

Uncertainty Over Causation
and the Determination of
Civil Liability

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Abstract. A model of situations in which there is uncertainty over the cause of harm is studied (e.g., where it is not known whether individuals' lung cancer was caused by their habit of smoking cigarettes, by their exposure to certain carcinogens discharged by a chemical plant, or by their exposure to medical x-radiation and the like). The chief interest is with identifying the effects on parties' incentives to reduce risk of various ways of treating uncertainty over causation in determining liability.

A summary of the conclusions from the model is as follows. First, use of a threshold probability of causation (e.g., 50%) as a criterion for determining liability may adversely affect incentives: Parties would face a diminished burden of liability and do too little to reduce risk if their probability of causation fell systematically below the threshold; on the other hand, parties would face excess liability and do too much to reduce risk if their probability of causation lay systematically above the threshold. Second, the best all-or-nothing criterion for determining liability -- a criterion under which a party is fully liable if at all liable -- is different in form from a threshold probability criterion. Third, liability in proportion to the probability of causation is superior to all other criteria and results in socially ideal behavior in the model.

Consideration of factors going beyond the model, however, leads to qualifications of these conclusions, the most important of which concerns administrative costs. These costs are likely to be higher if liability is in proportion to the probability of causation than if it is determined by the threshold probability criterion. This is because the volume of suits and trials and their average cost and complexity should be greater under the proportional approach. Hence, on net, the attractiveness of the proportional approach will be limited to areas where the likelihood of uncertainty over causation, and thus of incentive problems associated with the threshold probability criterion, is large. Accordingly, the proportional approach may have appeal in respect to certain environmental and health-related risks, but not in respect to the typical tort.

Uncertainty Over Causation and the Determination
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S. Shavell^{*}

I. Introduction

What is the importance to the working of the liability system of the possibility of uncertainty over the cause of accidents?¹ What is the importance, for example, of the possibility that it will not be known whether the carcinogenic substance discharged from a chemical plant or normal exposure to medical x-radiation and other risks caused an individual's lung cancer, or that it will not be clear whether a surgeon's careless use of a medical instrument, a nurse's mishandling of it, or a defect in its manufacture was responsible for a patient's injury?²

The present article studies such questions using a theoretical model of the occurrence of accidents and of the effect of liability on behavior. Because the chief concern is with the desirability of the incentives created to reduce accident risks, the measure of social welfare is assumed to depend only on the value of engaging in risky activities, on accident losses, and on prevention costs.

The conclusions reached in the model derive in essence from the familiar notion that for parties to be led to reduce accident risks appropriately, they should generally face probability-discounted or "expected" liability equal to

the increase in expected losses that they create. This, of course, is naturally the case in the absence of uncertainty over causation, for parties then face liability if and only if they cause losses. The conclusions that will be obtained may be summarized by three statements.

(i) The use of a threshold probability of causation as a criterion for the determination of liability has potentially adverse effects on behavior: According to this criterion, a liability rule is applied only if the probability that a party caused an accident exceeds the threshold probability; the usual more-probable-than-not test³ thus involves a threshold probability of one half. Given any threshold probability, two types of problem may arise. On the one hand, a party's probability of causation might be systematically less than the threshold in ambiguous cases. If so, he would escape liability in such cases, that is, face a diminished burden of liability, and might be inappropriately led to engage in risky activity or might fail to take desirable steps to reduce risk.⁴ On the other hand, a party's probability of causation in ambiguous cases might systematically exceed the threshold, meaning that he would always face liability in such cases; hence, the party would bear an extra burden of liability, and the opposite difficulties would arise.⁵

(ii) The best all-or-nothing criterion for determination of liability is different in form from a threshold probability criterion: An all-or-nothing criterion is defined as any criterion for deciding whether the relevant

liability rule applies which preserves the usual feature of liability that a liable party must pay damages fully equal to the injured party's losses. A threshold probability is therefore an example of an all-or-nothing criterion. The best all-or-nothing criterion takes into implicit account not only the probability of causation but also the magnitude of losses and the effect of liability on incentives.⁶ Even the best all-or-nothing criterion, however, does suffer from the same two types of defect as the threshold probability criterion.

(iii) Liability in proportion to the probability of causation would be superior to the best all-or-nothing criterion and, thus, in particular, to any threshold probability criterion: Under the proportional approach, the relevant rule of liability is always applied, but the measure of damages is set equal to the harm done multiplied by the probability that the liable party caused the harm.⁷ Use of the proportional approach eliminates all problems due to uncertainty over causation in the model; it results in parties' facing expected liability equal to the expected losses they impose⁸ and thus it leads to socially desirable behavior.⁹

These points¹⁰ will be developed in two types of situation. In the first, uncertainty over causation will involve a party versus natural, "background" factors (the chemical plant vs. normal exposure to medical x-radiation). In the second type of situation, the uncertainty will be over which

party among several was the author of harm (the surgeon, the nurse, or the manufacturer). Here it will be important whether the parties act independently or in concert; for if they act in concert, they will be led to behave desirably so long as they face joint liability equal to the victims' losses.¹¹ In each type of situation, both strict liability and the negligence rule will be considered, and both the choice whether to engage in an activity and whether to exercise care will be examined.

The article will conclude with a discussion of factors not taken into account in the analysis (administrative costs, allocation of risk, notions of fairness) and with brief remarks on its positive and normative interpretation.

II. The Model

The assumptions are as follows. (i) All outcomes are defined in terms of a single good, "wealth". (ii) Social welfare equals the expected value of the sum of parties' wealth. (Equivalently, it will be seen to equal the value of engaging in activities less, where relevant, the costs of care, and less expected accident losses.) (iii) Parties are risk-neutral in wealth; they act so as to maximize its expected value. (iv) Accidents--events involving a loss of wealth--occur with a probability depending on whether parties engage in risky activities and, possibly, on whether they take care; such decisions of parties are discrete.¹² (v) Each accident is caused by precisely one entity, that is, there is one and only one entity for which the following

statement is true: "The accident would not have occurred in the absence of the entity." (vi) When an accident occurs, there will be a chance that the entity that caused it will not be known to the court; such instances will be said to be of ambiguous origin; but the conditional probability that the entity caused the accident will be determined by the court and will be called the probability of causation.

(vii) Two types of legal treatment of cases of ambiguous origin will be investigated, as noted in the Introduction. The first involves the use of an all-or-nothing criterion, a function (of variables to be specified) determining whether the applicable liability rule (strict liability or negligence) shall be employed. The all-or-nothing criterion to which most attention will be paid is the threshold probability criterion, under which the applicable liability rule shall be employed if the probability of causation exceeds the threshold probability. The second type of treatment of ambiguous cases is to adopt use of proportional liability: always to employ the applicable liability rule, but to set the damages to be paid in the event of liability equal to the accident loss multiplied by the probability of causation.

We now analyze several versions of the model amplifying on the assumptions as we proceed.

A. Uncertainty Involves One Party vs. A Natural Agent

It is assumed here that there are two entities that might cause accidents, a party and a natural agent. For

convenience, we consider initially the case where the only decision of the party is whether to engage in the activity; then we consider the more general case where he decides also whether to take care.

1. case where party decides only whether to engage in his activity. Define the following notation.

v = value to the party of engaging in his activity; $v \geq 0$;

p = probability of accidents caused by the party's engaging in his activity;

n = probability of accidents caused by the natural agent; $0 < n < 1$; $p + n \leq 1$;

ℓ = loss if accident occurs; $\ell > 0$.

As the events that an accident is caused by the party and by the natural agent are mutually exclusive (assumption (v)), it is socially desirable for the party to engage in his activity if¹³

$$(1) \quad v > p\ell.$$

If the party does not engage in his activity, then all accidents are assumed to be known to be due to the natural agent.¹⁴ But if the party does engage in his activity, cases of ambiguous origin will arise, and to describe this, define

α = conditional probability that an accident caused by the party appears to be of ambiguous origin; $0 < \alpha \leq 1$;

β = conditional probability that an accident caused by the natural agent appears to be of ambiguous origin; $0 < \beta \leq 1$.

Hence, the probability of an accident known to be caused by the party will be

$$(2) \quad p(1 - \alpha);$$

the probability of an accident caused by the party but seen as of ambiguous origin will be

$$(3) \quad p\alpha;$$

the probability of an accident known to be caused by the natural agent will be

$$(4) \quad n(1 - \beta);$$

and the probability of an accident caused by the natural agent but seen as of ambiguous origin will be

$$(5) \quad n\beta.$$

Accordingly, the conditional probability that an accident of ambiguous origin was caused by the party, that is, the probability of causation, will be

$$(6) \quad c = p\alpha / (p\alpha + n\beta).$$

Note that c could equal any value in $(0,1)$.¹⁵ Assume that the court can observe p , n , ℓ , α , and β , and that while it cannot observe v , it knows its probability distribution,

$f(\cdot)$ = probability density of v ; f is positive over $[0, \bar{v}]$ and zero elsewhere; $\bar{v} > \ell$.

Finally, assume that the applicable liability rule is strict liability, according to which the party would simply be liable for losses in the absence of uncertainty over causation.¹⁶

Now consider the threshold probability criterion, where

t = threshold probability; $0 < t < 1$.

Under this criterion, in cases of ambiguous origin the party will be liable and pay ℓ in damages when

$$(7) \quad c > t.$$

(If the party is known to have caused an accident, then of course he also pays ℓ in damages.¹⁷) Let us now prove

Proposition 1. Use of the threshold probability criterion may lead to a socially undesirable outcome: the party might undesirably fail to engage in his activity or might undesirably engage in it.

Proof: If $c > t$ would hold in ambiguous cases,¹⁸ the party will be liable in all such cases. Thus, his expected liability were he to engage in his activity would be

$$(8) \quad p(1-\alpha)\ell + p\alpha\ell + n\beta\ell = p\ell + n\beta\ell,$$

so that he will engage in his activity if

$$(9) \quad v > p\ell + n\beta\ell.$$

Comparing this to (1), we see that the party will not engage in his activity when it would be socially desirable that he did if $p\ell < v < p\ell + n\beta\ell$.

On the other hand, if $c \leq t$ would hold in ambiguous cases, then the party will never be liable in such cases, so that his expected liability would be only

$$(10) \quad p(1-\alpha)\ell = p\ell - p\alpha\ell.$$

Hence, he will engage in the activity if

$$(11) \quad v > p\ell - p\alpha\ell,$$

implying that he will engage in the activity when that would be undesirable if $p\ell - p\alpha\ell < v < p\ell$. Q.E.D.¹⁹

Note that the likelihood here (and below) of undesirable outcomes is greater the higher are n , α , and β .

Consider next the best all-or-nothing criterion for determining liability in ambiguous cases. Under this criterion, the court uses all the information at its disposal-- p , n , ℓ , α , β , and $f(\cdot)$ --and determines whether there should be liability so as to maximize expected social welfare. Specifically, if the court would not hold the party liable, then social welfare would be

$$(12) \int_{p\ell - p\alpha\ell}^{\bar{v}} (v - p\ell) f(v) dv;$$

and if it would hold the party liable, then social welfare would be

$$(13) \int_{p\ell + n\beta\ell}^{\bar{v}} (v - p\ell) f(v) dv.$$

Hence the court would hold the party liable when (13) exceeds (12), or, equivalently, when

$$(14) \int_{p\ell}^{p\ell + n\beta\ell} (v - p\ell) f(v) dv < \int_{p\ell - p\alpha\ell}^{p\ell} (p\ell - v) f(v) dv.$$

The interpretation of the left-hand term in (14) is the "opportunity loss" that would be due to socially undesirable discouragement from the activity were there liability in ambiguous cases; the interpretation of the right-hand term is the loss that would be due to socially undesirable engagement in the activity were there not liability in ambiguous

cases. Under the threshold probability criterion, it is not (14) but the size of $c = p\alpha/(p\alpha + n\beta)$ versus t that determines whether there is liability.²⁰ This suggests

Proposition 2. The best all-or-nothing criterion is not equivalent to (and thus is superior to) a threshold probability criterion.

Remark. The best all-or-nothing criterion may still lead to both types of socially undesirable outcome as are possible under a threshold probability criterion.

Proof: Assume that the best all-or-nothing criterion is equivalent to a threshold probability criterion for some t and consider for example a p , n , α , and β such that $c = p\alpha/(p\alpha + n\beta) > t$. Then the party would be liable under the threshold criterion, but he might not be liable under the best all-or-nothing criterion; for, clearly, (14) might not hold (suppose that most of the probability mass of v is concentrated in the interval $(p\ell, p\ell + n\beta\ell)$). Thus the assumption that the criteria are equivalent is contradicted.

Also, with regard to the Remark, it is obvious that if (14) holds, the party might be undesirably discouraged from engaging in his activity; and if (14) does not hold, the party might be undesirably encouraged to engage in it.

Q.E.D.

Last, consider proportional liability. Under this approach, the party would pay $c\ell$ in all cases of ambiguous origin. We have

Proposition 3. Use of proportional liability leads to a socially desirable outcome.

Proof: If the party engages in his activity, his expected liability will be

$$(15) \quad p(1 - \alpha)\ell + (p\alpha + n\beta)c\ell = p(1 - \alpha)\ell +$$

$$(p\alpha + n\beta)[p\alpha/(p\alpha + n\beta)]\ell = p\ell.$$

Hence, the party will engage in his activity when $v > p\ell$, which is (1). Q.E.D.²¹

2. case where party decides whether to engage in his activity and, if so, whether to take care. Define

q = probability of accidents caused by the party's activity if he takes care; $0 < q < p$; and

x = cost of taking care; $0 < x$.

(If the party engages in his activity and does not take care, p is the probability of accidents.) Hence, if the party engages in his activity, it will be socially desirable for him to take care if

$$(16) \quad q\ell + x < p\ell.$$

Further, if (16) holds, then it will be socially desirable for the party to engage in his activity if

$$(17) \quad v > q\ell + x;$$

but if (16) does not hold, (1) will as before determine the social desirability of his engaging in his activity. Assume that the same conditional probabilities α and β of accidents

appearing ambiguous apply whether or not care is taken.²² Therefore, if the party engages in his activity and takes care, the probability of an accident known to be caused by him will be

$$(18) q(1 - \alpha);$$

the probability of an accident caused by him but seen as of ambiguous origin will be

$$(19) q\alpha;$$

and his probability of causation in cases of ambiguous origin will be

$$(20) c = q\alpha/(q\alpha + n\beta),$$

which, note, is lower than c if he does not take care (as $q < p$). Let us now proceed with the analysis, first assuming liability to be strict, and then to be based on the negligence rule.

situation under strict liability. We have

Proposition 4. Use of the threshold probability criterion may lead to a socially undesirable outcome: (a) The party might undesirably fail to engage in his activity or might undesirably engage in it; (b) if the party engages in his activity, he might undesirably fail to take care or he might undesirably take care.

Remark. With regard to (b), it will be shown (i) that the party might undesirably fail to take care precisely when $c \leq t$ whether or not care is taken; (ii) that the party might undesirably take care precisely when $c \leq t$ only if care is taken; and (iii) that the party will take care if

and only if that is desirable precisely when $c > t$ regardless of whether care is taken.

Proof: (a) The argument for this part is omitted, as it is analogous to that given in Proposition 1.

(b) Assume that the party is induced to engage in the activity and consider in turn the three possibilities in the Remark.

(i) $c \leq t$ regardless of whether care is taken: In this case, expected liability is $p\ell - p\alpha\ell$ if care is not taken and $q\ell - q\alpha\ell$ if it is, so that care will be taken if

$$(20) \quad q\ell - q\alpha\ell + x < p\ell - p\alpha\ell$$

or, equivalently, if

$$(20') \quad q\ell + x < p\ell - (p - q)\alpha\ell.$$

Comparing this to (16) and noting that $(p - q)\alpha\ell > 0$, it is evident that the party might undesirably fail to take care (but would not undesirably take care).

(ii) $c \leq t$ only if care is taken: In this case, expected liability is $p\ell + n\beta\ell$ if care is not taken and it is $q\ell - q\alpha\ell$ if it is, so care will be taken when

$$(21) \quad q\ell - q\alpha\ell + x < p\ell + n\beta\ell$$

or, if

$$(21') \quad q\ell + x < p\ell + (q\alpha + n\beta)\ell.$$

Comparing this to (16) and noting that $(q\alpha + n\beta)\ell > 0$, it is evident that the party might undesirably take care (but would not undesirably fail to do so).

(iii) $c > t$ regardless of whether care is taken: In this case, expected liability is $p\ell + n\beta\ell$ if care is not taken

and it is $q_l + n\beta_l$ if care is taken. Hence care will be taken if

$$(22) \quad q_l + n\beta_l + x < p_l + n\beta_l,$$

or if

$$(22') \quad q_l + x < p_l,$$

which is (16), so that care will be taken if and only if it is socially desirable. Q.E.D.

Proposition 5. The best all-or-nothing criterion is not equivalent to (and thus is superior to) a threshold probability criterion.

(The argument is analogous to that of Proposition 2 and is therefore omitted.)

Proposition 6. Use of proportional liability leads to a socially desirable outcome.

Proof. By the steps in (15), it is clear that if the party engages in his activity and does not take care, his expected liability will be p_l ; and if he does take care, it will be q_l . Hence, if he engages in his activity, he will take care if $q_l + x < p_l$. But this is (16), so that his decision about care will be socially desirable. Further, if he would wish to take care, then he will choose to engage in his activity when $v > q_l + x$; and if he would not wish to take care, he will choose to engage in his activity when $v > p_l$. These conditions are (17) and (1), so that the party's decision whether to engage in the activity will also be socially desirable. Q.E.D.

situation under the negligence rule Assume that a party would be found negligent if and only if he undesirably failed to take care, that is, if and only if he failed to take care when (16) held; thus, assume that when (16) does not hold, the party will never be found negligent.²³ If the party is found negligent, then under the negligence rule, he will be liable for the loss he has caused.

Let us review the properties of the negligence rule in the absence of uncertainty over causation so that we can see what difference such uncertainty makes. Thus, assume in this paragraph that an accident would be seen to be caused by the party if and only if it truly was caused by him. Now suppose the party has decided to engage in the activity and that the exercise of care is desirable. Then if the party failed to take care, he would be liable for all accidents he caused, implying that his expected liability would be $p\ell$; but he would never be liable if he took care. Hence, he will take care if $x < p\ell$. But, using (16),

$$(23) \quad x < q\ell + x < p\ell,$$

so that the party will indeed take care. On the other hand, if the exercise of care is not desirable, the party would never be found liable, so that he would not take care. In other words, the party will be induced to take care if and only if that would be desirable. However, the party will be led to engage in the activity too often: If taking care is desirable, then since he would be induced to do so and would never be liable, the party would decide to engage in the activity whenever

$$(24) \quad v > x$$

rather than only when $v > q_l + x$. And if taking care is not desirable, since he would never be liable and would not take care, he would engage in the activity whenever

$$(25) \quad v > 0$$

rather than only when $v > p_l$.

With these facts in mind, let us proceed.

Proposition 7. Use of the threshold probability criterion may lead to a socially undesirable outcome: (a) The problem of an excessive incentive to engage in the activity may be exacerbated; and (b) if the party engages in his activity, he might undesirably fail to take care.

Remark. The problem in (b) can arise only where $c \leq t$ if care is not taken; and the problem in (a) can arise only where the problem in (b) would arise.

Proof. (a) Suppose that the exercise of care would be desirable if the party were to engage in his activity. Then if the party would be induced to take care were he to engage in his activity, he would decide to engage in it when $v > x$, which is just (24), so in this case the problem of excessive incentives to engage in the activity would not be worsened. However, if the party would not be induced to take care were he to engage in his activity, then (as will be explained in the proof to (b)) he would engage in it whenever $v > p_l - \alpha p_l$; but since $p_l - \alpha p_l < x$ in this case, the problem of an excessive incentive to engage in his activity would be worsened.

Now suppose that the exercise of care would be undesirable. Then the party would never be found negligent, so he would engage in his activity if $v > 0$, which is (25), meaning that the problem of an excessive incentive to engage in his activity would not be altered.

(b) Assume that the party is induced to engage in his activity, that taking care is desirable, and consider the following two possibilities.

(i) $c \leq t$ if care is not taken: The party's expected liability will be $p_l - \alpha p_l$ if he fails to take care and 0 if he takes care. Hence he will take care if

$$(26) \quad x < p_l - \alpha p_l.$$

Comparing this to (16), it is evident that he might undesirably fail to take care. (This can occur whenever $q_l < \alpha p_l$: In that event, $p_l - \alpha p_l < p_l - q_l$, so that $p_l - \alpha p_l < x < p_l - q_l$ is possible. But this means that (26) is not satisfied even though the exercise of care is desirable.)

(ii) $c > t$ if care is not taken: In this case, the party's expected liability will be $p_l + n p_l$ if he fails to take care and 0 if he takes care. Hence he will take care if

$$(27) \quad x < p_l + n p_l,$$

but this is clearly true, since $q_l + x < p_l$. Thus the party will take care. Q.E.D.

Proposition 8. The best all-or-nothing criterion is different from (and thus is superior to) a threshold probability criterion.

(The argument is omitted, as explained before.)

Proposition 9. Use of proportional liability results in the same outcome that would be observed in the absence of any uncertainty over causation.

In other words, the decision regarding the exercise of care will be socially desirable, but there will be exactly the problem with excessive incentives to engage in the activity as was described in the paragraph preceding Proposition 7. This is obvious, as the party's expected liability would be pl if he failed to take care and care was desirable.

B. Uncertainty Involves One Party vs. Another Party

It is now assumed that the two entities that might cause accidents are two parties, designated A and B.²⁴ The situation regarding the occurrence of accidents and whether they are seen as of ambiguous origin is assumed to be analogous to that in Part A. Two possibilities regarding the parties' relationship to each other will be considered: they may act independently, in which case the outcome is assumed to be a (Nash) equilibrium, a situation such that neither party would wish to alter its behavior assuming the other's to be fixed. On the other hand, they may act in concert, in which case the outcome is taken to be that which results in the highest sum of their expected wealth. As in the last Part, it is convenient to consider first the case where levels of activity alone are variable.

1. case where parties decide only whether to engage in their activities. Let ℓ be as before and let

v_A, v_B = value to A and B respectively of engaging in their activities; $v_A, v_B > 0$;

p_A, p_B = probability of accidents caused respectively by A's and B's activities; $p_A, p_B > 0$; $p_A + p_B \leq 1$.²⁵

Hence, it will be socially desirable for A to engage in his activity if

$$(28) \quad v_A > p_A \ell,$$

and for B to do so if

$$(29) \quad v_B > p_B \ell.$$

If only one of the parties engages in his activity, all accidents will be assumed to be known to be caused by him. But if both A and B engage in their activities, cases of ambiguous origin may arise; specifically, let

α = probability that an accident caused by A appears to be of ambiguous origin; $0 < \alpha \leq 1$;

β = probability that an accident caused by B appears to be of ambiguous origin; $0 < \beta \leq 1$.

Thus, if both A and B engage in their activities, the probability of an accident known to be caused by A will be

$$(30) \quad p_A(1-\alpha);$$

the probability of an accident caused by A but seen as of ambiguous origin will be

$$(31) \quad p_A \alpha;$$

and the analogous probabilities for B will be $p_B(1-\beta)$ and $p_B\beta$. Thus, A's probability of causation in cases of ambiguous origin would be

$$(32) \quad c_A = p_A^\alpha / (p_A^\alpha + p_B^\beta)$$

and B's,

$$(32) \quad c_B = 1 - c_A = p_B^\beta / (p_A^\alpha + p_B^\beta).$$

Assuming liability to be strict, consider the threshold probability criterion. Under this criterion, the only statement to add from before by way of definition is that if both parties are liable in an ambiguous case--that is, if $c_A > t$ and $c_B > t$ --then A will be supposed to bear a fraction λ of the loss and B, a fraction $1-\lambda$.

Let us now prove

Proposition 10. Use of the threshold probability criterion may lead to a socially undesirable outcome: (a) If the parties act independently, then a party might undesirably fail to engage in his activity or might undesirably engage in it; (b) if the parties act in concert and no party would be liable for ambiguously caused accidents, then parties might undesirably engage in their activities. However, if some party (or parties) would be liable for all such accidents, then the parties will act in the socially desirable way.

Proof: (a) Suppose that $v_B > p_A^\ell + p_B^\ell$. As B would then choose to engage in his activity even were he liable for all accidents, he will definitely choose to engage in his activity. Suppose as well that if A also were to engage

in his activity, then $c_A > t$ and $c_B \leq t$. Thus A's expected liability were he to engage in his activity would be $p_A^\ell + p_B^\beta \ell$; he would thus do so only if $v_A > p_A^\ell + p_B^\beta \ell$; and comparing this to (28), it is clear that he might undesirably fail to engage in his activity. Now suppose that if A were to engage in his activity, then $c_A \leq t$ and $c_B > t$. Then A's liability would be $p_A^\ell - p_A^\alpha \ell$; he would thus engage in his activity if $v_A > p_A^\ell - p_A^\alpha \ell$; and comparing this to (28), it is evident that he might undesirably engage in his activity.

(b) If the parties act in concert, they will consider four possible strategies--neither engages in his activity, A alone does so, B alone does so, or both do so--and they will choose that strategy with the highest sum of values net of expected liability costs. Now if neither party engages in his activity, the sum is 0. If A alone does so, it is $v_A - p_A^\ell$. If B alone engages in his activity, the sum is $v_B - p_B^\ell$. If both engage in their activities, the sum is $v_A + v_B - (p_A^\ell + p_B^\ell)$ when one or both would be liable in ambiguous cases, but the sum is only $v_A + v_B - (p_A^\ell + p_B^\ell - (p_A^\alpha + p_B^\beta)\ell)$ when neither would be liable in ambiguous cases. (Neither being liable is possible if $t \geq 1/2$; suppose, for instance, that $c_A = c_B = 1/2$ if both engage in their activities.) Note that this statement is true regardless of the fraction λ paid by A if both happen to be liable in ambiguous cases, for the sum of A's and B's liability in such cases will be ℓ independent of λ .

With this in mind, let us consider the parties' decision assuming first that neither would be liable in ambiguous cases. Then it is possible that both A and B would engage in their activities when it is desirable for only one (or neither) to engage in his activity. Suppose, for example, that $v_A = 2p_A\ell$ and $v_B = p_B\ell - p_B\beta\ell$.²⁶ On the other hand, it is not possible that A or B would fail to engage in his activity when that would be desirable. To show this, observe that if it is desirable for A to engage in his activity, he would certainly do so: for then if A alone engages in his activity, the sum $v_A - p_A\ell$ will be positive; and the sum if both A and B engage in their activities minus that if B alone does so will be $v_A - p_A\ell + (p_A\alpha + p_B\beta)\ell$, which is also positive; hence either A alone will engage in his activity or both A and B will do so. Similarly, if it is desirable for B to engage in his activity, he would do so.

Now consider the possibility that one or both parties would be liable in ambiguous cases. Then if it is socially desirable for A alone to engage in his activity, this will be the outcome: for if A alone engages in his activity, the sum of parties' values net of liability costs will be $v_A - p_A\ell > 0$; if B alone engages in his activity, the sum will be $v_B - p_B\ell < 0$; if both A and B do so, it will be $v_A + v_B - (p_A\ell + p_B\ell) = (v_A - p_A\ell) + (v_B - p_B\ell) < v_A - p_A\ell$; hence the sum will be highest if A alone engages in his activity. Similarly, if it is desirable for B alone to engage in his activity, this will occur. And if it is desirable for both

A and B to engage in their activities, this will be the outcome: for then $(v_A + v_B) - (p_A^\ell + p_B^\ell)$ will exceed both $v_A - p_A^\ell$ and $v_B - p_B^\ell$. Q.E.D.

Proposition 11. The best all-or-nothing criterion is different from (and thus is superior to) a threshold probability criterion.

(The argument is omitted, as explained before.)

Proposition 12. Use of proportional liability leads to a socially desirable outcome.

Proof: Using the steps in (15), we know that A's liability will be p_A^ℓ if he engages in his activity, regardless of whether B engages in his activity; and B's will similarly be p_B^ℓ if he engages in his activity, regardless of whether A does so. Hence, if the parties act independently, A will engage in his activity if and only if $v_A > p_A^\ell$ and B will do so if and only if $v_B > p_B^\ell$, so that their decisions will be socially desirable. And if the parties act in concert, this again will be true. That is, the sum of the parties' positions will be $v_A - p_A^\ell$ if A alone engages in his activity, $v_B - p_B^\ell$ if B alone does so, and $v_A + v_B - (p_A^\ell + p_B^\ell)$ if both do so. Hence, the argument given at the end of the proof to Proposition 10 applies and shows that the parties will always choose the socially desirable outcome. Q.E.D.

2. case where parties decide whether to engage in their activities and, if so, whether to take care. Define

q_A, q_B = probability of accidents caused respectively
by A's and by B's activity if care is taken;
 $0 < q_A < p_A; 0 < q_B < p_B;$

x_A, x_B = costs of care for A and B respectively;
 $0 < x_A; 0 < x_B.$

The description of whether it is socially desirable for parties to take care and to engage in their activities is analogous to that given at the beginning of this Part.

Assume, similar to before, that the conditional probabilities α and β of accidents' appearing ambiguous apply whenever both parties engage in their activities and whether or not care is taken. Hence, for example, A's probability of causation if both he and B take care is $c_A = q_A\alpha/(q_A\alpha + q_B\beta)$; his probability of causation if he takes care and B does not is $c_A = q_A\alpha/(q_A\alpha + p_B\beta)$; and so forth. As the following Propositions are straightforward to demonstrate given the proofs to previous Propositions and would be tedious to set forth, we merely state the Propositions.

situation under strict liability

Proposition 13. Use of the threshold probability criterion may lead to a socially undesirable outcome:

(a) Suppose that the parties act independently. Then a party might undesirably fail to engage in his activity or

undesirably engage in it; and if both parties engage in their activities, then a party might undesirably fail to take care or undesirably take care. (b) Suppose instead that the parties act in concert. Then if no party would be liable in ambiguous cases, parties might undesirably engage in their activities and if so, they might undesirably fail to take care. If, however, some party (or parties) would be liable in ambiguous cases, the outcome will be socially ideal.

Proposition 14. The best all-or-nothing criterion is different from (and thus is superior to) a threshold probability criterion.

Proposition 15. Use of proportional liability leads to the socially desirable outcome.

situation under the negligence rule

Proposition 16. Use of the threshold probability criterion may lead to a socially undesirable outcome:

(a) Suppose that the parties act independently. Then a party might undesirably fail to take care, and the problem of an excessive incentive to engage in the activity may be exacerbated. (b) Suppose that the parties act in concert. Then if no party would be liable in ambiguous cases, the problems in (a) may arise. If, however, some party (or parties) would be liable in ambiguous cases, the outcome will be the same one that would be observed in the absence of any uncertainty over causation.

Proposition 17. The best all-or-nothing criterion is different from (and thus is superior to) a threshold probability criterion.

Proposition 18. Use of proportional liability results in the same outcome that would be observed in the absence of uncertainty over causation.

III. Concluding Remarks

In closing, let us discuss several factors that were omitted from the analysis and then comment on its interpretation.

omitted factors. Consideration of the factor of the administrative costs associated with use of the legal system appears to favor a threshold probability criterion over proportional liability. This can be seen by examining (as we do formally in the Appendix) what may be regarded as the three determinants of administrative costs. The first of these is the number of suits brought. This should be higher under the proportional approach, for while injured parties would be unlikely to initiate legal action where they believe the probability of causation is below the probability threshold, they might well do so under the proportional approach.²⁷

The second determinant is the likelihood that a suit, once brought, would result in litigation rather than settlement. This too should be higher under the proportional approach, for under it, the actual magnitude of the probability of causation is an issue of potential dispute between the parties; under the threshold probability criterion, by contrast, the actual magnitude of the probability is not relevant except with

respect to the question whether the probability is above or below the threshold. The third element affecting administrative costs is the expense per trial. This also should be higher under the proportional approach because, again, of the introduction of the magnitude of the probability of causation as a possible issue of dispute, and also because a greater number of defendants would often be involved in disputes. (Injured parties may find it worthwhile to name as defendants parties whose probabilities of causation are less than the threshold.) In summary, then, consideration of the volume of suits, the probability of litigation given suit, and the average expense of litigation all suggest that administrative costs would be higher under the proportional approach than under a probability threshold criterion.

Another factor omitted from the analysis is the desire of risk averse parties to be protected against risk. The main observation to be made here is that under the proportional approach, there is less variability in the amount paid to injured parties than under a probability threshold criterion; the loss is shared between liable defendants and injured parties rather than being either entirely shifted or not shifted at all. Thus, the general allocation of risk under the proportional approach seems better than under a probability threshold, but the relevance of this point is of course limited by parties' ownership of first party and of liability insurance.

An additional omitted factor is the notion that it is not fair for a party to be sanctioned for a harm unless he did it, and by extension, unless there is reasonable certainty that he did it. This principle of fairness is in perfect accord with use of a threshold probability criterion in the determination of liability. On the other hand, the principle would be violated by use of proportional liability, as a party would suffer some sanction even when it was unlikely that he caused a harm. Yet in assessing the importance of this consideration favoring the threshold probability criterion, the analyst should take into account two limiting factors. First, the appeal of the principle seems strongest in the criminal context, where actual punishment is meted out; in the civil context, where the sanction is monetary and frequently paid by a liability insurer, the significance of adherence to the principle seems diminished. Second, where the defendant parties are not individuals but (large) firms, the importance of the principle also seem reduced.

Last, and closely related to the principle of fairness, is the goal of minimizing the costs of error, defined as dollars paid where defendants were not truly the cause of losses plus dollars not paid where they were the cause of losses. As recent writers have shown, this goal implies the superiority of the more-probable-than-not threshold criterion over proportional liability.²⁸ Now while normally one cannot object to study of a particular social goal on logical grounds, in the present case one can. Minimizaton of the

above-defined error costs is presumably taken as the social objective because it is an analytically workable proxy for the true social objective of minimization of the undesirable consequences flowing from errors -- such as the number of cases of lung cancer -- not because of a belief that the particular notion of error cost matters in itself. If so, it is a mistake to take error cost minimization as the social goal, and a mistake which explains such anomalous implications as the recommended use of the more-probable-than-not threshold even where it would result in defendants' always escaping liability for harm done.

interpretation. The foregoing discussion of omitted factors, and especially of administrative costs, qualifies the theoretical advantages of proportional liability shown in the analysis. Thus, it seems that on net the appeal of proportional liability will be limited primarily to where the behavioral problems due to use of a probability threshold would be significant, which is to say, where the chance of uncertainty over causation is significant. An area where this is often the case is that involving health-related and environmental risks, for here proof of causation is frequently difficult: It may be hard to determine whether one type of product, a different type of product, or some "natural" agent really caused the harm; whether one seller of a particular product or another seller of that same product caused the harm; and so forth. Thus, for instance, there appears to be rationality in at least the thrust of a recent decision

finding companies liable in proportion to their market share where it was not ascertainable which company had sold the generic drug that caused the particular plaintiff's cancer.²⁹

In the context of most torts, however, there will be no real advantage of proportional liability, as the likelihood of uncertainty over causation is undoubtedly low.³⁰ Hence, it is quite understandable that the general approach of the law has been to adopt an all-or-nothing approach based on a threshold probability criterion.³¹

APPENDIX

The Incentive To Bring Suit and To Settle or Litigate
Under The Threshold Probability Criterion and Under Liability
in Proportion to the Probability of Causation

We briefly examine here a model of litigation focusing on uncertainty over causation. It is assumed in the model that the plaintiff and the defendant may have differing beliefs about the probability of causation, but they both agree that in the absence of such uncertainty the defendant would be liable. The plaintiff brings suit if and only if he would be willing to go to trial; and if he brings suit, he and the defendant settle if and only if there exists a settlement amount which both prefer to going to trial.³²

incentive to bring suit. Let t be the threshold probability; c_p be the plaintiff's estimate of the probability of causation; ℓ be the dollar amount of his loss; and k be the cost to the plaintiff of bringing suit. Then (i) if $c_p > t$, the following is true. Under the threshold criterion, the plaintiff would receive a judgment (or settlement) of ℓ , so he will bring suit if $\ell > k$. Under the proportional approach, he would receive $c_p \ell$, so he will bring suit if $c_p \ell > k$. Hence the plaintiff will bring suit under the threshold criterion but not under the proportional approach if $\ell > k > c_p \ell$. (ii) On the other hand if $c_p \leq t$, then the situation is simply that the

plaintiff will never bring suit under the threshold criterion but will do so under the proportional approach if $c_p \ell > k$.

Thus, the claim made in the text that suit is more likely under the proportional approach amounts to a claim that it is more likely that $c_p \leq t$ and $c_p \ell > k$ than it is that $c_p > t$ and yet $\ell > k > c_p \ell$.

settlement vs. litigation. We now assume that the plaintiff has brought suit and, as noted above, that he and the defendant will go to trial if and only if there does not exist a settlement amount which each would find preferable to going to trial. Let c_d be the defendant's estimate of the probability of causation, and k_p and k_d be respectively the plaintiff's and the defendant's costs of going on to trial. Consider three possible relationships that may exist between c_p and c_d . (i) Suppose that both c_p and c_d exceed t . In this case, under the threshold probability criterion, there will be a settlement, for both parties will agree a judgment for ℓ will result from trial; thus any amount in $(\ell - k_p, \ell + k_d)$ will be a mutually satisfactory settlement. However, under the proportional approach, there will be a trial if the plaintiff's estimate of the probability of causation exceeds the defendant's by enough. Specifically if $c_p \ell - c_d \ell > k_p + k_d$, then the plaintiff's minimum demand of $c_p \ell - k_p$ will exceed the defendant's maximum offer of $c_d \ell + k_d$ and there will be a trial. (ii) Suppose not that c_p exceeds t but c_d does not. Then under the threshold probability

criterion, the plaintiff will expect to win ℓ but the defendant will expect to pay nothing. Thus there will be a trial when $\ell > k_p + k_d$ (for this means the plaintiff's minimum demand of $\ell - k_p$ exceeds the defendant's maximum offer of k_d). Under the proportional approach, there will be a trial when $c_p \ell - c_d \ell > k_p + k_d$ (for this means $c_p \ell - k_p > c_d \ell + k_d$). But since $\ell > c_p \ell - c_d \ell$, there would be a trial more often under the threshold criterion. (iii) Last, suppose that c_p does not exceed t . In this case, under the threshold probability criterion, there will never be a trial, for the plaintiff would not expect any judgment. But under the proportional approach there will again be a trial if $c_p \ell - c_d \ell > k_p + k_d$.

The implication of the preceding is this. In cases (i) and (iii)-- whenever both the plaintiff's and the defendant's estimates of the probability of causation exceed the threshold or whenever the plaintiff's falls below it--there will be a settlement under the threshold probability criterion but there might be a trial under the proportional approach. Only in case (ii)--only when it happens that the plaintiff's estimate of the probability lies above the threshold and the defendant's lies below the threshold -- is it true that there is a greater likelihood of a trial under the threshold probability criterion than under the proportional approach. On balance, then, the suspicion is that the chance of litigation conditional on suit having been brought is greater under the proportional approach than under the threshold probability criterion.

FOOTNOTES

*Professor of Law and Economics, Harvard Law School. I wish to thank Lucian Bebchuk, Charles Nesson, Mitchell Polinsky, David Rosenberg, David Shapiro, Richard Stewart, and Arthur Von Mehren for comments and the National Science Foundation (Grant no. SES-8014208) for financial support. This article will provide the basis for a section of a book tentatively entitled A Theoretical Analysis of Tort Law, on which I am at work. A previous version of this article was released as Harvard Institute of Economic Research Discussion Paper 997, August, 1983.

1. "Accident" will refer to any instance in which harm is done and "liability" will refer mainly to tort liability, for it is in this area that problems of uncertainty over causation most often arise.
2. See generally the cases cited in William L. Prosser, Handbook of the Law of Torts (4th ed. 1971), at 241-244.
3. Id. at 241.
4. Suppose that the number of cases of lung cancer caused by the chemical plant's pollution would be 50, that the number caused by normal exposure to x-radiation would be 100, and that there is no way of determining the

cause of a particular individual's lung cancer. Then if a probability threshold of $\frac{1}{2}$ is employed, the plant would never be found liable, as its probability of causation in each case would be only $50/150 = 1/3$. Thus the plant would have no incentive to reduce the amount of pollution or to curtail the level of its activity.

5. Suppose that the chemical plant would cause 100 cases of lung cancer and x-radiation would cause 50. Then the plant's probability of causation in each case would be $100/150 = 2/3$, so that it would face liability for all cases of lung cancer given a threshold probability of $\frac{1}{2}$. Hence, the plant might be undesirably discouraged from engaging in its activity or be led to take excessive care.
6. It would take into account, for instance, how much additional risk would result from allowing chemical plants to escape liability as a consequence of uncertainty over causation.
7. An important example of liability in proportion to the probability of causation is liability in proportion to market share; see note 29 infra.

8. In the example where the chemical plant would cause 50 cases of lung cancer and x-radiation 100 cases, the plant would face liability for $1/3$ of a victim's losses in each of the 150 cases; this, desirably, is the same as the plant's facing liability for all of a victim's losses in only the 50 cases that the plant would truly cause. Similarly, in the example where the plant would cause 100 cases and x-radiation 50, the plant would face liability for $2/3$ of a victim's losses in all the cases; this is the same as its facing liability for all of a victim's losses in the 100 cases the plant would truly cause.
9. However, it will be suggested in the concluding remarks that the appeal of the proportional approach may be significantly limited by (among other reasons) its being associated with higher administrative costs than is the threshold probability criterion. See also the Appendix.
10. Point (i) concerning the possibility of a diminished or of an extra burden of liability has often been noted. See for example Laurence H. Tribe, Trial by Mathematics: Precision and Ritual in the Legal Process, 84 Harv. L. Rev. 1329 (1971) at 1350; Richard A. Posner, Economic Analysis of Law (2nd ed. 1977) at 430-433; David Kaye, the Limits of the Preponderance of the Evidence Standard:

Justifiably Naked Statistical Evidence and Multiple Causation, American Bar Foundation Research J. 487 (1982); William M. Landes and Richard A. Posner, Causation in Tort Law: An Economic Approach, 12 Journal of Legal Studies, 109 (1983) at 123-124; Steven Shavell, An Analysis of Causation and the Scope of Liability in the Law of Torts, 9 Journal of Legal Studies 463 (1980) at 494; David Rosenberg, The Causal Connection in Mass Exposure Cases, 97 Harv. L. Rev. 851 (1984); Charles Nesson, Foundations of Judicial Proof, forthcoming 98 Harv. L. Rev. (1985). Similarly, point (iii) on the desirability of the proportional approach has been noted. See, for example, Landes and Posner, Nesson, and, especially, Rosenberg; and see also the references mentioned in note 29 infra. Thus, the contribution of this article does not lie in any real novelty in respect to points (i) and (iii). (Point (ii), however, has not been previously stated.) Rather, it lies in the systematic, formal development of the points.

11. This follows because if they act in concert, they will behave in response to their joint liability. Their joint liability will equal victims' losses if some one of them would necessarily be liable or if a group of them would share liability. But their joint liability might be less than victims' losses; if, for instance, each could escape liability because his probability of causation was less than a threshold, then their joint liability would be zero.

12. The qualitative nature of our results would not be altered were we to study a model with the levels of activities and/or the levels of care continuously variable; see notes 19 and 21, infra.
13. Of course, if (1) holds with equality, it will not matter whether the party engages in his activity; for ease of exposition we will not comment hereafter on such possibilities of indifference.
14. This assumption will be maintained in the other versions of the model.
15. This is obvious. Suppose, for instance, that $p=n$. Then c reduces to $\alpha/(\alpha + \beta)$, which can clearly range over $(0,1)$.
16. The negligence rule cannot be studied in the present case because there is no variable interpretable as care.
17. This can be regarded as an implication of the criterion, for then $c = 1 > t$.
18. This is possible since we noted that c could range over $(0,1)$. (Hereafter, we will not bother to observe that various inequalities are possible, as this will be obvious.)

19. If the level of activity were continuously variable, Proposition 1 would still hold. We sketch the situation in this event. Let z be the level of activity, $v(z)$ be its value (where $v'(z) > 0, v''(z) < 0$), and $zp\ell$ be the associated level of expected accident losses. Then social welfare is given by $v(z) - zp\ell$ and the socially desirable level of activity, say z^* , is determined by $v'(z) = p\ell$. Now the probability of causation c is given by $c = c(z) = zp\alpha / (zp\alpha + n\beta)$, which is increasing in z . Let z_t be the z such that $c(z_t) = t$; thus $c(z) < t$ for $z < z_t$ and $c(z) > t$ for $z > z_t$. (If $c > t$ for all $z > 0$, define $z_t = 0$; if $c < t$ for all $z > 0$, define $z_t = \infty$). Hence, the party's choice of z will be determined by $\max_{z \leq z_t} (\max_{z \leq z_t} zp\ell - zp\alpha\ell, \max_{z > z_t} zp\ell + n\beta)$.
- From these facts the following may easily be established: If $z^* < z_t$, then the chosen z will be in $(z^*, z_t]$; in particular, z will be higher than is desirable. If $z^* > z_t$, then there are two possibilities. One is that the chosen z exceeds z_t , in which case $z = z^*$; but in the other case, z will equal z_t and will thus be below the socially desirable level. Hence Proposition 1 is indeed true.
20. Were the threshold t allowed to vary with $p, n, \ell, \alpha, \beta$, and $f(\cdot)$, then, trivially, the court could alter the threshold so as to achieve exactly the result under the best all-or-nothing criterion. (The court would merely

choose any $t < c$ when (14) holds, and it would choose $t \geq c$ when (14) does not hold.) But it does not seem natural to interpret such a variable probability threshold -- one which is in effect the more complicated criterion of (14) -- as a threshold probability criterion.

21. In the continuous case, proportional liability also results in the socially desirable outcome (as it does in the continuous case of subsequent versions of the model). Referring to the description of the continuous case in note 19, supra, we see from the steps used in (15) that the party's expected liability will equal zpt , so that he will maximize $v(z) - zpt$ and thus select z^* , the socially desirable z .
22. This seems the most natural assumption, but others are plausible. (For instance, taking care might alter the nature of accidents in such a way as to make them less easily confused with those caused by the natural agent; thus α and β might fall if care were taken.) It will be clear that our analysis could easily be modified to take into account such possibilities and that this could change some of our results (in the main, it could alter the nature of the departure from the socially desirable outcome under the threshold probability criterion), but we shall not discuss this matter.

23. Assume that there are no errors in observing x or in determining whether (16) holds.
24. It will be obvious how to extend the arguments to the case with three or more parties and/or with a natural agent, but it would be cumbersome to do so.
25. Analogous to the situation in Part A, the event that A causes an accident and the event that B causes an accident will be assumed mutually exclusive, etc.
26. In this case, it is desirable for A alone to engage in his activity, yet if both engage in their activities, the sum of their positions will be $(2p_A^{\ell} + p_B^{\ell} - p_B^{\beta\ell}) - (p_A^{\ell} + p_B^{\ell} - (p_A^{\alpha} + p_B^{\beta})\ell) = p_A^{\ell} + p_A^{\alpha\ell}$, which is positive, greater than what would be received if A alone were to engage in his activity (namely p_A^{ℓ}), and greater than what would be received if B alone were to engage in his activity (namely, $-p_B^{\beta\ell}$). Thus both A and B will engage in their activities.
27. There is, however, a competing consideration. If an injured party believes the probability of causation to be above the threshold, although he would often find it worthwhile to initiate legal action under the threshold criterion, he might not under the proportional approach because his damages would be less than complete. This

consideration seems less important than the one mentioned in the text, for it is precisely when the probability of causation exceeds the threshold that the proportional approach does not reduce damages much below the full amount. For details, see the Appendix.

28. Kaye, note 3a, supra, showed formally the superiority of the more-probable-than-not rule over all possible alternatives, and thus in particular over proportional liability. The idea behind the argument is that if it is known that a defendant is more-likely-than-not the cause of harm, then on average we will do best to make him pay for it. To make him pay only in proportion to his probability of causation would be to fail to insist that he pay some dollars which we know on average it is best to have him pay. And conversely, if we know that a defendant is more-likely-than-not innocent of having done harm, then on average we will do best to let him go. Suppose that there are 100 cases, in each of which the defendant is believed to have been the cause with probability 60%; that in 60 of these cases, defendants truly caused harm; that in 40 of these, they did not; and that the amount of harm in each instance is \$1,000. Then under the more-probable-than-not criterion, each defendant would pay \$1,000; thus in 60 cases, no errors will have been made; in 40 cases, \$1,000 will have been erroneously paid; hence the total error in dollars will

be \$40,000. By contrast, under the proportional approach, each defendant would pay \$600; thus in 60 cases \$400 will erroneously fail to be paid; in 40 cases, \$600 will erroneously be paid; hence the total error in dollars will be \$48,000--a higher dollar error than under the more-probable-than-not criterion.

In point of fact, Kaye's result can be strengthened. It turns out that whatever are the weights attaching to the two types of error--the dollars erroneously paid, and the dollars erroneously not paid--some threshold probability criterion (generally different from 50%) will be superior to proportional liability. (In Kaye's case, the weights attaching to the two types of error were equal.) To demonstrate this, let w_1 be the weight multiplying errors of the first type; let w_2 be the weight multiplying errors of the second; let p be the party's probability of causation; and let ℓ be the loss suffered. Then if the party pays ℓ in damages, the expected error cost is $(1-p)w_1\ell$, and if he does not pay anything in damages, the expected error cost is $pw_2\ell$. Hence, the expected error minimizing all-or-nothing rule is to make a party pay ℓ whenever $(1-p)w_1\ell < pw_2\ell$, or equivalently, whenever $p > w_1/(w_1 + w_2)$, which is a probability threshold criterion. (Note that this formula implies that when the weights are equal, the threshold is 50%, that when w_1 , the weight of the first

type of error (innocents' paying damages) is larger than w_2 , the threshold exceeds 50%, etc.) Under proportional liability, expected error costs are higher. Since under this approach the party pays $p\ell$ whatever is p , expected error costs are $(1-p)w_1p\ell + pw_2(\ell - p\ell)$. These expected error costs exceed the error costs under the optimal threshold criterion. To see this, observe that $(1-p)w_1p\ell + pw_2(\ell - p\ell) = p[(1-p)w_1\ell] + (1-p)[pw_2\ell] \geq \min[(1-p)w_1\ell, pw_2\ell] =$ error costs with a probability threshold of $w_1/(w_1 + w_2)$, and note that the inequality is strict so long as p is positive, unequal to the threshold, and less than 1.

29. We refer here to litigation over cancer caused by the drug DES. Millions of women used this drug during their pregnancies. This has created the risk of an often fatal cervical cancer in the women's prenatally exposed daughters, and the women have typically found it difficult or impossible to identify the producer (out of several hundred firms) of the DES that they purchased. In an influential decision in the DES litigation, Sindell v. Abbott Laboratories 163 Cal. Rptr. 132, 607 P. 2d 924 (1980), the California Supreme Court held that each defendant producer of the drug should be liable according to its share of the market, and thus ostensibly according to the probability that it sold the drug that caused the plaintiff's injury.

We note, though, that care must be taken in applying a market share formula for it to give the probability of causation. For one thing, all firms' shares of the market must be taken into account, including firms defunct at time of trial and foreign firms. Additionally, the possibility that different firms' products present different risks must be considered. If firm X and firm Y divide the market but firm Y's product is twice as risky as firm X's, then the likelihood that a loss of ambiguous origin was caused by firm Y is clearly more than its 50% share of the market; the likelihood is in fact 66.66% that Y caused the accident. More generally, suppose that n firms $i=1, \dots, n$ produce a total output of N units; that the maker of any particular unit cannot be identified; that s_i is the share of the market of firm i ; and that p_i is the probability of "failure" of a unit of firm i 's. Then if a unit fails, the likelihood that, say, firm j was

its maker equals $p_j s_j N / (\sum_{i=1}^n p_i s_i N) = p_j s_j / (\sum_{i=1}^n p_i s_i)$.

This may also be expressed as $(p_j/p_1) s_j / [s_1 + \sum_{i=2}^n (p_i/p_1) s_i]$,

that is, one may use "weighted" market shares, where the weights correspond to relative product risks.

On market share liability, see for example Comment, DES and a Proposed Theory of Enterprise Liability, 46 Fordham L. Rev. 963 (1978); Note, Market Share Liability:

An Answer to the DES Causation Problem, 94 Har. L. Rev. 668 (1981); Glen O. Robinson, Multiple Causation in Tort Law: Reflections on the DES Cases, 68 Va. L. Rev. 713 (1982); Richard Delgado, Beyond Sindell: Relaxation of Cause-In-Fact Rules for Indeterminate Plaintiffs, 70 Calif. L. Rev. 881 (1982); and David Rosenberg, note 10, supra.

30. If my neighbor's house burns down, the probability that there will be substantial uncertainty whether the cause was the fire I set to barbecue meat or one started by lightning will be slight. If an individual is struck from overhead by a piece of lumber when walking by a construction site, the likelihood of real uncertainty as to its source will be small.
31. See Prosser, note 2, supra at 241; and see for a description of the French and German Situation, A.M. Honoré, Causation and Remoteness of Damage, International Encyclopedia of Comparative Law, 191, at Sections 201-203.
32. Thus the model is a version of those in John Gould, The Economics of Legal Conflicts, 2 Journal of Legal Studies 279 (1973); Richard Posner, Economic Analysis of Law (2nd ed.), 1977, Chapter 21; and Steven Shavell, Suit, Settlement, and Trial: A Theoretical Analysis Under Alternative Methods for the Allocation of Legal Costs, 11 Journal of Legal Studies, 55 (1982).