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## OPTIMAL POLICY WITH HETEROGENEOUS PREFERENCES

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# Optimal Policy with Heterogeneous Preferences

Louis Kaplow\*

## *Abstract*

Optimal policy rules—including those regarding income taxation, commodity taxation, public goods, and externalities—are typically derived in models with homogeneous preferences. This article reconsiders many central results for the case in which preferences for commodities, public goods, and externalities are heterogeneous. When preference differences are observable, standard second-best results in basic settings are unaffected, except those for the optimal income tax. Optimal levels of income taxation may be higher, the same, or lower on types who derive more utility from various goods, depending on the nature of preference differences and the concavity of the social welfare function. When preference differences are unobservable, all policy rules may change. The determinants of even the direction of optimal rule adjustments are many and subtle.

JEL Classes D61, D62, D63, H21, H23, H24, H43, K34

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## 1. Introduction

Most analytical work in public economics that derives rules for optimal policy assumes that preferences are homogeneous. This characterization applies, for example, to most of the literature on optimal income taxation, commodity taxation, public goods provision, and the control of externalities. The reason for this simplification is tractability; indeed, the second-best problems are complex even when preferences are stipulated to be homogeneous.

It is natural to explore whether and how standard results are modified when preferences are heterogeneous. There may be ordinary differences in tastes: Some individuals may prefer chocolate, others vanilla; some may love nature and thus highly value improvements to national parks or pollution regulations that restore wilderness habitats, whereas others may desire city life and accordingly prefer enhancements to urban amenities. Differences may also arise on account of physical and mental disabilities.

These possibilities raise a number of questions. Do Atkinson and Stiglitz's (1976) result on uniform commodity taxation, the Samuelson rule for public goods, and the Pigouvian dictum that corrective taxes and subsidies should equal the marginal harms and benefits (respectively) from externalities survive the introduction of heterogeneity, and, if not, how do the results change? Likewise, how are standard first-order conditions for optimal income taxation affected? How do the answers to such questions depend on whether differences in preferences are observable? Finally, how do they depend on the manner in which preferences differ? In other words, in what respects is the problem of preference heterogeneity itself a heterogeneous phenomenon?

A related, more focused set of questions may be directed at a particular set of results in public economics. Certain policy rules have been demonstrated to be optimal because following them generates Pareto improvements, one consequence of which is that the optimal rules are independent of the particular choice of social welfare function (SWF). This approach is featured in generalizations of Atkinson and Stiglitz (1976) on commodity taxation as well as, in work on public goods and on externalities. See, for example, Kaplow (1996, 2006a, 2006b, 2006c, 2008), Konishi (1995), and Laroque (2005). Yet when preferences are heterogeneous, Pareto improvements are highly unlikely. Virtually any policy reform, no matter how desirable, is likely to generate some losers because certain individuals will have idiosyncratic preferences that are better served by the inefficient status quo. For example, even substantial reductions of a highly damaging pollutant at minimal cost may disfavor some whose bodies are insensitive to the pollutant and who especially enjoy polluting activities that need to be curtailed. Ng (1984b) has suggested that, as a consequence of heterogeneity, we might instead ask whether policies offer a "quasi-Pareto" improvement, by which he means a gain, on average, to individuals at every level of income (thereby addressing standard distributive concerns). Nevertheless, it is appropriate to explore formally and systematically how, if at all, the derivations and results in this literature need to be adjusted in light of preference heterogeneity.

This article undertakes a preliminary exploration of these questions. Section 2 presents a model with a nonlinear income tax and commodity taxes where individuals' preferences for

commodities are heterogeneous. In section 3, the model is analyzed for the case in which preference differences are observable. This case is of interest because it is more tractable and transparent, facilitates examination of the case with unobservable heterogeneity, and has some elements of realism (for example, certain physical disabilities, with associated differences in preferences, are observable). It is straightforward to show that the Atkinson-Stiglitz result concerning the optimality of uniform commodity taxation and a variety of extensions are preserved. Accordingly, the bulk of the analysis in this section focuses on optimal income taxation results, which do differ. For individuals with preferences that yield higher utility for a given level of disposable income, optimal levels of income taxation may be higher, the same, or lower, depending on the nature of preference differences and on the concavity of utility functions and of the SWF. An implication is that some results in prior literature attributed to preference heterogeneity are in fact a product of the choice to model heterogeneity in one particular way; other choices would have yielded different, even opposite, conclusions.

Section 4 considers the case in which preference differences are unobservable. It is demonstrated that uniform commodity taxation is no longer desirable. However, determinants of the direction and magnitude of deviations are complex and subtle. The basic explanation for the results is that differentials in commodity taxation are optimal because (and to the extent that) they have effects similar to those of the adjustments to the income tax (characterized in section 3) that would have been optimal if preference differences were observable.

Section 5 extends the results to policy rules for public goods and externalities, for which many of the conclusions are analogous to those for commodity taxation: When preference differences are observable, benchmark results on first-best policy rules (the Samuelson rule for public goods and the Pigouvian prescription for complete internalization of externalities) continue to hold. When preference differences are not observable, deviations depend on some (but not all) of the same sorts of factors as with commodity taxation and serve the same purpose of indirectly substituting for the redistributive adjustments to the income tax that would have been optimal in the case of observability.

Section 6 briefly considers pertinent issues in welfare economics, namely, concerning the concavity of the SWF and interpersonal comparisons of utility. The motivation is that many of the results concerning heterogeneity—including the signs of the optimal adjustments as well as the magnitudes—depend on these aspects of welfare aggregation. Moreover, when preferences and thus commodity demands are heterogeneous, the approach toward interpersonal comparisons of simply treating everyone as if they had the same utility function is unavailable. Concluding remarks are offered in section 7.

Prior work on heterogeneity is of a number of types. Saez (2002) introduces heterogeneity regarding previously established deviations from the Atkinson-Stiglitz uniform commodity tax result: Atkinson and Stiglitz (1976) had shown that relaxing labor separability favors nonuniformity and Mirrlees (1976) had shown that relaxing the assumption that preferences are independent of earning ability favors nonuniformity. Saez (2002) demonstrates that when these relationships are not deterministic but rather reflect mere correlations, similar

results obtain.<sup>1</sup> By contrast, the present article abstracts from these two considerations and examines instead effects of heterogeneity that pertain more directly to the marginal social value of redistribution than to the labor-leisure distortion caused by income taxation.

Some discussion of these other sorts of heterogeneity appears in prior literature—especially on public goods, e.g., Hylland and Zeckhauser (1979), Ng (1984b), Boadway and Keen (1993), and Kaplow (1996)—but does not suggest or demonstrate the results derived here. Other papers have examined certain technical aspects of the nonlinear income tax problem when additional dimensions are introduced. See Ebert (1988) and Tarkiainen and Tuomala (1999), and also Armstrong and Rochet (1999) on multidimensional screening more generally. Tarkiainen and Tuomala (1999) also present some simulations, but their example has special features and the means of the parameters in their two-dimensional case differ (often significantly) from the values in the one-dimensional case, so it is hard to interpret the simulation results regarding differences in optimal redistribution in the presence of heterogeneity. Boadway et al. (2002) focus on which self-selection constraints are binding in a model in which preferences are of two types, and Cremer et al. (2001) discuss modifications to the Atkinson and Stiglitz (1976) result when individuals differ in unobservable endowments.

Finally, some additional papers introduce specific forms of heterogeneity in certain variations of the income or commodity tax problem. Blackorby and Donaldson (1988) find that in-kind provision of medical care—or, if not feasible, differential taxation of medical care and other goods—may be optimal when there are unobservable differences in medical needs. Sandmo (1993) explores some optimal income tax implications of heterogeneous tastes for work in a model in which there are no differences in earning abilities and only one type of consumption. Marchand, Pestieau, and Racionero (2003) consider differences in the source of disutility to labor that are unobservable, are relevant to social welfare (a form of nonwelfarist SWF), and are related differently to demands for particular commodities. Hellwig (2004) addresses optimal pricing by a public monopolist with heterogeneous consumers. And Fleurbaey (2006) introduces preference heterogeneity along with a nonwelfarist SWF that features a *laissez-faire* criterion that favors noninterference with consumption choices.

As a whole, prior work does not address most of the questions examined here or how the answers depend on observability or on the policy context (commodity taxation, public goods, externalities, or pure income taxation). Of particular note, past inquiries (including by this author) typically speak of heterogeneity as if it were a unitary phenomenon when making general conjectures about how results may change with heterogeneity or when examining particular models. By contrast, the present analysis allows preferences among commodities (or public goods or external effects) to differ in various ways and finds that whether results change and, when they do, in what direction depends on the nature of the heterogeneity that is introduced.

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<sup>1</sup>Saez (2002) notes the existence of other forms of preference heterogeneity but does not analyze the relationship between different specifications of preferences and the resulting social welfare weights, which is the focus here.

## 2. Framework

### 2.1. Model

There are  $n$  commodities, indexed by  $i$ ; a particular commodity is denoted  $x_i$ . Individuals choose labor effort,  $l$ , and commodity vectors,  $x$ , to maximize utility  $u(x, l, \theta)$ , where  $\theta$  is a vector of preference parameters. The consumer price vector is  $q$ , taken to be the sum of fixed producer prices  $p$  and a commodity tax vector  $\tau$  (commodity taxes may be negative, i.e., subsidies). The nonlinear income tax  $T(wl)$  is a function of individuals' income,  $wl$ , the product of their unobservable wage rate (ability level)  $w$  and labor effort. The budget constraint is

$$(2.1) \quad qx = wl - T(wl).$$

The government chooses the commodity tax vector  $\tau$  (equivalently, the consumer price vector  $q$ ) and the nonlinear income tax  $T$  to maximize social welfare, given by

$$(2.2) \quad SW = \int W(v(w, \theta, \tau, T)) dF(w, \theta),$$

where  $W$  is a weakly concave function,  $v$  is an individual's indirect utility function—indicating the maximized value of  $u$  for ability level  $w$  and preference parameters  $\theta$  when the tax regime  $(\tau, T)$  is taken as given—and  $F$  is the distribution function. There is a revenue constraint—commodity tax plus income tax revenue must meet a fixed target  $R$ —that does not need to be examined explicitly here.

Much of the analysis will focus on the case in which the preference parameters  $\theta$  are observable. Accordingly, the pertinent nonlinear income tax schedules are  $T(wl, \theta)$ ; that is, for each preference type  $\theta$ , there is a separate income tax schedule. It will be assumed that commodity taxes,  $\tau$ , cannot be type-specific because of the possibility of resale. This restriction often will not matter (comments in a footnote will discuss implications when it does).

Taking commodity taxes as given, the optimal nonlinear income tax is characterized by a first-order condition derived, for example, in Atkinson and Stiglitz (1980), who make various simplifications (notably, that utility is separable between consumption and labor effort) and offer other caveats that will not be examined further here. Their condition can be expressed as:

$$(2.3) \quad \frac{T'(wl, \theta)}{1 - T'(wl, \theta)} = \frac{\int_w^{\infty} \frac{v_c(w, \theta)}{v_c(\omega, \theta)} \left( 1 - \frac{W'(v(\omega, \theta)) v_c(\omega, \theta)}{\lambda} \right) f(\omega, \theta) d\omega}{\varepsilon w f(w, \theta)}.$$

Primes denote derivatives;  $v_c$  is the marginal utility of consumption;  $\omega$  is the variable of

integration, indexing types with ability above  $w$ ;  $f$  is the density function derived from  $F$ ;  $\lambda$  is the marginal social value of a dollar (i.e., the shadow price on the government's revenue constraint); and  $\varepsilon$  (implicitly a function of  $w$  and  $\theta$ ) is related to the labor supply elasticity. (In stating expression (2.3), the fact that indirect utility,  $v$ , depends on  $\tau$ , which is taken as given, and on  $T$  is suppressed.)

The left side of expression (2.3) indicates the marginal income tax rate as a fraction of the untaxed proportion of marginal earnings. On the right side, the denominator indicates the marginal distortionary cost of a higher marginal income tax rate. It is the product of three components:  $\varepsilon$  (as noted, a term related to the labor supply elasticity);  $w$ , which measures the productivity lost by each unit reduction in labor supply (and likewise indicates the marginal revenue loss); and  $f(w, \theta)$ , which is the portion of the population that is distorted at the margin.

The integral in the numerator is related to the benefit from higher marginal income tax rates in terms of revenue raised on those earning more than  $wl$ , for whom a marginal income tax rate increase at income  $wl$  is inframarginal. (This integral is from the marginal type,  $w$ , who faces the marginal rate increase, to the upper limit of the distribution of ability types, taken here to be unbounded.) The key term for present purposes is that in the large parentheses in the integrand. The numerator of the latter component,  $W'v_c$ , is the marginal contribution to social welfare caused by a unit increase in utility of the pertinent type multiplied by that type's marginal utility of consumption—i.e., net-of-income-tax income or disposable income. This term is divided by  $\lambda$ , which converts units of social welfare into dollars. Accordingly, as the overall marginal contribution of individuals' consumption to social welfare is greater (ceteris paribus), the term in parentheses will be smaller, the value of the integral will be lower, and thus the optimal marginal tax rate will be lower.

A significant caveat to this analysis is that this first-order condition (2.3) characterizes optimal marginal tax rates but does not tell us the intercept of the schedule,  $T(0)$ , which typically is negative (a uniform grant). Suppose, for example, that there is a higher welfare weight on all individuals in some group,  $\theta$ . This weight will tend to favor a lower intercept (a larger transfer), which will tend to offset the aforementioned effect on optimal marginal tax rates. Accordingly, the discussion that follows will usually refer, for example, to a factor favoring a lower level of income taxation on a group, not disentangling the extent to which this result will optimally be effectuated through a lower intercept and lower marginal tax rates.

It is also important to keep in mind that, for the case in which  $\theta$  is observable, the first-order condition is separately stated for each  $\theta$ . These separate conditions are linked to the single social optimization problem by the common shadow price  $\lambda$ . When comparing groups with different values of  $\theta$ , it will be true, ceteris paribus, that those with higher marginal social valuations of consumption should be subject to lower levels of income taxation. This "ceteris paribus" statement is, however, highly problematic because  $\theta$  will affect other components in expression (2.3) as well. (Notably, a given ability type  $w$  with a different  $\theta$  may choose to supply a different level of labor effort,  $l$ , and thus earn a different level of income,  $wl$ .) Nevertheless, the standard practice of interpreting this sort of first-order condition term by term provides some valuable insight into this highly complicated problem.

The analysis will focus on utility functions that are at least weakly separable in labor. Specifically, attention will be confined to forms of  $u(x, l, \theta)$  that can be written as  $u(u^1(x, \theta), l, \theta)$ . This restriction is meaningful because  $x$  is a vector. The weak separability assumption means that, in allocating resources among the  $x_i$ 's, it is immaterial how much labor effort,  $l$ , was required to produce the individual's level of disposable income. Likewise, the allocation among the  $x_i$ 's will not directly influence the choice of labor effort,  $l$ ; all that will matter is the level of subutility,  $u^1(x, \theta)$ , that is obtained.

## *2.2. Types of Preference Heterogeneity*

As suggested in the introduction, individuals' preferences can vary in a number of qualitatively different ways. Before stating possibilities in terms of the functional form of the utility function and analyzing them formally, it is useful to articulate intuitively the sorts of heterogeneity that will be examined. Because heterogeneity may well be good-specific, the discussion throughout will be in terms of specific commodities rather than utility as a whole. Although some of the depictions may be somewhat elusive due to the informality, each type of heterogeneity corresponds to specific preference parameters that will be introduced momentarily and subsequently analyzed, at which point any residual ambiguity should be eliminated.

First (corresponding to  $\alpha$ , just below), individuals may vary in their sensitivity to a good's contribution to utility. For example, two individuals may enjoy ice cream and find that its consumption is subject to diminishing returns in the same manner, but one may have a higher overall sensitivity; perhaps ice cream, consumed at any given level, delivers twice as much utility to this individual. The individual with greater sensitivity will be shown to have a higher marginal utility of disposable income, *ceteris paribus*, as a result.

Second ( $\beta$ ), individuals may differ in their ability to convert raw commodities into utility. The difference may be physical (one is able to make use of the product with less waste), physiological, or psychological. For example, individuals with a greater body mass or rate of metabolism may convert calories to resulting states of being at different rates. Some readers may find this type of heterogeneity to be similar to the first, but, as will be seen, it is importantly different, specifically in terms of how this difference interacts with diminishing returns in consumption; see the discussion of parameter  $\rho$  below. As will be demonstrated, those who are more efficient converters of commodities into utility may have a higher or a lower marginal utility of income, *ceteris paribus*, depending on this curvature parameter.

Third ( $\gamma$ ), individuals may in essence start their consumption at different points along a utility curve. For example, one may need eyeglasses or corrective surgery to possess the same vision or other attribute that others come by naturally. Individuals who are relatively blessed in this regard will be shown to have a lower marginal utility of income, *ceteris paribus*, because their situation is equivalent to being endowed with greater initial income.

Fourth ( $\delta$ ), some individuals' utilities may have different base points. Perhaps due to a different psychological constitution, some may achieve higher or lower well-being for any given



level of consumption. Individuals better off in this respect have higher utility level but the same marginal utility of income as others do, *ceteris paribus*.

These types of heterogeneity are distinguished here because, due to their different effects on individuals' marginal utility of income, they have qualitatively different effects on behavior and on optimal policies. In order to understand these distinctions among types of heterogeneity more concretely and to derive their consequences, it is useful to explore a simple, specific utility function that embodies all four types in a transparent fashion. The functional form to be considered will be a five-parameter (by  $n$ ) function,  $\theta = (\alpha, \beta, \gamma, \delta, \rho)$ , where each of these parameters may take on a different value for each commodity. This utility function is<sup>2</sup>

$$(2.4) \quad u(x, l, \alpha, \beta, \gamma, \delta, \rho) = \sum_{i=1}^n \left( \alpha_i \frac{(\beta_i x_i + \gamma_i)^{1-\rho_i}}{1-\rho_i} + \delta_i \right) - z(l).$$

It is assumed that  $\alpha_i > 0$ ,  $\beta_i > 0$ ,  $\rho_i > 0$  (each for all  $i$ ),  $z' > 0$ , and  $z'' > 0$ . (For the case in which the concavity parameter,  $\rho_i$ , equals 1, the fractional expression is replaced by  $\ln(\beta_i x_i + \gamma_i)$ ; it will be apparent below that using this alternative will yield the same first-order conditions, so none of the analysis is affected.) It is helpful for most discussion to think of a base case in which, for any commodity  $x_i$ ,  $\alpha_i = \beta_i = 1$  and  $\gamma_i = \delta_i = 0$ . Relative to that case, a higher  $\alpha_i$  magnifies the contribution of  $x_i$  to utility without being subject to the curvature effect through  $\rho_i$ ; a higher  $\beta_i$  magnifies the contribution of  $x_i$  to utility but is subject to the curvature effect through  $\rho_i$ ;  $\gamma_i$  adds to utility in a manner that is subject to the curvature effect through  $\rho_i$ , and  $\delta_i$  adds to utility without being subject to the curvature effect through  $\rho_i$ . (The  $\delta_i$ 's are interchangeable—only their summation matters—but this notation is employed to maintain symmetry.) It is obvious that raising any of these four parameters raises utility; however, as will be explored, they have a qualitatively different effect on marginal utility and thus may have a different effect on optimal income taxation.

### 3. Analysis with Observable Types

#### 3.1. Optimal Commodity Taxation

For the case in which individuals' preference parameters are observable, the existence of preference heterogeneity has no significant impact on results concerning optimal commodity taxation in the presence of a nonlinear income tax. Specifically, weak separability of labor is

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<sup>2</sup>Readers will note a resemblance between the functional form in expression (2.4) and a number of standard utility functions. A CES (constant elasticity of substitution) utility function has  $\beta$ ,  $\gamma$ , and  $\delta$  equal to zero, the  $\alpha$ 's are typically weights that sum to one and do not vary across individuals,  $\rho$  varies neither across commodities nor individuals, and the entire summation would be raised to the power  $1/1-\rho$ , which is an immaterial difference for present purposes. Cobb-Douglas utility is equivalent to the special case described in the text following expression (2.4) in which  $\rho = 1$ . Finally, the parameter  $\gamma$  is suggestive of a Stone-Geary utility function, where it is typical to take each  $\gamma_i$  to be negative and to replace the summation with a product, so that the  $\gamma_i$  represent subsistence levels of each commodity (in that, if such level is not exceeded, total utility is zero regardless of how high is consumption of other commodities).

sufficient to generate the result that no differentiation is optimal, which implies that optimal commodity taxes may be taken to be zero. This result does not require that the income tax be optimal, and it can be extended to show the optimality of various partial commodity tax reforms, such as moving differentiated commodity taxes proportionally toward uniformity.

Upon reflection, this conclusion should not be surprising. The reason that results in the homogeneous case carry over in the presence of heterogeneity when preference differences are observable is that, as will now be elaborated, one can decompose the latter problem into a number of cases of the former. Furthermore, because the proof technique employed in certain papers in the relevant literature does not depend on the optimality of the income tax (which is affected by heterogeneity) or on any global properties of the system, the fact that these features may change in the presence of preference heterogeneity will not affect these results regarding optimal commodity taxation.

To see that this is indeed the case, begin by observing that the proof of all of the results on commodity taxation—including that no differentiation is optimal and that partial reforms moving proportionately toward neutrality are desirable—in Kaplow (2006a) (and in some other work, e.g., Laroque (2005)) requires only weak separability of labor and that the subutility function of commodities,  $u^1$  in the above formulation, be common. Regarding the latter, when preference parameters are observable, the problem can be analyzed as if preferences are identical because the nonlinear income tax can be preference-type-specific.

Rather than reconstructing the pertinent proofs, it should be sufficient to review their two key steps. The first step—the one that depends on homogeneous preferences—involves constructing an adjustment to the preexisting (arbitrary, i.e., not necessarily optimal) nonlinear income tax so that, when combined with the commodity tax reform (say, a move toward uniformity), everyone's utility is held constant. Specifically, this intermediate tax schedule,  $T^\circ(wl)$ , is defined such that  $V(\tau, T, wl) = V(\tau^*, T^\circ, wl)$  for all  $wl$ —where  $V$  is an indirect subutility function indicating the maximized value of what is here denoted  $u^1$ . For this construction to be feasible—i.e., for the same tax schedule to preserve subutility for all individuals—it is necessary that the underlying subutility functions be identical (but homogeneity in other respects is not required).<sup>3</sup> Here,  $u^1$  depends on  $\theta$ . However, when  $\theta$  is observable, a separate income tax schedule is applied for each  $\theta$ ; hence, a separate income tax adjustment may be employed as well. Thus, for each  $\theta$ , one can define  $T^\circ(wl, \theta)$  as the adjustment to  $T(wl, \theta)$  such that  $V(\tau, T, wl, \theta) = V(\tau^*, T^\circ, wl, \theta)$  for all  $wl$ .

At this point, the remainder of the proofs in Kaplow (2006a) and in other pertinent papers goes through. In particular, it was shown there that this tax adjustment, which holds utility constant for each level of labor supply, will in fact induce individuals to choose the same level of labor supply. (The reasoning is, in essence, that the tax adjustment, when combined with the contemplated commodity tax adjustment, produces the same mapping from  $l$  to total utility as

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<sup>3</sup>For further discussion, consider Boadway and Keen's (1993) examination of preference heterogeneity in the public goods context (in an analysis that employs first-order conditions for social welfare maximization rather than constructing Pareto improvements).

was produced initially, so whatever  $l$  maximized utility initially will continue to do so.) To complete the proofs, it was demonstrated that commodity tax reforms that were efficient in the narrow, conventional sense—including moves toward uniformity—produce a revenue surplus. (The intuition is that, since the income tax adjustment holds utility constant, yet there is an efficiency gain, it must be that the income tax adjustment is absorbing the dollar equivalent to that underlying efficiency gain.) This surplus, in turn, could be rebated pro rata, generating a Pareto improvement.<sup>4</sup>

### 3.2. Optimal Income Taxation

With observable differences in preferences, it is optimal to employ different nonlinear income tax schedules. For the present analysis, commodity taxes can be put to the side; specifically, they will be taken to be zero. The focus will be on how the marginal social contribution of disposable income to social welfare,  $W' v_c$ , depends on  $\theta$  when preferences have the form given by expression (2.4).<sup>5</sup> As already noted, it is obvious that utility *levels* rise with  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ . To confirm this, note that the partial derivative of  $u$  is clearly positive for each of these parameters, and the total derivative will be the same (the envelope theorem). If the SWF is utilitarian, which means that  $W'$  is constant, this feature is of no consequence. If instead the SWF is strictly concave, i.e.,  $W'' < 0$ , then a group with a higher value of any of the parameters should, ceteris paribus, face higher levels of income taxation because the marginal contribution of their utility to social welfare is lower on account of their already being better off.

Next, consider how raising any of these four parameters affects the *marginal* utility of consumption,  $v_c$ , which will be relevant to the marginal social welfare gain from additional disposable income regardless of the concavity of  $W$  (short of maximin, wherein the welfare weight on everyone but the least-well-off individual is zero). The key concept here is that parameters associated with a higher marginal utility of consumption will call for lower levels of taxation. (As mentioned in subsection 2.1, such lower taxation might be reflected in a more generous grant at zero income or lower marginal tax rates.)

Effects on marginal utility from the different types of heterogeneity are, as suggested in

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<sup>4</sup>Note that the requirement that commodity taxes be anonymous (i.e., not type-specific) is not restrictive because all types should face the same (zero) tax vector. If one relaxed the labor separability assumption, however, the optimal commodity tax problem would be complicated by the fact that the optimal differentiation, which reflects the degree to which various commodities are complements to or substitutes for labor, could depend on individuals' specific preferences. (Suppose, for example, that  $x_i$  was a complement and  $x_j$  was a substitute for type  $\theta_1$  but the opposite was true for type  $\theta_2$ .) Ideally, each type would be subject to distinctive commodity taxes and subsidies, but if this is infeasible the optimum would (put crudely) reflect a weighted average of what would be optimal for different types considered separately. When the optimum for each type is the same, which is true with separability, this complication does not arise.

<sup>5</sup>Analyzing policies' effects on different individuals by focusing on the marginal contribution to social welfare of a dollar distributed to different types of individuals is associated with Diamond (1975). Note further that the path of analysis articulated in the text is incomplete because of other aspects of the social first-order condition for optimal marginal income tax rates (2.3) that may differ, other standard caveats in interpreting first-order conditions to make conjectures about the optimum, and difficulties of comparing optimal income tax schedules across groups because individuals of a given earning ability  $w$  may choose to exert different labor effort  $l$  and thus earn different income  $wl$  if their preferences differ.

subsection 2.2, quite varied. To analyze them, begin with individuals' first-order conditions. When expression (2.4) is maximized subject to the budget constraint, we have, for each  $x_i$ ,

$$(3.1) \quad \alpha_i \beta_i (\beta_i x_i + \gamma_i)^{-\rho_i} = \mu q_i,$$

where  $\mu$  is the marginal utility of disposable income (the shadow price on the budget constraint; also equal to  $v_c$ , employing the notation of the indirect utility function from expressions (2.2) and (2.3)). It is useful to restate these conditions as

$$(3.2) \quad x_i = \frac{\left( \frac{\alpha_i \beta_i}{\mu q_i} \right)^{1/\rho_i} - \gamma_i}{\beta_i}.$$

Expression (3.2) indicates how each of the  $x_i$ 's is a function of the preference parameters for the corresponding good, the good's consumer price (which is taken here to be constant), and the marginal utility of consumption,  $\mu$ . The first-order condition for labor effort is

$$(3.3) \quad z'(l) = w(1 - T'(wl))\mu.$$

The task is to determine how  $\mu$  changes with each of the preference parameters. Let a generic preference parameter (aside from  $\rho$ ) for good  $k$  be denoted  $\phi_k$ . Differentiating the budget constraint (2.1) with respect to  $\phi_k$  yields

$$(3.4) \quad q_k \frac{\partial x_k}{\partial \phi_k} + \sum_{i=1}^n \left( q_i \frac{\partial x_i}{\partial \mu} \frac{d\mu}{d\phi_k} \right) = w(1 - T') \frac{dl}{d\mu} \frac{d\mu}{d\phi_k}.$$

Rearranging terms allows us to state

$$(3.5) \quad \frac{d\mu}{d\phi_k} = \frac{q_k \frac{\partial x_k}{\partial \phi_k}}{w(1 - T') \frac{dl}{d\mu} - \sum_{i=1}^n \left( q_i \frac{\partial x_i}{\partial \mu} \right)}.$$

To determine the sign of expression (3.5), begin with the denominator. Differentiating the first-order condition for labor effort,  $l$ , expression (3.3), with respect to  $\mu$ , and rearranging terms, produces

$$(3.6) \quad \frac{dl}{d\mu} = \frac{w(1 - T')}{z'' + \mu w^2 T''} > 0.$$

The inequality in expression (3.6) follows because the denominator must be positive according to the second-order condition for the choice of  $l$  (the denominator is the negative of the second derivative of the individual's Lagrangian with respect to  $l$ ). Differentiating expression (3.2) with respect to  $\mu$  yields

$$(3.7) \quad \frac{\partial x_i}{\partial \mu} = - \frac{\alpha_i}{\rho_i q_i \mu^2} \left( \frac{\alpha_i \beta_i}{\mu q_i} \right)^{\frac{1-\rho_i}{\rho_i}} < 0.$$

Expressions (3.6) and (3.7) together imply that the denominator of expression (3.5) is positive. Accordingly, the sign of  $d\mu/d\phi_k$  is the same as the sign of  $\partial x_k/\partial \phi_k$ .

This result is in accord with intuition. Suppose, for example, that  $\partial x_k/\partial \phi_k > 0$ . In this case, the direct effect of raising  $\phi_k$  is to induce the individual to purchase more of  $x_k$ , which through the budget constraint requires some combination of reductions in expenditures on the other  $x_i$ 's ( $i \neq k$ ) and an increase in  $l$ , both of which imply a higher  $\mu$ , which is apparent from expressions (3.2) and (3.3). (The same logic holds, mutatis mutandis, if  $\partial x_k/\partial \phi_k < 0$ .)

To sign the  $d\mu/d\phi_k$ 's, therefore, all that remains is to sign the  $\partial x_k/\partial \phi_k$ 's, which is straightforward from expression (3.2).

$$(3.8) \quad \frac{\partial x_i}{\partial \alpha_i} = \frac{\beta_i x_i + \gamma_i}{\rho_i \alpha_i \beta_i} > 0.$$

$$(3.9) \quad \frac{\partial x_i}{\partial \beta_i} = \frac{x_i}{\beta_i} \frac{1 - \rho_i}{\rho_i} + \frac{\gamma_i}{\rho_i \beta_i^2}.$$

$$(3.10) \quad \frac{\partial x_i}{\partial \gamma_i} = - \frac{1}{\beta_i} < 0.$$

$$(3.11) \quad \frac{\partial x_i}{\partial \delta_i} = 0.$$

Expression (3.8) implies that  $\partial \mu/\partial \alpha_i > 0$ . Because  $v_c = \mu$ , it follows that the overall effect on the social marginal value of consumption,  $W'v_c$ , of raising  $\alpha_i$  is ambiguous: utility rises, so  $W'$  falls if  $W$  is strictly concave; however, marginal utility rises, so  $v_c$  rises. Those with a higher  $\alpha_i$  get more out of consuming  $x_i$ , which has these two competing effects.

Although one might have expected the effect of  $\beta_i$  to be qualitatively similar, this is not the case. Expression (3.9) indicates that the sign of  $\partial\mu/\partial\beta_i$  is ambiguous. For convenience, interpretation will be confined to the benchmark case in which  $\gamma_i = 0$ . (When this is not so, the critical value of  $\rho_i$  for which the sign of the expression reverses would be adjusted accordingly.) If  $\rho_i < 1$ , then  $\partial\mu/\partial\beta_i > 0$ , and the results are indeed like those in the prior case. However, if  $\rho_i > 1$ , then  $\partial\mu/\partial\beta_i < 0$ , and the marginal utility effect combines with the effect on  $W'$  to reduce the social marginal valuation of consumption. The difference arises because the  $\beta_i$  coefficient in expression (2.4) directly multiplies  $x_i$  and thus is subject to the concavity of the subutility function, and when that concavity is sufficiently high ( $\rho_i > 1$ ), the diminishing returns effect dominates the efficiency effect in determining how marginal utility changes. (When  $\rho_i = 1$ , marginal utility is unchanged.)

Combining these two results, it can be seen that there are two senses in which an individual might be seen to get “more” out of a good  $x_i$  than do others. In the former case, the *contribution to utility* of good  $x_i$  is multiplied by the factor  $\alpha_i$ . In the latter case, there is a sense in which the *effective quantity* of good  $x_i$  is multiplied by the factor  $\beta_i$ . As mentioned in subsection 2.2, one might interpret the former as an individual enjoying a good more and the latter as an individual being able to use a good more effectively. These notions are similar, but as just indicated they are not the same.

Expression (3.10) indicates that  $\partial\mu/\partial\gamma_i < 0$ . A higher  $\gamma_i$  is thus associated with higher utility and lower marginal utility, which unambiguously reduces the marginal social welfare weight  $W'v_c$ . As suggested previously, a higher  $\gamma_i$  may be interpreted as it being as if the individual is naturally endowed with some of the good before purchasing any on the market. Or, put another way, a low  $\gamma_i$ —say, a negative value, relative to a benchmark value of 0—would be a type of disability, wherein the individual needs to purchase some amount of  $x_i$  to reach the same starting point as others. Clearly, an individual with such a low  $\gamma_i$  will have both lower total utility and higher marginal utility, which unambiguously implies a higher social welfare weight.

Finally, as expression (3.11) indicates (and is obvious),  $\partial\mu/\partial\delta_i = 0$ . A higher  $\delta_i$  implies a higher utility level but no difference in marginal utility, and thus decreases the social welfare weight  $W'v_c$  if and only if the SWF is strictly concave. Indeed, as is apparent from expression (2.4) and as noted previously, the parameter  $\delta_i$ , despite its subscript, is not commodity-specific. All of the  $\delta_i$ 's might be aggregated into a single parameter  $\delta$  for present purposes.

To summarize, the analysis in this subsection reveals that preference differences that all imply higher *utility levels* can nevertheless have qualitatively different implications for individuals' *marginal utility* of consumption. Moreover, marginal utility is directly relevant to the marginal social welfare weight and thus to determining how preference differences should affect optimal nonlinear income taxation. Indeed, with a utilitarian SWF, it is only the effect on marginal utility that matters. With strictly concave welfare functions, the utility level matters as well, and, depending on the type of difference in preference, its strength, and the concavity of utility and of the SWF, individuals who have preferences that generate greater utility for a given level of disposable income may receive higher or lower marginal social welfare weights on their

disposable income and thus optimally be subject to lower or higher levels of income taxation.<sup>6</sup>

#### 4. Optimal Commodity Taxation with Unobservable Types

The analysis in section 3 takes preferences to be observable, whereas here preference differences will be assumed to be entirely unobservable.<sup>7</sup> As a consequence, it will no longer be possible to redistribute across preference types through the income tax. When  $\theta$  is not observed, the income tax schedule  $T$  in expression (2.3) must be the same for all types. The terms on the right side in this first-order condition will accordingly represent weighted averages of sorts. Notably,  $W'v_c$  will be the product of the social welfare weight and marginal utility of consumption, averaged for all types at each pertinent level of earnings,  $wl$ .<sup>8</sup>

This limitation on across-preference-type redistribution through type-specific  $T$  schedules means that (even with weak labor separability) there is a potential role for differential commodity taxation to improve social welfare. Specifically, if individuals who have above-average demands for some commodity  $x_i$  would ideally (i.e., if types were observable) be subject to higher (lower) income taxation, then to some extent it will be optimal to tax (subsidize) that commodity relative to others. Favorable redistribution would result, and starting from the point of uniform commodity taxation there would be no first-order loss from consumption distortion. The optimal level of commodity taxes and subsidies would reflect a redistribution-distortion tradeoff, where here the redistribution is “horizontal” (across preference types) and the distortion is of commodity demands (rather than of the labor-leisure choice).

To explore which commodities should be taxed or subsidized, it is necessary first to ascertain the relationship between commodity demands and the preference parameters. Once again, it is helpful to refer to  $\phi_k$ , a generic preference parameter for good  $k$ . Differentiating the commodity demands (3.2) with respect to  $\phi_k$  yields

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<sup>6</sup>There is some ambiguity in interpreting any conclusion about how heterogeneity influences the level of optimal marginal income tax rates because there are different possible benchmarks for comparison. Ceteris paribus, a higher welfare weight on marginal dollars does favor lower income taxation, as discussed in the text. But, as note 5 mentions, heterogeneity has other effects, including on labor effort. Taking, for example, a case in which the marginal utility of consumption is higher, the analysis in the text indicates that labor effort will also be higher. Hence, for a given observed income level  $wl$ , the higher  $l$  implies a lower  $w$ . Moreover,  $w$  is in the denominator of the first-order condition (2.3) for optimal marginal income tax rates, so the lower  $w$  favors higher marginal income tax rates, the opposite implication of the higher marginal utility. Accordingly, if labor supply is highly elastic, the effects on optimal tax rates attributable to marginal utility differences could be offset or even reversed.

<sup>7</sup>Realistically, preferences are partially observable, in that age, physical disabilities, family status, or other factors signal preference differences. To an extent, one can interpret the present analysis as applicable to residual heterogeneity within an identifiable class of individuals, although there may be limits on the ability to employ different commodity taxes for different classes. Also note that, in addition to limitations due to feasibility and administrative cost, one can also imagine political constraints, for example, on the use of certain factors such as race that might correlate with preferences.

<sup>8</sup>In addition, in the denominator one would have an integral over types of the product  $\epsilon w f$ . Regarding  $w$ , note that different  $\theta$ 's may induce individuals of a given  $w$  to supply different  $l$ 's, so all those at a given income level  $wl$  need not be of the same earning ability  $w$ .

$$(4.1) \quad \frac{dx_k}{d\phi_k} = \frac{\partial x_k}{\partial \phi_k} + \frac{\partial x_k}{\partial \mu} \frac{d\mu}{d\phi_k}.$$

(In stating the derivative in expression (4.1), note that the first partial derivative term on the right side holds the marginal utility of income constant, which effect is taken into account in the second term.) Using expression (3.5) for  $d\mu/d\phi_k$ , expression (4.1) can be restated as

$$(4.2) \quad \frac{dx_k}{d\phi_k} = \frac{\partial x_k}{\partial \phi_k} \left( 1 + \frac{q_k \frac{\partial x_k}{\partial \mu}}{w(1-T') \frac{dl}{d\mu} - \sum_{i=1}^n \left( q_i \frac{\partial x_i}{\partial \mu} \right)} \right).$$

The denominator of the fractional term in large parentheses on the right side of expression (4.2) is the same as the denominator on the right side of expression (3.5) for  $d\mu/d\phi_k$ , and this was previously shown to be positive. Specifically, the first term of the denominator is positive and each term in the summation is negative. From the latter, it follows that the numerator in the fractional term is negative. Moreover, this numerator is equal to the  $k^{\text{th}}$  term in the summation in the denominator. Taken together, these features imply that the value of the term in large parentheses is in the interval (0, 1). Therefore, expression (4.2) indicates that the total derivative  $dx_k/d\phi_k$  has the same sign as the partial derivative  $\partial x_k/\partial \phi_k$  but is smaller in magnitude. This result is in accord with intuition: The total effect on demand is given by starting with the effect when  $\mu$ , the marginal utility of consumption, is taken to be constant and then dampening it by the resulting adjustment in  $\mu$ . (For example, if the partial derivative indicates that demand would rise, this rise in demand is financed partly by reducing demands for other goods and partly by increasing labor supply, both of which imply a higher  $\mu$ , and the higher  $\mu$  reduces the magnitude of the increase in demand, but not below zero.)

Because each of the partial derivatives  $\partial x_k/\partial \phi_k$  is signed in subsection 3.2, in expressions (3.8) through (3.11), we can now consider how preference heterogeneity bears on optimal differentiation of commodity taxes. Suppose initially that the only heterogeneity involves parameter  $\alpha_i$  for some good  $x_i$ . From expressions (3.8) and (4.2), it follows that individuals who have a higher  $\alpha_i$  will have a higher demand for  $x_i$ , ceteris paribus. Furthermore, the analysis in subsection 3.2 indicates that the direction of the ideal redistributive adjustment (i.e., the adjustment to  $T$  if  $\theta$  were observable) depends on the SWF since a higher  $\alpha_i$  implies a higher marginal utility  $v_c$  but also a higher utility level and thus a lower  $W'$  if the SWF is strictly concave.<sup>9</sup> For concreteness, suppose that the SWF is utilitarian, in which case  $W'$  is constant, so it is optimal to redistribute toward high- $\alpha_i$  individuals. Since they have higher demands for  $x_i$ ,

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<sup>9</sup>Here and throughout this section, one should keep in mind the qualification in note 6 regarding the direction and magnitude of optimal adjustments to income taxation when those adjustments derive from differences in marginal utilities of consumption.



some degree of subsidy would be optimal.<sup>10</sup> Note that the use of the term subsidy here refers to a relative subsidy (and likewise for later uses of the term tax); instead of subsidizing good  $x_i$ , one could instead (equivalently) tax all other goods and adjust  $T$  accordingly.

As one generalizes to cases involving preference heterogeneity relating to many commodities, the problem becomes more complex. For example, suppose that there was heterogeneity involving all of the  $\alpha_i$ 's. The preceding analysis suggests that one would like to relatively subsidize all goods, which is impossible—i.e., the effects of the different subsidies would be offsetting. Taking a more concrete example, suppose that some types have uniformly higher  $\alpha_i$ 's than do others. Under the utilitarian SWF, it would be ideal to redistribute income to them; however, this cannot be done through commodity taxation because, in this case, relative demands would be unaffected.

Combining the preceding points, it appears that it would be optimal to subsidize goods relatively preferred by types whose overall or average levels of the  $\alpha_i$ 's are higher. To illustrate, assume that individuals with generally higher  $\alpha_i$ 's have an  $\alpha_j$  that is relatively low for them. The prescription would be to relatively subsidize all goods except good  $x_j$ —i.e., to relatively tax good  $x_j$ , even if these individuals'  $\alpha_j$ 's are above the population average (although lower than the average of their own, other  $\alpha_i$ 's).

Alternatively, suppose that each of the  $\alpha_i$ 's is independently distributed. In that case, it would seem advantageous under a utilitarian SWF to relatively subsidize the goods with the greatest variance in the distribution of the  $\alpha_i$ 's, whereas a tax may be optimal if the SWF is sufficiently concave. This point is most easily seen in the limiting case in which the variance for one of the  $\alpha_i$ 's is large and that for all of the others approaches zero, which presents the original case in which there is heterogeneity with respect to only one of the  $\alpha_i$ 's.<sup>11</sup>

Similar analysis applies to the other parameters. For example, higher  $\gamma_i$ 's imply both lower marginal utility (see expression 3.10) and higher utility, so we would ideally like to redistribute away from individuals with atypically high  $\gamma_i$ 's. Furthermore, high- $\gamma_i$  individuals will have lower demands for corresponding goods  $x_i$ , so it would be ideal to subsidize goods for which some individuals have unusually high  $\gamma_i$ 's. (With the  $\alpha_i$ 's, commodity subsidies favored individuals with high values of the parameter; with the  $\gamma_i$ 's, subsidies favor individuals with low values.) But, as with the  $\alpha_i$ 's, one cannot relatively subsidize all goods, so one must consider relative differences in parameter values.

For the parameter  $\delta$ , higher values indicate no difference in marginal utility (see expression 3.11) but higher utility and thus a lower  $W'$  if the SWF is strictly concave. However, because the value of  $\delta$  has no influence on commodity demands, commodity taxes cannot be

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<sup>10</sup>Observe that any effects of such a subsidy across income levels (for example, with a normal good, higher-ability individuals who will earn higher incomes will spend more on the good and thus benefit more from the subsidy, even if they have the same  $\alpha_i$  as others) can be offset through the  $T$  schedule.

<sup>11</sup>The magnitude of the corresponding  $\rho_i$ 's would also matter, for  $\rho_i$  affects the magnitude of the utility level and marginal utility effects and also the magnitude of demand effects, including the demand elasticity, which is related to the distortionary cost being traded off against the redistributive benefit.

employed directly to accomplish any desired redistribution.

For the parameter  $\beta$ , the situation is more complicated. From expression (3.9), the sign of the marginal utility effect—and, accordingly, from expression (4.2), the sign of the demand effect—depends on the magnitude of the pertinent  $\rho_i$  (abstracting from the further adjustment required when the corresponding  $\gamma_i$  does not equal zero). For example, if heterogeneity concerns a particular  $\beta_i$ , and we consider the case in which  $\rho_i < 1$ , then high- $\beta_i$  individuals have higher marginal utilities, *ceteris paribus*, and under a utilitarian SWF it would be optimal to redistribute toward them. In this case, such individuals also have higher demands, so a subsidy would be optimal. If instead  $\rho_i > 1$ , then high- $\beta_i$  individuals have lower marginal utilities, so it would be optimal to redistribute away from them. In this alternative case, however, such individuals have lower demands, so again a subsidy (which would favor low- $\beta_i$ , high-demand individuals) would be optimal. In both instances, note that the ability to use commodity taxation to redistribute is limited the closer that  $\rho_i$  is to 1; when  $\rho_i = 1$ , there is no demand effect, so across-preference-type redistribution through commodity taxes is infeasible (and also would not be optimal with a utilitarian SWF, although it would be if the SWF was strictly concave).<sup>12</sup>

The optimal use of differential commodity taxation actually depends on the combination of all of the parameters. As a group, they determine whether it would be optimal to redistribute toward or away from an individual of a given overall preference type  $\theta$ . Furthermore, as a group they determine the direction of any differences in demand. Even for a specified SWF, such as a utilitarian one, the ideal direction of redistribution sometimes is in the same direction and sometimes is in the opposite direction of the corresponding demand effect. Accordingly, broad generalizations about preference heterogeneity and the signs of optimal deviations from uniform commodity taxation cannot be offered.

It may be possible, however, to identify some particular effects. It seems plausible that some individuals toward whom it would be optimal to redistribute do systematically demand more of certain commodities. This might include those with physical limitations (the goods might be types of medical care or disability accommodations such as wheelchairs) or mental infirmities (the goods might be certain drugs or psychiatric care). To some degree, the analysis of section 3 may be applicable because some of these differences in preferences correspond to observable differences, in which case heterogeneity-motivated redistribution should (in the benchmark case with separable labor) be accomplished entirely through the income tax. But some physical and mental infirmities may be more difficult to observe, so differential commodity taxation may play a useful role. In this regard, it should be noted that optimal differential taxation may be appropriate for more than the obvious goods, such as those already noted. For example, individuals with more hidden physical disabilities might engage less in physically strenuous activities (skiing) and more in gentler activities; those with certain psychological difficulties likewise may have atypically high demand for some commodities and low demand for others. What matters, it should be recalled, is various individuals' relative demands. Thus, some impaired individuals might have an unusually low  $\alpha_i$  for some commodity but that parameter

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<sup>12</sup>The existence of little or no demand effect due to differences in  $\beta_i$ 's does not, however, imply that there is no distortion in commodity demands from differential taxation.

value may still be relatively high for them, and this parameter value may in turn imply higher demand, which would favor a subsidy.

## 5. Public Goods and Externalities

### 5.1. Public Goods

To introduce public goods, one can modify the initial utility function to be  $u(x, g, l, \theta)$ , where  $g$  is a vector of  $m - n$  public goods. Likewise, the specific utility function (2.4) can be amended as follows:

$$(5.1) \quad u(x, l, \alpha, \beta, \gamma, \delta, \rho) = \sum_{i=1}^n \left( \alpha_i \frac{(\beta_i x_i + \gamma_i)^{1-\rho_i}}{1-\rho_i} + \delta_i \right) + \sum_{i=n+1}^m \left( \alpha_i \frac{(\beta_i g_i + \gamma_i)^{1-\rho_i}}{1-\rho_i} + \delta_i \right) - z(l).$$

(As before, for the case in which  $\rho_i = 1$ , the fractional expressions are replaced by  $\ln(\beta_i x_i + \gamma_i)$  and  $\ln(\beta_i g_i + \gamma_i)$  respectively.)

Consider first the case in which  $\theta$  is observable. With weak labor separability, which expression (5.1) exhibits, the proofs in Kaplow (1996, 2006c) on the optimality of the Samuelson rule for public goods provision—without adjustments for distribution or labor supply distortion—go through. (The reasoning is analogous to that in subsection 3.1, as the proofs in Kaplow (1996, 2006c) in relevant respects are analogous to that in Kaplow (2006a) for uniform commodity taxation.) Thus, in summing (integrating) individuals' marginal benefits (measured in dollars) from more of a public good, each individual's possibly idiosyncratic valuation would be employed; this sum, whatever it may be, would be compared with the marginal cost of increased provision, just as in the case with homogeneous preferences.<sup>13</sup>

Accordingly, the effect of preference heterogeneity regarding public goods on optimal policy is entirely through the income tax schedules  $T(wl, \theta)$ , which are customized for each preference type. These effects, however, are qualitatively different from those deriving from heterogeneous preferences for commodities. The reason is that, although differences in the  $\phi_k$ 's, for  $k > n$  (i.e., for the public goods) affect utility *levels* analogously (utility is increasing in each of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ ), differences in these  $\phi_k$ 's do not affect individuals' *marginal* utilities of consumption. Accordingly, for a utilitarian SWF, there would be no adjustment to the optimal income tax schedule, and for a strictly concave SWF, higher values of these parameters imply higher utility and thus a lower  $W'$ , so optimal levels of income taxation would be higher.

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<sup>13</sup>This sum, to be sure, may be influenced by heterogeneity itself. As it turns out, greater heterogeneity (with a constant mean) can raise or lower the sum of marginal benefits, depending on the preference parameter that varies (and the level of the corresponding  $\rho_i$ ).

This irrelevance result concerning marginal utilities is due to the separability between public and private goods embodied in the utility function (5.1). More generally, there could be marginal utility effects in either direction. For example, better roads may make automobiles more valuable, and one could imagine this arising in a manner that increased individuals' marginal utilities of consumption more for individuals who had a stronger preference for driving. Suppose further that this greater preference operated through the corresponding  $\alpha_i$  (perhaps  $\alpha_j g_j$  for roads multiplies this  $\alpha_i$ ). Then high- $\alpha_j$  individuals would have a higher "effective"  $\alpha_i$ , which, ceteris paribus, implies a higher marginal utility of consumption, so under a utilitarian SWF their optimal level of income taxation would be lower on this account.

Now consider the case in which  $\theta$  is unobservable. The analysis of the optimal income tax would be analogous to that described in section 4 on unobservable heterogeneity in preferences for commodities. It might appear that there is no implication corresponding to the nonuniform commodity tax results since all individuals necessarily receive the same amount of each public good—i.e., differences in consumption patterns do not exist and thus may seem to render differential treatment infeasible. But this supposition is incorrect. By providing more or less of the public good than the amount indicated by the Samuelson rule, different preference types can be favored and disfavored: Raising (lowering) the level of the public good disproportionately benefits those with a higher (lower) marginal utility for that good (assuming distribution-neutral finance of the sort described in subsection 3.1).

The remaining question is which goods should thus be over- or under-provided relative to the level that satisfies the Samuelson rule. Once again, with preferences separable between public and private goods and a utilitarian SWF, there is no basis for deviation since heterogeneity in preferences for public goods does not influence individuals' marginal utilities of consumption. If the SWF is strictly concave or if there are cross-effects, then the previous comments for the case of observable preferences would become relevant.<sup>14</sup> For example, with a strictly concave  $W$ , if individuals with overall detrimental preference parameters (i.e., those whose preference parameters generate less utility from public and private goods at a given income level) tend to derive atypically high marginal utility from public good  $j$ , then more of that public good should be provided. To be concrete, suppose that individuals who are unusually vulnerable to being mugged also tend to be worse off in other respects (which cannot readily be observed). In this case, greater police protection would have a favorable distributive effect. By contrast, if national parks are most enjoyed by more robust individuals, who otherwise tend to be better off on average, then lower provision would be distributively beneficial.

## 5.2. Externalities

To examine externalities instead of public goods (considering the two together would be straightforward), one can modify the initial utility function to be  $u(x, e, l, \theta)$ , where  $e$  is a vector of  $m - n$  externalities. For concreteness, one might take  $m = 2n$  and suppose that each externality is measured by the total consumption of the corresponding commodity. Analogous to expression

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<sup>14</sup>When there are public goods, optimal commodity taxation may also be influenced in related ways.

(5.1), the specific utility function (2.4) becomes:

$$(5.2) \quad u(x, l, \alpha, \beta, \gamma, \delta, \rho) = \sum_{i=1}^n \left( \alpha_i \frac{(\beta_i x_i + \gamma_i)^{1-\rho_i}}{1-\rho_i} + \delta_i \right) + \sum_{i=n+1}^m \left( \alpha_i \frac{(\beta_i e_i + \gamma_i)^{1-\rho_i}}{1-\rho_i} + \delta_i \right) - z(l).$$

(The modification for the case in which  $\rho_i = 1$  is analogous as well.) For a negative externality  $e_i$ , the corresponding  $\alpha_i$  would be negative. Observe that, in this formulation, externalities have a public good character: Each individual is exposed to the same levels of the  $e_i$ 's, although different preference types will be affected differently by any particular  $e_i$ . Thus, the sum (integral) of marginal external benefits or harms for purposes of applying the Pigouvian rule of setting commodity taxes and subsidies equal to marginal harm and benefit, respectively, is directly analogous to the sum of marginal benefits in the public goods case.

At this point it should be unsurprising that the analysis closely parallels that for public goods. When  $\theta$  is observable and there is weak labor separability, as in expression (5.2), the proof in Kaplow (2006b) indicating that there should be no adjustment on account of distribution or labor supply distortion to the Pigouvian rule goes through. (So do extensions for partial reforms that are analogous to those considered for commodity taxation.) Once again, heterogeneous preferences would only be relevant in setting the income tax schedules  $T(wl, \theta)$ , and the pertinent adjustments would follow the same sort of reasoning applicable for public goods. For example, in expression (5.2) in which private goods and externalities have separable effects, externalities—and thus differences in preference parameters regarding externalities—affect utility levels but not the marginal utility of consumption, so heterogeneous preferences regarding externalities would only be relevant to optimal income taxation to the extent that the SWF is strictly concave. Relaxing this separability assumption would allow for interactions.

When  $\theta$  is not observable, it may be optimal to depart from the pure Pigouvian rule. Although individuals all experience the same levels of the externalities, by regulating externalities more or less than indicated by the Pigouvian rule one can favor or disfavor different types of individuals.<sup>15</sup> For example, if some pollutant imposes greater marginal harm on more infirm individuals who also tend to be worse off in other (unobservable) respects, greater control of that pollutant would tend to be optimal under a strictly concave SWF. Likewise, just as it may have been optimal to reduce expenditures on national parks that disproportionately benefit more robust individuals who are otherwise better off on average, so it may be optimal to reduce (relative to the Pigouvian optimum) the control of pollution that primarily interferes with the

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<sup>15</sup>Since the regulation of externalities is taken here to be implemented through commodity tax adjustments, differences in preferences for commodities are also relevant to across-preference-type redistribution, but that is already captured by the analysis in section 4.

enjoyment of wilderness areas.

## 6. Welfare Economic Considerations

Analysis of optimal taxation, public goods provision, and control of externalities, like most policy analysis conducted by economists, is grounded in the welfare economic tradition—and, if the SWF is taken to be linear, utilitarianism in particular—all of which is controversial.<sup>16</sup> Certain issues are brought to the fore when heterogeneous preferences are introduced.

Problems of interpersonal comparisons of utility are often circumvented by treating all individuals as if they are identical. But when heterogeneity involves aspects of utility that give rise to different demands, this approach is no longer available, even if it could otherwise be defended. Furthermore, the formal meaning of ignoring differences is ambiguous and incomplete because the outcome will depend on what preferences are deemed normal; moreover, when taste differences are associated with, say, disabilities, many would find the case for recognizing rather than ignoring differences to be particularly compelling. It may well be that many taste differences can neither be observed nor well estimated, in which case it would be difficult for policies to reflect them, but when differences are observed or the nature of underlying heterogeneity can be determined to some extent, optimality does in principle require taking such differences into account.<sup>17</sup>

If preference differences are granted, the necessary interpersonal comparisons raise issues of cardinalization, which has been noted, for example, by Boadway et al. (2002) and Sandmo (1993). For example, if individuals are observed to have a certain disability, say blindness, it is necessary to determine how much this affects their utility level (if the SWF is strictly concave) and their marginal utility of consumption.<sup>18</sup> If there is to be a social response—and if it is to be based on how the disability actually affects individuals (rather than on some criteria that is independent of well-being)—such questions must somehow be answered, at least approximately.

Finally, unlike most analyses (other than of the optimal income tax itself), there is a need to specify the SWF in terms of the degree of concavity. Indeed, for setting commodity taxes (in the case in which preference differences are unobservable), even the sign of the optimal deviations from uniformity depend on the curvature of the SWF. Although notable economists, like Harsanyi (1953, 1955), and others, such as Rawls (1971) have famously staked out positions

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<sup>16</sup>For differing perspectives, see Hare (1981), Sen and Williams (1982), and Kaplow and Shavell (2002).

<sup>17</sup>An alternative approach is arguably implicit in Rawls (1971) and Sen (1985), who would determine allocations, respectively, based on conceptions of primary goods or of functionings and capabilities, rather than based on individuals' utilities, even if they could reliably be determined. But as Kaplow (2007) shows, these approaches conflict with the Pareto principle (as do deviations from welfarism more generally, on which see Kaplow and Shavell 2001).

<sup>18</sup>A utilitarian SWF requires what is referred to as cardinal unit comparability (von Neumann -Morgenstern utility functions are cardinal, due to rationality axioms; unit comparability requires the ability to relate the cardinal units of one person's utility to those of another), whereas a strictly concave utility function also requires level comparability, which together amounts to cardinal full comparability. Sen (1977). (A maximin SWF only requires level comparability since only the utility of the least-well-off individual matters; however, Ng (1984a) has shown that, with the axioms of individual rationality, this implies unit and thus full comparability in any event.)

on this question, debate has hardly subsided.

Some have reacted to prior drafts and presentations of this paper by suggesting that explicit analysis of optimal policy with heterogeneity bolsters the case against utilitarianism and perhaps welfare economics more broadly. I disagree. Having presented my views on these and related questions at length in Kaplow and Shavell (2001) and Kaplow (2008, chapters 13-15), I will offer no further elaboration here. What does seem apparent, however, is that analysis of heterogeneity—whether in the setting of physical and mental disabilities or otherwise—does prompt reflection on these fundamental normative questions.

## **7. Conclusion**

Heterogeneous preferences undoubtedly exist and in some instances may be important. Accordingly, it is useful to revisit optimal policy rules that have been derived in models in which preferences are taken to be homogeneous. The results depend greatly on whether preference differences are observable and on the nature of those differences.

With observability, a number of first-best policy prescriptions in benchmark cases (notably, with weak labor separability) continue to hold. This conclusion is true of the Atkinson-Stiglitz (1976) rule favoring uniform commodity taxation as well as generalizations that do not require the income tax to be optimal and that encompass partial reforms, such as proportional moves toward uniformity. Likewise, results regarding the Pareto optimality of moving public goods provision in the direction indicated by the Samuelson rule and Pigouvian taxes and subsidies toward full internalization of externalities—without regard to concerns for distribution and labor supply distortion—also extend to the present setting.

The characterization of optimal nonlinear income taxation, however, changes in important ways. Preference parameters indicating higher utility levels favor, *ceteris paribus*, higher levels of income taxation to the extent that the SWF is strictly concave. But different sorts of preference parameters have different effects on individuals' marginal utilities of consumption and thus parameters producing higher utility levels may favor higher or lower income taxation.

When differences in preferences are unobservable, a single income tax schedule must be applied to everyone, and optimal marginal income tax rates are determined similarly to the manner applicable with homogeneous preferences. (Roughly, weighted averages substitute for specific values, but the basic formula is the same.) In this case, it is the more specific policy rules that may differ qualitatively from what arises with homogeneous preferences. Differential commodity taxation, public goods provision that deviates from the levels implied by the Samuelson rule, and corrective taxes that over- or under-internalize externalities will be optimal to some degree in cases in which such deviations can indirectly accomplish some of the across-preference-type redistribution that would have been implemented through differentiated income tax schedules if preference differences were observable. Examples were offered that involve some possible manifestations of physical and mental disabilities.

Overall, departures from results in models with homogeneous preferences depend on a variety of factors. Different ways in which preferences might vary have qualitatively different effects, even in opposite directions. Furthermore, preference differences interact with each other and with the various policy instruments in complex and subtle ways. The problem of heterogeneous preferences is indeed heterogeneous.

Accordingly, conjectures and results in existing literature on heterogeneity need to be interpreted as special cases in ways that have not previously been recognized; indeed, somewhat different specifications of preference differences can reverse results. Nevertheless, some plausible conjectures can be offered when it is possible to ascertain the character of preference differences, even if they cannot be observed for each individual. These findings may have direct policy relevance, and they indicate the value of empirical research on preference heterogeneity.



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