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OPTIMAL DISTRIBUTION AND TAXATION OF THE FAMILY

Louis Kaplow

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Harvard Law School
Cambridge, MA 02138

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Abstract

Income tax burdens on family units are adjusted to reflect differences in ability to pay attributable to whether the unit consists of a single individual or a married couple and how many dependents are present. Substantial controversy exists over the appropriate forms of adjustment, and existing approaches to taxation of the family vary greatly across jurisdictions. This article derives equitable relative tax burdens for different family configurations from a utilitarian welfare function. The analysis considers how relative burdens should depend on the extent to which resources are shared among family members, the existence of economies of scale, the presence of altruism among family members, whether expenditures on children should be viewed as part of parents' consumption, and the possibility that some family members (children) have different utility functions from others.

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1. Introduction

How should tax burdens vary between single individuals and married couples? How should burdens reflect the number of dependents living in the family unit? These questions continue to be controversial in debates about the proper form of income taxation,[1] and the answers vary substantially among jurisdictions and have changed over time.[2]

The present investigation asks what distribution of tax burdens (and thus of income) maximizes the sum of utilities in a population with single individuals and families. The analysis focuses solely on distribution, and thus abstracts from incentive considerations.[3] It differs from prior work in attempting to ground distributive judgments in an explicit welfare function rather than upon refinements of intermediate concepts, notably ability to pay.[4]

Section 2 considers a number of cases. It begins briefly with the simplest situation, in which members of a family share income equally, realize no economies of scale, have no utility interdependence, and have identical

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[1] See, e.g., Groves (1963), Sunley (1977). It is familiar that the issues are similar for a consumption tax. Moreover, issues arising with the design of social security (both taxes and benefit formulas) and welfare programs are often similar. And the optimal design of each obviously depends on that of the others.

[2] See Pechman and Engelhardt (1990). For example, with regard to married couples, the United States began with individual filing, changed to income splitting, and now has separate schedules under which singles are treated more generously. Many countries use individual filing, with varying rules for unearned income and deductions. France allows partial splitting between parents and children.

[3] Previous analysis has considered how current tax rules discourage the second spouse (typically the wife) from undertaking market employment. See Boskin (1974), Boskin and Sheshinski (1983), Rosen (1976, 1977). The higher tax burdens faced by second earners (both directly and through the loss of untaxed imputed income) may be inefficient given that the wage elasticity is substantial for second earners, but not for first earners. In addition to labor force participation, taxation of the family affects incentives to get married and divorced and incentives to bear children.

[4] Familiar arguments include that income splitting is appropriate if (but only to the extent that) married couples share income, that couples tax burdens should be higher because they enjoy economies of scale, and that burdens should be lower (using deductions, credits, or some form of splitting) in the presence of dependents. See, e.g., Brannon and Morss (1973), Steuerle (1983), Vickrey (1947).

utility functions. The optimal rule in this instance involves simple income splitting: each of n family members should be taxed (or subsidized) as though each has 1/n of family income.

The bulk of the analysis considers variations on this simple case, often reaching conclusions that differ from conventional wisdom. When sharing is unequal, the optimal tax burden on the family may be higher or lower than when sharing is equal: that taxes will come disproportionately at the expense of the higher consuming members favors a heavier tax burden, but that those receiving a smaller share have a higher marginal utility of income favors a lighter burden. When economies of scale are present, the optimal burden on families might be higher or lower: higher because marginal utility is less but lower because the presence of economies of scale makes the family a more efficient producer of effective income and thus of utility. When family members are altruistic toward one another, a lighter burden is optimal because the utility of some individuals is counted more than once. When expenditures on children are viewed as part of parents' consumption, the analysis is similar to that for altruism. When some family members (e.g., children) reach the same utility level with less income, lighter taxation is optimal because such families are more efficient utility generators.

Section 3 examines how the results would be affected by changing various assumptions. First, it considers different evaluative criteria -- maximin and a libertarian approach. Second, it discusses how family tax burdens should reflect incentives to earn income, marry and divorce, and have children. Third, it addresses the view that the sharing of income within families should be analyzed as an exchange relationship. Section 4 offers brief concluding remarks.

2. Equitable Taxation of the Family

2.1. Framework for Analysis

A representative single individual (often referred to simply as an "individual") has utility function $u(\cdot)$ that is strictly concave in the individual's income y_I.[5] (Income refers to that which is available after payment of taxes or receipt of subsidies.) The representative family has n members, family income is y, and each member i has strictly concave utility $u_i(\cdot)$ and income of $\alpha_i y$, where $\sum_i \alpha_i = 1$. Because the analysis considers only questions of the equitable distribution of the tax burden, income and family composition are taken to be exogenous.

The social objective is to maximize the sum of utilities for all people. That is, a single individual's utility counts once and each family member's counts once.[6]

In the case in which there are only single individuals (i.e., the family has one member), the optimal tax-distribution policy is familiar:[7] income available for consumption for each individual should equate marginal utilities of income. If individuals have the same utility functions, incomes should be equalized. Translated into a tax-transfer policy, the tax rate on income above the mean is 100 percent and individuals with income below the mean receive a subsidy raising their disposable income to the mean.

For the case in which each family member has the same utility function as the individual ($u_i(\cdot) = u(\cdot)$ for all i) and in which family members share family income equally ($\alpha_i = 1/n$ for all i), the result is straightforward: income should be distributed so that $y_I = y/n$; the individual and each family member have the same disposable income and the same marginal utility of income. (In the remainder of the analysis, it often will not be feasible to

[5] Because this is a one-period model, no distinction is made between income and wealth, and important life-cycle considerations that are relevant as family membership changes over time are not considered.

[6] It is common to consider each family unit as having a single utility function. E.g., Boskin and Sheshinski (1983), Rosen (1976). If one defined $u_f = \sum_i u_i(\alpha_i y)$ (as is done for some purposes below), the results would be the same. The treatment here focuses on each member's utility function because it is hard to examine the effect of family income (y) on a family's utility (u_f) without making first determining how income is allocated among family members and what utility each member receives from his share of family income. Related, the standard normative premise is that individual's welfare is what counts. The distinction is particularly important where a family welfare measure reflects an average and family size varies. See Kondor (1975). (If instead one invoked a notion of a family's utility that was independent of the utility of its members, a different approach would be required.) For positive purposes, however, a family utility function approach will often be useful. See Willis (1973).

[7] See Pigou (1951).

adopt a redistributive policy that equalizes everyone's marginal utility of income, because marginal utility will not be equated within the family and it is assumed that society cannot enforce redistribution within the family.)

It will be convenient to let θ denote the ratio of individual to family income (y_I/y) when the distribution is optimal. (Thus, in place of y_I, the individual's income will be denoted θy.) In this example, the optimum is $\theta = 1/n$. In other cases, the optimum is characterized completely by the value of θ, which is sufficient to indicate how income is to be distributed between families and single individuals.

In addition, the analysis will consider the case in which $n = 2$, as all of the issues considered here can arise in a family with two members. For example, questions pertaining to how a married couple should be treated relative to a single individual are captured by the case in which the two members are the two spouses. Questions about the treatment of children can view member 1 as the representative parent and member 2 as the representative child. (Most of the cases below will be relevant to both situations.) Thus, the benchmark arising from this simple equal sharing example is that $\theta = 1/2$. If $\theta > 1/2$, families should be treated less generously than under a rule of income splitting, and conversely for $\theta < 1/2$.

2.2. Unequal Sharing

It is often suggested that when family members do not share income equally, the family should be treated less generously than under income splitting. Implicit is the idea that we should, ceteris paribus, treat the family members as if each were an individual with income equal to the share actually received. With progressive marginal rates, this implies a higher total tax burden than with income splitting, which from this perspective is seen to be grounded in the false assumption of equal sharing. In the model here, where the marginal rate is 100 percent throughout, or in any system with constant marginal rates, unequal sharing would have no implication for the family's tax burden from this perspective.

This viewpoint, however, is deficient. It implicitly assumes that if, for example, the family is assessed a tax (which may be negative, a subsidy) t_i on member i, that member i is the one who will bear the burden of the tax. It would seem more reasonable to assume instead that whatever produces the unequal shares in the first instance will determine how taxes are actually borne. Suppose, for example, that one family member consumes twice as much as the other. Then, taxing the family an amount greater than three times the levy on a single individual with income equal to that of the family member who consumes one-third of family income will result in that member having less disposable income than the single individual if the family allocates one-third of its tax burden to that member.

To examine this problem, assume that the shares of income in a family are simply given, and further that the shares are independent of the level of income. Let α denote the first member's share and assume that $\alpha > 1/2$. The family's total utility is

$$(1) \quad u_f = u(\alpha y) + u((1-\alpha)y).$$

To determine the optimal relative burden on families and individuals, one should equate the marginal utility of a dollar to the family (as it will actually be shared) with the marginal utility of a dollar to the individual:[8]

$$(2) \quad \alpha u'(\alpha y) + (1-\alpha)u'((1-\alpha)y) = u'(\theta y).$$

There are competing effects, making it indeterminate whether θ exceeds or is less than 1/2. The first term on the left side is the marginal utility of the first member from increasing family income by a dollar: the marginal utility per dollar actually received, $u'(\alpha y)$, is less than $u'(y/2)$, but the first member receives more than a dollar for each one dollar per capita that the family receives. The second member has a higher marginal utility than $u'(y/2)$ but receives less than a dollar for each dollar per capita increase in family income. Thus either term could be greater or less than $u'(\theta y)/2$. (At $\alpha = 1/2$, it is obvious that each term equals $u'(y/2)/2$.)

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[8] The second-order conditions for this and all other first-order conditions below hold.

To explore the matter further, differentiate the left side of (2) with respect to α:

$$(3)\quad \frac{d^2u_f}{d\alpha dy} = u'(\alpha y) + \alpha y u''(\alpha y) - u'((1-\alpha)y) - (1-\alpha)yu''((1-\alpha)y)$$

$$= u'(\alpha y)[1 - RRA(\alpha y)] - u'((1-\alpha)y)[1 - RRA((1-\alpha)y)],$$

where $RRA(x) = -xu''(x)/u'(x)$ -- the coefficient of relative risk aversion at income x. To interpret expression (3), consider the case of a constant-RRA utility function. Then (3) would reduce to $[u'(\alpha y) - u'((1-\alpha)y)](1 - RRA)$. The assumption that $\alpha > 1/2$ implies that the first component is negative, so this expression would be positive if and only if RRA exceeded one. If this derivative is positive, then the left side of the optimization condition (2) would be higher than when $\alpha = 1/2$, so it is optimal to set $\theta < 1/2$ -- i.e., to treat families more generously than if there were equal sharing. (If one follows Arrow's (1971, p. 98) suggestion of an RRA of 1 for a constant-RRA utility function, the two effects would be offsetting, meaning that no adjustment from $\theta = 1/2$ is appropriate on account of unequal sharing.[9]) The intuition is that if risk aversion is high (low), the benefit of giving additional income to the family is high (low) when sharing is unequal: even though the disfavored member gets a small share of the additional income, his marginal utility will (not) be particularly high relative to that of a single individual with income equal to the per capita family income.

Consider now the possibility that RRA increases with income, as Arrow (1971, pp. 96-98) speculates. Then, it is possible that the optimal θ exceeds (is less than) 1/2 when family income is low (high). Thus, it is possible that when income is low families should be treated less favorably than under income splitting and when income is high they should be treated more favorably.

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[9] For a survey of studies indicating plausible values of RRA, see Choi and Menezes (1992). They analyze a gamble involving the gain or loss of 1 percent of income and compute levels of RRA implied by different probabilities for the gain that produce indifference and suggest that RRA probably exceeds one substantially. Using their approach and a gamble involving 10 percent of income, a constant RRA of 1 (.5, 2) implies that the odds of winning would have to be approximately 52.5 percent (51.25, 55) for an individual to be indifferent.

Finally, consider the case in which shares vary with income. In particular, assume that the member with the larger share gets a greater share as income increases. Then, equating the marginal utility of a dollar to the family with that to the individual yields

$$(4) \quad (\alpha + y\frac{d\alpha}{dy})u'(\alpha y) + (1 - \alpha - y\frac{d\alpha}{dy})u'((1-\alpha)y) = u'(\theta y).$$

This expression differs from (2) in that the first term on the left side receives greater weight and the second correspondingly less. The marginal dollar more favors the higher-share family member than does the total (and thus the average) dollar. The result is that, for a given resulting share, the family should be treated less favorably than in the case in which the unequal shares were constant.[10]

2.3. Economies of Scale

To focus on economies of scale, this section will assume that the two members of the family share income equally.[11] Economies of scale in family consumption are represented as follows: $u_i(y) = u(\beta(y)/2)$, where $\beta(y) > y$. That is, in the family, income of y is "worth" $\beta(y)$. (It is assumed that single individuals do not realize these economies; alternatively, $\beta(\cdot)$ can indicate the additional economies that are possible when individuals are in the same family.[12])

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[10] A special case in which shares decrease with income is that in which, above a certain income level, additional income is consumed entirely by member 1 -- say, the representative adult. For families with income above that level, the optimal rule would provide a tax deduction equal to the amount spent on the children. (This rule would be too generous for lower incomes.) Observe that the optimal utilitarian rule in this context depends on what share children actually receive, not on any measure of what is required for their subsistence. The results differ if the sharing arises from altruism (section 2.4) or pure exchange (section 3.3).

[11] If shares were unequal, economies of scale would no doubt depend on the distribution of income in the family. In particular, one suspects that economies of scale would tend to be less the more unequal were the shares, because economies of scale cannot occur to the extent that one individual does all the consuming.

[12] Single individuals can achieve some economies of scale through joint living arrangements; differences between single individuals and families may arise due to differences in preferences, opportunities, and incentive problems that may be greater with joint living outside the family context. See also note 36. That remaining single involves a voluntary choice need not affect the present analysis. For example, being unable to find another with similar housing tastes or another whom one can love, and thus interact with in a manner where behavior causing negative externalities need not be feared, does

To further simplify, begin with the linear case: $\beta(y) = \beta y$, where $\beta > 1$. Then, $u_i(y) = u(\beta y/2)$. Thus, the requirement that the marginal utility of income to the family be equated with that to the individual is[13]

$$(5) \quad \beta u'(\tfrac{\beta}{2}y) = u'(\theta y).$$

There are two effects that cause θ to diverge from 1/2. First, the left side is weighted by β, indicating that an additional dollar is worth more than a dollar to the family due to economies of scale. This requires that θ be smaller. Second, the argument of $u(\cdot)$ on the left side is $\beta/2$, which exceeds 1/2, indicating that the marginal utility of an "effective dollar" (which costs less than a dollar) is lower for the family because, for a given level of actual income, it is better off already. This requires that θ be larger. If one differentiates (5) with respect to β, it can be shown that

$$(6) \quad \frac{d\theta}{d\beta} = \frac{-u'(\frac{\beta}{2}y)}{yu''(\theta y)}[RRA(\tfrac{\beta}{2}y) - 1].$$

Thus, $d\theta/d\beta$ is positive if and only if $RRA(\beta y/2) > 1$. The intuition is that if relative risk aversion is high, the second of the two effects is relatively more significant, which requires that $\theta > 1/2$. (Recall that, at $\beta = 1$, $\theta = 1/2$.) Equivalently, as risk aversion -- the diminishing marginal utility of income -- becomes unimportant, the greater productivity of the family dominates, and $\theta < 1/2$.

Consider a constant RRA utility function with risk aversion of 1: the two effects would be offsetting, meaning that no adjustment is appropriate on account of economies of scale. If, instead, RRA increases with income, the optimal θ would increase with income -- i.e., lower income families would get a greater relative preference or lesser relative disadvantage on account of

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not suggest any relevant difference in the marginal utility of income. To justify ignoring economies of scale achieved by families, individuals who remain single would have to be more efficient utility generators in a manner that offsets scale economies.

[13] Note that

$$u_f = 2u(\tfrac{\beta}{2}y).$$

scale economies, even if scale economies have the same proportionate effect on income for all income levels.

Consider briefly the more general case of scale economies -- when $\beta(y)$ need not be linear. Equating the marginal utility of a dollar to the family with that to the individual yields

$$(7) \quad \beta' u'\left(\frac{\beta(y)}{2}\right) = u'(\theta y).$$

The weight on the left side is β': it is the marginal scale economy effect that determines the productive benefit of giving the family an additional dollar. But the argument of $u'(\cdot)$ involves $\beta(y)$, the total (and thus the average) scale economy effect. If economies of scale fall with income, as many suspect, the marginal effect will be less than the average effect by a greater amount as income increases, implying that θ should rise with income.[14]

2.4. Altruism

Suppose that family members are altruistic toward one another.[15] In particular, assume that altruism takes the simple form that member i's total utility equals $u_i + \lambda_i u_j$ -- i.e., his own utility in consumption plus some positive portion of the other family member's.[16] In this case, the family's total utility is

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[14] Economies of scale may fall with income in large part because the most significant public good in many families is housing, and expenditures on it are a lower share of high incomes. But if higher incomes in large part buy power and prestige, as some suggest, economies of scale may rise with income, because sources of power and prestige (e.g., wealth accumulation) may be public goods for family members.

[15] See generally Becker (1991). If individuals are envious, the value of the λ_i would be negative and the same analysis would apply. If individuals are altruistic because other family members' having low levels of utility reduces their own utility, the analysis would be unaffected so long as the reduction did not affect the marginal utility of income. (For example, if i's total utility is $u_i - \lambda_i(u_0 - u_j)$, this is equivalent to the total utility formulation given in the text, minus a constant term $\lambda_i u_0$; all the results depend on the marginal utility of income and are thus unaffected, although the effect under a maximin criterion (see section 3.1) would change.)

[16] If one assumed instead that each member's utility were the sum of his own utility in consumption plus the other member's total utility (including his altruistic component), the expression for u_f (8) would be weighted by $1/(1-\lambda_1\lambda_2)$ and the analysis would be similar. (It must be that $\lambda_1\lambda_2 < 1$ for family utility to be bounded.) The greater effect of altruism would further deflate the individual's optimal income relative to the family's.

(8) $u_f = (1 + \lambda_2)u(\alpha_1 y) + (1 + \lambda_1)u(\alpha_2 y)$.

Consider now three possibilities for sharing: equal sharing of income (perhaps stipulated by law or custom), sharing that maximizes u_f, and selfish altruism by one family member (i.e., only one member is altruistic, and that member determines family members' shares in his self-interest).[17]

Assume first that $\alpha_1 = \alpha_2 = 1/2$. Let $\lambda = \lambda_1 + \lambda_2$. Then $u_f = (2+\lambda)u(y/2)$. Equating the marginal utility of a dollar to the family with that to the individual yields

(9) $(1 + \frac{\lambda}{2})u'(\frac{y}{2}) = u'(\theta y)$.

In this case, $\theta < 1/2$. The weight of $1+\lambda/2$ on the left side indicates that the family is a more efficient producer of utility on account of altruism. (The effect is parallel to the first effect noted with economies of scale.[18]) Differentiating (9) with respect to λ yields[19]

(10) $\frac{d\theta}{d\lambda} = \frac{-\theta u'(\frac{y}{2})}{2u'(\theta y)RRA(\theta y)}$.

The right side is negative: as one would expect, the greater the level of altruism, the more generous should be the treatment of the family relative to that of the individual. Also, the greater is risk aversion, the less the adjustment required to equate the marginal utility of income for the individual with the effective marginal utility for the family.

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17 The latter case corresponds to the idea that there might be a single family head who controls family resources. Prior analysis of such family units, as in Becker (1974), examines whether family production will be efficient, but is not concerned with evaluating the distribution of family resources or determining optimal tax burdens in light of the sharing that results.

18 The effects of economies of scale and altruism are similar, although the channels are different. When economies of scale arise because some commodities such as housing are public goods in the family, a given amount of income is able to produce utility to both family members. With altruism, income that directly produces utility to one member creates utility indirectly for the other.

19 One can note further that $y/2 > \theta y$ which implies that $u'(y/2) < u'(\theta y)$, so that (10) implies that $|d\theta/d\lambda| < 1/4RRA$. If, for example, $\lambda = 1/2$ and $u(\cdot)$ has constant RRA of 1, $\theta > .375$.

Now assume that the α_i are set to maximize family welfare. Let α denote the first member's share. The first-order condition for the family's optimization is

$$(11)\quad u'(\alpha y) = \frac{1 + \lambda_1}{1 + \lambda_2} u'((1-\alpha)y).$$

Using this result, one can determine the marginal utility of income to the family and equate it to that for the individual:[20]

$$(12)\quad (1 + \lambda_2)u'(\alpha y) = u'(\theta y).$$

The effect of altruism on the optimal rule is[21]

$$(13)\quad \frac{d\theta}{d\lambda_2} = \frac{-(1 + \lambda_1)u''((1-\alpha)y)\frac{d\alpha}{d\lambda_2}}{u''(\theta y)}.$$

(The result for $d\theta/d\lambda_1$ is symmetric.) Differentiating the family's first-order condition (11), it can be shown that $d\alpha/d\lambda_2 > 0$. Thus, the right side is negative, so again the greater the degree of altruism, the more generous the optimal rule is to the family. Consider the special case in which $\lambda_2 = 0$.[22] Expression (12) indicates that $\theta = \alpha$ -- i.e., the degree to which the family's optimization reduces the altruistic member's share (from 1/2) is the degree to which the social optimization reduces the individual's share.[23]

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[20] Equivalently, one can substitute from the family's first-order condition to yield

$$(1 + \lambda_1)u'((1-\alpha)y) = u'(\theta y).$$

[21] This result is obtained by differentiating (12) with respect to λ_2, and substituting for the numerator using the expression obtained from differentiating the family's first-order condition (11) with respect to λ_2.

[22] Another special case of interest involves $\lambda_1 = \lambda_2$, which produces (from the family's maximization) equal sharing. Then, expression (12) is the same as (9) from the case in which equal sharing was simply assumed, regardless of the relative magnitudes of the λ_i.

[23] The intuition is that the altruist's income in this special case has the same significance in the family's optimization as in the social optimization: it generates utility to a person that counts only once and it is a potential source of income to transfer to member 2, who is the object of altruism -- that the altruist is the person giving up the income is of no particular significance.

Finally, consider the case in which only member 1 is altruistic, and he selects α to maximize his welfare.[24] The family's (his) first-order condition (assuming that λ is sufficiently large for an interior solution to obtain) is

$$(14)\quad u'(\alpha y) = \lambda u'((1-\alpha)y).$$

The marginal utility of family income can be determined using (14) and equated to the marginal utility of income for the individual:

$$(15)\quad \frac{1 + \lambda - \alpha - y\frac{d\alpha}{dy}}{\lambda} u'(\alpha y) = u'(\theta y).$$

Consider two special cases. First, if $\lambda = 1$ -- the family head weights the other member's utility equally with his own -- income will be shared equally within the family and it can be shown that the weight on the left side equals 1.5.[25] The case is equivalent to the first case of altruism in which equal sharing was simply assumed. Thus, the family should be treated more generously than under income splitting. Second, as λ becomes small, there will not be an interior solution to the family's optimization problem, so that $\alpha = 1$. Equating the marginal utility of income to the family with that to the individual yields $u'(y) = u'(\theta y)$, so that $\theta = 1$. But, so long as λ is sufficiently large that some sharing occurs, it should be noted that expression (15) does not imply that θ is near 1 when λ is small. (If the weight on the left side of (15) equaled one, the result would be that $\theta = \alpha$, and α would be near 1, but the weight on the left side may exceed 1.[26])

2.5. Expenditures on Children as Parents' Consumption

Consider the case in which parents' (member 1's) allocation of income to children (member 2) arises because parents get direct consumption benefits from such expenditures. This section examines two possibilities: the

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24 See Kaplow (1990) (gift paper).

25 One can differentiate the first-order condition (14) with respect to y to derive an expression for $d\alpha/dy$. When $\lambda = 1$ and $\alpha = 1/2$, this derivative equals zero.

26 When λ is small so that α approaches 1, the weight exceeds $1 - (y/\lambda)d\alpha/dy$, and $d\alpha/dy$ can be negative, so long as $u''((1-\alpha)y)$ does not approach $-\infty$ (as it would if $u'(x)$ goes to ∞ as x approaches 0).

consumption benefits generate the same marginal utility of income as other expenditures (given the actual sharing that arises), and the consumption benefits are additively separable.

In the first case,

$$(16)\ u_f = u(y) + u(\alpha_2 y).$$

The requirement that the family's and single individual's marginal utility of an additional dollar be equated is

$$(17)\ u'(y) + \alpha_2 u'(\alpha_2 y) = u'(\theta y).$$

Obviously, it is optimal to set $\theta < 1$, which has the result that family member 1 is better off than the single individual. To explore the result further, note that, at $\alpha_2 = 1$, the left side of (17) reduces to $2u'(y)$ and the expression is equivalent to that for linear economies of scale (5), with $\beta = 2$. From (6), it therefore follows that, at $\alpha_2 = 1$, θ is greater (less) than 1/2 if RRA(y) is greater (less) than 1. In addition, one can differentiate (17) with respect to α_2, which yields

$$(18)\ \frac{d\theta}{d\alpha_2} = -\frac{u'(\alpha_2 y)}{yu''(\theta y)}[\mathrm{RRA}(\alpha_2 y) - 1].$$

Thus, as α_2 is reduced from 1 so some intermediate value, θ initially moves closer to 1/2 and may pass that level, being less (greater) than 1/2 if $\mathrm{RRA}(\alpha_2 y)$ is greater (less) than 1.

In the second case,

$$(19)\ u_f = u(\alpha_1 y) + v(\alpha_2 y) + u(\alpha_2 y),$$

where $v(\cdot)$ is member 1's utility from expenditures on member 2. For concreteness, consider the case in which $v(\cdot) = \lambda_1 u(\cdot)$. Then, the analysis is identical to that with altruism. (Compare expressions (19) and (8), substituting for $v(\cdot)$ in (19) and letting $\lambda_2 = 0$.)

2.6. Family Members' Utility Functions Differ

Suppose that family members' utility functions differ in a manner that some members (children) are able to reach a given level of utility with less income than is required by others (parents). In particular, assume that $u_i(\alpha_i y) = u(\alpha_i y/\gamma_i)$. In addition, assume that those requiring more income to achieve a given level of utility (parents) have the same utility of income functions as single individuals. This section will consider two cases: one with fixed sharing and one in which income shares are set so as to maximize the family's total utility.

For the case of fixed sharing, consider the simplest possibility, that $\alpha_i = \gamma_1$ for all i. (This sharing rule generates equal utility for each family member.) Let $\alpha_1 > \alpha_2$. Then, the individual's utility is $u(\theta y/\alpha_1)$ and family utility is $2u(y)$. Equating the marginal utility of an additional dollar for the family and the individual,

$$(20)\quad 2u'(y) = \frac{1}{\alpha_1}u'(\frac{\theta y}{\alpha_1}).$$

The assumption that $\alpha_1 > 1/2$ implies that $\theta < \alpha_1$ -- that is, the rule should be more generous to families than one that would equalize the total utility of each person (which is feasible in this case). In particular, the optimal rule results in family member 1 achieving greater utility than the single individual, even though they both have the same utility function. Because member 2 is a more efficient utility generator, the family is favored. Because member 1 is assumed to receive more than half of family income at the margin, the only way to channel more income to member 2 is to make member 1 better off than the single individual.

To further explore this case, one can differentiate expression (20) with respect to α_1, which yields

$$(21)\quad \frac{d\theta}{d\alpha_1} = \frac{\theta}{\alpha_1}[1 - \frac{1}{RRA(\frac{\theta y}{\alpha_1})}].$$

Thus, $d\theta/d\alpha_1$ is positive (negative) if $RRA(\theta y/\alpha_1)$ is greater (less) than 1. When $\alpha_1 = 1/2$, $\theta = 1/2$. If, for example RRA equals 1, θ remains at 1/2 as α_1 increases. If RRA is greater (less) than 1, θ rises (falls) with α_1 because the greater efficiency of member 2 in generating utility is relatively less (more) important than the reduction in marginal utility arising from the lower effective income received by member 1 and the single individual.

Consider now the case in which the family allocates income to maximize total family utility. It will be convenient to continue to use $1/\alpha_1$ as member 1's and the single individuals weight on available income (and thus $1/(1-\alpha_1)$ as member 2's weight). Let α denote the share of income allocated to member 1. The family's first-order condition is

$$(22)\quad u'\left(\frac{\alpha y}{\alpha_1}\right) = \frac{\alpha_1}{1-\alpha_1}u'\left(\frac{(1-\alpha)y}{1-\alpha_1}\right).$$

It is apparent that this rule allocates more income to the second individual than the previously stipulated fixed share (i.e., $\alpha < \alpha_1$), because the second individual is a more efficient utility generator. Using this expression, one can show that the condition equating the marginal utility of an additional dollar for the family and the individual is

$$(23)\quad \frac{1}{\alpha_1}u'\left(\frac{\alpha y}{\alpha_1}\right) = \frac{1}{\alpha_1}u'\left(\frac{\theta y}{\alpha_1}\right), \text{ or}$$

$$\alpha = \theta.$$

The intuition is that the family maximization equates the marginal utility of income for the family members and the social maximization equates the marginal utility of income to the single individual with that of the family members, and thus with that of the first family member. Since the first member and the individual have the same utility function, at the optimum they receive the same income. Note, however, that this result is still relatively generous to the family: because $\alpha < \alpha_1$ from the family maximization problem, it follows that $\theta < \alpha_1$. Because the family is a more efficient utility generator (due to member 2) than is the individual, the family gets more income than enough to allow each of its members to achieve the same utility level as that of the single individual.[27] In particular, member 1 is as well off as the single individual and member 2 is better off. (In the prior case, both family members were better off than the single individual.)

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[27] It is not possible to derive a simple condition for the sign of $d\theta/d\alpha_1$ as it was in the previous case. Nonetheless, a sufficient condition for $d\theta/d\alpha_1 > 0$ is that $RRA((1-\alpha)y/(1-\alpha_1)) > 1/\alpha_1$. The intuition parallels the previous case: if RRA is high, the reduction in effective income for member 1 and the single individual has a larger effect than the greater efficiency of member 2 in generating utility.

To illustrate these two cases, consider the example of a utility function exhibiting constant absolute risk aversion, $u(y) = -e^{-\eta y}$. In this case, it can be demonstrated that the optimal θ for the first case (with shares fixed to yield equal utility to each family member) is less than that for the second case (with shares set to maximize total family utility).[28] For more concrete results, suppose that $\alpha_1 = 2/3$, $y = 10{,}000$ and $\eta = .001$ or $\eta = .0001$.[29] In the former instance, with high risk aversion, the optimal values of θ for the two cases are approximately .647 and .651, indicating that the single individual's income is to be reduced only slightly (from a benchmark of $\alpha_1 = .667$). In the latter instance, with low risk aversion, the optimal values of θ for the two cases are approximately .475 and .513, indicating that the single individual is to be treated much less favorably. It can also be demonstrated that the divergence between θ and α_1 falls as income rises.[30]

3. Discussion

3.1. Alternative Welfare Criteria

The results on equitable taxation of the family presented here are quite sensitive to the welfare criterion, in a manner that is qualitatively different from that in which welfare criteria affect outcomes with the standard optimal income tax problem.[31] Consider the objective of maximizing the welfare of the individual with the lowest utility (maximin). When all individuals are identical, family income is equally shared, and there are no economies of scale, the result is the same: income is divided equally among the population.

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[28] In the equal total utility case, the optimal θ equals $\alpha_1(1 - \ln(2\alpha_1)/\eta y)$. In equal marginal utility case, the optimal θ equals $\alpha_1(1 - (1-\alpha_1)\ln(\alpha_1/(1-\alpha_1))/\eta y)$. In each case, $\theta = 1/2$ when $\alpha_1 = 1/2$. But, when $\alpha_1 > 1/2$, one can show that $d\theta/d\alpha_1$ is greater in the latter case.

[29] These values of η imply that a gamble that raises or lowers income by 1000, each with probability .5, reduces utility by an amount equal to that caused by taking away 434 (for $\eta = .001$) or 50 (for $\eta = .0001$) for certain.

[30] The amount by which θ differs from α_1 in both cases is an increasing function of $1/\eta y$. Thus, raising income has the same effect as higher risk aversion. (Relative risk aversion is given by ηy, which, as is familiar, increases with income with a constant-absolute-risk-aversion utility function.)

[31] See Atkinson (1973).

But in all other cases the results differ greatly, sometimes having the opposite characterization. When shares are unequal, a family should be treated more generously the more unequally it shares its income. Suppose, for example, that the shares are .99 and .01. Then, the family should receive 100 times the income of a single individual, so as to equalize the well-being of family member 2 and the single individual.[32] When economies of scale are present, families unambiguously should receive less income, because they need less to achieve the lowest utility level. Altruism also calls unambiguously for less generous treatment of the family: individuals who are altruistic are better off than otherwise, and thus should receive less income. That expenditures on children should be included in parents' consumption has no effect so long as children receive a lower share: the well-being of the less well-off family member is equated to that of the single individual, as in the case of unequal sharing. Finally, if family members need less income for some members to achieve a given level of utility, they should receive less.

Some of these contrary prescriptions (those pertaining to economies of scale and the case when some family members need less income) are commonly offered by commentators who do not appear to accept a maximin criterion.[33] Perhaps this reflects that invoking ability to pay often leads one to think about total utility rather than marginal utility.[34]

Consider briefly a libertarian approach. If people are entitled to what they earn, there is no direct role for a concept of ability to pay, and thus no obvious reason to determine equivalences between individual and family

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[32] This is somewhat like the "utility monster" example given to attack utilitarianism. The difference here is that there is instead an "income monster," who is allocated a grossly disproportionate share of income even though he does not get more benefit from it -- and, at the margin, gets far less.

[33] Since most take the opposite view on unequal sharing, arguing that it should lead to less generosity toward the family, it seems implausible that their implicit criterion is maximin.

[34] Thus, authors often refer to equivalence in standards of living between different family units. Such perspectives may be rooted in equitable principles such as equal sacrifice or equal proportional sacrifice, which have long been criticized by Pigou (1951), among others, as inferior to a principle of equal marginal sacrifice, equivalent to employing a utilitarian welfare function.

income. There may, however, be an indirect reason to measure income: if the normative principle requires benefits taxation and individuals' benefits vary with their income, an income tax might be appropriate. Note, however, that then the normatively relevant measure of income would not be concerned with marginal or total utility, but rather with providing the best proxy measure of an individual's or a family's benefits from the government.[35]

3.2. Incentives

Just as the familiar utilitarian rule of equalizing income is not fully (or even almost fully) implemented because of incentive considerations, the rules derived here in the family context would need to be altered in a complete optimal taxation analysis. Consider first incentives to earn income. The equitable rules -- stated in terms of relative after-tax incomes for individuals and families -- might hold as a first approximation, if the incentive effects of taxes were the same on both groups. It seems clear, however, that the incentive effects would differ. First, because the optimal shares need not correspond to relative pre-tax incomes, achieving optimal shares would require higher tax rates on some types of unit than others, which, ceteris paribus, would be more distorting at the margin than equal rates. Second, even at the same tax rates, wage elasticities will differ depending upon whether an earner is single or a member of a household. Thus, optimal rules that account for incentives to earn income could differ substantially from those offered here, although the distributive effects considered in the present analysis would have an important influence on the optimum.

Second, whenever the equitable rules treat individuals differently when they form a family unit -- as was true in virtually every instance considered

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[35] Thus, for example, if earnings from different sources were associated with different public benefits, different rates might be applied to different sources. Higher commuting expenses might lead to a higher tax (rather than a lower one, as some propose under an income tax) because they reflect heavier use of roads. The use of family equivalence scales derived from family budget studies for the purposes of determining relative tax burdens has been questioned on normative grounds. See, e.g., Pollak and Wales (1979). If, however, one's principle were benefits taxation, such measures might be more appropriate.

-- the incentives to marry and divorce will be affected.[36] One suspects that, as a result, it would be optimal to reduce the degree of differentiation from what would produce the most equitable distribution.

Third, because equitable rules vary with family size, they affect incentives to have children.[37] To assess the desirability of any such effect requires a welfare function that accounts for differences in population size.[38] As a result, it might be optimal to provide more or less differentiation than otherwise.[39]

3.3. Family Membership as an Exchange Relationship

Relationships among family members, particularly between spouses, are often viewed as involving the exchange of income and services.[40] In its pure form, there is no "sharing." Thus, if family income is divided evenly between spouses, it is either because each spouse earned an equal amount in the first instance or because the transfer of income between spouses was in exchange for the provision of services to the higher-earning spouse.

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[36] The discussion here has also ignored the important distinction between legal marriage and de facto marriage. One might analyze a regime in which individuals who would otherwise marry can opt for treatment as single individuals, and determine the optimal treatment of married couples in that setting. Such a regime is plausible only if single individuals cannot become married for tax purposes.

[37] One should include birth, adoption, as well as dependents who are not children. While it is often suspected that current tax provisions have little effect on the incentive to have children, the implicit alternative regime is often unspecified. Some of the contexts examined here call for substantial differentiation as family size changes. Also, it is commonly suspected that welfare programs -- where the benefits for changing family size are relatively greater than under most income tax systems -- do have some effect on decisions to have children.

[38] See Meade (1955), Mirrlees (1972), Nerlove, Razin, and Sadka (1986). For adopted children and other dependents, changing the composition of units would alter welfare, just as when considering incentives to marry and divorce.

[39] To illustrate, if the welfare criterion is maximizing total utility (rather than average utility), increasing family size may tend to increase social welfare. The question of optimal subsidies for children (which may take many forms -- e.g., welfare payments, tax incentives, provision of public education) cannot be separated from that of how taxes should reflect family size.

[40] See Becker (1991). Cox (1987) offers evidence that private transfers are in exchange for services rather than motivated by altruism; his data, however, do not include transfers among members of the same family unit, so the study's applicability in the present context is uncertain.

The exchange perspective is relevant to the tax treatment of the family for two reasons. First, it bears on the "sharing" that actually takes place: given the economic forces that determine marriage partners and the allocation of resources within the family, one could determine how tax burdens would be shared as well. Second, the exchange of income for services indicates that there is nonmarket (imputed) income in the family, so a tax base of market income mismeasures total income. Imputed income is likely to be most important when there are families precisely because they allow more specialization, making it likely that the amount of imputed income will differ among units of equal size and with similar market income. It is familiar that the problem of untaxed imputed income plagues attempts to achieve equitable and efficient taxation of the family. Note that the existence of exchange may allow a taxing authority to make inferences about the value of imputed income.[41] Even when imputed income can be measured, however, it may not always be appropriate simply to add it to market income, because the effect of nonmarket income on utility may not be equivalent to that of market income, and the marginal utility of market income determines the optimal distribution of tax burdens.[42]

4. Conclusion

From the perspective of an optimal utilitarian distribution of income, the tax burden on families in some cases should be higher and in others lower than that implied by simple income splitting or an approach that would equalize the welfare or even the marginal utility of income of a family head and a single individual. This indeterminacy often arises because factors indicating that

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[41] For example, if due to economies of scale it is most efficient for individuals to form families in which market income is shared approximately equally, the difference in each member's earned income will reflect the value of the nonmarket income produced by the member earning less (above a base level of nonmarket income supplied by both individuals).

[42] For example, if one spouse has high earnings and the other provides imputed income of equal total value, but that enters utility in an additively separable manner, the marginal utility of market income will be determined by the earnings alone. Note that in many instances imputed income can (roughly) be added to market income, because the nonmarket services are ones (say, cleaning) that could be obtained on the market. In others (raising children, companionship), there may be contributions to family members' utility that do not correspondingly reduce the marginal utility of income.

an additional dollar of per capita family income is worth more than a dollar to a given family member may have two effects: the greater weight directly favors giving more to the family while it sometimes indicates that the family member has higher effective income and thus a lower marginal utility. In addition, the results reflect that fact that it is the family that determines how its tax burden is borne by its members -- so, for example, tax burdens nominally designed to be borne by family heads may be borne in part by others. The analysis also identified how the strength of different effects may vary with income, thereby indicating when it would be optimal for family adjustments to rise or fall with income.

Many of the results contradict conventional wisdom, which often derives from the notion ability to pay.[43] For example, unequal sharing or the presence of economies of scale may favor lighter tax burdens on families. That expenditures on children should be included as part of parents' consumptions may favor providing higher after-tax incomes to parents than to single individuals. And when some family members (e.g., children) need less income to reach a given utility level, the optimal rule leaves them better off than single individuals and, more surprisingly, may leave other family members (parents) with the same utility functions as single individuals better off than single individuals.

Prescriptions for tax policy require further analysis. One must decide upon an objective function, incorporate incentive considerations, and obtain empirical evidence on such matters as actual sharing practices, economies of scale, and the degree of risk aversion.[44] One would also wish to extend the

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[43] The results are also inconsistent with Simons' (1938) view that, in principle, gifts should be nondeductible to donors and income to donees in defining taxable income (a view consistent with an exchange, altruistic, or expenditures on children as parents consumption view of the motivation for giving). The most significant instance of giving is within the family, so family sharing is intimately related to the appropriate tax treatment of gifts. The implication is that stipulated definitions of income, while often useful, need not be consistent with the optimal distribution of tax burdens. Optimality also may conflict with horizontal equity as it is often defined, as the optimal distribution need not preserve the pre-tax orderings of income or utility (although one might argue that different family configurations are simply not comparable).

[44] As suggested by the discussions of altruism and family membership as an exchange relationship, one's overall view of the economics of the family is

model by allowing for heterogeneity among families and incorporating a lifetime perspective, since most individuals will spend part of their lives as dependents, part as single individuals, and part as adult family members.[45]

quite important in answering these questions. General discussions include Becker (1991) and Sen (1974, ch. 16).

[45] For children, future circumstances may have limited relevance, at least from a utilitarian perspective, because consumption while a child is likely to depend primarily on what one's parents provide, although this may reflect in part what parents expect to give to or receive from their children in the future.

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