ISSN 1045-6333

A NOTE ON OPTIMAL FINES WHEN WEALTH VARIES AMONG INDIVIDUALS

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Discussion Paper No. 64

11/89

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The Program in Law and Economics is supported by
a grant from the John M. Olin Foundation

November, 1989

A NOTE ON OPTIMAL FINES WHEN WEALTH VARIES AMONG INDIVIDUALS

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Abstract: An important result in the economic theory of enforcement is that, under certain circumstances, it is optimal for a fine to be as high as possible -- to equal the entire wealth of individuals. Such a fine allows the probability of detection to be as low as possible, thereby saving enforcement costs. This note shows that when the level of wealth varies among individuals, the optimal fine generally is less than the wealth of the highest wealth individuals, and may well be less than the wealth of most individuals.

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* Stanford University and National Bureau of Economic Research; Harvard University and National Bureau of Economic Research, respectively.
Polinsky's research was supported by the John M. Olin Program in Law and Economics at Stanford Law School. Shavell's research was supported by the National Science Foundation (grant SES-8821400).

1. Introduction

An important result in the economic theory of enforcement is that, under certain circumstances, it is optimal to impose the highest possible fine -- equal to an individual's entire wealth -- with a relatively low probability of detection. The reasoning supporting this conclusion, which is usually attributed to Becker (1968), is well known. If the fine is not at its highest level, enforcement costs can be reduced without affecting deterrence. This can be done by raising the fine to its highest level and lowering the probability of detection proportionally, so that the expected fine -- and thus deterrence -- is left unchanged. Hence, according to this argument, it cannot be optimal for the fine to be less than an individual's wealth.

It is puzzling, of course, that this result differs so much from reality. Fines equal to an individual's wealth hardly ever are imposed. Several different explanations have been offered to reconcile Becker's theory with this fact. For example, it has been shown that if individuals are risk averse, fines less than their wealth generally are optimal.[1] This note adds a new explanation.

We will demonstrate that if, as is obviously realistic, the wealth of individuals varies, the optimal fine is less than the wealth of the highest wealth individuals, and may well be less than the wealth of most individuals. In particular, the optimal fine is such that only low-wealth individuals pay everything they have; all other individuals pay the fine, which is less than their wealth.

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[1] See Polinsky and Shavell (1979), Block and Sidak (1980), and Kaplow (1989). Another explanation is provided in Shavell (1989). See also Carr-Hill and Stern (1979, pp. 281-295) and Posner (1986, pp. 205-212).

To understand our conclusion, consider why the argument associated with Becker cannot be applied when wealth varies. Suppose that the fine is less than the wealth of the highest wealth individuals. If the fine is raised and the probability of detection is lowered proportionally, it is true that those who can pay the higher fine are deterred to the same extent. But those who cannot pay the higher fine are deterred less. For the latter reason, it generally is not optimal to raise the fine to the highest possible level.

To illustrate, a fine of $100 for speeding may be optimal because many drivers may have so little in savings that it is difficult to collect more than $100 from them. If a much larger fine were imposed with a much smaller probability, these drivers would be inadequately deterred. Thus, a fine of $100 may be optimal, which would mean that all drivers with wealth exceeding $100 pay a fine less than their wealth.

2. Analysis and Example

In the model, risk-neutral individuals contemplate whether to commit a harmful act. Each individual is identified by the benefit he would obtain from committing the act, and by his level of wealth. If an individual commits the harmful act, he will be made to pay a fine with some probability; this probability is determined by the enforcement expenditures of the state.

The following notation will be used.

h = harm caused if the harmful act is committed; $h > 0$;

b = benefit from committing the harmful act; $b \geq 0$;

$r(b)$ = probability density of b; r is positive for all $b \geq 0$;

w = wealth of an individual; $w \geq 0$;

$s(w)$ = probability density of w; s is positive for all $w \geq 0$;

$f(w)$ = fine for committing the harmful act for an individual whose wealth is w; $0 \leq f(w) \leq w$;[2]

e = enforcement expenditures of the state; $e \geq 0$;

$p(e)$ = probability of detection; $p'(e) > 0$.

The distribution of benefits is assumed to be the same for different levels of wealth; this assumption is not essential. Also, the probability of detection is assumed to be the same for individuals of different wealth. This assumption is crucial, as will be commented upon below.

Social welfare is the sum of the benefits obtained by individuals who commit the harmful act, less the harm done, and less enforcement expenditures. To determine social welfare, observe that an individual will commit the harmful act if and only if[3]

$$b \geq pf(w). \tag{1}$$

Hence, social welfare is

$$\int_0^\infty \int_{pf(w)}^\infty (b - h)r(b)db\,s(w)dw - e. \tag{2}$$

Let us first determine the optimal fine, $f^*(w)$, assuming that the probability of detection, p, is positive. Clearly, given p and any w, $f^*(w)$ is the f that maximizes

$$\int_{pf}^\infty (b - h)r(b)db \tag{3}$$

over f. The derivative of (3) with respect to f is $p(h - pf)r(pf)$, which is positive for $f < h/p$, 0 at $f = h/p$, and negative for higher f. Thus, the

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[2] Implicit in the assumption that $f(w) \leq w$ is the further assumption that an individual's wealth does not include the benefit he obtains from committing the harmful act. The latter assumption is made only for convenience.

[3] The assumption that an individual commits the act when $b = pf(w)$ is immaterial.

optimal f equals h/p if h/p is feasible, that is, if $h/p \leq w$; otherwise, the optimal f is w. In other words, $f^*(w) = \min(h/p, w)$.

This result can be restated as follows. <u>The optimal fine equals an individual's wealth for every individual with wealth less than h/p; for all other individuals, who have higher wealth, the optimal fine is h/p, which is less than their wealth.</u> Equivalently, the optimal fine is h/p for all individuals, but those who cannot pay this amount pay what they can. Note that those who can pay h/p are optimally deterred -- act in the first-best manner -- since the expected fine they pay equals the harm caused.[4]

Let us next consider the optimal probability. Because $f^*(w) = \min(h/p, w)$, (2) can be rewritten as

$$\begin{aligned} &\int_0^{h/p}\int_{pw}^{\infty}(b - h)r(b)db\,s(w)dw \\ (4)\qquad & \\ &\quad + \int_{h/p}^{\infty}\int_{h}^{\infty}(b - h)r(b)db\,s(w)dw \quad - \quad e. \end{aligned}$$

The first term relates to those who pay their wealth w when fined because they cannot pay h/p; the second term relates to those who have wealth of at least h/p and who therefore pay h/p and are optimally deterred.

Setting the derivative of (4) with respect to e equal to zero gives the relevant first-order condition,

$$(5)\qquad \int_0^{h/p} p'(e)(h - pw)wr(pw)s(w)dw \;=\; 1.$$

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[4] In Polinsky and Shavell (1984, pp. 96-97), we briefly considered optimal fines when there are <u>two</u> types of individuals who differ in wealth. It was shown there that the optimal fine for the low-wealth group is equal to their wealth and that the optimal fine for the high-wealth group is larger but not necessarily equal to their wealth. The analysis here generalizes that result and is consistent with it.

The left-hand side is the marginal benefit of raising e: individuals with wealth less than h/p are underdeterred since they cannot pay h/p; by raising e, p is raised and more such individuals are deterred; at the margin, there is a social gain of h - pw from deterring an individual with wealth w. The right-hand side is the marginal cost of raising e, namely 1. The optimal p is determined implicitly by the optimal choice of e from (5).

The preceding results can be illustrated by a simple example. Suppose that the harm h if the harmful act is committed is \$20; that the benefit b from committing the act is uniformly distributed between \$0 and \$25; that the wealth w of individuals is uniformly distributed between \$0 and \$100,000; and that the probability of detection p as a function of enforcement expenditures e is given by (\$75)e. Then it can be shown that the optimal probability of detection p* is .2 and that the optimal fine f* is \$100 (as expected, p*f* = h = \$20).[5] Thus, everyone with wealth less than \$100 pays their wealth, while everyone with wealth above this level pays the fine of \$100. Given the assumption that wealth is uniformly distributed between \$0 and \$100,000, the optimal fine is below the wealth of over 99% of the population and is equal to less than 1% of the wealth of the highest wealth individuals.

3. Comment

As noted above, the assumption that the probability of detection is the same for individuals with different wealth is central to our results. If the probability could be chosen independently for individuals with different levels of wealth, then, for each level of wealth w, the optimal fine would be the entire wealth w. The reason is that Becker's argument would apply

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[5] The optimal probability is determined by solving (5), which reduces in the example to $.00000711/e^2 = 1$.

for each w. For if $f^*(w) < w$, then by raising f to w and lowering p from $p^*(w)$ to the p such that $pw = p^*(w)f^*(w)$, deterrence would not be affected, but less would be spent on enforcement.

In fact, the probability of detection does seem to be largely independent of wealth. This may be because it is difficult to vary enforcement effort with respect to an individual's wealth. It would be impractical, for example, to vary the probability of detection of traffic violations according to the wealth of drivers. In these kinds of circumstances, a fine less than the wealth of many individuals generally will be optimal for the reasons explained in this note.

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