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A MODEL OF EFFICIENT DISCOVERY

Robert Mnookin*
Robert Wilson**

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Harvard Law School
Cambridge, MA 02138

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*Harvard Law School, Cambridge, MA.
**Stanford Business School, Stanford, CA.

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Robert Mnookin and Robert Wilson *

Dedicated to the memory of our friend and colleague Amos Tversky

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Corresponding Author: Prof. Robert Wilson
Stanford Business School
Stanford CA 94305-5015 USA

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Abstract

We construct a model of pre-trial discovery and settlement negotiations in which, in reverse order: the trial is a zero-sum game that each party incurs a cost to play; the parties can avoid trial and its costs by settling on a payment from the defendant to the plaintiff; and before negotiating settlement, each party can discover imperfect sample information about the facts known privately by the other, where such discovery imposes costs on both parties. We use a linear model of the trial-game value as a function of the parties' one-dimensional privately known parameters; we use a mechanism-design formulation to ensure that the settlement process maximizes the expected avoided cost of trial; we use a Normal diffusion process (Brownian motion) to model the acquisition of sample information via discovery; and we assume that the parties' privately known parameters are drawn independently from a diffuse Normal distribution.

The main result shows that the parties' *ex ante* expected gains from a joint plan of discovery are unaffected by their privately known parameters; hence, from a Coasian viewpoint, there is no intrinsic impediment to agreement initially on an efficient plan of discovery. That is, even though there are informational disparities between the parties, these have no effect on each party's calculation of the expected benefits and costs of a discovery plan, and therefore need not impede agreement on an efficient plan.

However, a dynamic process of discovery is problematic because the parties' incentives at later stages are affected by the outcomes of prior discoveries, and indeed the amounts of discoveries can be strategic substitutes or complements depending on the prior evolution of discoveries. This indicates the importance of agreement in the initial conference with the judge on an efficient plan of discovery, with subsequent enforcement based on contingent strategies in which excessive discovery by one party precipitates further discovery by the other. These results for this particular model indicate that discovery is not inherently inefficient, but they also indicate that 'wars of discovery' are possible, especially when discoveries are undertaken sequentially.

A subsidiary result is that each discovery benefits both parties (gross of the costs imposed) by reducing the risk of trial, although the discovering party tends to get the greater benefit because the discovery tilts the terms of settlement in his favor.

*Harvard Law School, Cambridge, MA.
**Stanford Business School, Stanford, CA.

1. Introduction

A striking characteristic of the American system of civil litigation is the process of pretrial discovery. The discovery rules in U.S. federal and state courts provide a variety of devices that broadly empower a litigant to acquire information from the other party before trial. When the Federal Rules of Civil Procedure were adopted in 1938, the primary justifications for liberal discovery were framed in terms of the fairness and efficiency of the subsequent trial. Many commentators felt that without discovery the outcomes of trials depended more on the skills of the lawyers than on the merits of the opposing claims. By assuring mutual knowledge of relevant facts, it was hoped that trials would be less a game of blind's man's bluff and more a fair contest with the basic issues and facts disclosed. This remains the prevailing view in law school courses on civil procedure.

But most litigation never reaches trial. The overwhelming majority of suits are withdrawn because the parties settle their disputes privately. A critical set of questions, therefore, concerns the relationship between discovery and settlement negotiations. If discovery were excluded, would pretrial settlements be affected? Does judicially enforced discovery promote pretrial settlements, or does it delay or impede settlements by encouraging expensive and wasteful combat? If discovery promote settlements, then how does it do so? Is discovery necessary in order to make a case "ripe" for settlement? These basic questions are highly contested. Surveys of lawyers suggest that the practicing bar believes, almost as an article of faith, that discovery promotes settlement. On the other hand, the few empirical studies question that conclusion (Glaser, 1968). Some recent theoretical work in law and economics also suggests that compulsory discovery may be unnecessary or even counterproductive (Shavell 1989; Cooter and Rubinfeld, 1993).

To examine these questions, this paper offers an economic model of the relationship between discovery and pretrial settlement. Our model starts with the premise that litigants "bargain in the shadow of the law." That is, their expectations about the trial outcome affect the chances and terms of a pretrial settlement. If they share similar expectations then they have a common motive to settle to avoid the costs of trial. However, if each party is optimistic about its prospects at trial then there may be no terms on which they can agree. Of course if both are optimistic then they cannot both be correct, so any model of settlement negotiations must account for the sources of conflicting expectations. Our model focuses on the role of private information; in particular, each party initially knows something that is relevant to the trial but unknown to the other party.

In this view, private information accounts for differing expectations that inhibit settlement. The situation is compounded by a "lemons" problem: each party recognizes that settling foregoes possible benefits at trial from the ultimate implications of information unfavorable to the other party, who therefore does not disclose it during negotiations. Discovery reduces these informational asymmetries by mandating exchanges of information. By reducing the divergence between the parties' expectations, discovery increases the prospects of settling. This gives operational meaning to the idea of "ripeness" for settlement. The parties are ready to negotiate seriously only after each sees that the cost of further discovery exceeds its expected benefit. There remains a possibility that negotiations fail and a trial ensues, but the chances are reduced as discovery progresses.

The model includes two effects from discovery. First, by reducing the private information held by the other side, the discovering party increases the probability of settling. This creates a mutual benefit because for both parties it reduces the chance of incurring the expense of trial. The second effect is distributional: discovery alters the expected terms of settlement in favor of the discovering party.[1] This reflects the general prediction from models of negotiation that a party with more precise information about the trial outcome captures the larger share of the gains from settling. This has important policy implications. For instance, discovery by one party can strengthen the other party's incentive for further discovery to redress this distributional asymmetry. In cases where substantial costs are imposed on the discovered party, both parties may undertake excessive discovery, like in a prisoners' dilemma. Thus, discovery wars can resemble an arms race.

To aid the reader, we first sketch the outlines of the construction. The subsequent sections then elaborate the model and derive its main implications.

2. Main Features of the Model

We view litigation as a sequential game played by the plaintiff and the defendant. Realistically, it consists of a sequence of recurrent cycles of discovery and other legal preparations, settlement negotiations, and judicial decisions until eventually the case is dropped, settled, or tried. Because settlements and trial judgments are essentially zero-sum transfer payments, the persistent stimulus to settle is avoidance of the costs of continuing

[1] This is reminiscent of Schelling's (1960, p. 43) description of the right to be sued as an asset, because it enables one to enter into contracts. Here, the 'right to be discovered' reduces informational disparities that would otherwise impede settlement; but like contracting, it risks an unfavorable distribution of the benefits. We are indebted to Avinash Dixit for this reference.

litigation, such as attorneys' fees and discovery expenses.

To capture the main elements, we simplify by dividing the game into two phases: an initial negotiation followed if necessary by a trial. Negotiation results either in a settlement that ends the game, or an impasse that initiates the trial phase. An impasse imposes a cost on each party to prepare for and conduct the trial. We ignore intermediate possibilities (such as summary judgment) that reduce costs via earlier resolution. The trial is a zero-sum subgame that we summarize simply by its value.[2] This value represents the product of two factors: the probability the defendant is found liable, and the expectation of the judgment, interpreted as a payment to the plaintiff from the defendant. Because we do not distinguish between these two factors, references to the value and the judgment will be used interchangeably.

The zero-sum aspect of the trial is a major simplifying feature of our model. We exclude external effects (e.g., liability in this case might affect other cases) and risk aversion for either party. Further, there is no social value from a trial (deterrence or precedence effects) nor its outcome; in particular, we ignore both the private and social values of the 'accuracy' of the trial outcome.

Negotiation resembles a zero-sum subgame in the sense that its outcome when they settle is a transfer payment to the plaintiff from the defendant. However, by avoiding an impasse, settling saves trial costs. So, the basic motive for negotiating is to obtain and split the "pie" or "gain from trade" that is the sum of the parties' avoided costs of trial. We ignore negotiation and settlement costs *per se*, but we do include costs of discovery incurred preparing for negotiations. Thus, the social value of settling consists entirely of the avoided costs of trial. Discovery costs offset this gain, but they are already sunk when settlement negotiations begin.

Negotiation is not easy, since the parties' common motive to avoid trial conflicts with their opposing interests to claim larger shares of the gain from settling. For instance, from the plaintiff's perspective, her personal gain from settling is the difference between the settlement and her expectation of the net value of a trial, which is the difference between the trial value and her cost. From either party's viewpoint, the settlement is essentially the price paid to the plaintiff by the defendant for withdrawing the suit. This price looms large in their calculations even though it is immaterial to the calculation of the gain from

[2] In the game-theoretic meaning of the term, the value of such a game is the plaintiff's expected judgment when the parties use either equilibrium or minmax strategies. Because the game is zero-sum, the value is unique.

trade. Thus, negotiations are strongly affected by distributional aspects: each seeks a settlement that garners a greater share of the gain from avoiding an impasse. To obtain better terms, each can threaten recourse to a trial.

We ignore the possibility that the plaintiff might withdraw the suit unilaterally; thus the parties' options are basically symmetric.[3] This assumption eliminates what would otherwise be a fundamental difference between the parties in their attitudes toward resolution of uncertainty. When the plaintiff has a withdrawal option, the value of this option increases with the extent to which uncertainty is resolved before settlement negotiations begin. Alternatively stated: the defendant prefers early settlement negotiations so that the plaintiff is forced to 'swallow' those events in which the judgment is insufficient to cover her trial costs. This effect is most evident in the extreme case that neither party has private information: in this situation, any resolution of their mutual uncertainty about the judgment increases the expected settlement paid by the defendant to the extent it might enable the plaintiff to withdraw an unprofitable suit. For example, the plaintiff and the defendant have directly opposing interests in deciding whether to conduct a mock trial, or to obtain preliminary rulings from the judge, that would reduce their common uncertainty about the judgment.

The important feature of our model is that each party initially has private information about the trial value. That is, each party knows privately some facts affecting the expectation of liability or judgment. Through disclosure or discovery, some of this information may be revealed to the other party. Nevertheless, typically the settlement negotiations are conducted with each party having some residual uncertainty about the other party's relevant information. This means in particular that each is uncertain about the expectation of the trial value perceived by the other party based on its residual private information. We therefore interpret the negotiations as a game with incomplete information.[4]

In particular, if there is incomplete disclosure or discovery then they negotiate with differing expectations of the judgment. Typically an impasse occurs if each party is

[3] See Bebchuk (1996) for an analysis of the effects of asymmetries in the parties' options to withdraw when litigation costs are incurred piecemeal over time.

[4] As mentioned above, in practice the process leading to settlement negotiations can also resolve uncertainties that are common to both parties; however, our model entirely omits this possibility, since the assumed symmetry between the parties' withdrawal options implies that it has no effect on the prospect that they avoid trial by settling. Of course the trial is also a game with incomplete information, but we account for this solely by conditioning the trial value on the parties' private information.

optimistic about its prospects at trial — even though one must be wrong. Moreover, negotiations are afflicted with the winner's curse: each anticipates that his or her offered settlement will be accepted only when the other is relatively pessimistic. Thus, the plaintiff foresees that the defendant's acceptance of her offer reveals that her prospects at trial are probably better than she expected beforehand. Similarly, she anticipates that the defendant will interpret her offer as a signal that reveals information about her expectation of the judgment, and therefore she must design her offer to take account of this compromising of her informational advantage.

Analyses of these winner's curse and signaling aspects are very complicated, and depend delicately on the outcome of prior discovery and the detailed 'rules' of the negotiation process, such as the sequencing of offers and counteroffers. However, to obtain our main conclusions it suffices to circumvent these complications by drawing on the theory of mechanism design. To use this theory, we need to assume only that the bargaining process is efficient, in the sense that its rules are designed to maximize the sum of the parties' expected gains, subject to incentive-compatibility constraints. These constraints reflect the supposition that in playing the negotiation game, each party uses a strategy that is an optimal response to the other's strategy; i.e., they use equilibrium strategies. Because our results depend only on the implications of incentive compatibility and efficiency, they are immune to the potential criticism that some other model of the negotiation process might reduce the likelihood of an impasse.

Relying on a mechanism-design formulation is both a strength and a weakness of the model. It brings the advantage that it provides explicit adaptation of the negotiation process to the altered information structure produced by discovery. On the other hand, the presumed efficiency need not be descriptive. We are sanguine about this matter because of our general view that development of efficient negotiation procedures is the likely result of competition among attorneys to serve the interests of their clients.[5]

Our main focus, however, is the preparation for negotiation, which we interpret entirely in terms of discovery. For this, we suppose that prior to settlement negotiations each party has options to acquire partial or sample information about facts known privately by the other party. Discovery is costly, possibly for both parties, yet its ex-

[5] We recognize that the theory of mechanism design is unfamiliar to most legal scholars. To ease the encounter with this novel approach, we provide in the Appendix a brief sketch of its application to a simple example. This example nicely illustrates our point that the procedural features of negotiations should be adapted to fit the uncertainties faced by the parties.

pected benefit can exceed its cost to the extent it reduces the likelihood of an impasse and thereby avoids trial costs. For a specific model of the sampling process, our analysis identifies circumstances in which the extent of discovery is efficient even when each party narrowly pursues its self-interest.

In analyzing the incentives for discovery, we presume throughout that each party's private information is 'negative' information; that is, its revelation would encourage the other party to be more optimistic about prospects at trial, and therefore to demand a more favorable settlement. This reflects a simplistic view that information is dichotomous: any positive information is disclosed voluntarily; all negative information is hidden. Further, we specifically avoid the 'unraveling' argument (Battigalli, 1995; Matthews and Postlewaite, 1985; Okuno-Fujiwara, Postlewaite, and Suzumura, 1990; Shavell, 1989) in the context of our model. We presume that credible disclosure of negative information is not feasible, or at least that any credible disclosures that are feasible leave a residual that is the subject of our analysis. We are mainly interested in a situation where a party's asserted disclosure that "here are all the folders from my files that pertain to this case" is *not* credible. The only way for the other party to verify the likely accuracy of this assertion is to sample the files via a discovery request. Such assertions are essentially vacuous because they do not deter discovery.

This approach to modeling discovery omits two major possibilities for strategic behavior. One is the possibility that the extent of discovery might be a signaling device, used to convey the discovering party's private information about the trial judgment. The other is impositional discovery as a threat in settlement negotiations, used as a prod to induce favorable offers from the other party (cf. Sobel, 1989). Unfortunately, including these aspects would take us too far afield from our main task.

Our model is limited by several technical features.

- Each party's initial private information is represented by a single real number. Thus, if the plaintiff knows x and the defendant knows y, then the conditional expectation of the judgment given *both* variables x and y is the bivariate function $v(x, y)$ that is the trial value. Each party knows this function, knows her or his number x or y, knows the probability distribution of the other's number, knows the other's probability distribution of one's own number, etc.[6]

[6] The representation of information as a single real number is not to be taken literally. If it were literal then we would have to consider the possibility that the amount of discovery undertaken by one party could signal accurately the magnitude of that party's

- The parties' numbers have independent probability distributions; e.g., the plaintiff's probability distribution of the defendant's number y is the same whether her number x is small or large.[7]
- The trial value depends only on the parties' private information and *not* on the results of their discoveries. That is, in computing the expectation of the judgment, the pair of initial private information parameters (x, y) is a sufficient statistic for the imperfect sample estimates of these parameters obtained from discoveries. In particular, even though each party's trial strategy might depend on discovery results, in determining the ultimate trial outcome the pair of true parameters outweighs estimates obtained from discoveries. This says essentially that the accuracy of the trial exceeds the accuracy of discoveries.
- Lastly, much of the analysis uses a particular linear formula for the trial value and a particular formulation in terms of Normal probability distributions to study the parties' incentives for discovery. These simplifications have been adopted to make the analysis tractable and the conclusions transparent.

We want eventually to relax these assumptions, but presently they are useful because they enable a straightforward analysis that conveys the main ideas.

3. The Negotiation Game

In this section we formulate and analyze the negotiation game. This assumes that disclosure and discovery are finished, and that if settlement fails then the trial ensues. For the plaintiff the trial yields the judgment less her trial cost, and the defendant pays the judgment plus his trial cost. Thus, each party hopes to settle to avoid its trial cost, but prefers the trial to a less favorable settlement. The complicating feature is that the parties have different information about the expectation of the judgment. Absent this informational disparity, they would share the same expectations and therefore settle. But with divergent expectations, they might each be optimistic about the trial and therefore reach an impasse.

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number. Especially when discovery is undertaken sequentially, the decision by one party to stop could reveal considerable information to the other.

[7] This assumption should not be confused with the peculiar independence assumption invoked by Cooter and Rubinfeld (1994). They assume that the parties have independent expectations of the judgment. We assume independent information about the judgment, recognizing that the judgment necessarily affects the parties oppositely: the plaintiff's receipts are paid by the defendant. Thus, one party's information about the judgment is invariably relevant to the other party.

First we specify the notation used, and then we derive the implications of our key assumption that the negotiation process is designed to be efficient subject to the parties' incentive-compatibility constraints. In the following sections, these implications are used to study the parties' incentives for discovery in preparation for settlement negotiations.

Notation

The ingredients of the game of settlement negotiation are defined below. All of the following is common knowledge between the parties except the actual realizations x and y of their privately observed random variables.

- **Information:** The two real numbers $x \in X$ and $y \in Y$ are statistically *independent* components of the judgment known privately by the plaintiff P and the defendant D, respectively. Their marginal densities are $f^P(x)$ and $f^D(y)$. Because they are independent their joint density is $f(x,y) = f^P(x)f^D(y)$. The corresponding distribution functions are $F^P(x)$ and $F^D(y)$, and their supports X and Y are intervals.

- **Trial and Judgment:** An impasse results in the trial judgment being paid to the plaintiff by the defendant. The conditional expectation of the trial judgment given both x and y is a function $v(x,y)$. In addition, the trial costs the parties $c^P(x,y)$ and $c^D(x,y)$ respectively. The total cost of the trial is denoted by $C(x,y) \equiv c^P(x,y) + c^D(x,y)$.

- **Settlement:** Settlement occurs with probability $p(x,y)$ and in that event the amount of the settlement is $q(x,y)$, payable to the plaintiff from the defendant. An efficient settlement process is designed by choosing the functions p and q so as to maximize the sum of the parties' expected gains compared to trial. This maximization is subject to incentive-compatibility and feasibility constraints specified below.

- **Payoffs:** If the parties settle then the plaintiff receives the payment $q(x,y)$ from the defendant. Similarly, if a trial occurs then she receives the judgment $v(x,y)$ less her trial cost $c^P(x,y)$, and the defendant pays the judgment $v(x,y)$ plus his trial cost $c^D(x,y)$. We interpret trial as the default outcome if settlement fails. Compared to this default outcome, the net expected gains from settlement are

$$\begin{aligned} \text{Plaintiff:} \quad & q(x,y) - [v(x,y) - c^P(x,y)], \\ \text{Defendant:} \quad & [v(x,y) + c^D(x,y)] - q(x,y). \end{aligned}$$

Thus, their expected net gains from negotiation are

$$U^P(x,y) = [+q(x,y) - v(x,y) + c^P(x,y)]p(x,y)\,,$$

$$\text{and} \qquad U^D(x,y) = [-q(x,y) + v(x,y) + c^D(x,y)]p(x,y)\,,$$

respectively. These are conditional on the pair (x,y), whereas their expected gains conditional on the information available initially to each are:

$$\bar{U}^P(x) = E[U^P(x,y) \mid x] \qquad \text{and} \qquad \bar{U}^D(y) = E[U^D(x,y) \mid y]\,,$$

respectively, where $E[\cdot \mid \cdot]$ denotes the conditional expectation.

This notation reflects a convention that we use throughout to avoid an excessive number of symbols. In each case a function denoted with a bar over the symbol is represented as a conditional expectation in which the conditioning parameters are shown as arguments of the function. Also, privately known parameters are separated from publicly known ones by a semicolon. For example, later references to $\bar{p}^P(x;s,t,\ldots)$ and $\bar{U}^P(x;s,t,\ldots)$ mean the probability of settling and the expected gain of the plaintiff P conditional on her private information x and the parameters $(s,t,\ldots)$ known to both parties.

*** Note: Some readers may prefer to skip the technical derivations in the following two subsections.

Formulation of the Efficient Mechanism Design Problem

We assume that an *ex ante* efficient mechanism is used for settlement negotiations. This means that the functions p and q are designed to maximize the sum

$$E[U^P(x,y) + U^D(x,y)]\,,$$

of the parties' *ex ante* expected net gains compared to trial. Using the formulas above, this is equivalent to maximizing their combined expected savings in trial costs

$$E[C(x,y)p(x,y)]\,,$$

from those events in which they settle. For instance, if the trial costs are fixed then the negotiation procedure is designed simply to maximize the probability of settling. This is an important restrictive feature of our model. The parties' collective interest in settling is solely to avoid trial costs. The only impediment to settling is the distributional effect: each seeks to bias the settlement amount in its favor. We take account of these divergent private interests by imposing the constraints enumerated below.

The maximization of the parties' expected gains is subject to three types of constraints.

(1) **Individual Rationality:**

$$\bar{U}^P(x) \geq 0\,,$$
$$\bar{U}^D(y) \geq 0\,.$$

These constraints say that in every event (x, y) each party must obtain no less in expectation, conditional on his private information, than opting for a trial. For the next constraints, we take advantage of the Revelation Principle, which says that a game of incomplete information, and one of its equilibria, induce another such game and equilibrium in which each party's optimal strategy is to report his private information truthfully to his agent (e.g., his attorney), who then implements the required actions in the ensuing negotiation.

(2) **Incentive Compatibility:**

$$\bar{U}_x^P(x) = E[[-v_x(x,y) + c_x^P(x,y)]p(x,y) \mid x]\,,$$
$$\bar{U}_y^D(y) = E[[+v_y(x,y) + c_y^D(x,y)]p(x,y) \mid y]\,,$$

where a subscript denotes a partial derivative with respect to that variable.

These are the necessary envelope conditions for truthful revelation in the associated direct-revelation game. (We omit here the second-order necessary conditions and more general sufficiency conditions because they play no role in the applications studied later.) The incentive-compatibility conditions ensure that each party cannot gain by imitating the behavior of a litigant with private information different from his own. This particular form of these conditions relies on the assumption that the components x and y are independent. [8]

(3) **Feasibility:**

$$E[\bar{U}^P(x) + \bar{U}^D(y)] \leq E[C(x,y)p(x,y)]\,.$$

This constraint ensures that no funds are injected into the game by a third party. All payments to the plaintiff must come from the defendant.

Characterization of an Efficient Mechanism

To address this constrained maximization problem, we first formulate the associated Lagrangian objective function by assigning Lagrange multipliers to each constraint. This

[8] Correlation between x and y requires a more complicated analysis that is described in Wilson (1993).

yields:

$$\int_X \int_Y \Big\{ (\bar{U}^P + \bar{U}^D + \mu[Cp - \bar{U}^P - \bar{U}^D]) f^P(x) f^D(y)$$
$$+\lambda^P(x)[\bar{U}_x^P - E[[-v_x + c_x^P]p \mid x]] + \lambda^D(y)[\bar{U}_y^D - E[[v_y + c_y^D]p \mid y]] \Big\} \, dx\, dy\,.$$

Because the settlement payment $q(x,y)$ drops out of the Lagrangian expression, we interpret the maximization as being with respect to the three variables $\bar{U}^P(x)$, $\bar{U}^D(y)$, and $p(x,y)$ for each pair (x,y).

The associated **Optimality Conditions** are:

- Settlement Events: The parties settle, namely $p(x,y) = 1$, if and only if

$$\mu C(x,y)p(x,y)f^P(x)f^D(y) + \lambda^P(x)[v_x(x,y) - c_x^P(x,y)]f^D(y)$$
$$-\lambda^D(y)[v_y(x,y) + c_y^D(x,y)]f^P(x) \geq 0\,.$$

- Lagrange Multipliers:

$$[1-\mu]f^P(x) - \lambda_x^P(x) \leq 0\,,$$

  and $= 0$ if $\bar{U}^P(x) > 0$, and $\lambda^P(x) = 0$ on a boundary of X where $\bar{U}^P(x) > 0$. Similarly for $\lambda^D(y)$.

The optimality conditions for the Lagrange multipliers can be used to derive more specific formulas. For this, assume that $v_x < 0$ and $v_y > 0$, and reversely for the trial cost functions.[9] Then

$$\lambda^P(x) = [\mu - 1][1 - F^P(x)]\,,$$

$$\lambda^D(y) = [\mu - 1][1 - F^D(y)]\,.$$

Therefore, we can rephrase the first condition above as:

[1] The parties settle, namely $p(x,y) = 1$, whenever

$$C(x,y) \geq \alpha \cdot \left\{ -[v_x(x,y) - c_x^P(x,y)]H^P(x) + [v_y(x,y) + c_y^D(x,y)]H^D(y) \right\}\,,$$

[9] These properties, together with the Lagrangian conditions above, ensure that, say, $\bar{U}^P$ cannot be everywhere positive and therefore λ^P is everywhere zero, unless $\mu = 1$ and $\alpha = 0$, in which case none of the incentive-compatibility conditions would be binding. In the usual case studied below in which they are binding, it is necessary that $\bar{U}(\min X) = 0$, indicating that the least type of the plaintiff always expects trial or is indifferent between settlement and trial.

where we define:

$$\alpha \equiv [\mu - 1]/\mu \geq 0\,,$$

$$H^P(x) \equiv [1 - F^P(x)]/f^P(x) \qquad \text{and} \qquad H^D(y) \equiv [1 - F^D(y)]/f^D(y)\,.$$

This key condition for an optimal bargaining procedure is central to the subsequent analysis.

To apply these optimality conditions, one must determine μ or α from the feasibility condition (3). Integrating by parts in the left side of the feasibility condition, and substituting the incentive-compatibility condition, yields:[10]

$$\mu E[g(x,y,1) \mid g(x,y,\alpha) \geq 0] \cdot \bar{p} \geq \bar{U}^P(x_*) + \bar{U}^D(y_*)\,,$$

where $x_* \equiv \min X$ and $y_* = \min Y$, and

$$g(x,y,\alpha) \equiv C(x,y) - \alpha \cdot \left\{ -[v_x(x,y) - c_x^P(x,y)]H^P(x) + [v_y(x,y) + c_y^D(x,y)]H^D(y) \right\}\,.$$

Here, the *ex ante* probability $\bar{p} \equiv E[p(x,y)]$ that the parties settle is the probability that $g(x,y,\alpha) \geq 0$. The usual case that $\bar{p} > 0$ (so settlement is possible), $\bar{U}^P(x_*) = \bar{U}^D(y_*) = 0$, and $0 < \alpha < 1$, is identified by the equation

[2] $$E[g(x,y,1) \mid g(x,y,\alpha) \geq 0] = 0\,,$$

which determines the constant α to ensure that the feasibility constraint (3) is satisfied as an equality.[11] In this usual case, the individual-rationality and incentive-compatibility conditions imply that

$$\begin{aligned} \bar{U}^P(x) &= \bar{U}^P(x_*) + \int_{x_*}^{x} \bar{U}_x^P(x')\,dx' \\ &= \int_{x_*}^{x} E[[-v_x(x',y) + c_x^P(x',y)]p(x',y) \mid x']\,dx'\,, \end{aligned}$$

assuming as before that $v_x < 0$, and analogously for the defendant. From these equations and the definition of $\bar{U}^P(x)$ and $\bar{U}^D(y)$ one can derive the conditions that must be satisfied by the expected payment functions $\bar{q}(x) = E[q(x,y) \mid x]$ and $\bar{q}(y) = E[q(x,y) \mid y]$. Typically there are many payment functions q satisfying these conditions.

[10] This is because $p(x,y) = 1$ if and only if $g(x,y,\alpha) \geq 0$.

[11] Generally one needs to assume that $g(\cdot,\cdot,\alpha)$ is monotone for this to be true. Typically, α is smaller if the disparity of information between the parties is less.

Implications of an Efficient Negotiation Mechanism

To develop the implications of the optimality conditions [1] and [2], we address hereafter a specific example that conveys the main ingredients.

The first simplification is to assume that trial costs are independent of the litigants' private information; that is, c^P, c^D, and $C = c^P + c^D$ are constants. This seems harmless for present purposes.

The second simplification is to assume that v_x and v_y are constants too, say $v_x = -a$ and $v_y = b$. This amounts to specifying $v(x,y) = A + by - ax$. In that case, there is no loss of generality in assuming further that $A = 0$ and $a = b = 1$, so that $v(x,y) = y - x$.[12] The rationale for the choice of signs is discussed in detail in our expository article. Basically, the motivation derives from the supposition that each party voluntarily discloses favorable information; it is only unfavorable information that is hidden from the opposing party and that is potentially the object of discovery.

With these two simplifications, the efficient negotiation mechanism has the parties settle whenever

[1] $$C/\alpha \geq H^P(x) + H^D(y) .$$

For interpretive purposes it is useful to think of this condition for efficient incentive-compatible settlement in the form $V^P(x) + V^D(y) \geq 0$, where

$$V^P(x) = c^P - \alpha H^P(x) \qquad \text{and} \qquad V^D(y) = c^D - \alpha H^D(y)$$

represent components of the two parties' expected gains from settling compared to trial. Note that typically $V^P(x) < c^P$ and $V^D(y) < c^D$, so negotiations fail in some circumstances where they would succeed if the parties had the same information.

We assume further that each of the functions H^P and H^D is decreasing and convex. This assumption is satisfied for the wide class of probability distributions having

[12] All of the subsequent derivations extend straightforwardly to the more general linear case. A multiplicative formula is more useful in some contexts. For instance, $v(x,y) = y \cdot d/x$ captures the idea that the defendant has superior information about his probability y of being found liable due to negligence, and the plaintiff has superior information about the judgment's deflation x of her damage d due either to contributory negligence or an interest rate signifying impatience for receipt of the judgment. Other interesting specifications include ones that specify an expected judgment that is the maximum of zero and $y - x$, or that is a fixed amount if $y > x$ and zero otherwise; these recognize the plaintiff's burden of establishing the defendant's liability.

increasing hazard rates. For instance, this class includes the uniform and Normal distributions; e.g., $H^P(x) = a - x$ for the uniform distribution on the interval $X = [0, a]$.

Besides the optimality condition [1], we use the following properties in our analysis of discovery.

[3] Each party's expected net gain is an increasing and convex function. That is:

$\bar{U}^P(x)$ and $\bar{U}^D(y)$ are increasing functions of x and y respectively.

That $\bar{U}^P(x)$ is increasing follows from the incentive-compatibility condition (2), since $\bar{U}^P_x(x) = E[p(x,y) \mid x] \equiv \bar{p}^P(x) \geq 0$. That it is convex follows from the fact that it is the upper envelope of the family of convex functions indexed by all the reports $\hat{x}$ that the plaintiff might provide to her agent. The convexity property says that a plaintiff or defendant hiding a larger value of x or y obtains a disproportionately larger expected gain from negotiations. One with nothing to hide, say $x = x_*$ or $y = y_*$, gains nothing from settling and would as soon risk a trial, so $\bar{U}^P(x_*) = 0$ and $\bar{U}^D(y_*) = 0$.

[4] Each party's conditional probability of settling is an increasing function. That is:

$\bar{p}^P(x) \equiv E[p(x,y) \mid x]$ and $\bar{p}^D(y) \equiv E[p(x,y) \mid y]$

are increasing functions of x and y, respectively.

That $\bar{p}^P(x)$ is increasing follows from the fact that it is the derivative of the convex function $\bar{U}^P(x)$. This property says that a litigant with more to hide has a greater chance of settling.

[5] The set of pairs (x, y) for which the parties settle is convex and open above.

That this set S is convex follows from the optimality condition [1] for settlement, using the assumption that H^P and H^D are convex functions. That it is open above, namely if $(\hat{x}, \hat{y}) \gg (x, y) \in S$ then also $(\hat{x}, \hat{y}) \in S$, follows from the assumption that H^P and H^D are decreasing functions. Note that there is no possibility of settling if x is so small that $H^P(x) > C/\alpha$, and similarly if y is too small. Thus, a party hiding nothing prefers a trial to settling with an opponent who might be hiding negative information.

[6] Any development (such as discovery) that enlarges the set of events in which the parties settle benefits all potential types of both parties.

Such a development increases the conditional probabilities $\bar{p}^P(x)$ and $\bar{p}^D(y)$ of settling for each party, regardless of his type x or y. Further, this increases his expected net gain because

$$\bar{U}^P(x) = \int_{x_*}^{x} \bar{p}^P(x')\, dx' ,$$

and similarly for the defendant. This property says that increasing the prospects of settling is uniformly a good thing for both parties. In part this reflects merely the fact that settling is a 'public good' in the sense that it avoids both parties' trial costs, but in fact the result is appreciably stronger: the conclusion that their expected gains also increase says that the distributional effects do not outweigh either party's private benefit from the public good produced. Of course, it need not be that the gain exceeds the cost of discovery, so it is to that issue that we turn next.

4. A Model of Discovery

We interpret discovery by one party as a means to obtain partial information about the otherwise hidden information known by the other. To represent the coercive aspect of discovery, we assume that the discovered party cannot select the items (documents, depositions, interrogatories, etc.) obtained by the other. This differs fundamentally from disclosure, in which the disclosing party selects the items provided. Further, because the discovering party is ignorant about the other's private information, the selectivity of discovery requests is inherently limited. The combined effect of these features is that discovery is typically a hit-or-miss process in which the discovering party examines a sample of items that provide only imperfect indications of what the discovered party knows precisely.

To obtain concrete predictions about the parties' incentives for discovery, we use a specific model of the sampling process. A further motive for using a specific model is that it illustrates that the convenient assumptions that might be imposed to enable a general analysis are not satisfied by plausible models of the sampling process. For instance, it would be convenient for a general analysis simply to assume that discovery by the plaintiff tends to reduce the defendant's term $H^D(y)$ that affects whether they settle; that is, we might assume directly that by reducing the plaintiff's uncertainty about the defendant's hidden information, discovery promotes settlement. In fact, however, our experience with various models indicates that such an assumption is very restrictive.

The model is motivated by the following scenario. Suppose that y is a numerical measure of the defendant's privately known negligence, culpability, or other source of liability for damages to the plaintiff; and similarly x measures the plaintiff's contributory negligence or melioration of damages. In the defendant's files, evidence about the extent of his negligence accumulates stochastically at an average rate proportional to his actual negligence y. Consequently, if the plaintiff uses her privilege of discovery to examine a number t of his files then she obtains an imperfect estimate of his liability, and the precision of this estimate increases as t increases.

We use the model of a Brownian motion to formalize this scenario. According to this model, a sample of length t produces an observation with a Normal distribution whose mean and variance are proportional to t. Thus, conditional on the true value of y the observation is a random variable $Y(t) = yt + \tilde{\eta}\sqrt{t/h^D}$, where $\tilde{\eta}$ is a standard Normal variate with mean 0 and variance 1, and h^D measures the sampling precision [reciprocal of the variance]. Using Bayes' rule for conditional probabilities, this specification of the sampling process implies that if the plaintiff's prior distribution of y was Normal with mean m_o^D and precision h_o^D [i.e., variance $1/h_o^D$], then after observing the realization of $Y(t)$ her posterior distribution has mean $m^D(t)$ and precision $h^D(t)$, where

$$m^D(t) = [m_o^D h_o^D + Y(t)h^D]/[h_o^D + th^D],$$
$$h^D(t) = h_o^D + th^D.$$

In the following paragraphs we apply this model of discovery to the model of negotiation developed in the previous section. We impose three restrictions on its application.

- The discovered party also observes the results of discoveries, and therefore knows the mean and variance of the discovering party's posterior distribution. Here, the defendant observes the statistic $(t, Y(t))$ obtained by the plaintiff from her discovery of his files.
- As mentioned previously, the trial value $v(x,y)$ continues to depend only on x and y, rather than depending jointly on the private information (x,y) and the pair of observations $(s, X(s); t, Y(t))$ obtained from discoveries. That is, we assume that for the conduct of the trial the true pair (x,y) is a sufficient statistic for the pair of estimates obtained from sample observations. This means that the parties' actual information ultimately outweighs the sample estimates in determining the trial outcome. This assumption is explained further below, but its restrictiveness is evident in the fact that it precludes a role for discovery results to affect the accuracy of the ultimate outcome of the trial.
- The prior precision h_o^D is small relative to the sampling precision h^D. This implies that the posterior mean $m^D(t)$ differs only slightly from the sample mean $Y(t)/t$ and the posterior precision $h^D(t)$ is nearly proportional to the sample size t. We view this assumption as realistic in that it ensures that discoveries outweigh *a priori* beliefs; however, it is adopted here mainly to simplify calculations.

The next step is to determine how the parties' conditional probabilities of settlement are affected by the extent of the discovery conducted by each. Suppose that after discovery the plaintiff's probability distribution of the defendant's y is Normal with mean $m^D(t)$ and precision $h^D(t)$; and similarly, the defendant's probability distribution of the plaintiff's x is Normal with mean $m^P(s)$ and precision $h^P(s)$, where t and s measure the sample sizes of the discovery undertaken by the plaintiff and the defendant. Because these probability distributions are Normal, the optimality condition [1] can be rephrased in terms of the standard Normal distribution. Let $H(z) \equiv [1 - F(z)]/f(z)$ denote the so-called Mill's ratio calculated from the standard Normal distribution function F and density f with mean 0 and variance 1; as required, it is a decreasing and convex function.[13] Also, let

$$\sigma \equiv 1/\sqrt{h^P(s)} \qquad \text{and} \qquad \tau \equiv 1/\sqrt{h^D(t)},$$

be the respective standard deviations, and let

$$\xi \equiv [x - m^P(s)]/\sigma \qquad \text{and} \qquad \eta \equiv [y - m^D(t)]/\tau,$$

be the respective standard Normal variates, each as perceived by the other party. Then,

$$H^P(x) = \sigma H(\xi) \qquad \text{and} \qquad H^D(y) = \tau H(\eta),$$

In this notation, the optimality condition [1] says that the parties settle in each event (ξ, η) for which

[1] $$C/\alpha \geq \sigma H(\xi) + \tau H(\eta).$$

This translation of notation corresponds to rewriting the trial value and settlement payment as the functions

$$v(x, y) = [m^D(t) - m^P(s)] + \tau\eta - \sigma\xi \qquad \text{and} \qquad q(x, y) = [m^D(t) - m^P(s)] + q(\xi, \eta),$$

[13] It is worth mentioning here that Mill's ratio has the property that

$$\frac{d}{ds}\sigma H([x - m^P]/\sigma) = [H(\xi) - \xi H'(\xi)]\frac{d\sigma}{ds},$$

where $d\sigma/ds < 0$. This derivative is negative only if $\xi > -0.84$. Thus the defendant's discovery that increases only s is sure to increase the probability of settlement only if x is not too small relative to the defendant's perception of the posterior mean m^P. This illustrates that discovery need not assure greater chances of settling. It depends on the actual outcome of the discovery.

and rewriting the parties' settlement probabilities and expected gains

$$\bar{p}^P(x) = \bar{p}^P(\xi) \qquad \text{and} \qquad \bar{p}^D(y) = \bar{p}^D(\eta)\,,$$
$$\bar{U}^P(x) = \bar{U}^P(\xi) \qquad \text{and} \qquad \bar{U}^D(y) = \bar{U}^D(\eta)\,,$$

as functions of the standardized variates ξ and η — and of course implicitly also the public parameters $(s, t, m^P(s), m^D(t))$. This reflects the fact that the constant term representing the difference in means will be accounted for in any settlement, so it is irrelevant for the subsequent analysis.

The plaintiff can assess the probability of settling from two perspectives, one after observing the outcome $Y(t)$ of her discovery, and one before. We shall see that a special simplifying feature of the Brownian model is that these two probabilities are the same; that is, the plaintiff's probability of settling depends only on the amount of her discovery, not the actual observation it produces.

From the plaintiff's perspective after discovery, but before negotiations, $\eta = [y - m^D(t)]/\tau$ is a standard Normal random variable with mean 0 and variance 1. Therefore, at this point her perceived marginal probability of settling is the probability that $C/\alpha \geq \sigma H(\xi) + \tau H(\eta)$, where she knows ξ but not η. This probability can be written as

$$\bar{p}^P(\xi; s, t) = \Phi([C/\alpha(s,t) - \sigma H(\xi)]/\tau)\,,$$

where $\Phi(z) \equiv 1 - F(H^{-1}(z))$ is a strictly increasing (and S-shaped) function for $z > 0$, and $\Phi(z) \equiv 0$ for $z \leq 0$. Notice that this formula does not depend on the posterior means $(m^P(s), m^D(t))$ nor on the actual observations $(X(s), Y(t))$ that the discoveries produced. This feature is reflected further in the fact that the Lagrange multiplier μ, and therefore also the parameter α, depend only on (s,t). Therefore, this formula is valid also as a description of her perceived probability of settling before conducting her discovery, provided only that she knows ξ and the sample size t that she will use. This reveals an important feature of the Brownian model: the plaintiff (and analogously the defendant) can anticipate the probability of settling that a given sample size will produce, provided only that she knows the *net* outcome ξ of the defendant's discovery.[14]

Consider next the consequences for the plaintiff of discovery of her files in the amount s by the defendant. Conditional on knowing x but not the observation $X(s) =$

[14] Due to this feature, there is no further gain from sequential sampling in which each additional discovery request depends on the outcome of the previous ones.

$xs + \tilde{\xi}\sqrt{s/h^P}$ he will observe (e.g., due to random sampling of her files), she perceives his posterior statistic $\xi = [x - m^P(s)]/\sigma$ as a Normal random variable with mean and standard deviation

$$\sigma[x - m_o^P]h_o^P \qquad \text{and} \qquad \sigma\sqrt{sh^P}\,.$$

Thus, by taking the expectation of the previous formula for $\bar{p}^P(x;s,t)$ with respect to this random variable she can calculate the expected effect of his discovery on her chances of settling. Recall, however, that above we assumed for simplicity that each party's prior precision, in this case the defendant's prior precision h_o^P, is negligible compared to the precision h^P of his sample information obtained from discovery of the plaintiff. In this case, she perceives the posterior statistic ξ as a standard Normal random variate with mean 0 and variance 1. Thus, before knowing the outcome $X(s)$ of the defendant's discovery, her probability of settling is the expectation of her probability of settling after knowing the net outcome ξ. This expected probability of settling is

$$\begin{aligned}\bar{p}^P(s,t) &\equiv E[\bar{p}^P(\xi;s,t)]\,,\\ &= \int_{-\infty}^{\infty} \Phi([C/\alpha(s,t) - \sigma H(\xi)]/\tau)\,dF(\xi)\,.\end{aligned}$$

An important observation is that this formula does not depend on P. In fact, it and the analog for the defendant's expected probability of settling

$$\bar{p}^D(s,t) \equiv \int_{-\infty}^{\infty} \Phi([C/\alpha(s,t) - \tau H(\eta)]/\sigma)\,dF(\eta)\,,$$

are just two different ways of writing the *ex ante* probability of settling. Specifically, this common probability of settling is $\bar{p}(s,t) \equiv E[p(\xi,\eta;s,t)]$.

5. The Effect of Discoveries on Settlement

We now use the characterizations obtained above from the Brownian model to establish several propositions about the consequences of discovery. We refer only to the plaintiff, but symmetric conclusions are valid for the defendant.

[7] The plaintiff's perceived probability of settling is an increasing function of the amount that she discovers the defendant. That is:
$\bar{p}^P(x;s,t,m^P(s),m^D(t))$, $\bar{p}^P(\xi;s,t)$, and $\bar{p}(s,t)$ are increasing functions of her discovery amount t.

That each of these functions increase with t is clear from the preceding formulas once one knows that $\alpha(s,t)$ is a decreasing function — which is true but we do not prove it here. This same fact implies:

[8] The plaintiff's *ex ante* probability of settling is an increasing function of the amount that the defendant discovers her. That is:

$\bar{p}(\xi; s, t)$ and $\bar{p}(s,t)$ are increasing functions of his discovery amount s.

Both of these results reflect the basic fact that the set S of events (ξ, η) in which the parties settle is enlarged by discovery. The effect is substantially asymmetric however: the primary effect of discovery is to enlarge the set S mainly in the direction of the discovered party's variable. For instance, increasing t and thereby reducing τ reduces the lower bound $H^{-1}(C/\alpha\tau)$ on those types η of the defendant who might settle *and* the lower bound $H^{-1}(C/\alpha\sigma)$ on those types ξ of the plaintiff who might settle, but the former effect is much greater than the latter because t reduces τ but not σ. This is illustrated in Figure 1, which shows how the boundary of the set of settlement events shifts downward in response to a decrease in τ.

These results about the settlement probabilities imply that in expectation both parties benefit from discovery, although of course the discoverer benefits more than the discovered party.

[9] The plaintiff's expected gain is an increasing function of the amount of her discovery. That is:

$\bar{U}^P(x; s, t, m^P(s), m^D(t))$, $\bar{U}^P(\xi; s, t)$, and $\bar{U}^P(s,t)$ are increasing functions of t.

To see this, recall that her expected gain from negotiation is

$$\bar{U}^P(x; s, t, \ldots) = \int_{-\infty}^{x} \bar{p}^P(x'; s, t, \ldots)\, dx' .$$

An increase in t increases every probability $\bar{p}^P(x'; s, t, \ldots)$, so it also increases the integral. As mentioned in footnote 8, this kind of reasoning does *not* apply directly to the case that s is increased. However, it does apply *ex ante* to the expectation of the outcome of the defendant's discovery. In this context, it is the expected probability of settling that is relevant, and as we have seen above the probabilities $\bar{p}^P(\xi; s, t)$ and $\bar{p}(s,t)$ are indeed increasing in s. Thus:

[10] The plaintiff's *ex ante* expected gain is an increasing function of the amount of the defendant's discovery. That is:

$\bar{U}^P(\xi; s, t)$ and $\bar{U}^P(s,t)$ are increasing functions of the defendant's discovery s.

6. Private Incentives and the Social Value of Discovery

We turn now to an examination of the parties' incentives for discovery. To provide a benchmark, we compare these with the incentives implied by the efficiency criterion used to design the negotiation process.

Recall that the social gain from settling is the avoided cost of a trial. The expected social gain is therefore the avoided cost times the probability of settling, less of course some overall combined discovery cost $D(s,t)$ incurred by the parties. When the discovery amounts are s and t, therefore, the net social gain is

$$C\bar{p}(s,t) - D(s,t).$$

where $\bar{p}(s,t) = E[p(\xi,\eta;s,t) \mid s,t]$ is the *ex ante* probability of settling conditional on only the two amounts of discovery.[15] In such a static context, the necessary conditions for socially optimal amounts of discovery are:

$$C\bar{p}_t(s,t) = D_t(s,t) \qquad \text{and} \qquad C\bar{p}_s(s,t) = D_s(s,t).$$

The parties' private incentives for discovery are possibly different than the social incentives. One difference is that the social cost of discovery is typically divided between the parties, as say $d^P(s,t)$ incurred by the plaintiff and $d^D(s,t)$ incurred by the defendant, where $D(s,t) = d^P(s,t) + d^D(s,t)$. Here, however, we adopt the view that distortions in the discovery costs borne by the parties can be corrected by policy measures. Instead, our focus is on the possibility of a divergence between the social benefits of discovery and the private benefits perceived by each party separately.

Presumably the plaintiff (and analogously for the defendant) is interested only in choosing t to maximize her expected net gain

$$\bar{U}^P(x;s,t) - d^P(s,t),$$

given the defendant's discovery amount s — but not yet knowing the outcome of either discovery. The formula for the plaintiff's benefit $\bar{U}^P(x;s,t)$ can be derived as follows.

[15] There is a seeming complication here in the fact that the plaintiff chooses t knowing x but not y, and the defendant chooses s knowing y but not x. However, if s and t are chosen initially, each without knowing the outcome of the other's discovery, then both parties choose without knowing either of the net effects ξ or η. Thus, even though they have different private information, they face the same uncertainty as regards the consequences of discovery on the probability of eventually settling.

Recall that after the discovery results become known the plaintiff's expected gain will be

$$\bar{U}^P(x; s, t, m^P(s)) = \int_{-\infty}^{x} \bar{p}^P(x'; s, t, m^P(s))\, dx' \,,$$

where

$$\begin{aligned} \bar{p}(x'; s, t, m^P(s)) &= \Phi([C/\alpha(s,t) - \sigma M(\xi')]/\tau)\,, \\ \xi' &= [x' - m^P(s)]/\sigma\,, \\ m^P(s) &= x + \sigma\zeta\,, \end{aligned}$$

and before knowing the outcome of the defendant's discovery she perceives the net outcome of the defendant's discovery as the standard Normal variate ζ. Consequently, taking the expectation with respect to ζ yields:[16]

$$\begin{aligned} \bar{U}^P(x; s, t) &\equiv E[\bar{U}^P(x; s, t, m^P(s) = x + \sigma\zeta) \mid x, s, t]\,, \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{x} \Phi([C/\alpha(s,t) - \sigma M([x' - x]/\sigma - \zeta)]/\tau)\, dx'\, dF(\zeta)\,, \\ &= \int_{-\infty}^{\infty} \Phi([C/\alpha(s,t) - \sigma M(\xi)]/\tau)[1 - F(\xi)]\, d\xi\,, \\ &= \int_{-\infty}^{\infty} \bar{U}^P(\xi; s, t)\, dF(\xi)\,, \\ &\equiv \bar{U}^P(s,t)\,, \end{aligned}$$

where in the fourth line,

$$\bar{U}^P(\xi; s, t) = \int_{-\infty}^{\xi} \Phi([C/\alpha(s,t) - \sigma M(\xi')]/\tau)\, d\xi' \,.$$

The important observation about this calculation is that the plaintiff's expected benefit $\bar{U}^P(x; s, t) = \bar{U}^P(s, t)$ does *not* depend on her private information x. An analogous result is true for the defendant. This leads to a significant implication of this model:

[11] If the parties select their discovery amounts *ex ante*, and their prior precisions are negligible compared to their sampling precisions, then the difference in their private information (x or y) is irrelevant to their selections of the optimal amounts of discoveries.

This result indicates that, for this particular model, private information presents no fundamental impediment to an agreement between the parties to undertake efficient amounts of discovery.

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[16] Each of the third and fourth lines is obtained from the preceding line using integration by parts.

Of course this result is derived from a very special model. But we do not think that it is an irrelevant model: it provides an instance in which there cannot be any presumption that the parties will choose either excessive or insufficient discovery amounts. It is true that a poorly designed allocation of the costs of discoveries might distort the parties' incentives, but this too can be the subject of negotiation or judicial intervention.[17] The key point is that one cannot argue that inefficiencies stem from informational disparities, since in this model there are none that are relevant at the *ex ante* stage when the discovery amounts are selected.

We conclude this section by examining the implications of the feasibility constraint (3). Recall that the Lagrange multiplier μ, and therefore also the parameter α, is determined to ensure equality between the expected social gain and the sum of the parties' expected gains. Since this must be true also at the *ex ante* stage, we know that

$$C\bar{p}(s,t) = \bar{U}^P(s,t) + \bar{U}^D(s,t) .$$

This implies that the necessary conditions for efficient discovery amounts can be written in the form

$$\bar{U}_t^P(s,t) + \bar{U}_t^D(s,t) = D_t(s,t) \qquad \text{and} \qquad \bar{U}_s^P(s,t) + \bar{U}_s^D(s,t) = D_s(s,t) .$$

As mentioned previously, most of the benefit of the plaintiff's discovery t typically accrues to the plaintiff, and little to the defendant. That is, $\bar{U}_t^P(s,t)$ is typically much larger than $\bar{U}_t^D(s,t)$. These necessary conditions therefore imply a presumption that the plaintiff should bear most of the marginal cost of her discovery. That is, it is conducive to efficiency to allocate costs so that $d_t^P(s,t) \approx D_t(s,t)$, and analogously for the defendant. In particular, this argues against allowing a cost allocation that is impositional, in which the discovered party bears a substantial share of the marginal cost.[18]

In sum, from a Coasian perspective, the absence *ex ante* of incentive effects from informational disparities indicates that the parties can reach agreement on an efficient plan of discovery; e.g., an allocation of discovery costs that would otherwise lead to inefficient amounts of discovery can be circumvented by an agreement that reallocates these costs appropriately.

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[17] A relevant case occurs when the costs are impositional; that is, it is the discovered party who bears most of the cost of the other's discovery. In such a case it is clearly possible and even likely that the discoveries are excessive.

[18] This conclusion must, of course, be tempered with the consideration that the situation could have other asymmetries, such as sampling precisions that differ greatly, which would invalidate the supposition that the discovering party obtains the greater benefit.

7. Dynamics of Discovery

An alternative view of discovery recognizes that it can be a dynamic process in which the parties alternate sampling of each other's files. In this context, it is important to recognize that the history of the sample observations can have significant effects. We have seen in [9] that the plaintiff's gain from discovering the defendant is positive regardless of the history of previous observations. The new consideration in a dynamic context is that the magnitude of this gain can depend on the outcome of the defendant's previous discoveries of her files, depending on whether his sample observation produced an under- or over-estimate $m^P(s)$ of the true value of her private information x. Intuitively, it is evident that a plaintiff dealing with a defendant who has erroneously overestimated her contributory negligence anticipates that the terms of a settlement would be unfavorable and that a trial will be necessary; therefore, further discovery of his files might be useless in promoting settlement. To examine the main ingredients of these effects, we focus on how the plaintiff's marginal benefit from discovery is changed by the defendant's discovery.[19]

The key fact is that the plaintiff's perceived settlement probability $\bar{p}^P(x; s, t, m^P)$ is *not* an increasing function of the defendant's discovery s unless $\xi > -0.84$ and therefore x is sufficiently large; viz., $x > m^P - 0.84\sigma$. This condition requires x to be above the 0.2 fractile of its probability distribution as perceived by the defendant. Similarly, increasing the amount of the defendant's discovery enlarges the set of events (x, y) in which the parties settle only if the plaintiff is not too optimistic about trial compared to the defendant's posterior mean m^P.

[12] In the region $x > m^P - 0.84\sigma$ where the plaintiff's conditional probability $\bar{p}^P(x; s, t, m^P)$ of settling is an increasing function of s, the cross-partial derivative $\frac{\partial^2}{\partial s \partial t}\bar{p}^P(x; s, t, m^P)$
is positive or negative as $\bar{p}^P(x; s, t, m^P)$ is less or more than $\approx 1/3$. The signs are reversed when $x < m^P - 0.84\sigma$.

For this approximation it suffices to ignore the small effects on $\alpha(s, t)$. With this omission, direct calculation of the cross-partial derivative shows that its sign is the prod-

[19] We are indebted to Alessandro Lizzeri for pointing out that the following analysis omits consideration of the fact that the extent of discovery is itself informative. That is, when discovery is undertaken sequentially, the amount t of discovery by the plaintiff, and similarly the amount s by the defendant, are generally functions of their private information x and y respectively. Consequently, the amount of discovery by each party is a signal about its private information.

uct of the signs of the factors $[\Phi'(z) + z\Phi''(z)]$ and $[H(\xi) - \xi H'(\xi)]$, where $z \equiv [C/\alpha(s,t) - \sigma H(\xi)]/\tau$. Numerical evaluation of the first factor shows that it is positive or negative as z is less or more than 0.92044, which corresponds to $\bar{p}^P(x;s,t,m^P) = \Phi(z)$ being less or more than 0.33554. The second factor is positive when $\xi > -0.84$, which is the range in which $\bar{p}^P(x;s,t,m^P)$ is an increasing function of s.

This conclusion can be rephrased as follows.

- Suppose first that x is actually above the 0.2 fractile of its probability distribution as perceived by the defendant.
    - If without further discovery the plaintiff's probability of settling is large (more than 1/3), then: The plaintiff's expected increase in the chance of settling produced by her further discovery *decreases* as the defendant invokes more discovery.
    - Alternatively, if without further discovery the plaintiff's probability of settling is small (less than 1/3), then: The plaintiff's expected increase in the chance of settling produced by her further discovery *increases* as the defendant invokes more discovery.
- If x is actually below the 0.2 fractile then the hypotheses of these two conclusions are reversed.

This indicates that there are several regimes possible. The most likely (with probability $0.64 = 0.8 \times 0.8$) is that neither party thinks its parameter has been grossly overestimated and that they are likely to settle, in which case further discovery by one weakens the incentive for further discovery by the other. In the terminology of game theory, this is described by saying that in terms of the settlement probability, discovery amounts are strategic substitutes. On the other hand, they are strategic complements when each party anticipates trial due to a gross overestimate by the other; in this circumstance, discovery by one stimulates discovery by the other.[20] In general, however, one must expect a rocky road in the early stages as initial overestimates by either party stimulate vigorous reciprocal discovery by the other, who expects eventually to settle but is strongly motivated to redress its otherwise adverse terms. Even in the last stages when one party (or a judge) thinks that negotiation would likely produce a settlement, the other party might insist on further discovery to counter the effects of what it knows to be an overestimate. These ambiguous aspects of dynamic discovery seem to be intrinsic whenever

[20] This feature is an instance of a general pattern studied by Dixit (1987).

the discovery process produces imperfect sample information that the discovered party might view as misleading.

An alternative view of sequential discovery is that it offers new opportunities to attain efficiency, especially when discovery costs are heavily impositional. As described in the previous section for the static case, impositional costs can lead to excessive discovery if each party foresees a large gain from a small marginal cost of additional discovery. As the economics literature has emphasized, however, such dynamic games have multiple equilibria, and some of these yield efficient outcomes (Fudenberg and Tirole, 1986; Spence, 1978). The one that is relevant here invokes contingent strategies of the following form, based on an initial understanding between the parties about what the efficient amounts of discoveries are:

> "I will discover you no more than the efficient amount, provided you do not discover me more than the efficient amount; otherwise, I will discover all that is self-interestedly optimal for me — even if most of the cost is imposed on you."

This strategy punishes the other party for inefficient behavior. Moreover, once excessive discovery by either party precipitates such a war of discovery, each finds it optimal to carry through. As we argue in our expository article, the initial discovery conference with the judge or magistrate offers an opportunity to establish the initial understanding between the parties that enables such contingent strategies to be used.

Appendix
Examples of Optimal Negotiation Procedures

The purpose of this appendix is to illustrate the construction of an optimal negotiation procedure using the results established in Section 2 of the text. For this purpose we use the simplest example in which each party's privately known parameter has a probability distribution that is uniform on an interval. In this example, the optimal procedure depends on the locations of these intervals. Interpreting these locations as the results of previous discoveries by the parties, we infer that in general the optimal negotiation procedure is sensitive to the outcome of the discovery process. This justifies our reliance on a mechanism-design formulation to study incentives for discovery; for, an analysis that assumed some particular negotiation procedure would fail to recognize that the parties should, and presumably would, adapt the procedure to the results of discoveries.

The example supposes that the plaintiff's parameter x is perceived by the defendant to be uniformly distributed on the interval $[0, a]$, and similarly the defendant's parameter y is uniformly distributed on the interval $[0, b]$. Thus, $F^P(x) = x/a$ and $F^D(y) = y/b$, and therefore $H^P(x) = a - x$ and $H^D(y) = b - y$. As in the text, we assume that the trial value is $v(x, y) = y - x$. Consequently, according to [1] an optimal procedure has the parties settle when $C/\alpha \geq [a - x] + [b - y]$. The subsequent analysis divides into six cases that depend on the relative magnitude of the trial cost C compared to a and b. Here, we address only two cases that illustrate the key idea. We omit various complicated formulas.

Case 1

Suppose first that C is small, but not so small that they never settle. This corresponds to the case that the set of those events (x, y) in which they settle, namely $x+y \geq a+b-C/\alpha$, is a triangle occupying the northeast corner of the rectangle $[0, a] \times [0, b]$. In this case α is determined by the feasibility requirement (3) that the conditional expectation of $C - [a - x] - [b - y]$ on this triangle should be zero. From this requirement one finds that $\alpha = 2/3$. Thus, an optimal procedure must produce a settlement whenever $x + y \geq a + b - [3/2]C$. In fact, one such procedure is a so-called settlement escrow: the parties simultaneously submit offers $X(x)$ and $Y(y)$, each dependent on her or his private information x or y, and they settle for the payment $q(x, y) = Q(X(x), Y(y)) =$

$[1/2][X(x)+Y(y)]$ if $X(x) \leq Y(y)$. That is, they settle if the plaintiff demands no more than the defendant offers. There is an equilibrium of this negotiation game in which the strategies $X(x)$ and $Y(y)$ are simple linear functions of x and y respectively, and as required for optimality, these functions are such that the parties settle whenever (x, y) lies in the designated triangle.

Case 2

Now consider the other extreme case in which C is large, but not so large that they always settle. This corresponds to the case that the set of those events (x, y) in which they do *not* settle, namely $x + y < a + b - C/\alpha$, is a triangle occupying the southwest corner of the rectangle $[0, a] \times [0, b]$. In this case α is determined by the feasibility requirement that the conditional expectation of $C - [a - x] - [b - y]$ outside this triangle should be zero. From this requirement one finds that α must satisfy a cubic equation whose relevant root lies in the interval $1/2 \leq \alpha < 2/3$. Figure 2 provides a graph of this root for the situation in which $a = b$ as a function of $1/a$. Note that the probability of settling is 1 if $a = b \leq 1$.

In this case the negotiation procedure constructed for Case 1 will not suffice, since the settlement escrow still settles only in the events for which $x + y \geq a + b - [3/2]C$. However, it can be modified to obtain an optimal procedure as follows. One party, say the defendant, makes an initial offer to the plaintiff and if she accepts then they settle on those terms; if it is rejected, they then proceed with a settlement escrow as in Case 1. Note that the plaintiff accepts the defendant's initial offer if her parameter x is sufficiently large that his offer is better than taking her chances with the settlement escrow; and anticipating this, the defendant designs his offer optimally, taking account of the probability it will be accepted as well as the complementary probability that the settlement escrow will ensue. A key fact is that the defendant's optimal offer signals nothing about his parameter y; all types of the defendant prefer the same optimal offer. Moreover, the net result of this two-stage procedure is that the parties settle in precisely the right events required by the optimal mechanism. Essentially, this is because the effect of the first stage is to exclude from the settlement escrow those types of the plaintiff in an interval $[a^*, a]$. The combined effect of acceptances by the plaintiff's types $x \geq a^*$ in the initial stage, and the contingent settlement escrow that settles when $x \leq a^*$ and $x + y \geq a^* + b - [3/2]C$ is to implement the optimal value of $\alpha < 2/3$.

Thus, this example conveys the main point that a proper analysis of discovery must take account of the prospect that the procedural aspects of settlement negotiations will be adapted *ex post* to the peculiar features of the uncertainties that the two parties face after the results of discoveries become known. An analysis of discovery that presumes the parties are committed to a particular procedure for settlement negotiations necessarily precludes realization of some of the potential benefits from negotiation.

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