THE OPTIMAL PROBABILITY AND MAGNITUDE OF FINES FOR ACTS THAT ARE DEFINITELY UNDESIRABLE

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ABSTRACT

Even when society would wish to deter all acts of some type, such as tax evasion and many common crimes, the benefits from deterrence often will be insufficient to justify the expenditures on enforcement that would be required to deter everyone. If some individuals are not deterred, however, they will bear risk when fines are employed as a sanction. As a result, it may be optimal to reduce total risk-bearing costs by reducing the number of individuals who bear any risk. This can be accomplished by increasing enforcement above the level that would be justified considering only the benefits of deterrence and the direct costs of enforcement. Another possibility is that it may be optimal reduce the risk borne by those who act, by employing fines below the maximum feasible level. This latter result constitutes an instance in which the well-known implication of Becker's analysis that it is optimal to employ extreme sanctions for all offenses is invalid.

The Optimal Probability and Magnitude of Fines for Acts That Are Definitely Undesirable

Louis Kaplow*

Society wishes to deter all individuals from committing some types of acts, such as tax evasion and many common crimes, for which the harm done exceeds any legitimate private benefits. Because enforcement is costly, however, complete deterrence often will not be desirable, at least on deterrence grounds alone. Individuals who are not deterred will bear the risk of sanctions -- here fines. If individuals are risk-averse, risk-bearing costs are incurred and must be taken into account in determining optimal enforcement policy.

One strategy would be to reduce the risk borne by those who are not deterred. Thus, it may be optimal to employ fines below the maximum feasible level, perhaps at the same time spending more on enforcement to maintain deterrence. This latter prescription is contrary to the well-known suggestion, with roots in Becker's (1968) analysis, that optimal enforcement involves achieving the appropriate expected sanction through a fine that is as high as possible (equal to a person's entire wealth) and a probability of detection that is correspondingly low, so as to economize on enforcement costs.[1] This motivation for less-than-extreme sanctions supplements the previously developed argument of Polinsky and Shavell (1979) addressed to the case in which some individuals ideally are not deterred, such as those who efficiently breach contracts or who are subject to strict liability for their acts. Their argument and that developed here each offer reasons why, even when enforcement is optimal, individuals will commit acts and thus bear risk.

Another strategy to reduce total risk-bearing costs would be to reduce the number of individuals who bear any risk. This may be accomplished by increasing enforcement beyond the level that would be justified based upon a consideration of only the benefits of deterrence and direct costs of

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[1] See Stern (1978).

enforcement. In some cases, complete deterrence may be optimal in light of the benefits of eliminating all risk-bearing costs.

Section 1 presents the model. Sections 2 and 3 examine, respectively, cases when individuals are risk-neutral and when they area risk-averse. Concluding remarks are offered in section 4.

1. The Model

Individuals decide whether to commit a harmful act based on whether it will maximize their expected utility. Individuals' benefits from committing the act differ, and are not observable by the social authority. Acts are subject to fines, which are imposed with some probability.[2] Enforcement is financed by fine revenues and a lump-sum tax. The following notation is employed:

h = total harm caused by each act, borne evenly by all individuals.[3]

b = benefit to an individual from committing the act.[4]

$f(\cdot)$ = continuous distribution of b on $[0,h]$.

$F(\cdot)$ = cumulative distribution function for $f(\cdot)$.

U = utility of wealth.

y = initial wealth.

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[2] It is assumed that individuals cannot insure against the fine. If they could insure at an actuarially fair rate, individuals intending to act would purchase complete coverage, making the analysis as in the risk-neutral case. This may increase achievable welfare by eliminating risk-bearing costs. (The demonstration of the sixth claim of Proposition 5 illustrates this: the risk-averse case involves lower deterrence and incurring substantial risk-bearing costs or substantial harm -- or some combination of the two -- while enforcement costs, even if lower, entail only trivial savings.) In addition, the availability of insurance may reduce achievable welfare, because risk-aversion allows a given level of deterrence to be achieved at a lower probability of detection. (The demonstration of the second claim of Proposition 5 illustrates this result.)

[3] One can imagine an externality with such a property or that the harm is borne with equal probability by all individuals, with individuals purchasing actuarially fair insurance.

[4] The analysis, for the most part, assumes that both b and h are monetizable -- that is, they are to be taken into account in computing the maximum feasible fine. It will be seen in the proof of Proposition 1 that this complicates the argument (because the maximum feasible fine changes with the probability of being fined in more complicated ways). All the propositions hold if either or both factors are nonmonetizable.

π = fine.

p = probability that an individual who commits the act is fined.

c(p) = cost of enforcement; $c' > 0$, $c'' > 0$.

t = lump-sum tax.

Begin by considering an individual's maximization problem. The expected utility of an individual of type b from committing the act is given by

$$(1) \quad U_a = (1-p)U(y - t - (1 - F(b^*))h + b) + pU(y - t - (1 - F(b^*))h + b - \pi),$$

where b* denotes the benefits of the individual who is just indifferent as to whether to commit the act. If the act is not committed, utility is given by

$$(2) \quad U_n = U(y - t - (1 - F(b^*))h).$$

The marginal individual is the type b* for whom (1) equals (2).[5] Individuals with greater benefits will commit the act -- (1) is increasing in b and two is independent of b -- and those with benefits less than b* will not commit the act.

The social authority, which does not observe individuals' types,[6] chooses p and π to maximize social welfare, defined as the sum of individuals' utilities,

[5] Clearly, for $b > b^*$, (1) exceeds (2), so all such individuals will act, and, for $b < b^*$, (2) exceeds (1), so no such individuals will act.

Note that, in general, there need not exist or be a unique b* that equates (1) and (2). (Existence is the lesser complication, as, for $b^* = 0$, (2) is greater than (1), for $b^* \geq \pi$, (1) is greater than (2), so there will exist a b* with the properties described in text even if that b* does not equate (1) and (2).) For the risk-neutral case and for the constant absolute risk aversion utility function used to prove each of the results, there exists a unique b*.

[6] It is obvious that, when it would be ideal for some individuals to engage in an activity, it would be optimal to excuse them from sanctions whenever sanctions are costly -- and imposing risk is a relevant cost. The results here imply that, for the same reason, if individuals' types could be observed, it would tend to be optimal to excuse even those who, ideally, should not act, if their benefits are high enough that they are not deterred at the optimum. In addition to avoiding the risk-bearing costs at a given p and π, one could raise π and lower p, thus achieving more cost-effective deterrence.

$$(3)\quad W = \int_0^{b*} U_n f(b)db + \int_{b*}^{h} U_a f(b)db,$$

subject to the constraint that the lump-sum tax finances the required enforcement expenditures net of the fine revenue collected.

$$(4)\quad t = c(p) - (1 - F(b*))p\pi.$$

2. Individuals are Risk-Neutral

When individuals are risk-neutral, one can without loss of generality write $u(w) = w$. Using the tax constraint (4) to substitute for t in the expression for social welfare (3) in this case gives the following:[7]

$$(5)\quad W = y - c(p) - (1 - F(b*))h + \int_{b*}^{h} bf(b)db.$$

That is, social welfare equals initial wealth, minus enforcement costs and harm, plus the benefits of those who act. Finally, note from (1) and (2) that, in the case of risk-neutral individuals, $b* = p\pi$.

A. No wealth constraint limiting maximum possible fine

The first-order condition for the optimal fine is

$$(6)\quad \frac{dW}{d\pi} = hf(b*)\frac{db*}{d\pi} - b*f(b*)\frac{db*}{d\pi} = 0.$$

Recalling that $b*=p\pi$,

$$(7)\quad \frac{dW}{d\pi} = p(h - p\pi)f(b*) = 0.$$

As a result, $h = p\pi = b*$.[8] Consider now the condition for the optimal probability.

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[7] All the terms involving π cancel, as the distributive effect of the fine is irrelevant to welfare in the risk-neutral case.

[8] Because f is continuous, even if it were to equal zero for some b, it cannot equal zero for b* at the optimum. If it did, one could reduce the probability slightly causing, at the margin, no additional harm while saving a positive cost.

$$(8) \quad \frac{dW}{dp} = -c' + hf(b^*)\frac{db^*}{dp} - b^*f(b^*)\frac{db^*}{dp}.$$

The second and third terms offset, leaving

$$(9) \quad \frac{dW}{dp} = -c'.$$

Because $c' > 0$, no optimum exists; a lower p (and a correspondingly higher fine, so that $p\pi = h$) increases welfare further -- so long as $p > 0$. This is the familiar result motivated by Becker's (1968) analysis.

B. Wealth constraint limits maximum possible fine

If, for any p, the fine necessary to satisfy (7) is infeasible due to a wealth constraint on the maximum possible fine, it is clear that the optimal fine will be the highest possible.[9] And, from (9), the expression for dW/dp when there is no wealth constraint, it is clear that the wealth constraint will bind. (If it did not, one could raise π slightly and lower p a corresponding amount, which would increase welfare.)

Consider, then, the case in which the constraint is binding. The first-order condition for p can then be written as

$$(10) \quad c' = (h - b^*)f(b^*)\frac{db^*}{dp}.$$

The marginal cost of increasing enforcement is equated to the marginal harm prevented by increasing enforcement, as $h - b^*$ is the net social harm imposed by the marginal individual and $f(b^*)db^*/dp$ is the rate at which such individuals are deterred as p is increased.[10]

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[9] For any fine less than h/p, $dW/d\pi > 0$.

[10] Note that the maximum feasible π may be a function of p. Thus

$$\frac{db^*}{dp} = p\frac{d\pi}{dp} + \pi.$$

The reason is that p affects available wealth. Most directly, p affects the required lump-sum tax (as it affects both enforcement costs and fine revenues). In addition, the level of harm and of b* are affected; if harm or benefits are monetizable (rather than nonmonetary, with h and b merely expressing monetary equivalents), the maximum feasible fine would vary -- in the case of h, for all individuals, and, in the case of b, for the marginal type b*, which is relevant for determining who can be deterred.

To interpret this further, let $\bar{b}$ denote the highest b for which $f(b) > 0$. If $\bar{b} = h$, complete deterrence cannot be optimal, for it would imply $c' = 0$, which is ruled out by assumption. The intuition here is that deterring those with benefits just equal to h produces no social benefit at the margin, but entails a positive cost, as a higher level of p must be financed. (This case is assumed for section 3.) On the other hand, if $\bar{b} < h$, it is possible that complete enforcement is optimal. If $f(\bar{b}) > 0$, enforcing just enough to deter those with the highest private benefits will entail a positive marginal gain, which may (or may not) exceed the marginal cost of raising p just enough to accomplish this.[11]

3. Individuals are Risk-Averse

A. No wealth constraint limiting maximum possible fine

It is clear that the optimum will be as in the risk-neutral case: an infinite fine and an infinitesimally small probability of enforcement. Because all individuals are deterred, no risk is borne.

This result contrasts with that of Polinsky and Shavell (1979) because they examined acts for which some individuals benefited by more than the harm caused; in that case, the optimum in the risk-neutral case involved a sanction sufficient to deter only those for whom the act caused more harm than benefit. When individuals are risk-averse, the risk-neutral solution -- in which individuals with high benefits continue to act -- entails such individuals bearing risk. Taking this cost into account can thus lead to a different optimum. It might appear that the existence of "desirable illegal acts" was necessary to their result. In fact, however, similar results may follow solely from the existence of a constraint on the maximum possible sanction.

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[11] The lower is $\bar{b}$ relative to h, the greater the density of the population at the maximum level of harm, and the greater the rate at which b^* increases with p (taking into account that a higher p may increase or reduce the maximum feasible π for the marginal type b^*, see note 10), the more likely it is that complete deterrence would be optimal.

B. Wealth constraint limits maximum possible fine

When the maximum possible fine is constrained, the character of the optimum may change substantially when individuals are risk-averse. Consider the result when the probability of detection is set as in (11) and the fine is set at the maximum feasible level -- the optimum for the risk-neutral case. Because individuals are risk-averse, the level of deterrence (and thus the resulting b*) will be greater because the expected utility cost of the probabilistic fine is greater. (The optimal probability in the risk-neutral case will achieve complete deterrence if individuals are sufficiently risk-averse.) A lower probability could achieve the partial level of deterrence present at the optimum for the risk-neutral case. But there is no reason to believe that the result would be optimal, as individuals who engage in the act bear risk. Adjusting p affects risk-bearing costs as well as the cost of enforcement and level of deterrence, so a different p may be optimal. In addition, lowering π from the maximum feasible level reduces deterrence but it also reduces risk-bearing costs, so it is not clear a priori that the optimum involves the maximum feasible fine.

The first four propositions to follow characterize possible results when individuals are risk-averse. The final proposition notes how the results for the risk-neutral and risk-averse cases may diverge. Each claim is proved in the appendix by constructing examples that have the asserted properties.[12] The intuition behind each result is sketched briefly here.

Proposition 1: A probability of detection sufficient to deter all individuals will, for some parameter values, be optimal even if $\overline{b} = h$; in such a case, the lowest probability for which complete deterrence is feasible will be optimal and the optimal fine at that probability, which will be the maximum feasible fine, will deter all individuals. Consider this hypothesized optimum. In the risk-neutral case, a slightly lower probability was preferred because enforcement costs were reduced but, at the margin, no net harm results. When individuals are risk-averse, however, a marginal reduction in

[12] The examples involve the continuous model presented here. For simpler, discrete examples, see Kaplow (1989b).

the probability will impose positive risk-bearing costs on those who engage in the activity. If the savings in marginal enforcement costs are less than the resulting risk-bearing costs, a small reduction in the probability from the lowest threshold probability will reduce welfare.[13] Although substantially lower probabilities may entail little or no risk-bearing costs, positive net harm will result because of the lost deterrence, and this harm will exceed total enforcement costs at the threshold probability if enforcement costs are sufficiently low.

Proposition 2: No deterrence may be optimal. Even in the risk-neutral case, it is clear from (11) that if $c'(0)$ is sufficiently high, no deterrence would be optimal (so long as $f(0)$ is bounded). When individuals are risk-averse, this is also the case.[14]

Proposition 3: Partial deterrence may be optimal. If enforcement costs at the lowest threshold probability (with the fine at the maximum feasible level) are sufficiently high, complete deterrence cannot be optimal. (For example, enforcement costs at that level could exceed total harm if no one were deterred, indicating that no enforcement would dominate complete deterrence.) Consistent with this assumption, enforcement costs at low levels of enforcement may be sufficiently low that substantial harm can be prevented at little cost, considering both enforcement and risk-bearing costs. (The example in the appendix chooses an $f(\cdot)$ such that most of the mass is near $b=0$, so most harm is prevented and few individuals incur risk-bearing costs at a relatively low probability.)

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[13] Clearly, even if some individuals benefits exceed harm, but not by a significant amount, it may be optimal to deter them. This result and that in text is analogous to the suggestion in Polinsky and Shavell (1984) that when imprisonment (which is socially costly to impose) is the sanction, overdeterrence may be optimal because it decreases sanctioning costs.

[14] On one hand, this result may be more likely (that is, for lower levels of c', no deterrence may be optimal) due to risk-bearing costs. For example, even at $p=0$, the marginal risk-bearing costs of increasing p are positive unless the fine equals zero, and in that case no deterrence is achieved so it could not be optimal to increase p. On the other hand, a given level of deterrence can be achieved with a lower probability when individuals are risk-averse.

Proposition 4: If partial deterrence is optimal, it may not be optimal to impose the maximum feasible fine. Consider the case from (3) with an optimal p that is intermediate, and assume that the fine is at the maximum feasible level. If the fine is decreased slightly and p is raised a corresponding amount so that the marginal type b* is unchanged, there will be a reduction in risk-bearing costs and an increase in enforcement costs. One can construct examples with marginal enforcement costs near the optimal p sufficiently low and the portion of individuals who, at the optimum, are not deterred and thus bear risk sufficiently high so that some reduction in the fine will be optimal.

Proposition 5: Optimal enforcement when individuals are risk-averse may involve a (1) higher or (2) lower probability of detection, a (3) higher or (4) lower fine, and a (5) higher or (6) lower level of deterrence than when individuals are risk-neutral. The optimal probability may be higher when individuals are risk-averse for the same reason that the optimal fine may be lower than the maximum feasible fine (which is always optimal in the risk-neutral case): achieving a given level of deterrence with a higher probability and lower fine reduces risk-bearing costs. On the other hand, the optimal probability may be lower because a given level of deterrence can be achieved with a lower probability. The optimal fine may be higher, despite the fact that the risk-neutral case always uses the maximum feasible fine, because the maximum feasible fine itself may be higher when individuals are risk-averse and in some instances, as illustrated in Proposition 1, it will be optimal to use the maximum feasible fine even when individuals are risk-averse (because, when all are deterred, there are no risk-bearing costs). That the optimal deterrence may be higher when individuals are risk-averse is demonstrated in Proposition 1, which contrasts with the result in the risk-neutral case that full deterrence is never optimal. Finally, when individuals are risk-averse, optimal deterrence may be lower, as the savings in risk-bearing costs may exceed the additional harm from reduced deterrence.

4. Concluding Remarks

(a) For many crimes, tax evasion, and some other acts, it would be ideal (or, at least, approximately so) for no individuals to commit the acts. It is often the case, however, that a probability sufficient to deter all individuals would be extremely expensive. When partial deterrence is thus optimal, undeterred individuals bear risk. Optimal enforcement policy will take this cost into account. One strategy involves reducing the number of individuals who bear risk by increasing enforcement above the level that would be warranted based on considerations of deterrence and enforcement costs alone. As in one of the examples presented, it even may be optimal on these grounds to expend resources to deter individuals whose acts produce benefits equal to their social costs.

Another strategy involves reducing the risk borne by individuals who are not deterred. Thus, it may be optimal to employ fines below the maximum feasible level, possibly with corresponding increases in enforcement expenditures. This result is similar to that in Polinsky and Shavell (1979), where the reason some individuals optimally were not deterred was that socially beneficial acts as well as harmful ones were subject to sanctions (as in the case of breach of contract and acts subject to strict liability). For most offenses, one indeed observes fines far less than most violators' wealth. While consistent with the results presented here and those in Polinsky and Shavell, these practices are unlikely to be justified by the concern for avoiding risk-bearing costs in contexts in which the fine is so low that the marginal risk-bearing costs from a slightly higher fine would be trivial.[15]

(b) It will be difficult to determine optimal policy in practice. Most notably, all the stated effects can be in one or the opposite direction, depending upon parameters that often will be difficult to measure: marginal enforcement costs, the distribution of individuals' benefits from engaging in the illegality activity,[16] the degree to which individuals are risk-averse,

[15] This point applies to the level of the fine relative to wealth, not relative to the severity of the offense.

the harm caused by the activity, and individuals' initial wealth. Moreover, this model involved individuals identical in all respects except for their private benefit from the act.

(c) In light of this analysis, it seems inappropriate when analyzing optimal enforcement policy simply to assume, as is commonly done, that the fine is fixed at some stated level -- whether an absolute amount or some rate to be multiplied by a measure of the severity of the infraction.[17] The optimal fine will be a function of the other instruments under study, and the entire character of the optimum with regard to other instruments (here, the probability of detection) may differ when the fine is set optimally rather than stipulated.

(d) Many explanations have been offered for the commonly observed practice of using fines far below individuals' wealth. Most prominent is the notion that fines must in some sense be proportional to the harm caused by an activity. See Stern (1978). Usually, little normative defense is offered for such claims and the appropriate proportion is unstated. Actual practice varies widely in this respect. For example, a \$10 fine for failing to put 10¢ in a parking meter is a fine of 10,000%; civil tax fraud (not mere mistake or negligence) -- which most would consider a worse offense -- has a fine, in addition to repayment, of under 100%. Double parking may be subject to a fine of \$30, an infinite percent of the actual harm caused in many instances. If a serious accident results and one is successfully sued for negligence, one might have to pay \$1,000,000 -- which is no penalty beyond actual harm caused but a penalty of, perhaps, more than 3,000,000% of the expected harm. Thus, it is extremely difficult to infer from actual practice the content of this

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[16] When there is little dispersion in individuals' benefits, as may be the case with many categories of tax evasion -- the benefit being the amount of tax evaded minus the cost of evasion -- the optimum would probably involve nearly complete deterrence or no deterrence, in which case little risk would be imposed.

[17] It is not obvious that a wealth constraint would justify a constant penalty rate. Moreover, when there is no limit to the severity of the infraction, or the only limit (as with tax evasion) is one's wealth, a constant penalty rate is infeasible.

proportionality concept. Nonetheless, it seems that such beliefs are widespread.

A less frequent, but quite plausible explanation for moderate penalties is that some offenses are committed by mistake.[18] One may forget to put money in a parking meter or not understand the income tax rule one violated in filing a tax return. In such instances, high fines may have little additional deterrent effect but impose great risk on violators. If there is a significant group who make such mistakes -- or, alternatively, who act intentionally but mistakenly underestimate the fine or probability of detection -- a very high fine could produce substantial risk-bearing costs. The better policy may be to expend more resources on detection and impose lower penalties, even to such a degree that some intentional and informed violators will remain undeterred. Note that this explanation has much in common with that explored in the model here. One could, in principle, expend enough resources to educate the entire population so that few made mistakes.[19] But this often would be quite costly. The uninformed group is much like those individuals in the model here that had high private benefits and thus remain undeterred at the optimum, even though they were assumed to be fully aware of the consequences.

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[18] See Kaplow (1989a) on this case. Another reason sometimes offered for moderate fines is that marginal deterrence must be preserved. This is at best a weak argument even against fines approaching total wealth, as imprisonment remains available. In addition, for fines well less than total wealth but far above currently observed fines, it is difficult to understand the marginal deterrence problem. Since the fine is only imposed when individuals are detected, and in many instances apprehension and collection could follow almost immediately and automatically, one does not confront the problems that arise in such cases as the kidnapper who, faced with capital punishment for the offense, may be left with little incentive not to kill the victim.

[19] Higher penalties might induce individuals to be more careful or to learn more about applicable rules, but such responses are themselves costly and may be incomplete.

Appendix

Proof of Proposition 1

Choose parameters and functions as follows:

$h = 1$.

$f(\cdot) = 1$ (uniform distribution on [0,1]).

$U(w) = -e^{-\alpha w}$, $\alpha = .1$ (constant absolute risk aversion utility function).

$y = 12$.[20]

$c'(1) = .001$.[21]

For these parameters and functional forms, define $\overline{p}$ as the lowest "threshold" probability (that is, a probability for which for which complete deterrence, $b^* = h$ is feasible).[22] It will now be demonstrated in a series of steps that $\overline{p}$, with the fine sufficient to deter everyone, produces higher welfare than any lower p.

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[20] This guarantees that a low threshold probability exists. In particular, since y is reduced by taxes at most by c(1), which is less than .001, and by harm -- if monetizable -- equaling no more than 1, there is more than 10 in wealth left over. Thus, comparing (1) and (2), any probability equal to or exceeding .1 and the maximum feasible fine will necessarily deter everyone, since for all individuals $b \leq 1$.

[21] This implies that $c(p) < .001$ and $c'(p) < .001$ for any $p < 1$. (Recall that $c''(p) > 0$. The intuition is that for c, c' close to zero, one approximates the case of costless enforcement, in which event complete deterrence is optimal; in the risk-neutral case, full deterrence is not optimal even for small c' due to even smaller social benefits from deterring types for whom b is sufficiently close to h. Here we add sufficient risk-bearing costs to provide a social benefit greater than the cost of deterring the high b types.

[22] A threshold probability (less than .1) exists. See note 20. Moreover, the greatest lower bound to the set of threshold probabilities is contained in that set. As indicated by the continuity argument in the Lemma below, see note 28, the maximum feasible b* varies continuously in p, as do all the relevant terms -- notably the maximum feasible π. Thus, the probability that is the greatest lower bound $\overline{p}$ must also be a threshold probability: if it is not, there will exist a neighborhood N of probabilities around $\overline{p}$ such that, for any $p \in N$, $b^*|_p < h$. This contradicts that $\overline{p}$ is the greatest lower bound.

For any $p \leq \overline{p}$, $h-b^*|_p \geq 10(\overline{p}-p)$.[23] That is, as p falls below $\overline{p}$, the portion of the population who act is at least as great as the degree to which p falls weighted by a positive constant. To demonstrate this, one can examine $\hat{b}$ -- the highest b* that is feasible given p. Clearly, $\hat{b}$ must equal or exceed the b* that arises from imposing the optimal fine given p. A lower bound will now be derived for the rate at which $\hat{b}$ falls as p falls.

LEMMA: For the stated parameters and for $p \leq \overline{p}$, $d\hat{b}/dp > 10$.

Proof: The marginal type, b*, is determined by equating (1) and (2) for the specified utility function. To simplify, let $x = y - t - (1-F(b^*))h$. Then

(A1) $-e^{-\alpha x} = -(1-p)e^{-\alpha(x+b^*)} - pe^{-\alpha(x+b^*-\pi)}$, or

(A2) $e^{\alpha b^*} = 1 - p + pe^{\alpha\pi}$.

Let $\hat{\pi}$ refer to the maximum feasible π given p. It then will be the case that

(A3) $e^{\alpha\hat{b}} = 1 - p + pe^{\alpha\hat{\pi}}$.[24]

Differentiating with respect to p, we have

(A4) $\alpha\hat{b}'e^{\alpha\hat{b}} = e^{\alpha\hat{\pi}}(1 + p\alpha\hat{\pi}') - 1$.

(A single prime indicate the derivative with respect to p.) To determine $\hat{\pi}'$, begin with the definition of $\hat{\pi}$:

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23 $b^*|_p$ denotes the b* that arises given p, when the optimal π given p is employed.

24 If the maximum feasible b* required that $\pi > \hat{\pi}$, it would not in fact be feasible. If the maximum feasible b* could be induced with $\pi < \hat{\pi}$, it would not in fact be the maximum feasible b*: As will be demonstrated in note 28, the maximum feasible π for any given p is a differentiable, and thus continuous function of b*, and that b*, for a given p, is a continuous and increasing function of π. Thus, if, at $\hat{b}$, $\pi < \hat{\pi}$, there exists a feasible fine $\pi < \tilde{\pi} < \hat{\pi}$ such that the resulting b* exceeds $\hat{b}$, contradicting that $\hat{b}$ is the maximum feasible b*.

$$\text{(A5)}\quad \hat{\pi} = y - c(p) + (1-F(\hat{b}))p\hat{\pi} - (1-F(\hat{b}))h + \hat{b}.^{25}$$

This simply is initial wealth minus the tax and harm imposed, plus the benefit of the marginal individual.[26] This formulation assumes that the harm and benefit are both monetizable.[27] Rearranging,

$$\text{(A6)}\quad \hat{\pi} = \frac{y - c(p) - (1-F(\hat{b}))h + \hat{b}}{1 - p(1-F(\hat{b}))}.$$

Differentiating with respect to p yields

$$\text{(A7)}\quad \hat{\pi}' = \frac{-c' + f(\hat{b})\hat{b}'h + \hat{b}' + \hat{\pi}(1 - F(\hat{b}) - pf(\hat{b})\hat{b}')}{1 - p(1-F(\hat{b}))}.$$

Substituting this into (A4) $\hat{b}'$ and solving for $\hat{b}'$ yields[28]

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[25] (A5) can be defined using $\hat{b}$ because, if $b^* > \hat{b}$, it is by definition infeasible and, if $b^* < \hat{b}$, π must be lower than the maximum feasible level because $\hat{\pi}$ is increasing in b^* (see note 28).

[26] It is assumed that the maximum feasible fine for the marginal type b^* is applied to all individuals, not just to the marginal type. For those with $b < b^*$, the assumption is inconsequential even though this fine is not feasible. Individuals would pay all their wealth, which would clearly be sufficient to deter them. (From (1), the second term is evaluated at the same wealth as it is for type b^* and the first term is evaluated for lower wealth.) Since these individuals do not act, they never pay any fine, and thus fine revenue is unaffected by their inability to pay $\hat{\pi}$. If $b > b^*$, individuals could pay a greater fine than $\hat{\pi}$, but even charging them all their wealth would not be sufficient to deter them. (In (1), the wealth for the first term is higher than for type b^* and that in the second term is the same.) If one did apply a greater fine to this group, the fine revenue thus generated and the risk-bearing costs would be greater than assumed in the text.

[27] This may be appropriate for instances such as tax evasion, where the harm may derive from higher payments required from some other tax and the benefits may be saved taxes net of resources expended in evasion. A similar (and actually simpler) derivation along the lines of that in the text is possible for the case where harm or benefits are not monetizable. See note 31.

[28] The analysis to follow requires that $\hat{b}'$ $(d\hat{b}/dp)$ and $\hat{\pi}'$ $(d\hat{\pi}/dp)$ exist for $p \leq \bar{p}$. From (A3), one can write

$$\hat{b} = (1/\alpha)\ln(1 - p + pe^{\alpha\hat{\pi}}).$$

The function ln(z) is continuously differentiable in p if $z > 0$ and z is continuously differentiable in p. Clearly, $1 - p + pe^{\alpha\hat{\pi}} > 0$. Also, this term is continuously differentiable in p if $\hat{\pi}$, expressed in (A6), is continuously differentiable in p. In examining (A6), the denominator is nonzero for $p < 1$, and $\hat{\pi}$ will be continuously differentiable in p if $\hat{b}$ is continuously differentiable in p.

$$\text{(A8)}\quad \hat{b}' = \frac{e^{\alpha\hat{\pi}}[1 + \frac{p\alpha}{1-p(1-F(\hat{b}))}(-c' + (1-F(\hat{b}))\hat{\pi})] - 1}{\alpha e^{\alpha\hat{b}} - \frac{e^{\alpha\hat{\pi}}p\alpha}{1-p(1-F(\hat{b}))}(f(\hat{b})h + 1 - pf(\hat{b})\hat{\pi})}.$$

Begin by examining the denominator, which is now shown to be bounded from above and positive. To establish an upper bound, note first that $f(\hat{b}) = h = 1$, $p < .1$,[29] and $\hat{\pi} < y + \hat{b} \leq y + h = 13$.[30] Thus, $(f(\hat{b})h + 1 - pf(\hat{b})\hat{\pi}) > 2-.1\times 13 > 0$. The numerator and denominator of the

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To complete this argument, consider, for a given $p \leq \bar{p}$, the function $\hat{b}(\pi)$ -- defined by (A3), where π replaces $\hat{\pi}$ -- and the function $\hat{\pi}(b)$ -- defined by (A6), where b replaces $\hat{b}$. First, examine $\hat{b}(\pi)$. At $\pi = 0$, $\hat{b} = 0$. In addition, for $p \in (0,\bar{p}]$ and any feasible π, $0 < d\hat{b}/d\pi < .37$. (The former inequality is obvious and the latter can be established by taking the derivative and substituting the relevant lower and upper bounds on parameter values; these bounds are discussed in the text, below.) This in turn implies that $\hat{b}^{-1}(b)$ (which, by the inverse function theorem, exists and is continuously differentiable) has a slope greater than 2.7. Now, consider $\hat{\pi}(b)$. At $b = 0$, $\hat{\pi} > 0$. (The text below demonstrates that, for any b, $\hat{\pi} > 10$.) In addition, $0 < d\hat{\pi}/db < 2.3$. (The former inequality is obvious from (A5), since in the relevant range it must be that $p\hat{\pi} < h$. The latter inequality can be established by taking the derivative, using (A6), and substituting appropriate bounds on the parameters.)

As a result, there will exist a single intersection of $\hat{b}(\pi)$ and $\hat{\pi}(b)$, defining uniquely values for $\hat{b}$ and $\hat{\pi}$. (If there is no intersection, or none at which $b \leq 1$, this would imply that, at the given p, $b = 1$ can be induced by a fine π below the maximum feasible level. Since $\hat{\pi}(b)$ is continuous in p, there will exist a lower p for which there exists a feasible fine inducing $b = 1$, which contradicts the range restriction that $p \leq \bar{p}$.)

Finally, note that $\hat{b}(\pi)$ (and thus $\hat{b}^{-1}(b)$, since $d\hat{b}(\pi)/d\pi \neq 0$) and $\hat{\pi}(b)$ are continuously differentiable in p. In addition, the above slope restrictions indicate that the derivative of $\phi(b) = \hat{\pi}(b) - \hat{b}^{-1}(b)$ is negative in the relevant range. As a result, one can use the inverse function theorem to establish that ϕ^{-1} exists and is continuously differentiable. (In particular, note that $\phi^{-1}(0) = \hat{b}$, so one can examine a neighborhood around 0 in the domain of ϕ^{-1}.) Thus, the unique values for $\hat{b}$ and $\hat{\pi}$ are continuously differentiable in p.

[29] See note 20.

[30] The lump-sum tax may involve a rebate if the fine revenues exceed the cost of enforcement. But fine revenues equal $p\hat{\pi}(1-F(\hat{b}))$ whereas harm equals $\hat{b}(1-F(\hat{b}))$; because $p\hat{\pi} \leq \hat{b}$ (otherwise type $\hat{b}$ will strictly prefer to comply and thus some for whom $b > \hat{b}$ will also be deterred, contradicting that $\hat{b}$ is the maximum feasible b^*), $\hat{\pi} < y + \hat{b}$.

factor preceding this component are both positive, the entire subtracted term in the denominator is positive, and the value of the denominator must exceed that of the first term. For the first term, note that $\hat{b} \leq h = 1$ and $\alpha = .1$. Thus,

(A9) $D < .1e^{.1} \approx .11 < .12.$

To demonstrate that the denominator is positive, first consider the highest value that can be taken by the subtracted term.[31] Using the preceding observations regarding the parameters, $\hat{\pi} < 13$, $p < .1$, and $(f(\hat{b})h + 1 - pf(\hat{b})\hat{\pi}) \leq 2$. In addition, note that $\hat{b} \geq 0$ and $1-p(1-F(\hat{b})) > .9$. Thus, one has

(A10) $D > .1 - (e^{.1\times 13}\times .1\times .1/.9)\times 2 \approx .018 > 0.$

Consider now the numerator. From the posited conditions on the parameters, $p < .1$, $\alpha = .1$, $c' < .001$, and $\hat{\pi} > y - c - h > 12 - .001 - 1 > 10$. Thus,

(A11) $N > e^{.1\times 10}[1 + (.1 \times .1/.9)\times(-.001+0)] - 1 \approx 1.72 > 1.7.$

(Note: if a smaller c' is chosen, the numerator is larger; thus, the additional limitation on the maximum permissible value of c' presented below does not affect this result.)

Combining these results, for $p \leq \bar{p}$, $\hat{b}' > 10$, as the Lemma asserts. Q.E.D.

Finally, observe that, at $\bar{p}$, $\hat{b} = h$.[32] Thus, for any $p \leq \bar{p}$, $h-b^*|_p \geq 10(\bar{p}-p)$. Q.E.D.

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[31] Note that if the harm and benefit were nonmonetizable, $f(\hat{b})h + 1$ would not be present in the final component of this term, and it would immediately be obvious that the denominator was positive. An upper bound would exist, and, although it would be larger than that in (A9) -- calculations show that .164 would be an upper bound -- the claim of the lemma that $\hat{b}' > 10$ would still hold.

[32] $\hat{b} \geq h$ follows by the definition of $\bar{p}$. If $\hat{b} > h$, there would exist a $p < \bar{p}$ for which $\hat{b} = h$ (see the continuity argument in note 28. This would contradict that $\bar{p}$ is the lowest threshold probability.

If p is optimal and $p \leq \overline{p}$, total risk-bearing costs are greater than $.002(\overline{p}-p)$. Assume that p is optimal, and all who act will be punished by an optimal fine π. First, one can demonstrate that $\pi \geq h/2$. If it is not, $b^*|_p < h/2$.[33] That is, at least half the population causes harm, which reduces average expected wealth relative to that when all are deterred by at least

$$\text{(A12)} \quad \int_{h/2}^{h} (h-b)f(b)db = h/8.$$

The only factor that makes average expected wealth higher at p, π than at $\overline{p}$ (combined with a fine sufficient to deter everyone) is that enforcement costs are lower, but this savings is less than the wealth reduction arising from additional harm, as $c(\overline{p}) - c(p) < c(\overline{p}) < c(1) < .001 < 1/8 = h/8$.[34] Thus, there exists a regime with greater average wealth than p, π and that also distributes wealth equally, so p, π cannot be an optimum.

Because $\pi > h/2$ and $p \in (0,1)$,[35] any individual who acts bears a positive risk, given the strict concavity of the utility function. In particular, the reduction in utility arising from the bearing of risk for an individual of type b, $b > b^*|_p$ (that is, for any who act) can be expressed as an wealth-equivalent ρ that corresponds to the certain reduction in wealth beyond $p\pi$ (the expected fine if one acts) that produces utility equal to that of acting and bearing the risk:

$$\text{(A13)} \quad -e^{-\alpha(x+b-p\pi-\rho)} = -(1-p)e^{-\alpha(x+b)} - pe^{-\alpha(x+b-\pi)}, \text{ or}$$

$$\text{(A14)} \quad \rho = (1/\alpha)\ln(1 - p + pe^{\alpha\pi}) - p\pi.^{36}$$

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33 If $\pi < h/2$ and $b = h/2$, it is clear that the value of (1) exceeds that of (2).

34 That $c(1) < .001$ follows because $c'(1) = .001$ and $c'' > 0$.

35 The threshold probability is less than one and the immediately preceding argument rules out p=0 as an optimum because $b^*|_{p=0} = 0$.

36 Thus, ρ is independent of b; in particular, from (A2), $\rho + p\pi = b^*$. This is a consequence of the constant absolute risk aversion utility function.

Let $\tilde{p}$ be a positive lower bound on the probability for which it is feasible to induce at least h/2 of the population to comply. The lowest such probability is clearly bounded away from zero given the upper bound on π. In particular, for the stated parameters, one can use the bound $\tilde{p} = .019$.[37] Now, one could compute the value of (A14) for every $p \in [.019, .1]$ (recalling that $\bar{p} < .1$) and $\pi \in [.5, 13]$ (recalling that h = 1 and the maximum feasible fine is less than 13).[38] The greatest lower bound of the risk-bearing cost for any individual at any p and π in the stated ranges must, due to the strict concavity of the utility function, is positive. In particular, it can be demonstrated that, in the stated range, ρ everywhere exceeds .0002.[39] We know from the earlier result that $h-b^*|_p \geq 10(\bar{p}-p)$, where, given the uniform distribution with h = 1, $h-b^*|_p$ is the portion of the population that bears risk-bearing costs greater than .0002. Q.E.D.

<u>The lowest threshold probability, $\bar{p}$, dominates any $p < \bar{p}$.</u> Assume $p < \bar{p}$ is an optimum. First, construct the following hypothetical regime, which will be shown to pareto dominate the result for p. Allow no individuals to act. Impose a tax (that need not be equal, or positive, for all individuals) that raises revenue equal to $c(\bar{p})$. Expected wealth in this regime can be lower than that at the assumed optimal p by at most $c'(1)(\bar{p}-p) = .001(\bar{p}-p)$.[40] Set the tax (subsidy) for each individual as follows: Those who act in the regime

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[37] With a fine of 13, which exceeds the maximum feasible fine, this probability induces $b^* \approx .495$.

[38] For this construction, one need not determine what π is optimal given each p or what b* results.

[39] First, note that, for any p in the stated range, ρ is minimized at $\pi = .5$. From (A14),

$$\frac{d\rho}{d\pi} = (1/\alpha)\frac{\alpha p e^{\alpha\pi}}{1-p+pe^{\alpha\pi}} - p = p(e^{\alpha(\pi-b^*)} - 1) > 0.$$

The inequality holds because, from (1) and (2), $p < 1$ implies $b^* < \pi$. Given this result, one can demonstrate that ρ is minimized at $p = .019$. From (A14),

$$\frac{d\rho}{dp} = (1/\alpha)\frac{e^{\alpha\pi}-1}{1-p+pe^{\alpha\pi}} - \pi = (1/.1)\frac{e^{.1\times.5} - 1}{1-p+pe^{.1\times.5}} - .5 \approx \frac{.513}{1+.051p} - .5 > 0.$$

The inequality holds because the denominator of the first term is highest at p = .1, which gives a value for that term of approximately .51. Thus, ρ is lowest in the stated range for $\pi = .5$, $p = .019$, and for those parameters $\rho \approx .00024$.

with p are to have, after tax (subsidy), expected wealth in the constructed regime equal to that they had in the regime with p minus the wealth-equivalent of the risk-bearing cost they bore in the regime with p. This will give them the same utility in both regimes. Note, however, that the total wealth required to accomplish equal utility for this group in the constructed regime is less than the total expected wealth this group receives in the regime with p by at least $.002(\overline{p}-p)$. As a result, the remaining wealth is greater in the constructed regime than for the regime with p, by an amount exceeding $.001(\overline{p}-p)$. Set the tax (subsidy) for those who do not act in the regime with p so as to distribute all remaining wealth equally among them. This group will have greater utility in the constructed regime. Thus, the constructed regime pareto dominates the regime with p.

Second, note that welfare can only be higher in a regime with $\overline{p}$ and the maximum feasible fine than in the constructed regime because both have the same expected wealth and the former regime distributes it equally. Q.E.D.

<u>If a probability sufficient to deter all individuals is optimal, the optimum probability is $\overline{p}$ and the optimal fine will deter all individuals.</u> A regime with $\overline{p}$[41] and a fine sufficient to deter all individuals entails lower enforcement costs than any regime with $p > \overline{p}$. Because this regime with $\overline{p}$ deters all harmful activity, total expected wealth is higher than for any regime with a greater probability. Moreover, at the maximum feasible fine, no risk is imposed with $\overline{p}$. Thus, any $p > \overline{p}$ entails lower welfare. Finally, at $\overline{p}$, a fine that does not deter all individuals lowers total expected wealth (because benefits of acting are less than the harm caused), imposes risk on

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40 This follows because $c'(1) = .001$ and $c'' > 0$. Even if expected wealth is higher at p, the difference must be less than stated in the text on two accounts. First, the difference in enforcement costs is less than given by the stated expression because $c'' > 0$. Second, all individuals are not deterred in the regime with p, so it entails the imposition of some net harm that is avoided in the constructed regime.

41 If the lower bound were not contained in the set of threshold probabilities -- note 22 proves it is in the case examined here, but not for any case in which full deterrence is optimal -- then the argument in text shows that, if p_1 and p_2 are threshold probabilities, p_1 is superior to p_2 if and only if $p_1 < p_2$.

those who act (unless $\pi = 0$), and produces no savings in enforcement costs compared to a fine sufficient to deter all individuals. Q.E.D.

<u>The the maximum feasible fine is required to deter all individuals at $\overline{p}$.</u> If it were not, one could raise the fine and lower the probability slightly and still deter all individuals,[42] which would contradict that $\overline{p}$ was the lowest threshold probability. Q.E.D.

Proof of Proposition 2

Choose parameters and functions as in the proof of proposition 1, except for c, which instead is to satisfy $c'(0) > 40$.

The method of proof is to show that, for any $p > 0$, the rise in each individual's wealth (over that at $p = 0$) due to the savings in harm is less than the rise in per capita enforcement cost. To establish an upper bound on the harm averted, the first step is to derive an upper bound for the level of b^* at any p and then to derive an upper bound for the savings in harm at any p.

Recall from the proof of proposition 1 that

(A3) $e^{\alpha\hat{b}} = 1 - p + pe^{\alpha\hat{\pi}}$,

where $\hat{b}$ refers to the maximum feasible b^* given p and $\hat{\pi}$ refers to the maximum feasible π given p. Let $\overline{\pi}$ be a constant that is greater than $\hat{\pi}$ for any p (recall from the proof of proposition 1 that, for any p, $\hat{\pi} < 13$), and define $\overline{b}$ as b^* given p and $\overline{\pi}$. That is,

(A15) $e^{\alpha\overline{b}} = 1 - p + pe^{\alpha\overline{\pi}}$.

Differentiating (A15) with respect to p yields

(A16) $\overline{b}' = \dfrac{e^{\alpha\overline{\pi}} - 1}{\alpha e^{\alpha\overline{b}}}$.

$\overline{b}'$ is highest when $\overline{b} = 0$, in which case

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[42] See the continuity argument in note 28.

$$\text{(A17)}\quad \bar{b}' < \frac{e^{.1\times 13} - 1}{.1} \approx 26.69 < 27.$$

It is apparent that $\hat{b} < \bar{b}$, so it follows that, for any $p > 0$,[43] $b^* < \bar{b} < 27p$.

For the stated parameters, the harm from individuals' acts is

$$\text{(A18)}\quad \int_{b^*}^{1} 1db = 1-b^*.$$

The derivative of (A18) with respect to b^* is -1.

Combining these results, the savings in harm for any $p > 0$, compared to the level of harm at $p = 0$, is less than $27p$. In contrast, the total enforcement cost for any $p > 0$ exceeds $pc'(0)$, which equals $40p$.

Finally, for $p > 0$, consider separately the expected utility of individuals of type $b \leq b^*$ and those of type $b > b^*$, in each case comparing their expected utility to that at $p = 0$. The former group is deterred, and thus receives (from (2))

$$\text{(A19)}\quad U_n|_p = U(y - t - (1 - F(b^*))h),$$

while, at $p = 0$, this group acts, and thus receives (from (1))

$$\text{(A20)}\quad U_a|_0 = U(y - h + b).$$

In addition, recall that

$$\text{(4)}\quad t = c(p) - (1 - F(b^*))p\pi.$$

The preceding argument demonstrates that the savings in harm, $F(b^*)h$, in (A19) is less than the enforcement cost, $c(p)$, which is part of the tax term in (A19), by more than $13p$. The tax t is reduced by the fine revenue component, $(1-F(b^*))p\pi$. Recalling that $\pi < 13$, this reduction is less than $13p$; that is, the tax t exceeds $27p$, which in turn exceeds the savings in harm. Thus, expected utility in (A19) is lower than that in (A20). (It is also the case that, in (A20), wealth is higher by the benefit b.)

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[43] From (A2), at $p = 0$, $b^* = 0$.

Now, consider (using (1)) the utility of those who are not deterred at $p > 0$:

$$\text{(A21)} \quad U_a|_p = (1-p)U(y - t - (1 - F(b^*))h + b) + pU(y - t - (1 - F(b^*))h + b - \pi).$$

At $p = 0$, utility for this group is given by (A20). As with the other group, wealth (whether or not penalized) is reduced by the tax by more than it is increased by the savings in harm. The benefit b is received by this group under either regime. The additional difference is that, in (A21), individuals pay the penalty π with probability p, which further lowers their expected utility relative to that in (A20).

Thus, for all individuals, expected utility is higher at $p = 0$ than at any $p > 0$. Q.E.D.

Proof of Proposition 3

Choose parameters and functions as in the proof of proposition 1, except for c and $f(\cdot)$:

$c'(.01) = .1$; $c'(.011) = 100$.

$$f(b) = \begin{cases} 18-180b, & \text{for } 0 \le b \le .1 \\ 0, & \text{for } .1 < b < .9 \\ 20b-18, & \text{for } .9 \le b \le 1. \end{cases}$$[44]

<u>Welfare at $p = .01$ and $\pi = 10$ is greater than welfare at $p = 0$.</u>[45] Inserting these parameter values into (A2) indicates that $b^* \approx .17$; that is, all individuals (.9 of the population) with $b \le .1$ are deterred and none of the individuals (.1 of the population) with $b \ge .9$ are deterred.

Consider first the utility of those who are deterred.

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[44] With this distribution, .9 of the population has $b \le .1$ and .1 of the population has $b \ge .9$.

[45] Given that $y = 12$, $h = 1$, and $c(.01) < .001$, it is clear that $\pi = 10$ is feasible.

$$\text{(A21)}\quad U_n|_{.01} = U(y - t - .1h) > U(y - .101).$$

The equality follows from (2) and the inequality because $h = 1$ and $t < c(.01) < .001$. In contrast, at $p = 0$ (in which case all individuals act), their utility is

$$\text{(A22)}\quad U_a|_0 = U(y - h + b) \leq U(y - .9).$$

The equality follows from (1) and the inequality because $h = 1$ and, for this group of individuals, $b \leq .1$. Thus, all individuals in this group are better off in the regime with $p = .01$.

Now consider the utility of individuals who are not deterred with $p = .01$.

$$\text{(A23)}\quad U_a|_{.01} = .99U(y - t - .1h + b) + .01U(y - t - .1h + b - \pi)$$
$$> .99U(y - .101 + b) + .01U(y - .101 + b - 10).$$

The equality is from (1) and the reasoning for the inequality is as with (A21). In contrast, at $p = 0$, their utility is

$$\text{(A24)}\quad U_a|_0 = U(y - h + b) = U(y - 1 + b).$$

Comparing (A23) and (A24), there are two differences. First, (A24) has wealth reduced by 1 rather than .101, a difference of .899. Second, (A23) entails a .01 probability of a reduction of 10 in wealth. The certainty equivalent of this loss is less than .2.[46] As a result, this group also has greater utility in the regime with $p = .01$.

<u>Welfare at $p = .01$ and $\pi = 10$ is greater than welfare at any probability sufficiently large to deter all individuals.</u> First, observe that a probability exceeding .021 is required to deter all individuals. (This follows from the fact that at $p = .021$ and $\pi = 13$ -- which is greater than the maximum feasible π -- $b^* \approx .51$, while .1 of the population has $b \geq .9$.) Given that $c'(.011) = 100$,, $c(.021) > 1$.

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[46] Paralleling the derivation of the risk premium for the proof of proposition 1, (A13) and (A14), with the given constant absolute risk aversion utility function, the certainty equivalent can be expressed as $\rho + p\pi = b^* \approx .17$.

If all individuals are deterred, no one acts and (certain) wealth for each is y - t, which is less than y - 1.[47] Compare welfare at $p = 0$, in which case all individuals act and taxes are zero. If an individual did not act (which yields less utility than that at $p = 0$, because all individuals will prefer to act and receive b), certain wealth would be y - 1, which exceeds the level of wealth when all are deterred. Thus, welfare is lower when all individuals are deterred than at $p = 0$, which is less than welfare at $p = .01$ by the preceding argument. Q.E.D.

Proof of Proposition 4

Choose parameters and functions as in the proof of proposition 3, with the following addition: $c'(.009) = .001$. This restriction is consistent with the assumptions used in the proof of proposition 3, so the analysis there is applicable. In particular, the optimal probability is greater than zero but less than a level sufficient to deter all individuals.

Denote the optimal probability and fine as p and π. It will now be shown that the optimum entails a level of deterrence that is sufficiently low ($b^* \leq .1$) and a probability sufficiently high ($p > .0085$) so that π must be less than the maximum feasible fine.

<u>At the optimum, $b^* \leq .1$.</u> First, if $.1 < b^* < .9$, one could improve welfare by reducing the fine slightly (by a small enough amount so that one still has $b^* > .1$). Behavior would not change (there are no individuals in this range), and the reduction in fine revenue constitutes a redistribution of wealth toward the lowest wealth (highest marginal utility of wealth) circumstances from others.[48]

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[47] This is because $c(p) > 1$ and there is no fine revenue because all are deterred.

[48] Although fines are paid by the highest b types, a fine sufficient to deter individuals for whom $b = .1$ must be at least 3. (From the proof of proposition 3, the optimal probability is less than .021, and a probability of .021 requires a fine of about 3.9 to deter type .1 individuals.) Thus, individuals who act and are fined have wealth *in that state* below that when they are not fined and below that of individuals who do not act.

Second, the possibility that $b^* \geq .9$ can be ruled out by a slight extension of the argument in the proof of proposition 3 indicating that $b^* = 1$ was not the optimum. The necessary enforcement costs exceed 2.[49] Fine revenue is less than .1.[50] Thus, the tax is greater than 1.9. Compare the level of welfare at $p = 0$: the tax is zero; the harm is 1; and each individual receives the benefit from acting, b. Thus, individuals who do not act at p, π have greater certain wealth at $p = 0$. Individuals who do act at p, π, would have greater wealth at $p = 0$ even when they are not fined (and all the more so when they are). Thus, $p = 0$, which implies $b^* = 0$, dominates this case.

<u>$p > .0085$.</u> If not, this p and π are dominated by $\tilde{p} = .009$ and $\tilde{\pi}$ selected to induce the same b^* as p, π. (Note that $c(\tilde{p}) < c'(.009)\times.009 = .000009$.) This is demonstrated in four steps. First, from the proof of proposition 3, for $p < .01$, at most one can deter the group of individuals for whom $b \leq .1$.

Second, at p, π, $b^* \geq .05$. To demonstrate this, consider $b^* < .05$. The greatest possible fine revenue in this regime will be less than that possible if all individuals (regardless of whether they act) were subject to the fine and the probability and fine were such that $b^* = .05$.[51] At $b^* = .05$, the greatest fine revenue is with the highest possible probability.[52] Since we are considering an optimum with $p \leq .0085$, consider $p = .0085$. The fine just sufficient to induce $b^* = .05$ is somewhat less than 4.64. The maximum possible fine revenue is thus less than .04, so the lowest possible tax is greater than -.04.[53] (That is, there may be a lump sum subsidy of an amount

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[49] At $p = .031$ and $\pi = 13$, which exceeds the maximum feasible fine, $b^* \approx .8$. Thus, if $b^* \geq .9$, enforcement costs exceed $(.031-.011) \times c'(.011) = .02 \times 100 = 2$.

[50] At most .1 of the population -- those for whom $b \geq .9$ -- act and pay the fine. The expected fine must be less than 1 for fine revenue to be positive: if it were not, all would be deterred -- as $b \leq 1$ for all individuals and utility is concave -- and there would be no fine revenue.

[51] If one had $b^* < .05$, one could increase the probability or fine, keeping $b^* \leq .05$, which, under the hypothetical assumption that all are subject to the fine, would increase fine revenue.

[52] The strict concavity of the utility function implies that, for a lower probability, one need not raise the fine proportionately to produce the same b^*. And, at $p = .0085$, the constraint on the maximum feasible fine is not binding. (As noted in the text to follow, the necessary fine is less than 10, which in turn is less than the maximum feasible fine at any probability.)

less than .04.) Compare this to $\tilde{p} = .009$ and $\pi = 7.8$: this induces $b^* \geq .1$[54] for a lump-sum tax of less than -.007 (that is, a subsidy exceeding .007)[55] and reduces total harm, in moving from $b^* = .05$ to $b^* = .1$, by .225. Examining just these effects from the change in harm and the tax, the wealth of all individuals is greater by at least .19[56] than for any possible optimum yielding $b^* < .05$. Thus, individuals who do not act under either regime have greater utility with $\tilde{p} = .009$, $\pi = 7.8$ than with p, π. Those who act with p, π but who are deterred with $\tilde{p} = .009$, $\pi = 7.8$ (individuals with $b^*|_{p,\pi} \leq b \leq .1$) lose their benefit from acting, which is at most .1, which in turn is less than the wealth increase of at least .19.[57] And those who act in both regimes (individuals with $b \geq .9$) face a greater probability and possibly a greater fine with $\tilde{p} = .009$, $\pi = 7.8$ than with p, π, but the reduction in certain wealth corresponding to $\tilde{p} = .009$, $\pi = 7.8$ is less than .11,[58] which in turn is less than the wealth increase of at least .19. Therefore, any p, π such that $b^* < .05$ cannot be optimal.

Third, compare the fine revenue at any optimum with $p \leq .0085$ to that at $\tilde{p} = .009$, $\tilde{\pi}$ (the fine that induces the same b^* as p, π). First, fine revenue for p, π cannot exceed that at a probability of .0085 and a fine just sufficient to induce $b^*|_{p,\pi}$.[59] Second, analysis of the utility function indicates that fine revenue per individual who acts at $\tilde{p} = .009$, $\tilde{\pi}$ exceeds that at $p = .0085$, π by a greater amount the higher is b^*.[60] Computing the fines that induce $b^* = .05$ (a lower bound on the optimal b^*, by the preceding

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53 This ignores enforcement costs, which only reinforces the claim.

54 $b^* \approx .106$.

55 Fine revenue per individual who acts and is thus subject to the fine is .0702; .1 of the population is subject to the fine, yielding revenue of .00702; and the enforcement cost is less than .000009.

56 $(.225 + .007) - .04 = .192$.

57 They also gain by avoiding the possibility of paying π.

58 As noted in the proof of proposition 3, with the given constant absolute risk aversion utility function, the certain wealth cost of the probabilistic fine, $\pi + p\pi$, simply equals b^*, which is given in note 54.

59 As explained in note 52, for a given b^*, fine revenue is greater when the probability is increased and the fine correspondingly reduced, because less of the deterrent effect is produced by risk-bearing costs.

argument) at probabilities of .0085 and .009, one obtains the result that the difference in fine revenues per individual who acts between the two regimes exceeds .0003 (favoring the regime with a $\tilde{p}$ = .009).[61]

To complete this part of the argument, note that, by construction, behavior is the same under the two regimes. As a result, it is clear that individuals for whom $b \leq b^*$ are better off in the regime with $\tilde{p}$ = .009 by a wealth equivalent of at least .00002 (the only difference is that the tax is lower).[62] Individuals for whom $b \geq b^*$ also benefit from the lower tax. They do, however, pay a correspondingly greater expected fine. Yet, because the fine with $\tilde{p}$ was selected to yield the same b^* as at p, π, this group is indifferent between the two regimes with regard to the utility cost of the probabilistic fine. (Given the constant absolute risk aversion utility function used here, the utility cost of the probabilistic fine is equal in the two cases and independent of b, because risk-bearing costs are independent of the level of wealth.[63]) Thus, all individuals are better off in the regime with $\tilde{p}$ = .009, $\tilde{\pi}$, so $p \leq .0085$ cannot be an optimum.

The maximum feasible fine cannot be optimal. At p = .0085, a fine of 8 is sufficient to induce $b^* > .1$; for $p > .0085$, a lower fine will suffice. As the maximum feasible fine exceeds 10 for any p, the maximum feasible fine induces $b^* > .1$ at the optimal p, which exceeds .0085. This is not optimal. Q.E.D.

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60 The difference is fine revenue per individual who acts is given by the expression

$$.009\times\frac{1}{\alpha}\ln\left[\frac{e^{\alpha b^*}-1 + .009}{.009}\right] - .0085\times\frac{1}{\alpha}\ln\left[\frac{e^{\alpha b^*}-1 + .0085}{.0085}\right].$$

The derivative of this expression with respect to b^* is positive for $b^* > 0$.

61 The fine for p = .0085 is under 4.64 and that for $\tilde{p}$ = .009 is over 4.42. The difference in fine revenue per individual exceeds .0003.

62 Fine revenues per individual who acts and is thus subject to the fine are higher by more than .0003; at least .1 of the population is subject to the fine (at p = .0085, a fine of 13, which exceeds the maximum feasible fine, $b^* \approx .224 < .9$), so total fine revenues are higher by more than .00003; and total enforcement costs are less than .000009.

63 For any p, π, the expected utility of an individual who acts is reduced (compared to utility with p = 0) by an income equivalent of $\pi + p\pi$ (A13), which simply equals b^* (A14, A2).

Proof of Proposition 5

Proof of (1): a higher probability of detection. Use the parameters from the proof of proposition 4. In the risk-neutral case, a probability of .0084 is sufficient to induce $b^* > .1$. (At this b^*, the maximum feasible fine will exceed 12, and, in the risk-neutral case, $b^* = p\pi$.) In addition, given the high marginal cost for any probability exceeding .011 and that the maximum feasible fine does not exceed 13, it clearly will not be optimal to deter any individuals of type $b \geq .9$.[64] Thus, the optimal probability for the risk-neutral case with these parameters is less than .0084. The proof of proposition 4 established that, for the risk-averse case, the optimal probability exceeds .0085. Q.E.D.

Proof of (2): a lower probability of detection. Use the parameters in proposition 1. For the case in which individuals are risk-averse, consider $p = .05$, $\pi = 12$. This induces a b^* exceeding one, and this π is feasible when all are deterred. Thus, the minimum threshold probability, which is optimal, is less than .05.

For the risk-neutral case, note that any probability less than .05 and any fine less than 13 (which exceeds the maximum feasible fine for any p) induces $b^* < .65$. (Recall, again, that, in the risk-neutral case, $b^* = p\pi$.) Any such outcome is dominated by $p = .085$, $\pi = 12$: This regime induces $b^* > 1$ (which makes $\pi = 12$ feasible); the savings in net harm is $\frac{1}{2}(.35)^2 = .06125$; and the increased enforcement cost is less than the total enforcement cost, which in turn is less than .001. Thus, the optimal probability must be greater than or equal to .05. Q.E.D.

Proof of (3): a higher fine. The optimal fine may be higher in the case involving risk-aversion even though the optimum for the risk-neutral case involves use of the maximum feasible fine, because the maximum feasible fine at the optimal probability when individuals are risk-averse may exceed the maximum feasible fine at the optimal probability in the risk-neutral case.

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[64] To induce $b \geq .9$ will require $p > .061$, which entails enforcement costs of more than 5; even complete deterrence reduces total harm by only 1.

This is true for the parameters in the proof of proposition 1. In the specified case in which individuals are risk-averse, all are deterred and, as the proof of (2) demonstrates, this involves a lower probability than when individuals are risk-neutral. As a result, the maximum feasible fine is greater in the risk-averse case: Enforcement costs are lower (which makes taxes lower), harm is less, and b^* is greater, all three of which contribute to a greater maximum feasible fine. (Fine revenue is less, implying a lower tax rate for a given level of enforcement costs, but this effect, which equals $(1-F(b^*))p\pi$, is dominated by the reduction in harm, as the total harm equals $(1-F(b^*))h$ and $p\pi < h$.[65]) Since the optimum when individuals are risk-averse

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[65] This argument assumes that h is monetizable. Assume that both h and b are nonmonetizable. (If b is monetizable, the result would still hold; solving with the parameters below, the optimal fine in both cases rises by about the same amount -- slightly more when individuals are risk-averse because the optimal b^* is slightly greater in that case -- leaving the gap of at least .05 largely unaffected.) Choose parameters and functions as in the proof of proposition 1, except for α, c, and $f(\cdot)$:

$$\alpha = 1.$$

$$c'(.0001) = .0001;\ c'(.0002) = 10;\ c'(.01) = 11.$$

$$f(b) = \begin{cases} 9, & \text{for } 0 \le b \le .1 \\ 49.5\text{-}405b, & \text{for } .1 < b < 11/90 \\ 0, & \text{for } 11/90 \le b \le 1. \end{cases}$$

Consider first the case when individuals are risk-averse. Deterrence of all individuals is feasible, for example, with $p = .0000008$, and a fine of 11.999, which is less than the maximum feasible fine which exceeds 12 - .0001. (At this probability and fine, $b^* \approx .12228$, which exceeds 11/90.) In addition, a necessary condition for less than complete deterrence to be optimal is that the resulting harm be less than the savings in enforcement costs. (If this is not the case, complete deterrence involves greater total wealth, and that wealth is distributed equally.) This implies an optimal b^* in excess of .12146. (At this b^*, total harm exceeds .000117, which is greater than .0001, which in turn exceeds total enforcement costs at $p = .0001$.) For such a b^* to be optimal, the probability must be less than .0000008 for there to be any enforcement cost savings. At $b^* = .12146$ and $p = .0000008$, the required fine exceeds 11.99. (For a higher b^* or lower p, the necessary fine would be even higher, as is clear from (A2).) Thus, the optimum when individuals are risk-averse involves a fine exceeding 11.99.

When individuals are risk-neutral, note first that at the optimum $b^* \ge .1$. If this is not the case, the first-order condition (10) is violated (and welfare is increasing with the probability of detection). Assume (10) holds:

$$(10)\quad c' = (h - b^*)f(b^*)\frac{db^*}{dp}.$$

For the stated parameters, $(h - b^*) > 1-.1 = .9$ and $f(b^*) = 9$. In addition, it can be seen that $db^*/dp > 11$: Begin by recalling that, when individuals are risk-neutral, involves imposing the maximum feasible fine, it involves a higher fine than when individuals are risk-neutral. Q.E.D.

Proof of (4): a lower fine. Using the parameters in the proof in proposition 4, it is established, for the case when individuals are risk-averse, that the optimal fine is less than 8. The risk-neutral case uses the maximum feasible fine for the given probability, which must exceed 10. Q.E.D.

Proof of (5): a higher level of deterrence. Proposition 1 establishes that complete deterrence is optimal for some cases involving risk-aversion, but complete deterrence is never optimal when individuals are risk-neutral and there is a limit on the maximum feasible fine. Q.E.D.

Proof of (6): a lower level of deterrence. Use the parameters in the proof of proposition 3. By choosing c'(.009) sufficiently low, the optimal level of deterrence when individuals are risk-averse, b_{ra}, will be below that

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$$\frac{db^*}{dp} = p\frac{d\pi}{dp} + \pi.$$

At p = .01, c(p) < .11, so the maximum feasible fine exceeds 11.89, which is sufficient to induce b* > .1189. Thus, b* < .1 implies π > 11.89 and p < .01. With h and b nonmonetizable, the maximum feasible fine is given be

$$\pi = \frac{y - c}{1 - p(1-F(b^*))}.$$

Taking the derivative and using the first-order condition (10) to substitute for $db^*/d\pi$ yields

$$\frac{d\pi}{dp} = \frac{-c' - p\pi c'/(h-b^*) - \pi F(b^*) + \pi}{1 - p(1-F(b^*))}.$$

A lower bound (greatest negative magnitude) can be given at c' = 11, p = .01, b* = .1 for the numerator and b* = 0 for the denominator, and π = 12.1. (For the latter, a larger π increases the magnitude, and the maximum feasible π must be less than 12 + $p\pi(1-F(b^*))$, where $p\pi$ < .1 and $(1-F(b^*)) < 1$.) Using these values, $d\pi/dp > -11.4$. This, in turn, implies $db^*/dp > -.114 + 11.89 > 11$. Thus, the right side in the first order condition (10) exceeds 89.1, while, for $p \leq .01$, $c' \leq 11$. This contradiction establishes that $b^* \geq .1$ when individuals are risk-neutral.

One can now establish an upper bound on the maximum feasible fine. First, at most .1 of the population is not deterred and fine revenue per individual cannot exceed .13 (fine revenue per person is $p\pi$, which equals b* in the risk-neutral case, which cannot involve full deterrence -- i.e., $b^* = 11/90 \approx 1.22$). Thus, ignoring enforcement costs, the maximum feasible fine cannot exceed 12.013. At such a fine, the probability of detection would have to exceed .0083 for $b^* \geq .1$. Such an enforcement level costs more than $.008 \times 10 = .08$. With fine revenue less than .013, the lump-sum tax would have to exceed .06, so the maximum feasible fine is less than 11.94. In contrast, the optimal fine when individuals were risk-averse was seen to exceed 11.99.

when individuals are risk-neutral, b_{rn}. The intuition is that, if marginal enforcement costs at p = .009 are sufficiently low, the risk-neutral optimum involves deterrence very close to $b^* = .1$. Reducing deterrence slightly (say, by reducing π) increases harm at a rate close to zero ($f(.1) = 0$), but, in the case when individuals are risk-averse, it reduces risk-bearing costs for portion of the population who act -- at least .1 of the population, who are of type $b \geq .9$ -- at a positive rate.

Assume the contrary: that is, if $c'(.009) > 0$, $b_{ra} \geq b_{rn}$. Define

(A25) $\tilde{b} = b_{ra} - (.1 - b_{rn}) = b_{rn} - (.1 - b_{ra})$.

Note, by the proof of proposition 4, $b_{ra} \leq .1$. Moreover, it should be clear that $b_{ra} \neq .1$: If $b_{ra} = .1$, a small decrease in π results in no additional harm at the margin but decreases risk-bearing costs.[66] Therefore,

(A26) $\tilde{b} < b_{rn}$.

The next step is to compare the welfare at p_{ra}, π_{ra} to that at p_{ra}, $\tilde{\pi}_{ra}$, where p_{ra} and π_{ra} are the optimal p and π in the case with risk aversion, and $\tilde{\pi}_{ra}$ is the fine that induces $\tilde{b}$ at p_{ra}.[67] The method will be to demonstrate that $dW/d\pi$, given p_{ra}, is negative for $\tilde{\pi}_{ra} \leq \pi \leq \pi_{ra}$ when $c'(.009)$ is sufficiently low. Differentiating (3),

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[66] From (A31), below, $dW/d\pi$ is negative at the π that induces $b^* = .1$: the first term in brackets -- the marginal increase in harm -- is zero, and the second (subtracted) term -- reflecting risk-bearing costs -- is positive.

[67] From (A2), b^* is continuous and increasing in π, for a given p, and $\pi = 0$ induces $b^* = 0$, so there exists $\tilde{\pi}_{ra} \in (0, \pi_{ra})$ that induces $\tilde{b} \in (0, b_{rn}$. (Note: $b^* > .09$ -- at $b^* \leq .09$, harm is at least .009 higher than at $b^* = .1$, which is feasible with additional enforcement costs that are less than total enforcement costs of less than .000009. Thus, $\tilde{b} > 0$.)

$$\text{(A27)}\quad \frac{dW}{d\pi} = \int_0^{b*} \Big[(1-F(b*))p - p\pi f(b*)b*' + hf(b*)b*'\Big]U'_n f(b)db + U_n|_{b*}f(b*)b*'$$

$$+ (1-p)\int_{b*}^{1} \Big[(1-F(b*))p - p\pi f(b*)b*' + hf(b*)b*'\Big]U'_{a1} f(b)db$$

$$+ p\int_{b*}^{1} \Big[(1-F(b*))p - p\pi f(b*)b*' + hf(b*)b*' - 1\Big]U'_{a2} f(b)db - U_a|_{b*}f(b*)b*',$$

where $b*'$ denotes $db*/d\pi$, U' denotes the marginal utility of wealth, subscripts n, a1, and a2 on U' indicate that marginal utility is evaluated at wealth equal to x, $x + b$, and $x + b - \pi$, respectively (with $x = y - t - (1-F(b*))h$), and subscripts n and a on $U|_{b*}$ refer to (1) and (2) evaluated for type $b*$. In examining (A27), first note that the second and fifth terms offset, as the individual of type $b*$ equates U_n and U_a. Next, the term under each integral $-p\pi f(b*)b*'$ is negative (or zero, for $b* = .1$), so dropping it can only increase the value of the right side.[68] Thus, one can write

$$\text{(A28)}\quad \frac{dW}{d\pi} \le [(1-F(b*))p + hf(b*)b*']\bar{U}' - p\int_{b*}^{1} U'_{a2} f(b)db,$$

where, with the utility function $-e^{-\alpha w}$,

$$\text{(A29)}\quad \bar{U}' = \int_0^{b*} \alpha e^{-\alpha x} f(b)db + \int_{b*}^{1} [(1-p)\alpha e^{-\alpha(x+b)} + p\alpha e^{-\alpha(x+b-\pi)}]f(b)db$$

$$< \alpha e^{-\alpha x}\Big[F(b*) + (1-F(b*))e^{-\alpha b*}(1 - p + pe^{\alpha\pi})\Big] = \alpha e^{-\alpha x}.$$

The inequality follows because $e^{-\alpha b}$ is greatest in the range $b \in [b*,1]$ at $b*$. From (A2), $1 - p + pe^{\alpha\pi} = e^{\alpha b*}$, so the entire term in brackets equals 1. Returning to (A28), note that, in the range $b \in [b*,1]$,

$$\text{(A30)}\quad U'_{a2} = \alpha e^{-\alpha(x+b-\pi)} > \alpha e^{-\alpha(x+1-\pi)}.$$

Using (A29) and (A30) in (A28),

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[68] The expression derived below for $b*'$ indicates that it is positive.

$$(A31)\ \frac{dW}{d\pi} < \alpha e^{-\alpha x}\Big[hf(b^*)b^{*\prime} - (1-F(b^*))p(e^{\alpha(\pi-1)} - 1)\Big].$$

For the first term in brackets, note that, in the range $b^* \in [\breve{b},.1]$, $f(b^*)$ is at a maximum at $\breve{b}$; $h = 1$; and, from (A2), $b^{*\prime} = pe^{\alpha(\pi-b^*)} < \bar{B}$, where $\bar{B}$ is an upper bound (which exists, as $p < 1$, $\alpha = .1$, and, as demonstrated in previous proofs, $\pi < 13$). For the second term in brackets, note that $F(b^*) \leq .9$ in the range $b^* \in [\breve{b},.1]$. For $c'(.009) \leq .001$, $p > .007$ by the proof of proposition 4. In addition, one can show that $\pi > 2$.[69] Thus, there exists a positive lower bound, $\underline{B}$, for the second term in brackets.

Using these bounds, one can deduce from (A31)

$$(A32)\ \frac{dW}{d\pi} < \alpha e^{-\alpha x}(f(\breve{b})\bar{B} - \underline{B}).$$

The term $\alpha e^{-\alpha\pi}$ is positive.[70]

It will now be demonstrated that the term in parentheses in (A32) is negative for $c'(.009)$ sufficiently small. Note first that, as $c'(.009)$ approaches zero, b_{rn} approaches .1: If this were not the case, for $c'(.009)$ arbitrarily small, one would have b_{rn} bounded away from .1. At such a b_{rn}, total harm would exceed that at .1 by an amount bounded away from zero. But with $c'(.009)$ sufficiently small, the total enforcement cost necessary to finance $p = .009$, which is sufficient to induce $b^* = .1$,[71] would be less than that level of harm. Next, from (A25), as b_{rn} approaches .1, $\breve{b}$ approaches b_{ra}. Finally, by assumption, $b_{rn} \leq b_{ra}$, and it was demonstrated that $b_{ra} < .1$, so, as b_{rn} approaches .1, b_{ra} approaches .1. Thus, $\breve{b}$ approaches .1. This implies that, in (A32), $f(\breve{b})$ goes to zero.

This argument demonstrates that, for $c'(.009)$ sufficiently small, the expression must be negative in the range $\tilde{\pi}_{ra} \leq \pi \leq \pi_{ra}$. Because welfare is

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69 The proof of proposition 4 also shows that $b^* \geq .05$ and the analysis in proposition 3 indicates that, at the optimum, $p < .021$. The minimum fine sufficient to induce $b^* = .05$ at $p = .021$ is approximately 2.14.

70 x is bounded, so the term cannot equal zero.

71 To see that $p = .009$ is sufficient to induce $b^* = .1$, note that the maximum feasible fine at $b^* = .1$ exceeds $12 - .009 + .09 - .9 > 11.18$, and $.009 \times 11.18 = .1006$.

decreasing in π throughout this range, the assumption that $b_{ra} \geq b_{rn}$ is contradicted. (Any such b_{ra} is dominated by a $\tilde{b}$ given by (A25), with $\tilde{b} < b_{rn}$.) That is, when $c'(.009)$ is sufficiently small, optimal deterrence must be less when individuals are risk-averse than when individuals are risk-neutral.
Q.E.D.

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