# A NOTE ON THE OPTIMAL USE OF IMPRISONMENT TO INCAPACITATE

Steven Shavell

Discussion Paper No. 14

10/85

Program in Law and Economics
Harvard Law School
Cambridge, MA 02138

The Program in Law and Ecnomics is supported by a grant from the John M. Olin Foundation.

# A Note on the Optimal Use of Imprisonment to Incapacitate

S. Shavell

*Abstract*. The incapacitative function of imprisonment is examined in a model in which individuals cause a fixed amount of harm each period that they are free. (The assumption that the amount of harm is fixed is made to abstract from considerations of deterrence.) In this model, the optimal system of sanctions is determined: an apprehended individual should be imprisoned for life if his dangerousness exceeds a threshold level; otherwise he should go free. The optimal probability of apprehending individuals is also determined.

A Note on the Optimal Use of Imprisonment to Incapacitate

S. Shavell*

One of the functions of imprisonment is to remove individuals from the population to prevent them from doing harm in the future. This incapacitative function of imprisonment[1] is examined below in a model in which individuals cause an exogenously determined amount of harm each period that they are free. The assumption that the harm done each period by a free individual is fixed is made to abstract from the role of imprisonment as a deterrent.[2]

The conclusions reached in the analysis may be summarized by two statements.[3] First, it is not optimal to imprison individuals if the harm they cause per period is less than a threshold level (equal to the per period cost of imprisonment); otherwise it is best to imprison them for life, since if it is worthwhile imprisoning an individual for some length of time due to his dangerousness, it is worthwhile continuing to do so. Second, the optimal probability of apprehending individuals balances appropriately the considerations that raising the probability increases the number of individuals who are imprisoned and thus stopped from doing further harm, but also involves greater enforcement costs.

## 2. The model

An equal number of finite-lived individuals enter the population every period, so that in the steady state the population is comprised of equal numbers of individuals of each age cohort. As noted, individuals cause fixed amounts of harm every period that they are free, where the amount of harm varies by the individual. Individuals are apprehended each period with a probability, and if apprehended, may be imprisoned. Specifically, let

$h$ = harm done each period by an individual if not imprisoned; $0 \leq h \leq \bar{h}$;

$f(h)$ = probability density of individuals of type h entering the population each period;4/

$n$ = number of periods that individuals live;

$p$ = probability of apprehension of individuals per period;

$s(h)$ = length of imprisonment sentence imposed on an individual of type h if apprehended.

Society bears certain costs in apprehending and imprisoning individuals; let

$c$ = cost of imprisoning an individual per period;[5] $c > 0$;

$e(p)$ = enforcement expenses associated with maintaining the probability of apprehension; $e'(p) > 0$, $e''(p) > 0$.

The social problem is to choose a system of sentences and the probability of apprehension to minimize expected social costs, defined as the expected sum of harm, the costs of imprisonment, and enforcement expenses.

The optimal system of sentences is clear: <u>an apprehended individual for whom $h>c$ should be imprisoned for life</u>, <u>but one for whom $h \leq c$ should be set free</u>. This is because individuals for whom $h>c$ do more harm <u>any</u> period that they are free than they cost to imprison, and conversely if $h \leq c$.

Therefore, social costs are minimized by imprisoning the former for life and by allowing the latter to go free.[6]

Note that the optimal sentences do not depend on p or on n, but they do depend on c.

Given that sentences are optimal, social costs per period as a function of the probability of apprehension are

$$(1) \quad n\int_0^c hf(h)dh \quad + \quad q(p)\int_c^{\bar{h}} hf(h)dh$$

$$+ \quad (n - q(p))\int_c^{\bar{h}} cf(h)dh + e(p),$$

where

$$(2) \quad q(p) = (1 - p) + (1 - p)^2 + \ldots + (1-p)^n,$$

that is, the sum of the probabilities that individuals in different age cohorts have not been apprehended, and thus where the sum of the probabilities of individuals in the different cohorts who have been apprehended is

$$(3) \quad [1 - (1 - p)] + \ldots + [1 - (1-p)^n] = n - q(p).$$

<u>The first-order condition determining the optimal p is therefore</u>

$$(4) \quad e'(p) = -q'(p)\int_c^{\bar{h}} (h - c)f(h)dh,$$

namely, marginal enforcement expenses must equal the reduction in harm (net of the cost of imprisonment) due to imprisonment of additional individuals.

It follows from (4) that the optimal probability rises with n (since if individuals do harm for a longer time, it is more important to incapacitate them);[7] with a rightward shift in the distribution of h (for similar reasons);[8] and with a decline in c (since this means it is optimal to imprison a greater percentage of individuals who are apprehended, and thus the social payoff from raising the probability is enhanced).[9]

## 3. Comments

(a) The conclusion that if it is optimal to imprison individuals for any length of time, it is optimal to imprison them for life would obviously not hold if the model were generalized to allow for the dangerousness of individuals to decline with age[10] or as a result of a rehabilitative effect of imprisonment. If the dangerousness of an imprisoned individual fell below the cost c of imprisonment, it would be best to release him.[11]

(b) Optimal sentences where incapacitation is the only goal are different from optimal sentences where deterrence is the goal.[12] In the latter case, the magnitude of the optimal sentence depends on the ability to deter; and if this is small (as for instance with the insane), a low sentence would be indicated, whereas a high sentence might be called for to incapacitate. Also, the magnitude of the sentence that is best for deterrence rises continuously with the harm done and is generally larger the smaller the probability of apprehension (rather than being independent of the probability), which in turn affects the optimal choice of the probability.

Footnotes

* Professor of Law and Economics, Harvard Law School. I wish to thank Louis Kaplow and A. Mitchell Polinsky for comments and the National Science Foundation (grant no. SES-8420226) for financial support.

1. Imprisonment is no doubt the most important incapacitative sanction today, but it should be observed that the death penalty and deportation also have incapacitative effects. (Monetary sanctions, however, plainly do not.)

2. While the deterrent effect of sanctions has of course been much investigated by economists (see the references in note 12 below), the incapacitative role of sanctions does not appear to have been the focus of a theoretical study.

3. Following the analysis, a qualifying remark is made about these conclusions and they are contrasted with the conclusions drawn in models of deterrence.

4. The total number of individuals entering the population each period is normalized at 1.

5. This may be interpreted as including not only the costs associated with the building and operation of prisons, but also the foregone production of the imprisoned individual and the disutility he suffers.

6. Of course, in the case where $h = c$, it does not matter whether individuals go free, but for simplicity we adopt the convention that they do.

7. Since $-q'(p) = 1 + 2(1 - p) + \ldots + n(1 - p)^{n - 1}$ rises with n, p must rise to maintain equality in (4).

8. Let the harm done by a person of type h be h + t, where t is a parameter. Then the right hand side of (4) becomes $- q'(p) \int_{c-t}^{\bar{h}} (h + t - c)f(h)dh$. Differentiating this with respect to t gives $-q'(p)$(Probability that h exceeds c - t), which is positive. Hence, again, p must rise to maintain equality in (4).

9. Differentiating the right hand side of (4) with respect to c gives $q'(p)$(Probability that h exceeds c), which is negative.

10. There is strong evidence that dangerousness does decline with age; see for instance pp. 32-33 of Report [1983].

11. If h is a function purely of age, it is clear that if an individual is apprehended, it would be optimal to imprison him when and only when $h > c$; in particular, he should be released if h falls below c. If h were lowered due to a rehabilitative effect of imprisonment, however, it might be optimal to imprison an individual even if $h < c$ before releasing him. For instance, if h would be lowered to 0 on account of a single period of imprisonment, it might be advantageous to imprison someone of type $h = c/2$ for one period and then free him, since this would save social costs of $c/2$ for each remaining period of his life.

12. See Becker's original paper [1968] on the use of sanctions to deter, and see also Carr-Hill and Stern [1979], Polinsky and Shavell [1984], and Shavell [1985].

## References


Becker, Gary "Crime and Punishment: An Economic Approach," Journal of Political Economy, 1968, Vol. 76, pp. 169-217.

Carr-Hill, R. and Nicholas Stern, Crime, The Police, and Criminal Statistics, Academic Press, London, 1979, pp. 280-309.

Polinsky, A. Mitchell and Steven Shavell, "The Optimal Use of Fines and Imprisonment," Journal of Public Economics, 1984, Vol. 24, pp. 89-99.

Report to the Nation on Crime and Justice: The Data. U.S. Department of Justice, NCJ-87068, October, 1983.

Shavell, Steven "The Optimal Use of Nonmonetary Sanctions as a Deterrent," Law and Economics Discussion Paper No. 10, Harvard Law School, 1985.