that the maximum total value at date 1 is given by $v(\underline{S},\underline{A}|x) \equiv V(x)$, say. The first-best overall social surplus is then

$$\underset{x}{\text{Maximum}} \quad W(x) \equiv V(x) - \sum_{i=1}^{I} C_i(x_i).$$

Let the maximum be attained at $x = x^*$. By assumptions (A1) and (A2), x^* is unique, and is characterized by the first order conditions

$$\frac{\partial}{\partial x_i} V(x) \Big|_{x=x^*} = v^i(\underline{S},\underline{A}|x^*) = C_i'(x_i^*) \quad \text{for all } i. \qquad (2.1)$$

We shall be interested in a noncooperative situation where agent i chooses x_i at date 0, anticipating that at date 1 the value $V(x)$ will be divided among the I agents according to their Shapley values.[6] In order to compute each agent's payoff we first need to know who controls which assets.

The Control Structure

We represent the control structure by a mapping α from the set of subsets of $\underline{S}$ to the set of subsets of $\underline{A}$ -- where $\alpha(S)$ is the subset of the assets $\{a_1, \ldots, a_N\}$ that the coalition S <u>controls</u>. For any partition $S \cup (\underline{S} \setminus S)$ of the agents $\underline{S}$, each of the assets is controlled by at most one of the subsets S, $(\underline{S} \setminus S)$. Hence,

$$\alpha(S) \cap \alpha(\underline{S} \setminus S) \quad = \quad \phi. \tag{B1}$$

Also, the assets controlled by any subset S′ of a coalition S must also be controlled by the whole coalition:

$$\alpha(S') \quad \subseteq \quad \alpha(S). \tag{B2}$$

Finally,
$$\alpha(\phi) \quad = \quad \phi. \tag{B3}$$

<u>Definition</u> A (deterministic) <u>control structure</u> is a mapping α from the set of subsets of $\underline{S}$ to the set of subsets of $\underline{A}$ satisfying (B1)-(B3).

Examples of control structures are where one person i owns asset a_n (i.e., $a_n \in \alpha(S) \iff i \in S$); or person i has a share (or vote) $\sigma_n(i)$ in asset a_n and majority rule applies (i.e., $a_n \in \alpha(S) \iff \sum_{i \in S} \sigma_n(i) > .5$). Other arrangements are possible, however. In the next Section, we shall want to consider <u>stochastic</u> control structures: a stochastic control structure is simply a random mapping $\tilde{\alpha}$, whose realization α is a deterministic control structure (for example, each individual could be the sole owner of asset a_n

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[6]See Shapley (1953). For a recent summary of the Shapley value literature and further references, see S. Hart (1988). For examples of applications of Shapley value, see Aumann and Kurz (1977) and Rydqvist (1987).

with probability (1/I)).

Given a (deterministic) control structure α, the date 1 value of a coalition S is $v(S,\alpha(S)|x)$. As noted above, we suppose that agent i's share of V(x) is given by his Shapley value

$$B_i(\alpha|x) \equiv \sum_{S|i\in S} p(S) \; [\; v(S,\alpha(S)|x) - v(S\setminus\{i\},\alpha(S\setminus\{i\})|x) \;],$$

where

$$p(S) = \frac{(s-1)!\;(I-s)!}{I!} \quad \text{and } s = |S|, \text{ the number of agents in S.}$$

In words, Shapley value gives agent i his expected contribution to a coalition, where the expectation is taken over all coalitions to which i might belong. In particular, we can imagine that the agents $\underline{S}$ are ordered randomly, with each ordering being equally likely. If agent i is placed s^{th} from the end, followed by the other members of coalition S, we say that i belongs to coalition S (this happens with probability p(S)); agent i's contribution to the coalition is then given by the difference $[v(S,\alpha(S)|x) - v(S\setminus\{i\},\alpha(S\setminus\{i\})|x)]$ and Shapley value is simply the expectation of this over all random orderings. Note that saying that i belongs to a particular coalition is a manner of speaking; the statement should not be taken literally since Shapley value is predicated on the idea that the grand coalition $\underline{S}$ forms and distributes surplus efficiently.

In Appendix A, we give a brief noncooperative justification for the use of Shapley value. Suffice it to say here that we believe that none of our results depends on this precise allocation of total value V(x) at date 1; all that is needed is that the division reasonably reflects the bargaining power (i.e., value) of all the potential coalitions. Of course the assumption that there is costless bargaining at date 1 leading to an ex-post efficient allocation is itself very strong. We discuss this assumption further in the Conclusion.

At date 1, agent i chooses x_i to maximize $B_i(\alpha|x) - C_i(x_i)$. From assumptions (A1) - (A3), the Nash equilibrium $x^e(\alpha)$, say, for x is characterized by the first order conditions

$$\frac{\partial}{\partial x_i} B_i(\alpha|x) \Big|_{x=x^e(\alpha)} = \sum_{S|i\in S} p(S)\, v^i[S,\alpha(S)|x^e(\alpha)]$$

$$= C_i'(x_i^e(\alpha)) \quad \text{for all } i. \tag{2.2}$$

From assumption (A6) and condition (B2), we know that the LHS of (2.2) is less than or equal to

$$\sum_{S|i\in S} p(S)\, v^i(\underline{S},\underline{A}|x^e(\alpha)) = v^i(\underline{S},\underline{A}|x^e(\alpha)) = \frac{\partial}{\partial x_i} V(x) \Big|_{x=x^e(\alpha)} .$$

Comparing (2.1) and (2.2), then, <u>for a given</u> $x_{-i} = (x_1,\dots,x_{i-1},x_{i+1},\dots,x_I)$, agent i's private marginal return on investment is less than the socially efficient level. In Proposition 0 we show that assumptions (A1) - (A4) together imply that in fact the entire vector $x^e(\alpha)$ will be less than the first best x^*. Moreover, any change in control structure α that increases each agent's private marginal return on investment will unambiguously improve welfare.

<u>Proposition 0</u> For any control structure α, there is underinvestment: the unique Nash equilibrium $x^e(\alpha)$ satisfies

$$x_i^e(\alpha) \le x_i^* \quad \text{for each } i.$$

Moreover, if the control structure α changes to $\hat{\alpha}$, say, so that every agent's marginal return on investment increases, i.e., if for each i

$$\frac{\partial}{\partial x_i} B_i(\hat{\alpha}|x) \ge \frac{\partial}{\partial x_i} B_i(\alpha|x) \qquad \text{for all } x,$$

then equilibrium investment increases, $x^e(\hat{\alpha}) \ge x^e(\alpha)$, and welfare increases, $W(x^e(\hat{\alpha})) \ge W(x^e(\alpha))$.

<u>Proof</u> See Appendix B.

The underinvestment occurs, of course, because of an externality: when agent i invests more, some of his increased productivity will be dissipated in bargaining at date 1. In fact, it can be seen from (2.2) that he will only receive the full marginal return from his investment if he is first in the random ordering of the agents $\underline{S}$ at date 1;[7] in all other cases some of the benefits will flow to other agents.[8]

In the next section, we will investigate the control structures α that maximize welfare $W(x^e(\alpha))$. Then in Section 4, we will apply our general results to some special cases in order to understand better what factors determine the boundaries of a firm.[9]

Before proceeding, though, some simplification in notation would be useful at this point. Rather than carry the last argument x of the functions $B_i(\alpha|x)$, $v(S,A|x)$ and $v^i(S,A|x)$ throughout the rest of the paper, we will drop it, and write simply $B_i(\alpha)$, $v(S,A)$ and $v^i(S,A)$. Also, we define the function $B_i^{\ i}(\alpha)$ by

$$B_i^{\ i}(\alpha) \quad \equiv \quad \frac{\partial}{\partial x_i} B_i(\alpha|x),$$

where here, and throughout the paper, we use the equivalence sign, $\equiv$, to denote "for all x".

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[7] This is because from assumption (A6) and condition (B2), we know that $v^i(S,\alpha(S)|x^e(\alpha))$ is greatest when $S = \underline{S}$.

[8] In Grossman and Hart (1986), overinvestment is also a possibility. The reason for the difference is that in the latter paper the status quo point in the date 1 bargaining sometimes involves excessive trade (for a given x), whereas in the present paper it always involves too little trade (in the sense that $v^i(S,A|x) \leq v^i(\underline{S},\underline{A},x)$ for all S,A).

[9] We will take the point of view that efficient trading at date 0 will lead to a control structure which maximizes $W(x^e(\alpha))$. That is, if the intital α does not maximize $W(x^e(\alpha))$, someone will propose a new α and a set of sidepayments such that everyone is better off.

3. GENERAL RESULTS

Our aim in the following Propositions is to give a partial characterization of an optimal control structure α: one which will provide the agents with the second-best incentives to invest at date 0.[10] Of course, if there are no investments then the control structure will be unimportant; this follows from our assumption that there is costless bargaining at date 1, leading to ex-post efficiency.

Proposition 1 If only one agent i has an investment, then he should own all the assets (i.e. for any coalition S, $a_n \in \alpha(S)$ if and only if $i \in S$).

Proof If only agent i has an investment, then we want to to choose a control structure α that maximizes his marginal return on investment

$$B_i^i(\alpha) = \sum_{S|i\in S} p(S)\, v^i(S,\alpha(S)).$$

From (A6), this will be maximized by putting $\alpha(S) = \underline{A}$ for all S containing i.

Q.E.D.

The idea is simple. The best way to induce the agent to invest is to give him use of all the assets, no matter to which coalition he may belong -- for then he will have the greatest marginal incentive to invest to improve his bargaining position at date 1. That is, by giving him control over all the assets, the classic hold-up problem can at least partly be alleviated.

Proposition 2 Take any coalition of agents. Then each asset should be controlled either by the coalition or its complement.

Proof Suppose that $a_n \notin \alpha(S) \cup \alpha(\underline{S}\backslash S)$ for some asset a_n and coalition S. Then consider a new control structure $\hat{\alpha}$ which is the same as α except that S,

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10 Note that many of the Propositions are stated in the form: a control structure should have a certain property. This should be understood to mean that, in general, the property will be true of at least one (but not necessarily all) of the α's that maximize welfare. Typically though, the optimal control structure will be unique -- in which case our results are clear-cut.

and all supersets S^+ of S, now control a_n. It is straightforward to confirm that $\hat{\alpha}$ satisfies requirements (B1)-(B3). (Since α satisfies (B2), we know that $a_n \notin \alpha(\underline{S}\backslash S^+)$ for all supersets S^+, and hence $\hat{\alpha}$ does not violate (B1)).

The change $B_i^{\,i}(\hat{\alpha}) - B_i^{\,i}(\alpha)$ in marginal return on investment for some agent i in S will be

$$\sum_{S^+ | S \subseteq S^+} p(S^+) \; [\; v^i(S^+, \alpha(S^+) \cup \{a_n\}) \;-\; v^i(S^+, \alpha(S^+)) \;]$$

-- which is nonnegative by (A6). For no agent will the change in marginal return be negative. From Proposition 0, welfare will be higher under the new control structure $\hat{\alpha}$ than under α.

Q.E.D.

Again, the idea is simple. At the margin, an agent needs to be in a strong bargaining position at date 1 if he is to be induced to invest at date 0. This means giving control of as many assets as possible to the coalitions to which he might belong. It is therefore wasteful not to give control of an asset a_n to either S or $\underline{S}\backslash S$.

Proposition 2 is a useful simplifying result. In particular, we can rule out control structures where more than one agent has <u>veto power</u> over an asset -- i.e., where no coalition can control the asset unless all those agents with veto power belong to the coalition. In other words, we can rule out certain types of joint ownership.

<u>Proposition 3</u> More than one agent should not have veto power over an asset.

<u>Proof</u> Suppose agents i and j both have veto power over an asset a_n. Then take any coalition S which has agent i as a member, but not agent j. Since i and j both have veto power, a_n is not contained in $\alpha(S)$ or in $\alpha(\underline{S}\backslash S)$. But this contradicts Proposition 2.

Q.E.D.

Proposition 3 reflects the fact that in our model, investment is in <u>human</u> rather than <u>physical</u> capital (see assumption (A3)). The model can be generalized to include this latter possibility but then, inter alia, Proposition 3 may no longer hold.[11] For simplicity, in this paper we have chosen not to pursue the possibility of relaxing (A3).

There are two further cases (in addition to Proposition 1) where we can be sure that an agent should own a particular asset: namely, where the asset is idiosyncratic to him, and where he is indispensable to the asset.

Definition An asset a_n is idiosyncratic to an agent i if for all other agents the asset is irrelevant to their marginal benefit. That is, for all agents j in any coalition S, and for all sets A of assets containing a_n,

$$v^j(S,A) \equiv v^j(S,A\backslash\{a_n\}) \quad \text{for all } j \neq i.$$

Proposition 4 If an asset is idiosyncratic to an agent then he should own it.

Definition An agent i is indispensable to an asset a_n if, without agent i in a coalition, asset a_n has no effect on the marginal product of investment for the members of that coalition. That is, for all agents j in any coalition S, and for all sets A of assets containing a_n,

$$v^j(S,A) \equiv v^j(S,A\backslash\{a_n\}) \quad \text{if } i \notin S.$$

Proposition 5 If an agent is indispensable to an asset then he should own it.

If an asset is idiosyncratic to an agent, then a fortiori he is indispensable to that asset. So it is enough to prove Proposition 5.

Proof Suppose agent i is indispensable for an asset a_n, but under control structure α he does not own it. Then consider a new control structure $\hat{\alpha}$, which is the same as α, except that a_n is now owned by i. It is straightforward to confirm that $\hat{\alpha}$ satisfies requirements (B1)-(B3).

[11] For example, suppose that there are just two agents, and at date 0 they together "invest" in building a physical asset ready for sale at date 1. Whoever owns the asset at date 1 will get the proceeds. Here, ownership by one agent may clearly be a bad arrangement, since the other agent would have no incentive to invest. It may be better to have joint ownership, so that they split the proceeds from the sale at date 1.

The change $B_j^j(\hat{\alpha}) - B_j^j(\alpha)$ in marginal return on investment for some agent $j \neq i$ will be

$$\sum_{S \,\middle|\, \substack{i,j \in S \\ a_n \notin \alpha(S)}} p(S) \, [\, v^j(S,\alpha(S)\cup\{a_n\}) - v^j(S,\alpha(S)) \,]$$

$$- \sum_{S \,\middle|\, \substack{i \notin S;\ j \in S \\ a_n \in \alpha(S)}} p(S) \, [\, v^j(S,\alpha(S)) - v^j(S,\alpha(S)\backslash\{a_n\})].$$

The second summation is zero since i is indispensable for a_n. The first is nonnegative by (A6). So agent j's marginal incentive will not be reduced by the change in control structure.

Agent i's own marginal incentive to invest cannot fall as a result of now owning asset a_n. Hence by Proposition 0, welfare will be higher under the new control structure $\hat{\alpha}$ than under α.

Q.E.D.

We saw this result in the example of the Introduction. There agent 2 was indispensable, and agent 3 was dispensable -- and it was optimal to make agent 2 the owner of the asset.

The intuition behind the result is that if agent i is indispensable to an asset a_n, then in order for an agent j to derive marginal benefit from this asset he has to be in a coalition that <u>both</u> contains i <u>and</u> controls a_n. Not making i the owner of a_n would only serve to reduce the number of such coalitions. And this would reduce j's incentive to invest -- because at the margin j invests to improve his bargaining position at date 1. Of course making i the owner of a_n will enhance his own incentive to invest since then the asset would always be controlled by any coalition of which he is a member.

Proposition 5 is interesting because it tells us that the importance of an agent's action is only one force determining an agent's ownership rights. A second force is the agent's importance as a trading partner. In particular, if a subset of agents has investment decisions, one <u>cannot</u> conclude that ownership rights of assets should be concentrated only on this subset. In fact, if some agent outside the subset is indispensable, Proposition 5 tells us that it is better to give all ownership rights to this agent.[12]

Proposition 5 begs the question: what happens if a whole group of agents are indispensable to an asset? In this case, one (or more) of the group should always be given control over the asset. In the extreme, where all the agents are indispensable to all the assets, the control structure is unimportant. The reason for this is that an agent's marginal product of investment will only be enhanced by an asset if he is in the grand coalition $\underline{S}$ (in any subcoalition, there will be one or more indispensable agents missing). And $\underline{S}$ will control all the assets, whatever the control structure.[13]

If we think of the asset(s) as a firm, or project, it would be rather an extreme case if an entire group of agents were literally indispensable. More realistically, we might suppose that there is a group of agents who are key to the success of a project -- in the sense that they are particularly skilled or knowledgeable -- but it would not a calamity if a few of them (say less than half) were absent.

Definition A group G of agents are a key group to an asset a_n if more than half of them are needed to generate marginal product from a_n. That is, for all agents i in any coalition S, and for all sets A of assets containing a_n,

$$v^i(S,A) \equiv v^i(S,A\backslash\{a_n\}) \quad \text{if S contains less than or equal to half of the agents in G.}$$

Suppose a group G is key to an asset. Then consider any coalition S that contains more than half of the agents in G. The complementary coalition, $\underline{S}\backslash S$, would have too few members of the key group to benefit from the asset, and so control of it is best not given to them, but instead given to S. This suggests a possible explanation for partnerships, where control is decided by majority rule. If each of the members of G is given a vote, the right control

[12] An exception is where only one agent has an action, in which case Propositions 1 and 5 tell us that it doesn't matter if this agent or the indispensable agent has ownership rights.

[13] There is a caveat: $\underline{S}$ must control all the assets $\underline{A}$. However, we know from Proposition 2 and condition (B3) that this should indeed be the case.

structure is ensured: only those coalitions S which contain a majority of G get control over the asset. Hence:

<u>Proposition 6</u> If a group of agents are key to an asset, then control of it should be decided by simple majority voting among them.

Next, we turn from grouping agents to the question of grouping assets. There are a host of examples of assets which ordinarily are owned or controlled together: e.g., a window of a house and the house itself; a lock and a key; the engine of a truck and its chassis; a list of clients' names and the list of their addresses; a baseball field and the spectator stand; the two ends of a pipeline; a railroad track and the rolling stock; power stations and the distribution grid. Why are these assets usually paired together, even though in principle they could be owned or controlled separately?

<u>Definition</u> Two assets a_m and a_n are (strictly) <u>complementary</u> if they are unproductive unless they are used together. That is, for all coalitions S, and for all sets A of assets containing a_m and a_n,

$$v^i(S,A\backslash\{a_m\}) \equiv v^i(S,A\backslash\{a_n\}) \equiv v^i(S,A\backslash\{a_m,a_n\}) \qquad \text{if } i \in S.$$

<u>Proposition 7</u> If two (or more) assets are (strictly) complementary, they should be owned or controlled together.

<u>Proof</u> Suppose that a_m and a_n are complementary, but that under control structure α for some coalition S, $a_m \in \alpha(S)$ and $a_n \notin \alpha(S)$. Consider a new coalition structure $\hat{\alpha}$, which is the same as α except that whenever <u>any</u> coalition controls a_m under α, that same coalition controls both α_m <u>and</u> α_n under $\hat{\alpha}$. It is straightforward to confirm that $\hat{\alpha}$ satisfies requirements (B1)-(B3).

The change $B_i^i(\hat{\alpha}) - B_i^i(\alpha)$ in marginal return on investment for some agent i will be

$$\sum_{S \,\middle|\, \substack{i \in S \\ a_m \in \alpha(S);\; a_n \notin \alpha(S)}} p(S)\ [\, v^i(S,\alpha(S)\cup\{a_n\}) - v^i(S,\alpha(S))\ ]$$

$$-\sum_{S \,\middle|\, \substack{i \in S \\ a_m \in \alpha(\underline{S}\backslash S);\; a_n \in \alpha(S)}} p(S)\ [\ v^i(S,\alpha(S)) - v^i(S,\alpha(S)\backslash\{a_n\})\ ].$$

The second summation is zero since a_m and a_n are complementary. By (A6), the first summation is nonnegative. So agent i's marginal incentive to invest will not be reduced. Hence by Proposition 0, welfare will be higher under the new control structure $\hat{\alpha}$ than under α.

Q.E.D.

We saw this result in the example in the Introduction. Recall the intuition: in order for an agent to derive marginal benefit from either asset he must be in a coalition that controls both. Separating control of the assets would therefore reduce the number of such coalitions. And this would reduce his incentive to invest.

We now consider the opposite extreme of Proposition 7. Supposing that there are no synergies between assets, can we conclude that they should be owned or controlled separately? More generally, should ownership extend across agents who are economically independent? And related to this last question, should control be given to an agent who is "dispensable" in the following sense?

<u>Definition</u> An agent k is <u>dispensable</u> if the other agents' marginal product of investment is unaffected by whether or not he is a member of their coalition (assuming the coalition controls a given set of assets). That is, for all coalitions S containing agents k and j ($\neq$ k), and for all sets A of assets,

$$v^j(S,A) \equiv v^j(S\backslash\{k\},A).$$

We encountered the notion of dispensability in the Introduction. The idea is that if agent k were not a member of a coalition, the coalition members could hire an outside agent to replace him. And <u>at the margin</u>, in this coalition, their individual product of investment would be unchanged. But of course there would in general be a (fixed) loss associated with having to hire someone who is not as skilled as agent k.

Definition An agent k has some control rights if there is an asset a_n and a coalition S containing k such that $a_n \in \alpha(S)$ but $a_n \notin \alpha(S \setminus \{k\})$.

Proposition 8 An agent who is dispensable and who has no investment should not have any control rights if stochastic control is possible.

Proof See Appendix B.

An obvious and important corollary of Proposition 8 is that outside parties should not have any control rights, where we define:

Definition An outside party, agent 0, say, is an agent who is economically independent from the other agents. That is, for any set A of assets, and any coalition S not containing agent 0,

$$v(S \cup \{0\}, A) \equiv v(S, A) + v(0) \quad \text{if } i \in S.$$

Corollary. An outside party should not have any control rights if stochastic control is possible.

At the intuitive level, this corollary to Proposition 8 is compelling: if outside parties have control rights then the date 1 surplus is shared among a larger set of agents at date 1 -- and this dilution of the returns on investment only serves to reduce incentives at date 0. The same argument would seem to apply to anyone who has no investment; but as we know from Proposition 5, if an agent is indispensable he should be given full control even though he may have no investment. Hence Proposition 8 requires not only that an agent has no investment, but also that he is dispensable.

It is perhaps surprising that one cannot rule out control by outside parties without having to resort to stochastic schemes. The reason is that if one is forced to use a deterministic control structure, then outside parties can play a useful role in dividing up ownership. For example, if there are just two agents, 1 and 2, in $\underline{S}$, then by introducing an outside party and (say) majority rule among the three of them, agents 1 and 2 can be given more balanced incentives than if one of them owns. However, a stochastic control structure in which ownership is randomly allocated between 1 and 2 will be better, because it avoids the dissipation of surplus at date 1.[14,15]

Proposition 8 implies that ownership should not extend across agents who are economically independent: and if there are no synergies between assets, they should not be owned or controlled together. Owing to the importance of this result, we have written it out as a separate Proposition 9. It is a direct application of Proposition 8, and so no separate proof is needed.

<u>Definition</u> $(\hat{\underline{S}},\hat{\underline{A}})$ and $(\hat{\hat{\underline{S}}},\hat{\hat{\underline{A}}})$ are <u>economically independent</u> if (i) the two groups $\hat{\underline{S}}$ and $\hat{\hat{\underline{S}}}$ each have access to an arbitrary number of outside parties (for the purpose of dividing ownership if there is no stochastic control); and (ii) for all coalitions $\hat{S},\hat{\hat{S}}$, where $\hat{S} \subseteq \hat{\underline{S}}$, $\hat{\hat{S}} \subseteq \hat{\hat{\underline{S}}}$, and for all sets $\hat{A},\hat{\hat{A}}$ of assets, where $\hat{A} \subseteq \hat{\underline{A}}$, $\hat{\hat{A}} \subseteq \hat{\hat{\underline{A}}}$,

$$v^i(\hat{S}\cup\hat{\hat{S}},\hat{A}\cup\hat{\hat{A}}) \equiv v^i(\hat{S},\hat{A}) \qquad \text{if } i \in \hat{S}$$

$$\text{and } v^i(\hat{S}\cup\hat{\hat{S}},\hat{A}\cup\hat{\hat{A}}) \equiv v^i(\hat{\hat{S}},\hat{\hat{A}}) \qquad \text{if } i \in \hat{\hat{S}}.$$

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[14] Consider a symmetric case in which agents 1 and 2 work with a single asset, $\underline{A} = \{a\}$. Assume the asset is essential to both agents (in the sense that their marginal product of investment is zero if they don't have access to it), but they are both dispensable. Without introducing an outside party, there is in effect only one kind of deterministic control structure possible: viz., one of them owns the asset. Let this be agent 1. Then their marginal returns on investment are respectively $v^1(\underline{S},\underline{A})$ and $(1/2)v^2(\underline{S},\underline{A})$. Now it may be that $(1/2)v^2(\underline{S},\underline{A})$ provides agent 2 with too low an incentive to invest. A control structure which instead uses majority rule among $\{0,1,2\}$, where 0 is an outside party, changes their marginal returns to $(2/3)v^1(\underline{S},\underline{A})$ and $(2/3)v^2(\underline{S},\underline{A})$. However, a stochastic control structure will do better: e.g., a 50:50 allocation of the asset at date 1 will raise their marginal returns to $(3/4)v^1(\underline{S},\underline{A})$ and $(3/4)v^2(\underline{S},\underline{A})$.

[15] One can show that, even if stochastic control is ruled out, an outside party should not be the sole owner of an asset which is "essential" to another agent -- see before Proposition 10 for the full definition of an essential asset.

Definition $(\hat{\underline{S}},\hat{\underline{A}})$ and $(\hat{\hat{\underline{S}}},\hat{\hat{\underline{A}}})$ are independently controlled if no agent in $\hat{\underline{S}}$ has control rights over any of the assets in $\hat{\hat{\underline{A}}}$, and no agent in $\hat{\hat{\underline{S}}}$ has control rights over any of the assets in $\hat{\underline{A}}$.

Proposition 9 With economic independence there should be independent control.

Finally in this section we consider the effect of a change in ownership on an agent who works with an asset that is essential to him -- for example, a worker at a firm. The important point is that, outside the firm, the worker's specific investment is unproductive.

Definition An asset a_n is essential to an agent i if the marginal product of investment for the agents in a coalition will not be enhanced by agent i unless the coalition controls a_n. That is, for all coalitions S containing j, and for all sets A of assets,

$$v^j(S,A) \equiv v^j(S\backslash\{i\},A) \qquad \text{if } a_n \notin A.$$

Notice that, in the light of assumption (A3), the definition implies that for agent i himself (i.e., for j = i),

$$v^i(S,A) \equiv 0 \qquad \text{if } a_n \notin A.$$[16]

Proposition 10 Suppose an agent with an investment has an essential asset which is owned by a second agent. Then, ignoring the effects of changes in other agents' investment levels, the first agent's incentive to invest will be increased if the second agent takes ownership of all the assets.

Proof See Appendix B.

The intuition behind this result is that if an asset a_n is essential to an agent i, but another agent j owns it, then i's marginal return on

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[16] Strictly speaking, Proposition 10 only requires that $v^i(S,A) \equiv 0$ if $a_n \notin A$. However, the full definition of an essential asset is used elsewhere in the paper.

investment will be zero unless he is in a coalition with j. Given that j owns a_n, i's incentive to invest will be highest if j owns all the other assets too -- because that way, i will have access to all those assets whenever his investment is productive, and at the margin this will enable him to strengthen his bargaining position at date 1.

Proposition 10 can help us to rule out rather strange control structures such as: agent i owns an asset a which is essential to a group of workers, {w}, but at the same time an agent j owns an asset a^* which is essential to agent i. This corresponds to a hybrid employment structure where workers are employed at one firm, while the boss of that firm is working for another firm.[17]

Proposition 10 suggests that it may be better to gather the two assets under common ownership. Specifically, the control structure should be arranged so as to have only one boss -- agent j -- at the top of a single firm $\{a, a^*\}$. However, there would be a snag with this arrangement if, for example, agent i were indispensable, because then under the new ownership structure the workers {w} could only be productive in a coalition that contained both i and j. If i is dispensable, then this snag disappears, and we can rule out the hybrid arrangement:

Proposition 11 Suppose an asset a is essential to a group of workers {w}, and an asset a^* is essential to an agent i, who is dispensable. Then ignoring the effects of changes in investment by outside parties, agent i should not own a if a^* is owned by someone else.

Proof See Appendix B.

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[17] Arguably, this kind of "layered firm" is part of what is meant by a wholly-owned subsidiary: j is an agent representing the parent company, i is an agent representing the subsidiary, and the workers {w} work for i.

4. APPLICATIONS

We now apply and develop the results from the previous section. We begin with the case of a single asset.

4.1 One Asset

Suppose that J workers, w1,...,wJ, work with a single asset, a, to supply a service to K consumers, c1,...,cK. For simplicity assume that the asset is essential to the workers in the sense of Section 3, and that only they have investment decisions. Who should have ownership rights over the asset?

We have seen in Section 3 that an individual is more likely to have ownership rights if his investment is important and/or if he is an important (in the limit, an indispensable) trading partner. Specifically, if only one of the workers has an investment, or one of the consumers, say, is indispensable, then that one should be the owner or boss (Propositions 1 and 5). More generally, if there is a group G of "key" agents such that for any worker's marginal product of investment to be positive he must be a member of a coalition containing a majority of members of G (different majorities may do for different workers), then it is optimal to give each member of G a vote and adopt majority rule (Proposition 6). Such an arrangement can be interpreted as a partnership if G is a subset of {w1,...,wJ}; as a worker cooperative if G = {w1,...,wJ} (i.e., a majority of all workers is required to realize a positive marginal product of investment); and as a consumer cooperative if G = {c1,...,cK}.

To get some insight into the case where there is no group of key agents, consider the opposite extreme where every agent is dispensable. According to Proposition 8, ownership rights should then be allocated over those agents that invest. Denote this subset by H. Since everyone is dispensable, $v^i(S,\{a\}) = v^i(\underline{S},\{a\})$ for all i in any coalition $S \cap H$. So the FOC (2.2) for agent i's investment x_i simplifies to

$$p_i \, v^i(\underline{S},\{a\}) \;=\; C_i'(x_i),$$

where, recalling our discussion of Shapley value in Section 2, p_i is the probability that i belongs to a coalition that owns a. For example, if agent 1 owns a then $p_1 = 1$ and $p_i = \frac{1}{2}$ for all $i > 1$ (since the probability of appearing before 1 in the random ordering is just (1/2)).

If agent 1 owns a,

$$\sum_{i \in H} p_i = 1 + \frac{1}{2}(|H| - 1), \tag{4.1}$$

assuming 1 lies in H. In fact this equality holds for <u>all</u> ownership structures that give control either to a subset of H or to its complement in H (i.e. that satisfy Proposition 2 restricted to H).[18] Thus, if every agent is dispensable, a control structure amounts to an allocation of the p_i's subject to the constraint (4.1).

We can say more about the ownership structure in two special cases. First, if there is symmetry within the group H, i.e., each agent's investment has the same importance, we might expect the optimum to have $p_i = [1 + \frac{1}{2}(|H|-1)]/|H|$ for all i in H. It is interesting to note that if $|H|$ is odd, this can be achieved by giving each member of H a vote and adopting majority rule.

Secondly, suppose that there is one "big" worker and many "small" workers, in the sense that the big worker's investment is an order of magnitude more important than a small worker's (i.e., the social value of a dollar increase in the big worker's investment is comparable to the social value of a simultaneous dollar increase in all the small workers' investments). Then the above analysis tells us that the big worker should have all the control rights. (We are retaining the assumption that everyone is dispensable, but it is easy to see that what follows continues to hold if the big worker is not dispensable.) The reason is that, according to (4.1), it is impossible to allocate control rights to the small workers so as to yield p_i's significantly above $\frac{1}{2}$ for more than a small fraction of them. But this means that their aggregate investment will hardly differ from what it would be if the big worker were made sole owner and $p_i = \frac{1}{2}$ for each small worker. On the other hand the big worker's investment will be strictly higher if he is the owner. Hence giving all control rights to the big worker will be

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[18] To see this, consider any random ordering of members of H and let $\xi_i = 1$ if i belongs to a coalition which owns a, and $\xi_i = 0$ otherwise. Now reverse the ordering and denote the new ξ_i's by $\hat{\xi}_i$. Then $\sum_{i \in G} \xi_i + \sum_{i \in G} \hat{\xi}_i = |H| + 1$. Taking the expectation over all orderings and their reverses yields (4.1).

optimal in such a situation.

In general, we expect to find something in between the extremes where either there is a key group of agents that everyone needs to trade with or agents are dispensable. Rather than analysing this general situation, however, we turn to a model with two assets.

4.2 Two Assets

Consider now a situation where there are two assets, a_1 and a_2, and assume that for each asset there are workers to whom the asset is essential. One can imagine that the workers of asset a_1 use a_1 to supply some service to the workers of asset a_2, who then use it together with a_2 to produce a final good for consumers. For simplicity we do not model final consumers (or any other input suppliers) explicitly; one can suppose that the good is sold (and other inputs are purchased) on a spot market.

To simplify further, we will suppose that each asset a_i, $i = 1,2$, has one "big" worker i and many "small" workers {wi}, in the sense of subsection 4.1 above, where each small worker is dispensable. (To avoid heavy notation, we simply give the small workers the generic description w1 or w2; there is no presumption that they are identical.) Under these conditions, we know that it will not be optimal to give the small workers control rights.

Hence the issue is whether (i) a_1 should be owned by 1 and a_2 should be owned by 2 -- which can be interpreted as nonintegration, with 1 as the boss of the first firm and 2 as the boss of the second firm; or whether (ii) 1 should own both assets -- integration with 1 as boss of the integrated firm; or whether (iii) 2 should own both assets -- integration with 2 as boss. Actually, there is a fourth possibility: (iv) 1 owns a_2 and 2 owns a_1 -- but we will show at the end of this subsection that this is not optimal.

We start with (i) and (ii), and write down the first-order conditions (FOCs) for the agents' investment levels. Bearing in mind that the workers w1 and w2 are dispensable, we can simplify notation and omit reference to them in the coalitions of agents. For example, $v^1(12,\{a_1,a_2\})$ equals 1's marginal product of investment in any coalitions $\{1,2\} \cup S_1 \cup S_2$ that control assets a_1 and a_2 -- where S_1 and S_2 are, respectively, arbitrary sets of workers w1 and workers w2. And for example, $v^{w1}(1,\{a_1\})$ equals a certain worker w1's marginal product of investment in coalitions $\{1\} \cup S_1 \cup S_2$ that control asset a_1 -- where now S_1 must contain this particular worker.

(i) Nonintegration: 1 owns a_1, 2 owns a_2

$$\text{FOC for 1:} \quad \tfrac{1}{2}v^1(12,\{a_1,a_2\}) + \tfrac{1}{2}v^1(1,\{a_1\}) = C_1'(x_1) \tag{4.2a}$$

$$\text{FOC for typical w1:} \quad \tfrac{1}{3}v^{w1}(12,\{a_1,a_2\}) + \tfrac{1}{6}v^{w1}(1,\{a_1\}) = C_{w1}'(x_{w1}) \tag{4.2b}$$

$$\text{FOC for 2:} \quad \tfrac{1}{2}v^2(12,\{a_1,a_2\}) + \tfrac{1}{2}v^2(2,\{a_2\}) = C_2'(x_2) \tag{4.2c}$$

$$\text{FOC for typical w2:} \quad \tfrac{1}{3}v^{w2}(12,\{a_1,a_2\}) + \tfrac{1}{6}v^{w2}(2,\{a_2\}) = C_{w2}'(x_{w2}) \tag{4.2d}$$

(ii) Integration: 1 owns a_1 and a_2

$$\text{FOC for 1:} \quad \tfrac{1}{2}v^1(12,\{a_1,a_2\}) + \tfrac{1}{2}v^1(1,\{a_1,a_2\}) = C_1'(x_1) \tag{4.2e}$$

$$\text{FOC for typical w1:} \quad \tfrac{1}{3}\, v^{w1}(12,\{a_1,a_2\}) + \tfrac{1}{6}\, v^{w1}(1,\{a_1,a_2\}) = C_{w1}'(x_{w1}) \tag{4.2f}$$

$$\text{FOC for 2:} \quad \tfrac{1}{2}v^2(12,\{a_1,a_2\}) = C_2'(x_2) \tag{4.2g}$$

$$\text{FOC for typical w2:} \quad \tfrac{1}{3}v^{w2}(12,\{a_1,a_2\}) + \tfrac{1}{6}v^{w2}(1,\{a_1,a_2\}) = C_{w2}'(x_{w2}) \tag{4.2h}$$

We now use these conditions[19] to compare the marginal incentive to invest under nonintegration and integration, for each agent in turn.

Not surprisingly, comparing (4.2e) with (4.2a), we see that agent 1 has a greater incentive to invest under integration since he then always has access to both assets. Correspondingly, comparing (4.2c) with (4.2g), we see that agent 2 has a greater incentive to invest under nonintegration, since he then always has access to a_2.

What is more interesting is the effect of integration on the employees {w1} and {w2}. Comparing (4.2f) with (4.2b), we see that, ignoring the effects of changes in the other agents' investment levels, a typical w1's incentive to invest is unambiguously greater under integration (Proposition 10). The reason is that under either nonintegration or integration, w1 has to be in a coalition with 1 to be productive at the margin since, by assumption, he requires access to a_1. But under integration, every time he is in coalition with 1, he also has access to a_2 and is therefore more productive.

The situation for a typical w2 is less clear since, in contrast to w1, his boss 2 loses control when switching to integration. On the positive side, w2 is more likely to have access to both assets since this will happen whenever he is in a coalition with his new boss 1; previously he had to be in a coalition with both 1 and 2 for this to occur. On the negative side, under integration, w2 is less likely to find himself in a coalition with his old

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[19] To understand these conditions, it may be worth recapping on the "random ordering" explanation of Shapley value that we gave in Section 2. Consider the case of nonintegration. With probability (1/2) agent 1 appears before 2 in the random ordering at date 1 and receives his full marginal product $v^1(12,\{a_1,a_2\})$. And with probability (1/2) he appears after 2, in which case he receives $v^1(1,\{a_1\})$ -- bearing in mind that he owns a_1 but not a_2. This explains (4.2a). In contrast, a worker w1 has a positive marginal product only if he appears before 1 in the random ordering (otherwise he is in a coalition which does not have access to his essential asset a_1). With probability (1/3) he is before 1 and 2, in which case he receives $v^{w1}(12,\{a_1,a_2\})$; and with probability (1/6) he is before 1 but after 2, in which case he receives $v^{w1}(1,\{a_1\})$. This explains (4.2b). (The missing terms are $\frac{1}{6}v^{w1}(2,\{a_2\})$ and $\frac{1}{3}v^{w1}(\phi,\phi)$, which are both zero.) The six other conditions (4.2c) - (4.2h) follow similarly.

boss 2 and asset a_2; and if this is a powerful combination this can cause a loss. For example, if 2 is <u>indispensable</u> to a_2 (implying $v^{w2}(1,\{a_1,a_2\}) = 0$), a comparison of (4.2h) with (4.2d) shows that w2's incentive to invest falls under integration. (We know from Proposition 5 that if agent 2 is indispensable to an asset then he should own it, which means that it cannot be optimal for 1 to acquire a_2.)

To summarize, there are two effects from integration on employees. On the one hand, there is a positive effect due to increased coordination: agents now have to negotiate with only one person to get access to both assets and this reduces the fraction of surplus dissipated via bargaining. (This positive effect also applies to the boss of the acquiring firm: he does not now have to bargain with anyone to get access to both assets.) On the other hand, there is a negative effect due to the fact that an employee of the acquired firm now has to negotiate with two people to have access to both his essential asset and his old boss. (This negative effect also applies to the boss of the acquired firm.)

In some special cases, we can say which effect will dominate. First, if 2 is <u>dispensable</u>, the negative effect on the employee w2 disappears:[20] $v^{w2}(1,\{a_1,a_2\}) \geq v^{w2}(2,\{a_2\})$. In this case, then, the only cost of integration is the negative incentive effect on agent 2.

Secondly, if assets a_1 and a_2 are <u>economically independent</u> in the sense that $v^i(S_1 \cup S_2,\{a_1,a_2\}) = v^i(S_1,\{a_1\})$ for all $i \in S_1$, and $= v^i(S_2,\{a_2\})$ for all $i \in S_2$ -- where S_1 and S_2 are any subcoalitions of {1 + workers w1} and {2 + workers w2} respectively -- then nonintegration <u>always</u> dominates integration (Proposition 9). [The FOCs (4.2a,b,e,f) imply that the incentives of 1 and w1 are unchanged under integration, while (4.2c,d,g,h) imply that the incentives of 2 and w2 are reduced.] This simply reflects the fact that in the absence of a synergy between assets a_1 and a_2, making 1 the owner of asset a_2 is like bringing in an outside party -- its only effect is to cause surplus to be

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[20] This is similar to the effect in Proposition 11. Recall that, in the context of that Proposition, an asset a ($\equiv a_2$) is essential to an agent w ($\equiv$ w2). The agent's incentive to invest does not fall when agent j ($\equiv$ 1) takes over the ownership of a from agent i ($\equiv$ 2) -- given that agent i is dispensable.

dissipated (Proposition 8).

Thirdly, if assets a_1 and a_2 are (strictly) <u>complementary</u> in the sense that $v^i(S,\{a_1\}) = v^i(S,\{a_2\}) = v^i(S,\phi) = 0$ for all i in any coalition S, then integration <u>always</u> dominates nonintegration (Proposition 7). This follows from the fact that $v^2(2,\{a_2\})$ and $v^{w2}(2,\{a_2\})$ are both zero in (4,2c) and (4.2d), and hence the negative incentive effects of integration on 2 and w2 disappear. In words, suppose two assets are (at the margin) useless unless used together, but they are owned by two different people. Then making one of these people the owner of both assets will help outside agents who now only have to negotiate with one person, rather than two people, in order to use them. (And of course the owner himself can now use both assets without having to negotiate with anyone else.) The person who used to own one of the assets doesn't lose out since he couldn't use the asset without reaching an agreement with the other person anyway. Of course, this argument does not tell us that 1 should be the owner rather than 2 -- it only says that the assets should be under common ownership.

Once we leave these special cases, the costs and benefits are less easy to evaluate. The main forces are summarized in Figure 1.

As noted before, there are two other cases to consider: (iii) 2 can own both assets, or (iv) "reverse" nonintegration where 1 owns a_2 and 2 owns a_1. Obviously similar considerations to those above determine when it is better to concentrate both assets in 2's hands than to have them separately owned. As for the choice between making 2 the owner of both assets or 1, this will be determined by factors such as who has the more important investment, and/or who is the less dispensable. Finally, it is easy to show that reverse nonintegration is never optimal -- it is dominated by making 1 the owner of both assets. This follows from the fact that ownership by 1 will raise the incentives of 1 and the workers w1, and cannot lower the incentives of 2 and the workers w2 since these agents had to reach agreement with 1 anyway to have access to a_2.

4.3 <u>Three Assets</u>

New effects arise when there are more than two assets. We do not have space to be comprehensive, but we would like to point out certain key effects which can be seen in a three-asset model.

Forces determining whether 1 and 2 should integrate (1 should own 2)

Force	Tendency towards Integration	Tendency towards Nonintegration	Proposition
agent 1's investment is important	x		1
agent 2's investment is important		x	1
asset a_2 is idiosyncratic to agent 2		x	4
asset a_2 is idiosyncratic to agent 1	x		4
agent 2 is indispensable to asset a_2		x	5
agent 1 is indispensable to asset a_2	x		5
assets a_1 and a_2 are complementary	x		7
agent 2 is dispensable	x		8
assets a_1 and a_2 are economically independent		x	9
agent wl's investment is important	x		10

FIGURE 1

To simplify, we ignore employees now and suppose that each of the three assets $\{a_1,a_2,a_3\}$ has just one ("big") agent working on it -- agents 1,2,3, respectively. As above, asset a_i is assumed to be essential for agent i. We further simplify by assuming that all three agents are dispensable. We can interpret the situation as one in which a_1 supplies a_2 who then supplies a_3, or in which a_1 supplies a_2 and a_3 (or, equivalently, a_2 and a_3 supply a_1).

As a final simplification, in this subsection we only consider control structures α in which each asset has a sole owner. (In the following subsection, we will briefly discuss multiple ownership in a model which has more assets.)

The first finding is that if an agent does not own the asset which is essential to him, then he should not own any other asset. To see why, note that such an arrangement would be equivalent to one of the following (subject to relabelling): 2 owns a_1 and 3 owns a_2; or 2 owns a_1 and 1 owns a_2. Both arrangements would be dominated by having 3 owning a_1, a_2 and a_3 (he may already own a_3): obviously 3's incentive to invest would increase, but so too would 1's and 2's since each had to be in a coalition with one other person to be productive anyway. (In fact we know from Proposition 11 that the first arrangement is not optimal -- given that 2 is dispensable.)

This leaves us with three possibilities (subject to relabelling): (i) <u>nonintegration</u> -- 1 owns a_1, 2 owns a_2, 3 owns a_3; (ii) <u>partial integration</u> -- 1 owns a_1 and a_2, 3 owns a_3; and (iii) <u>integration</u> -- 1 owns a_1, a_2 and a_3.

The effects that we have already seen in subsections 4.1 and 4.2 carry over and generalize to this three-asset model. First, there are several variations on Proposition 9. Most obviously: if the three assets are economically independent then nonintegration is optimal. Also: if there is no synergy between a_3 and the pair $\{a_1,a_2\}$ then integration will be dominated either by nonintegration or by partial integration. Slightly less obvious: if there is no synergy between a_1 and a_2 then partial integration as in (ii) is dominated by another form of partial integration in which 3 owns $\{a_2,a_3\}$ and 1 owns a_1. That is, if anyone other than 2 is to own a_2 then it should be 3, not 1 -- and the same is true reversing 1 and 2.

Secondly, Proposition 7 readily generalizes to this three-asset model: if the three assets are (strictly) complementary, then integration is optimal.

Partial integration is the distinctive new control structure in a three-asset model. The main new effect is that if 1 acquires a_2, then this has an impact on 3. So how are 3's incentives affected by a change from (i) nonintegration to (ii) partial integration? The FOCs for 3's choice of investment x_3 in these two cases are written out below. Since all three agents are dispensable, we can omit reference to the coalitions in the marginal products, v^i, and simply write, say, $\underline{v}^3(a_1,a_2,a_3)$ for any one of $v^3(3,\{a_1,a_2,a_3\})$, $v^3(13,\{a_1,a_2,a_3\})$, $v^3(23,\{a_1,a_2,a_3\})$ or $v^3(123,\{a_1,a_2,a_3\})$.

<u>(i) Nonintegration</u>

FOC for 3: $\frac{1}{3}\underline{v}^3(a_1,a_2,a_3) + \frac{1}{6}\underline{v}^3(a_1,a_3) + \frac{1}{6}\underline{v}^3(a_2,a_3) + \frac{1}{3}\underline{v}^3(a_3) = C_3'(x_3)$.

<u>(ii) Partial Integration</u>

FOC for 3: $\frac{1}{2}\underline{v}^3(a_1,a_2,a_3) + \frac{1}{2}\underline{v}^3(a_3) = C_3'(x_3)$.

Thus, ignoring the effects of changes in the other agents' investment levels, agent 3 will have a greater incentive to invest under partial integration than under nonintegration, provided

$$\underline{v}^3(a_1,a_2,a_3) + \underline{v}^3(a_3) > \underline{v}^3(a_1,a_3) + \underline{v}^3(a_2,a_3) \tag{4.3}$$

In both of our earlier interpretations of the model, this could be argued to hold. If on the one hand a_1 supplies a_2 who then supplies a_3, then arguably 1 and 3 have only an indirect relationship with each other and would get limited synergy in the absence of 2: i.e.,

$$v(13,\{a_1,a_3\}) \cong v(1,\{a_1\}) + v(3,\{a_3\}) \quad \Rightarrow \quad \underline{v}^3(a_1,a_3) \cong \underline{v}^3(a_3)$$
$$\Rightarrow \quad (4.3) \text{ holds.}$$

If on the other hand a_1 supplies a_2 and a_3 (or, equivalently, a_2 and a_3 supply a_1), then arguably 2 and 3 have only an indirect relationship with each other and would get limited synergy in the absence of 1: i.e.,

$v(23,\{a_2,a_3\}) \cong v(2,\{a_2\}) + v(3,\{a_3\}) \quad \Rightarrow \quad \underline{v}^3(a_2,a_3) \cong \underline{v}^3(a_3)$

$\Rightarrow$ (4.3) holds.

There are, however, circumstances in which agent 3 will have a greater incentive to invest under nonintegration than under partial integration. For example, if the assets a_1 and a_2 are close substitutes then $\underline{v}^3(a_1,a_2,a_3)$ is likely to be close to both $\underline{v}^3(a_1,a_3)$ and $\underline{v}^3(a_2,a_3)$, and (4.3) will not hold. Here, 3 prefers that a_1 and a_2 are separately owned, so that their owners 1 and 2 have to compete with each other for access to his asset a_3.

4.4 Many Assets

In subsection 4.3, we focussed on the case where each of the (three) assets had a single owner. In order to obtain some insight into the circumstances in which multiple ownership will be selected, consider the following model.

There are N assets (firms) $\underline{A} = \{a_1,\ldots,a_N\}$, and N agents $\underline{S} = \{1,\ldots,N\}$. We make the standard assumption that asset a_i is essential to agent i. We also assume that all the agents are dispensable.

Agent 1 uses a_1 to supply firms $a_2,\ldots,a_N$. The particular interpretation that we have in mind is that a_1 is an oil pipeline and $a_2,\ldots,a_N$ are oil refineries (this case is discussed in Klein et. al. (1978)). A fairly natural assumption to make then is that the only synergies are bilateral, between a_1 and each of $a_2,\ldots,a_N$. Note that $a_2,\ldots,a_N$ are useless in the absence of a_1. Thus for all $i\neq 1$ in any coalition S, and for all sets A of assets, we are assuming

$$\begin{aligned} v^i(S,A) &= v^i(\underline{S},\{a_1,a_i\}) && \text{if } i \in S \text{ and } a_1,a_i \in A \\ &= 0 && \text{otherwise.} \end{aligned}$$

(Note the implicit assumption that there is no capacity constraint in the pipeline.)

Since $a_2,\ldots,a_N$ are useless without a_1, we expect some common ownership to be optimal here. If 1's investment is particularly important, there is a case for 1 owning all the assets (integration downstream). On the other hand, if 1's investment can be ignored, but those of $2,\ldots,N$ are important (as is arguably the case in the pipeline example), then $2,\ldots,N$ will share control rights in a_1 (Proposition 8).

Suppose that 1's investment x_1 can be ignored. Given our assumptions, agent i's FOC for his investment x_i is

$$p_i v^i(\underline{S},\{a_1,a_i\}) = C_i'(x_i) \quad \text{for } 2 \le i \le N,$$

where p_i is the probability that i belongs to a coalition that owns a_1. Now as in (4.1),

$$\sum_{i=2}^{N} p_i = 1 + \frac{1}{2}(N-2). \tag{4.4}$$

Hence an optimal control structure amounts to an allocation of the p_i's across the agents $2,\ldots,N$, in accordance with the responsivenes and importance of their investments, subject to (4.4). One way to achieve such an allocation is to give (not necessarily equal) shares or votes in a_1 to each agent and adopt simple majority rule (this guarantees that Proposition 2 holds); obviously, an agent's p_i will be increasing in the number of votes he receives.

Arangements like these are in fact observed in the case of oil pipelines, with an agent's share being linked to his use of the pipeline (see Klein et.al (1978)). To the extent that usage is in turn related to investment, this finding provides some support for the model presented here.[21]

The multiasset analysis can be extended in several ways. First, the assumption of dispensability can be dropped. In the example just discussed, this would have the consequence that allocating control rights in a_1 to agent i rather than agent 1 will have a negative effect on j's incentives to the extent that access to agent 1 as well as asset 1 is important for j. Secondly, the assumption that there is only one agent per asset can be relaxed. We leave such generalizations for future work.

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[21]Another application of these ideas might be to the case of joint ownership of bridges by different railroads in the nineteenth century (see Chandler (1977), p.124).

5. EXTENSIONS

In the model of Section 2, agents make investments in human capital at date 0 which affect their productivities at date 1. The results given in Sections 3 and 4 hold under a variety of other circumstances, however. The crucial feature is that there should be a link between actions in one period and payoffs in subsequent periods, and agents are unable to write complete long-term contracts which specify precisely current and future actions.

In this section, we will sketch two further models in which this feature is present: learning by doing, and signaling type through choice of action.

Learning By Doing

Let us interpret x_i now as the amount that agent i works during period 0 (at a cost of $C_i(x_i)$). Suppose that a coalition $S \subseteq \underline{S}$ of agents, which controls assets $\alpha(S) \subseteq \underline{A}$, generate a value in period 0 given by

$$\hat{v}(S,\alpha(S)|x) \equiv \sum_{i\in S} \hat{w}_i(S,\alpha(S)|x)$$

where $\hat{w}_i(S,\alpha(S)|x)$ is the private benefit accruing to i from the coalition. Assume that each of the functions $\hat{w}_i$ satisfy the equivalent of (A2)-(A6), rewritten with $\hat{w}_i$ instead of with v.[22]

Assume that in period 1, the agents' productivities are affected by whatever they have learned through their work in period 0. In particular, let x_i represent the productivity of agent i in period 1. Suppose further that in period 1 a coalition S can generate a value given by

$$v(S,\alpha(S)|x) \equiv \sum_{i\in S} w_i(S,\alpha(S)|x),$$

where $w_i(S,\alpha(S)|x)$ is the private benefit accruing to i from the coalition. Again, assume that the w_i's satisfy the equivalent of (A2)-(A6). In this period the x_i's are no longer choice variables, but are fixed from period 0:

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[22] We continue to assume that C_i satisfies (A1), and α satisfies (B1)-(B3).

notice that, because we are not dealing with a many-period model, we ignore any further choice of action in period 1.

In both periods, since the value functions are superadditive (they satisfy (A5)), total surplus is maximized by forming the grand coalition $\underline{S}$. As before, we suppose that this surplus is divided according to the agents' Shapley values.

The order of events is as follows. After appropriate transfer payments have been made at the start of period 0 (to divide the anticipated total value from both periods), each agent i privately chooses how much to work, x_i, during period 0 and enjoys his private benefit $\hat{w}_i$ at cost C_i. As in Section 2, we assume that the agents cannot contract in period 0 about transactions in period 1. So at the start of period 1, there are further transfer payments to divide the total value $v(\underline{S},\underline{A}|x)$ from that period according to Shapley value.

Let the first-best social surplus be attained at $x = x^*$. From (A1) and (A2), x^* is characterised by the first order conditions

$$\hat{v}^i(\underline{S},\underline{A}|x^*) \quad + \quad v^i(\underline{S},\underline{A}|x^*) \quad = \quad C_i'(x_i^*) \qquad \text{for all } i \tag{5.1}$$

where $\hat{v}^i(\underline{S},\underline{A}|x) \equiv \frac{\partial}{\partial x_i}\hat{v}(\underline{S},\underline{A}|x)$, and likewise for v^i.[23]

Acting noncooperatively, agent i will choose x_i in period 0 to maximize

$$w_i(\underline{S},\underline{A}|x_i,x_{-i}) \quad - \quad C_i(x_i)$$

$$+ \quad \sum_{S|i\in S} p(S) \; [\; v(S,\alpha(S)|x_i,x_{-i}) - v(S\backslash\{i\},\alpha(S\backslash\{i\})|x_i,x_{-i}) \;]$$

-- where x_{-i} is the vector $(x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_I)$ of the other agents' choices of work level, and p(S) is the probability function specified in Section 2. From (A1)-(A3), the Nash equilibrium $x_i^e(\alpha)$, say, for x is

[23] Notice that there is no discount factor since we are measuring everything in date 1 dollars.

they conclude that agent i has ability $\theta_i = y_i - x_i^e(\alpha)$.[24] (In contrast to previous sections, we suppose that x_i remains unobservable to agents other than i.) Agent i can exploit this assessment to get a larger share of the surplus at date 1. The important point is that the extent to which he tries to fool the others in this way will depend on how much his bargaining position would be improved by his being perceived as having higher ability; and this in turn will depend on the allocation of assets at date 1. A well-chosen control structure α will induce more effort from the agents in period 0 as they each attempt to boost their reputations at date 1.

We can retain much of the formalism of the above learning-by-doing model. Let the net private marginal benefit in period 0 accruing to some agent i in a coalition S be $\hat{w}_i(S,\alpha(S)|x + \hat{\theta}) - C_i(x_i)$, where x is the vector of actions $(x_1,\ldots,x_I)$ and $\hat{\theta}$ is the vector of true abilities $(\hat{\theta}_1,\ldots,\hat{\theta}_I)$. The coalition thus generates a gross value $\hat{v}(S,\alpha(S)|x + \hat{\theta})$ equal to $\sum_{i\in S} \hat{w}_i(S,\alpha(S)|x + \hat{\theta})$.

In period 1, assume that the agents make no effort.[25] Let the private marginal benefit accruing to agent i in a coalition S be $w_i(S,\alpha(S)|\hat{\theta})$. We will assume that w_i is independent of agent i's own ability, so that we can write $w_i = w_i(S,\alpha(S)|\ 0, \hat{\theta}_{-i})$.[26] Accordingly, the private benefit that agent i in coalition S <u>believes</u> will accrue to him in period 1 is

$$w_i(S,\alpha(S)|0,\theta_{-i}) \tag{5.3}$$

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[24] We assume that the distribution of θ_i has full support, in the sense that no observed y_i is ever a zero probability event.

[25] In a many period-model, the agents would make an effort in period 1 to maintain their later reputations.

[26] We also assume that (i) C_i satisfies (A1); (ii) $\hat{w}_i$ satisfies (A2)-(A6), with $\hat{w}_i$ replacing v and $(x + \hat{\theta})$ replacing x; and (iii) w_i satisfies (A2)-(A6), with w_i replacing v, θ_{-i} replacing x_{-i}, and 0 replacing $\bar{x}_i$.

characterised by the first order conditions

$$\hat{w}_i^{\,i}(\underline{S},\underline{A}|x^e) \;+\; \sum_{S|i\in S} p(S)v^i(S,\alpha(S)|x^e) \;=\; C_i'(x_i^{\,e}) \quad \text{for all } i, \tag{5.2}$$

where $\hat{w}_i^{\,i}(\underline{S},\underline{A}|x) \;\equiv\; \frac{\partial}{\partial x_i}\hat{w}_i(\underline{S},\underline{A}|x)$.

Comparing (5.1) and (5.2), we see from assumption (A6) and condition (B2) that each agent i's private marginal return from work is less than the social marginal return for a given x_{-i}. It is straightforward to show that a generalization of Proposition 0 holds: i.e., $x^e(\alpha) \le x^*$, and any change in the control structure α which improves the agents' private marginal returns from work will raise $x(\alpha)$ and welfare.

Clearly, then, in this model all the propositions and results from Sections 3 and 4 go through unchanged.

Signaling Type Through Choice of Action

This is a variant on a model of reputation due to Holmstrom (1982). The idea is that each agent has a characteristic -- ability -- which neither he nor the outside world knows. The other agents receive a signal which is an amalgam of his ability and how much effort he exerts; by putting in more effort he can try to convince people that he is of high ability, i.e. he can try to boost his reputation. Of course, in equilibrium the other agents are able assess his optimal effort level, and discount it accordingly, so they are not fooled. But this does not mean that he can slack; he still has to work hard so as not to convey a poor signal about his ability and lose his reputation.

This general idea can be applied to our framework as follows. Consider a two-period model. Each agent i has a true ability θ_i, drawn from a publicly-known ex ante distribution. In period 0, agent i (knowing the distribution of θ_i but not its value) takes an action x_i, and we suppose that all agents observe $y_i = (x_i + \theta_i)$. This means that, given a period 0 Nash equilibrium of actions $x_i^{\,e}(\alpha)$, when the other agents observe $y_i = x_i^{\,e}(\alpha) + \theta_i$, where $\theta_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_I)$ is his assessment of other agents' abilities. The perceived benefits w_i given by (5.3) add up to the value of the coalition in period 1:

$$v(S, \alpha(S) | \theta) = \sum_{i \in S} w_i(S, \alpha(S) | 0, \theta_{-i}).$$

The assumption that w_i is independent of agent i's own ability allows us to avoid asymmetric information in the bargaining at the start of period 1: if agent i chooses to deviate from $x_i^e(\alpha)$ in period 0 (and thus convey a false signal) the bargaining at date 1 will still in essence be bargaining with complete information.[27] Thus we can continue to assume that bargaining leads to division according to the agents' (perceived) Shapley values.

The first-best social surplus is attained at $x = x^*$, where from (A1) and (A2), x^* is characterized by the first order conditions

$$\frac{\partial}{\partial x_i} E_\theta \{\hat{v}(\underline{S}, \underline{A} | x + \theta)\} \Big|_{x=x^*} = C_i'(x_i^*) \quad \text{for all i.} \tag{5.4}$$

From (A1)-(A3) the Nash equilibrium $x = x^e(\alpha)$ is characterised by

$$\frac{\partial}{\partial x_i} E_\theta \{\hat{w}_i(\underline{S}, \underline{A} | x+\theta)\} \Big|_{x=x^e(\alpha)} + E_\theta \left\{ \sum_{S | i \in S} p(S) \sum_{j \in S \setminus \{i\}} \frac{\partial}{\partial \theta_i} w_j(S, \alpha(S) | 0, \theta_{-j}) \right\}$$

$$= C_i'\left(x_i^e(\alpha)\right) \quad \text{for all i.} \tag{5.5}$$

Comparing (5.4) and (5.5), we see that the equilibrium vector $x^e(\alpha)$ of effort in period 0 is not necessarily below the first-best level x^*. For example, if period 0 were short relative to period 1, then agents may overwork

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[27] We can explain this in terms of our noncooperative model for Shapley value given in Appendix A. If agent i is to make an offer, he will take into account the other agents' assessments of his ability in determining their reservation prices. But any offer that they make him will be independent of their assessment of his ability since, by assumption, his own payoff and reservation price are independent of his true ability. Therefore no breakdown in bargaining occurs (if agent i deviates from $x_i^e(\alpha)$) as a result of other agents offering him the "wrong" price.

at maintaining their reputations: i.e., the second term in the LHS of (5.5) -- the reputation-building effect -- may dominate. However, if we assume that the inefficiency in period 0 more than outweighs the reputation-building effect, then $x(\alpha)$ will indeed be below x^*. It is straightforward to confirm that in these circumstances, changing the control structure α so as to give each agent a greater incentive to build a reputation will increase welfare. All the propositions and results from Sections 3 and 4 then apply.

6. CONCLUDING REMARKS

The purpose of this paper has been to provide a framework for analyzing the boundaries of a firm. Our approach is based on the idea that an important force motivating an economic agent is the future reward that the agent can anticipate from his current actions. In a world of transaction costs and incomplete contracting, this reward will depend on the agent's future bargaining power, which in turn will be sensitive to who owns and controls the assets that the agent requires access to in order to be productive. We have used this idea to understand the optimal assignment of assets. We have shown that an agent is more likely to own an asset if his action is sensitive to whether he has access to the asset and is important in the generation of surplus (Proposition 1); or if he is a crucial trading partner for others whose actions are sensitive to whether they have access to the asset and are important in the generation of surplus (Proposition 5). In addition, starting from a situation in which some agent 1 owns an asset a_1, worked on by workers {w1} and some agent 2 owns an asset a_2 worked on by workers {w2}, a move to common ownership of both assets by agent 1 is likely to increase overall efficiency to the extent that (i) the assets are strongly complementary; (ii) 1 is an important trading partner for 2 and the workers {w2}; (iii) 2 is dispensable and/or his investment is not particularly important; and (iv) 1 has an important investment and/or the workers {w1} have important investments (in aggregate). On the other hand, it is likely to decrease overall efficiency to the extent that (i) the assets are economically independent; (ii) 2 is an important trading partner for the workers {w2}; (iii) 1's investment is not particularly important; and (iv) 2 has an important investment and/or the asset a_2 is idiosyncratic to him.

An important assumption underlying the analysis is the idea (borrowed from Grossman-Hart) that a firm can be identified with some physical assets and that a key right provided by ownership is the ability to exclude people from the use of these assets. We have argued that this authority over physical assets provided by ownership translates into authority over human assets: party 2 will put more weight on party 1's objectives if 2 is an employee of 1 working with assets owned by 1 than if 2 is an independent contractor working with his own assets. We have emphasized the role of tangible assets such as machines or location (or inventories or client lists) in the analysis, but we suspect that the ideas may generalize to intangible assets such as good will. Some non-human assets are essential for the argument, however, and in fact we suspect that they are an important ingredient of any theory of control or authority or of the firm. The reason is that in the absence of any non-human assets, it is unclear what authority or control means. (Authority over what? Control over what? Surely not over workers' human capital, in the absence of slavery?) It is true that even in the absence of physical assets in some special cases an optimal contract may give one party so much power that it is as if that party had authority over the relationship.[28] However, we do not see why the granting of authority should be a general feature in such a situation. In contrast, some individual or group must be given (residual rights of) control over physical assets; that is, in the presence of physical assets, some notion of authority seems almost inevitable.

This view of physical assets and authority can shed light on the well-known criticism that Alchian and Demsetz (1972) made of Coase's (1937) paper. Coase argued that the key difference between an employer-employee relationship and a relationship between independent contractors is that whereas an employer can tell an employee what to do, one independent contractor must persuade another independent contractor to do what he wants through the use of prices.[29] Alchian and Demsetz criticized this view, arguing that an employer typically cannot force an employee to do what he wants, he can only ask him and fire the employee if he refuses; which is no different from one independent contractor firing another (quitting their relationship) if he is unhappy with the latter's performance (an employer can "fire or sue, just as I can fire my grocer by stopping purchases from him or sue him for delivering faulty products" (Alchian and Demsetz, p.777)). Our approach in some sense provides a reconciliation of these two positions. While it follows Alchian and Demsetz in not distinguishing between the contractual form or nature of sanctions in the two relationships, it is able to capture the idea that one agent is more likely to do what another agent wants if they are in an employment relationship than if they are independent contractors. That is,

[28] This is the approach followed in Rabin (1988). For an early discussion of authority, see Barnard (1938).

[29] Similar descriptions of the employment relation have been given by Simon (1951) and Arrow (1974).

the reason the manager of Alchian and Demsetz's grocery store will be more likely to follow their wishes if he is an employee of theirs than if they are mere customers is that in the former case his future livelihood depends on them (they control the assets the manager intends to work with), whereas in the latter case it does not.

Let us turn to some of the other assumptions underlying the analysis. An important one is the idea that in a world of incomplete contracts ex-post bargaining is efficient and that inefficiencies arise only with respect to ex-ante actions. This assumption may not be unreasonable when small numbers of people are involved in the bargaining process, but it stretches credulity when large numbers are required to consummate the gains from trade. At the same time it makes the analysis tractable and it is also difficult to relax: almost all symmetric information bargaining models lead to ex-post efficiency, and the introduction of asymmetric information raises a whole host of new questions. Furthermore, our feeling is that in stressing ex-ante inefficiencies, we may be picking some of the same effects that an ex-post inefficiency model would generate. For example, an ad hoc way of introducing ex-post inefficiencies is to assume that any ex-post trade or agreement between two individuals will with some small probability not be consummated even if it is mutually advantageous. This assumption implies roughly that assets should be allocated in such a way at date 0 as to minimize the number of new agreements that have to be reached at date 1. This seems likely to lead to the conclusion that an agent who is crucial for the generation of surplus should have ownership rights (agreement will have to be reached with the agent anyway and so why increase the number of agreements necessary by giving ownership rights to others); and that highly complementary assets should be owned together (since coordination of these assets is crucial, the total number of agreements is reduced by having one person control both of them). But these conclusions are also implications of the ex-ante analysis. To put it another way, the ex-ante analysis appears able to capture some of the coordination issues that are central to an ex-post perspective. This is not to deny, of course, that in future work it would be very desirable to analyze the consequences of ex-post inefficiencies in a systematic matter.

There are a number of other extensions of the analysis that are called for. The model that we have presented ignores pay-off uncertainty; risk aversion; and wealth constraints. This means that the issue of how an investment should be financed, and how ownership rights should be allocated between those who finance it and those who manage it play no part in our analysis. Yet we believe this issue is important to an understanding of many firms.[30] In addition we have ignored problems having to do with the dissemination of information. That is, we have avoided the issue of how coordination takes place between individuals with different sources of information, but possibly similar goals.[31] An indication of this is that in our model an employer never has to tell an employee what to do: the employee simply figures it out himself and acts accordingly. It seems very desirable to relax this assumption in future work.

Two other assumptions are also worth relaxing. First, we have supposed that each agent's investment or action enhances his own productivity, but not that of the assets he works with (see footnote 11). There is no reason, however, why the analysis could not be generalized to allow for asset-enhancing investments. Secondly, we have supposed that an agent's action or investment is a scalar. In addition, due to the nature of the externality imposed on others, agents underinvest. Asset ownership is a way of mitigating this underinvestment. In fact if there were no constraints on ownership, underinvestment would be minimized and social surplus maximized by giving each agent 100% ownership of every asset. In a richer setting, where an agent chooses what <u>type</u> of action to take as well as what level, this may no longer be the case. For example, if agent 1 takes an action and later trades with agent 2, then it may be positively undesirable to give 1 ownership of the asset he works with. The reason is that, while this would encourage 1 to invest a lot, it may also cause him to choose the wrong type of investment -- he may try to improve his bargaining position with 2 by investing in such a way that he can supply 2′ as well as 2. Granting 2 ownership rights may avoid this since agent 1 knows that he must bargain with 2 whatever happens.

In spite of the many restrictive assumptions that we have made, we believe that our analysis has identified some of the forces which determine

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[30] Recent papers that investigate the financing issue are Aghion and Bolton (1988), Grossman and Hart (1988) and Harris and Raviv (1988).

[31] This is the focus of the team theory literature; see Marschak and Radner (1972).

the boundaries of the firm. For example, it can explain why a firm faces first increasing and then decreasing returns to scale (increasing returns due to the positive benefits of coordinating complementary assets (Proposition 7) and decreasing returns due to the inefficiencies of centralized control as new managers with economically independent roles must be brought in (Proposition 9)). And, as we have seen in Section 4, it can throw light on ownership arrangements in situations where several firms depend on a single supplier for an input (e.g. an oil pipeline). At the same time, there are other phenomena that the model in its present state cannot explain; e.g. the delegation of control or the determination of hierarchical structure within a firm. Our hope is that such factors can be understood in generalizations of the model. In addition we hope that the model or its extensions will be useful in throwing light on integration decisions in particular empirical contexts.[32] Our purpose in writing this paper has been to provide a framework for analysis; much remains to be done, however, in the application of this framework and its further development.

[32] As discussed in Chandler (1977), say; for a survey of recent empirical work, see Joskow (1988).

APPENDIX A

In this Appendix we sketch a noncooperative game whose unique equilibrium payoff vector assigns each agent their Shapley value (for a different approach, see Gul (1986)).

Preliminary Stage The agents $\{1,2,\ldots,I\}$ are lined up in a random order:

$$j_1 \quad j_2 \quad \ldots \quad j_I, \text{ say.}$$

Stage 1 Agent j_1 makes an offer to agent j_2. The offer is in the form of a contract for agent j_2 to sign. No restrictions are placed on the type of contract which can be offered. Among other things, the contract may specify what offer agent j_2 has to make to agent j_3 at stage 2. Once the offer has been made, it is either accepted (signed) by agent j_2 or rejected; and in either event the game then moves on to Stage 2.

Stage 2 Agent j_2 makes an offer to agent j_3. Again, the offer is in the form of a contract for agent j_3 to sign, and no restrictions are placed on the type of contract which can be offered. Among other things, the contract may specify what offer agent j_3 has to make to agent j_4 at stage 3. Once the offer has been made, it is either accepted (signed) by agent j_3 or rejected; and in either event the game then moves on to Stage 3.

...

Stage I-1 Agent j_{I-1} makes an offer to agent j_I. Again, the offer is in the form of a contract for agent j_I to sign, and no restrictions are placed on the type of contract which can be offered. Agent j_I can either accept by signing the contract, or reject.

Final Stage Production and trade are carried out according to the agreed contracts.

We will show that in any (subgame perfect) equilibrium of this game, agent j receives his Shapley value

$$B_j = \sum_{S|j\in S} p(S) \ [\ v(S) - v(S\backslash\{j\})\ ]$$

where $v(S)$ = the (maximum) value that a coalition S can generate, and

$$p(S) = \frac{(s-1)!\ (I-s)!}{I!}, \text{ where } s = |S|, \text{ the number of agents in S.}$$

Suppose that a particular line-up $j_1, j_2, \ldots, j_I$ is realized at the Preliminary Stage. Then we will prove by backward induction ($i = I, I-1, \ldots, 1$) that in the subgame starting after Stage i-1, if agent j_i refused to sign the contract offered at Stage i-1 then he has an equilibrium payoff

$$v(\{j_i, \ldots, j_I\}) - v(\{j_{i+1}, \ldots, j_I\}).$$

We adopt the convention that $v(\{j_{i+1}, \ldots, j_I\})$ equals $v(\phi) = 0$ when $i = I$.

First, consider agent j_I. If he refused to sign the contract offered at Stage I-1, then he is left to his own resources and obtains $v(\{j_I\})$. The first step in the induction is thus proved.

Next, suppose that the induction hypothesis holds for agents $j_{i+1}, \ldots, j_I$ in the subgames starting after Stages $i, \ldots, I-1$. At Stage i, agent j_i wants to agree a contract with agents $j_{i+1}, \ldots, j_I$ which generates the (maximum) value $v(\{j_i, \ldots, j_I\})$. But by hypothesis we know that the least agent j_i can offer agent j_{i+1} at Stage i is $v(\{j_{i+1}, \ldots, j_I\}) - v(\{j_{i+2}, \ldots, j_I\})$. Moreover, if agent j_{i+1} is to sign, then by hypothesis we know the least he in turn can offer agent j_{i+2} at Stage i+1 is $v(\{j_{i+2}, \ldots, j_I\}) - v(\{j_{i+3}, \ldots, j_I\})$. And so on for agent j_k -- $j+2 \le k \le I-1$ -- offering a contract to agent j_{k+1} at Stage k. Thus at Stage i, if agent j_i is ultimately going to elicit the cooperation of $j_{i+1}, \ldots, j_I$ in generating the value $v(\{j_i, \ldots, j_I\})$, he must be prepared to give a total share of at least

$$\sum_{k=i+1}^{I} [v(\{j_k, \ldots, j_I\}) - v(\{j_{k+1}, \ldots, j_I\})\ ] = v(\{j_{i+1}, \ldots, j_I\})$$

to the other agents in the coalition $\{j_i, \ldots, j_I\}$. This leaves agent j_i

himself with at most $v(\{j_i,\ldots,j_I\}) - v(\{j_{i+1},\ldots,j_I\})$.

The rules of the game prevent agent j_i from contracting directly with any agent other than j_{i+1}. Nevertheless, he can obtain his maximum payoff of $v(\{j_i,\ldots,j_I\}) - v(\{j_{i+1},\ldots,j_I\})$ by offering to agent j_{i+1} at Stage i a multi-layered contract of the form:

> "We agree to participate in the generation of the value $v(\{j_i,\ldots,j_I\})$. And in return for your share $v(\{j_{i+1},\ldots,j_I\}) - v(\{j_{i+2},\ldots,j_I\})$, you agree to offer agent j_{i+2} at Stage i+1 a contract of the form:
>
> > "We agree to participate in the generation of the value $v(\{j_i,\ldots,j_I\})$. And in return for your share $v(\{j_{i+2},\ldots,j_I\}) - v(\{j_{i+3},\ldots,j_I\})$, you agree to offer agent j_{i+3} at Stage i+2 a contract of the form:
> >
> > .... etc.

Now from the induction hypothesis, each of the agents will in turn just be willing to accept their contract. Thus agent j_i can indeed obtain $v(\{j_i,\ldots,j_I\}) - v(\{j_{i+1},\ldots,j_I\})$, as claimed. This completes the proof by induction.

Now that this has been established, it immediately follows from simple probability theory that for each agent j the equilibrium payoff of the entire game -- starting at the Preliminary Stage -- is given by the Shapley value B_j. (Recall that the agents are assumed to be risk neutral.)

APPENDIX B

In this Appendix, we prove Propositions 0,8 10 and 11.

Proof of Proposition 0 The Nash equilibrium investments $x_i^e(\alpha)$ are characterised by (2.2), which using assumption (A3) can be re-written (restricting attention to those agents i who have investments, $\bar{x}_i > 0$):

$$\nabla g(x;\alpha) \Big|_{x=x^e(\alpha)} = 0,$$

$$\text{where} \quad g(x;\alpha) \equiv \left[\sum_{\text{all } S} p(S)v(S,\alpha(S)|x) - \sum_{i=1}^{I} c_i(x_i) \right].$$

Now consider the change in control structure from α to $\hat{\alpha}$ given in the Proposition. By assumption,

$$\nabla g(x;\hat{\alpha}) \geq \nabla g(x;\alpha) \quad \text{for all } x.$$

Define $f(x;\lambda) \equiv \lambda g(x;\hat{\alpha}) + (1-\lambda)g(x;\alpha)$ for $\lambda \in [0,1]$. And define $x(\lambda)$ to solve $\nabla f(x;\lambda) = 0$. Totally differentiating we obtain

$$H(x;\lambda)dx(\lambda) = -[\nabla g(x;\hat{\alpha}) - \nabla g(x;\alpha)]d\lambda,$$

where $H(x;\lambda)$ is the Hessian of $f(x;\lambda)$ (with respect to x). By assumptions (A1) and (A2), $H(x;\lambda)$ is negative definite. Also, by assumption (A4), the off-diagonal elements of $H(x;\lambda)$ are nonnegative. So $H(x;\lambda)^{-1}$ is a nonpositive matrix (see, for example, page 393 of Takayama (1985), Theorem 4.D.3, (III′′) and (IV′′)). Hence $\partial x(\lambda)/\partial\lambda \geq 0$, and $x(1) \geq x(0)$, or $x^e(\hat{\alpha}) \geq x^e(\alpha)$.

Similar reasoning can be used to show that $x^e(\alpha) \leq x^*$. Simply replace $g(x;\hat{\alpha})$ in the above argument by the social surplus $W(x)$, and replace $f(x;\lambda)$ by $\lambda W(x) + (1-\lambda)g(x;\alpha)$. As was pointed out in Section 2, assumption (A6) and condition (B2) together imply

$$\nabla W(x) \geq \nabla g(x;\alpha) \quad \text{for all } x.$$

And by assumptions (A1) and (A2), the Hessian of $W(x)$ is negative definite. The rest of the argument is the same; hence $x^* \geq x^e(\alpha)$.

Finally, since $\nabla W(x^e(\hat{\alpha})) \geq \nabla g(x^e(\hat{\alpha})) = 0$ and $x^e(\hat{\alpha}) \geq x^e(\alpha)$, it follows from the concavity of $W(x)$ that $W(x^e(\hat{\alpha})) \geq W(x^e(\alpha))$. Q.E.D.

Proof of Proposition 8 In this Proposition, a control structure α may be stochastic. Suppose that under some realisation of α, an agent k has some control rights even though he is dispensable and has no investment. Then consider a new stochastic control structure $\tilde{\alpha}$, which is the same as α except that (in this realisation) agent k's control rights are now allocated randomly at date 1 to the other (I-1) agents, on an equal basis. It is straightforward to confirm that each realisation of $\tilde{\alpha}$ satisfies (B1)-(B3), as required.

The expected change $EB_i^{\,i}(\tilde{\alpha}) - B_i^{\,i}(\alpha)$ in marginal return on investment for some agent $i \neq k$ will be (letting $s = |S|$),

$$\sum_{S \,|\, i \in S,\ k \notin S} \frac{s}{(I-1)}\, p(S) \;[\, v^i(S,\alpha(S \cup \{k\})) \;-\; v^i(S,\alpha(S)) \,]$$

$$-\sum_{S \,|\, i \in S,\ k \in S} \{1 - \frac{(s-1)}{(I-1)}\}\, p(S) \;[\, v^i(S,\alpha(S)) \;-\; v^i(S,\alpha(S\backslash\{k\})) \,].$$

Since agent k is dispensable, this equals

$$\sum_{S \,|\, i \in S,\ k \notin S} q(S,k) \;[\, v^i(S,\alpha(S \cup \{k\})) \;-\; v^i(S,\alpha(S)) \,]$$

-- where
$$\begin{aligned} q(S,k) &= \frac{s}{(I-1)}p(S) - \{1 - \frac{s}{(I-1)}\}p(S \cup \{k\}) \\ &= \frac{s!\,(I-s-1)!}{I!}\,[\,\frac{(I-s)}{(I-1)} - \{1 - \frac{s}{(I-1)}\}\,] \\ &= \frac{s!\,(I-s-1)!}{I!}\,[\,\frac{1}{(I-1)}\,] \;>\; 0. \end{aligned}$$

Hence by (A6), the marginal incentive to invest by any agent $i \neq k$ will not be reduced by the change in control structure. And by assumption, agent k himself has no investment. So from Proposition 0, welfare will be higher under the new control structure $\tilde{\alpha}$ than under α. Q.E.D.

Proof of Proposition 10 Suppose that asset a_n is essential to agent i, and that under some control structure α another agent j owns a_n. Then consider a new control structure $\hat{\alpha}$ in which j owns all the assets $\underline{A}$. $\hat{\alpha}$ trivially satisfies requirements (B1)-(B3).

The change $B_i^{\,i}(\hat{\alpha}) - B_i^{\,i}(\alpha)$ in marginal return on investment for agent i will be

$$\sum_{S \,\middle|\, \substack{i \in S \\ j \in S}} p(S)\, v^i(S,\underline{A}) \quad - \sum_{S \,\middle|\, \substack{i \in S \\ j \in S}} p(S)\, v^i(S,\alpha(S))$$

$$+ \sum_{S \,\middle|\, \substack{i \in S \\ j \notin S}} p(S)\, v^i(S,\phi) \quad - \sum_{S \,\middle|\, \substack{i \in S \\ j \notin S}} p(S) v^i(S,\alpha(S)).$$

The last summation is zero, because $j \notin S$ implies $a_n \notin \alpha(S)$ and, by assumption, a_n is essential to agent i. Also, (A6) implies that the first summation is no less than the second. So, ignoring the effects of changes in other agents' investment levels, agent i's marginal incentive to invest will be increased by the change in control structure.

Q.E.D.

Proof of Proposition 11 Suppose α is a control structure in which agent j owns asset a^*, and agent i owns asset a. Define a new control structure $\hat{\alpha}$ which is the same as α except that now agent j exercises all the control that agent i used to exercise under α. It is straightforward to confirm that $\hat{\alpha}$ satisfies requirements (B1)-(B3).

In what follows, we ignore the effect of changes in the level of investment by agents other than than agents i,j and {w}. Clearly, agent j's marginal incentive to invest will be higher under $\hat{\alpha}$ than under α, since he now controls more assets. And agent i's incentives will not change: under both α and $\hat{\alpha}$ he has to be in a coalition with j in order to gain access to a^*, so it does not matter that j has his control rights.

The change $B_w^{\,w}(\hat{\alpha}) - B_w^{\,w}(\alpha)$ in marginal return on investment for a worker w will be

$$\sum_{S \mid \substack{j,w \in S \\ i \notin S}} p(S)\, v^{w}(S,\alpha(S\cup\{i\})) \quad - \quad \sum_{S \mid \substack{i,w \in S \\ j \notin S}} p(S)\, v^{w}(S,\alpha(S))$$

$$+ \quad \sum_{S \mid \substack{i,w \in S \\ j \notin S}} p(S)\, v^{w}(S,\alpha(S\setminus\{i\})) \quad - \quad \sum_{S \mid \substack{j,w \in S \\ i \notin S}} p(S)\, v^{w}(S,\alpha(S)).$$

The last two summations are both zero, because if a coalition does not contain agent i, then under α it does not control a which is essential to w. And the first summation is no less than the second, because of (A6) together with the facts that (i) agent i is dispensable, and (ii) p(S) is a function only of the size, not the composition, of coalitions S. Thus w's marginal incentive to invest will be increased by the change in control structure.

Q.E.D.

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