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MECHANISM DESIGN UNDER COMMON AGENCY

Lars A. Stole*

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Program in Law and Economics
Harvard Law School
Cambridge, MA 02138

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*Former John M. Olin Fellow in Law and Economics at Harvard Law School, Assistant Professor, Graduate School of Business, University of Chicago.

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Abstract

This paper considers the contracting problem facing multiple principals, each of whom desires to contract with the same agent. If the agent has private information regarding his gains from the contracting activity and the contracting activities in the principal-agent relationships are substitutable (complementary), the principals will typically extract less (more) information rents in total and induce less (more) productive inefficiency in the contracting equilibrium than if there were a single principal contracting over the same activities. This analysis is subsequently applied to various environments, including joint ventures, exclusive-dealing relationships, and regulation between conflicting agencies.

'The question is,' said Humpty Dumpty, 'which is to be master that's all.'

— *Lewis Carroll*

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ABSTRACT. This paper considers the contracting problem facing multiple principals, each of whom desires to contract with the same agent. If the agent has private information regarding his gains from the contracting activity and the contracting activities in the principal-agent relationships are substitutable (complementary), the principals will typically extract less (more) information rents in total and induce less (more) productive inefficiency in the contracting equilibrium than if there were a single principal contracting over the same activities. This analysis is subsequently applied to various environments, including joint ventures, exclusive-dealing relationships, and regulation between conflicting agencies.

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1. INTRODUCTION

Mechanism design has proven to be a fertile area of research for the economist studying the role of information in economic exchange. Since the methodology was first developed by Mirrlees [1971], it has been applied to numerous contexts. Theorists have subsequently extended the use of mechanism design to problems with multi-dimensional type spaces[1], multiple agents[2], and informed principals.[3] But to date, we know very little about the problem of mechanism design with multiple principals and a single agent – what has been termed the problem of *common agency.*[4]

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[1] See Rochet[1985], Laffont, Maskin, and Rochet [1987], and McAfee and McMillan [1988].

[2] See Myerson[1981], Demski, Sappington [1984], Demski, Sappington, and Spiller [1988], and Ma, Moore, and Turnbull [1988].

[3] See Myerson[1983], and Maskin and Tirole [1990a,1990b].

[4] David Martimort [1991] has independently studied many of the issues in this paper and obtained similar conclusions.

Common agency contracting under adverse selection is ubiquitous. Wherever hidden information and some degree of competition among principals exists for a set of agents, we will generally find an environment where mechanism design under common agency is appropriate. Often the assumption that a single principal completely controls the contracting environment with an agent is not realistic as the following examples illustrate:

- Multiple regulators. Several agencies may have authority to promulgate regulations affecting a single agent. To the extent that each regulator (principal) wishes to extract the agent's information rents, an analysis of mechanism design under common agency is appropriate.[5]
- Common Marketing Agency. Manufacturers frequently choose to use the same marketing agency for their wares. Such agencies typically have private information about marketing and distribution costs, as well as their effort levels.[6]
- Price discrimination. Duopolists selling differentiated products to the same consumers may find it optimal to employ second-degree price discrimination, but must take into account the effect of their rival's nonlinear screening contract.[7]
- Exclusive Supply Contracts and Joint Ventures. Firms may decide to form joint ventures with one another to create an exclusive input supplier for members of the venture. In one sense, a joint venture allows firms to coordinate their separate contracts into a single cooperative contract with an agent. In the absence of a joint venture (or alternatively an exclusive supply contract) the firms may non-cooperatively contract with the same agent and fail to take into account the externalities which they impose on one another. An analysis of common agency illuminates some of the benefits of joint ventures and exclusive supply contracts.[8]

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[5] Related research by Baron [1985] considers a Stackelberg game of regulating a public utility with emission abatement regulation by the EPA (the leader) and rate regulation by a local public utility commission (the follower). This paper extends Baron's approach to a large class of simultaneous contracting games.

[6] This situation was originally considered by Bernheim and Whinston [1985] in an environment of moral hazard. A more general treatment of common agency under conditions of moral hazard is found in Bernheim and Whinston[1986]. Recent work by Villas-Boas [1990] examines the information costs of firms using the same advertising agency, where an agent may tell the "secrets" of one principal to the other. Neither, however, considers adverse selection with common agency. Gal-Or [1989] has also examined a special case of common agency between two principals using the same marketing agent where the utility the agent derives from the relationship with one of the principals is independent of the contract with the other principal. This case is briefly considered in Section 2.3.

[7] Competition with nonlinear tariffs was considered by Oren, et al. [1983], but in a more limited framework where players are restricted to taking the choices of the agent from the rival principal's contract as given.

[8] Related models which have examined organizational and market structures from a common agency perspective with moral hazard are Braverman and Stiglitz [1982], which considers sharecroppers responsible to both landlords and creditors, and Stiglitz [1985], which considers corporate managers as agents to both stockholders and corporate creditors.

- Franchise Contracts. Franchisors frequently contract with nonexclusive franchisees, such as automobile dealerships, which have contracts with multiple franchisors. The nature of the equilibrium contracts in the nonexclusive environment sheds light on the benefits of exclusive control.

- State and Federal Taxation. Following Mirrlees [1971], an obvious extension of the optimal theory of taxation would consider the effects of two principals (State and Federal revenue departments), each attempting to minimize the distortion introduced by its taxation while maximizing its own objective.

Following the work of Bernheim and Whinston [1986] on common agency under moral hazard, we note that environments with common agency can either be delegated or intrinsic. Under *delegated* common agency, the choice of contractual relationship is delegated to the agent who can choose whether to contract with both, one, or none of the principals. This is a natural setting for examining such phenomena as second degree price discrimination by duopolists, where the consumer ultimately decides from whom to purchase. Alternatively, when common agency is *intrinsic*, the agent's choice is more limited: the agent can choose only between contracting with both principals or contracting with neither. A common example of such a setting is industrial regulation by multiple regulators. The regulated firm's only choice beside regulation is to leave the market and forego profits altogether.

The distinction between these two environments is less important when the contracting activities of the two principals are complementary in terms of the common agent's utility: In any equilibrium where the agent finds it attractive to contract exclusively with either principal, the agent will find it desirable to contract with both. Although this is not the case when the activities are substitutes, we choose to focus on intrinsic common agency as a first step toward a more general theory on common agency under adverse selection. Nonetheless, as the applications in this paper demonstrate, a large set of interesting economic questions are addressable within this class of models.

The main focus of this paper is twofold. First, we develop techniques for studying common agency contracts with mechanism design. Second, using these new tools, we consider some of the economic ramifications of a common agency setting. Section 2 of this paper introduces a general model of contracting under common agency, and proceeds by characterizing the contracts for two benchmarks: the cooperative (or single principal) solution and the case of contractual independence (where the agent's marginal utility derived from the contract with one principal is unaffected by the contract with the other).

Two fundamental problems are encountered when one attempts to apply traditional mechanism design tools to common agency problems in absence of contractual independence. First, the simple characterization of incentive compatibility and participation constraints used in single principal contracts is no longer available. Instead, we find a more complicated analog in our two-principal setting when we consider common agency implementability in Section 3. With two principals, each of whom observes only the report meant for her, we require more than that the agent finds it incentive compatible to report truthfully to principal i given he reports truthfully to principal j: It must also be the case that lying to both principals (with perhaps differing reports) is not beneficial to the agent. A significant contribution of this research is to explicitly characterize the set of commonly implementable contracts. Second, when searching for a Nash equilibrium in contracts among principals, one cannot invoke the revelation principle without exercising care. Each principal will typically find it rational to attempt to induce the agent to report falsely to a rival and thereby extract a larger share of the agent's information rents. Of course in equilibrium, all contracts are incentive compatible so that such attempts are useless, but their possibility imposes constraints on the set of equilibrium contracts. This problem is also taken up in Section 3.

Section 4 analyzes the set of pure-strategy differentiable Nash equilibria in the contract game for the cases of contract complements. Section 5 analogously considers equilibria with contract substitutes. We find that the presence of common agency results in each principal creating a contractual externality. When the contracting activities are complementary, equilibria in the simultaneous contracting game have each principal introducing too much distortion in an effort to extract rents from the agent. With substitutes, the reverse typically occurs and too little distortion is introduced from each principal's point of view. The results are in accord with our notions of Nash equilibria in prices between competing duopolists in a differentiated product market. When the goods for sale are complements, each duopolist prices excessively relative to the monopoly solution; when the goods are substitutes, each duopolist sets prices closer to marginal cost, introducing a smaller distortion. In Section 6 several applications of common agency contracting in environments of adverse selection are presented as a motivation to the preceding analysis. Section 7 concludes.

2. The Model

2.1 The Contracting Framework

For simplicity we consider a contracting environment with two principals, $i = 1, 2$, and one agent. Although our model is quite general, for exposition we take each principal i as a potential purchaser of some good, x_i, which the agent produces. The agent has private information, or *type*, θ in some compact set Θ, which we take to be the interval $\Theta \equiv [\underline{\theta}, \overline{\theta}]$. Furthermore, it is common knowledge among the principals that θ is distributed according to the differentiable density function $f(\theta)$, where $f(\theta) > 0$, $\forall\theta \in \Theta$, with corresponding cumulative distribution function $F(\theta)$, and with $\frac{1-F}{f}$ nonincreasing in θ. Without loss of generality, we consider direct revelation mechanisms in which the agent announces his type to each principal separately, although as indicated care must be taken in this regard when considering deviations by each principal from the equilibrium.

We assume that each principal observes only the report meant for her, and denote the reports for each principal as $\hat{\theta}_1$ and $\hat{\theta}_2$, respectively. Various motivations exist to justify this approach. First, antitrust laws might deal harshly with collusive activities to coordinate contracts and reports from the agent, particularly given our results in Section 5 regarding the potential anticompetitive effects of such coordination. Second, even if principals could jointly observe the agent's report, the possibility of secret side contracts between each principal and the agent before the agent's type is announced may render such joint observations useless.[9] Finally, at least in the regulatory context, it may be legally impossible for one agency to contract on the decision variable of another, even though it may be publicly observed (e.g., the local public utility commission cannot make allowed rates of return an arbitrary function of pollution abatement and the EPA cannot choose levels of allowable pollution as a function of local rate making).

Each principal chooses an *allocation* or contract, $y_i(\cdot)$, which consists of a *decision*, $x_i(\cdot)$, that belongs to a compact, convex, nonempty subset $\mathcal{X} \subset \Re_+$, and a monetary transfer, $t_i(\cdot)$, paid by the principal to the agent: $y_i(\hat{\theta}_i) = \{x_i(\hat{\theta}_i), t_i(\hat{\theta}_i)\}$. We suppose the decision choice of each principal's contract is one-dimensional to simplify the analysis although, as in Guesnerie and Laffont [1984], it is possible to generalize the results to choices over vectors of decisions.

The principals have von Neumann-Morgenstern utility functions that are given by $V^i(x_1, x_2, t_i)$, $i = 1, 2$, which are thrice continuously differentiable, decreasing in t_i, and have partial derivatives up to the third order which are uniformly bounded on any given compact subset of $\mathcal{X}^2 \times \Re_+$. Initially, we let V^i depend upon x_j as in the case where each principal i buys inputs x_i from the agent and sells them in the same downstream product market.

We have chosen to model each principal's utility as a function only of the two contract variables and the transfer to the agent. The agent's type does not affect the principal's welfare. It is straightforward to make each principal's utility a function of θ as well as x_1 and x_2, although the assumptions used in this paper must be modified to ensure concavity in the principal's problem and monotonicity in the resulting menus of allocations. Such an extension would be appropriate, for example, in the multiple regulators context. In such circumstances, each regulator may place some weight on the agent's welfare (e.g., a public utility's profits may have a positive weight of less than one attached to it), which renders principal i's payoff a function of x_1, x_2, θ, and t_j as well. Nonetheless, we make the simplifying assumption for ease in exposition. Because each principal's utility depends upon both x_1 and x_2, the contract between the agent and one of the principals will directly affect the well being of the other principal. More interestingly, to the extent that $U_{x_1 x_2} \neq 0$, one principal's contract will affect the agent's marginal utility, and therefore indirectly affect the cost of contracting with the other principal. Later in this paper we will make a further simplification that each V^i is independent of x_j in order to focus on this second affect.

[9] If, however, the side contracts are negotiated under asymmetric information, a role may nonetheless exist for common contracts. See the work of Caillaud, Jullien, and Picard [1990], which shows in a multi-principal and multi-agent framework that if secret contracts are feasible, initial contracts may be useful when asymmetric information exists during side contract negotiation.

We assume the agent has a von Neumann-Morgenstern utility function given by

$$U(x_1, x_2, t_1 + t_2, \theta),$$

which is also thrice continuously differentiable, strictly increasing in aggregate transfers, $t_1 + t_2$, and has uniformly bounded partial derivatives up to the third order on any given compact subset of $\mathcal{X}^2 \times \Re$. We also suppose there are no fixed costs of production by the agent: $U(0,0,0,\theta) = 0$.

We normalize the agent's outside opportunities to zero and assume that the principals have all of the bargaining power and simultaneously offer take-it-or-leave-it contracts. Because we analyze intrinsic agency, we suppose that the agent is forced either to accept both contracts or to refuse to contract with both principals.

Given a contract pair, $\{y(\hat{\theta})\} = \{y_1(\hat{\theta}_1), y_2(\hat{\theta}_2)\}_{\hat{\theta}_i \in \Theta, i=1,2}$, we can represent an agent's indirect utility as a function of reports and type by

$$U(\hat{\theta}_1, \hat{\theta}_2, \theta) \equiv U(x_1(\hat{\theta}_1), x_2(\hat{\theta}_2), t_1(\hat{\theta}_1) + t_2(\hat{\theta}_2), \theta),$$

which we will frequently use when no confusion should result. Additionally, subscripts denote partial derivatives with respect to direct arguments and primes denote derivatives with respect to a single argument at all points where such derivatives exist.

2.2 The Cooperative Benchmark

As a comparison, we initially consider the situation where both principals choose contracts that depend upon a single report by the agent and that maximize their joint utilities.[10] [The reader familiar with the theory of mechanism design may wish to skip to Section 2.3.] Alternatively, we can think of the situation as one of a single principal that contracts over both activities of the agent. As a consequence, we can restrict ourselves to a simple mechanisms $y(\hat{\theta}) = \{t(\hat{\theta}), x_1(\hat{\theta}), x_2(\hat{\theta})\}$, where $\hat{\theta}$ is the single report by the agent. Given an allocation, we may denote the agent's utility as a function of type and report by $U(\hat{\theta}, \theta) \equiv U(x_1(\hat{\theta}), x_2(\hat{\theta}), t(\hat{\theta}), \theta)$.

Definition 1 *A decision function, $x : \Theta \mapsto \mathcal{X}^2$, is implementable if there exists a transfer function $t(\cdot)$ such that the contract satisfies the incentive compatibility (IC) constraint:*

$$U(x_1(\theta), x_2(\theta), t(\theta), \theta) \geq U(x_1(\hat{\theta}), x_2(\hat{\theta}), t(\hat{\theta}), \theta), \ \forall(\theta, \hat{\theta}) \in \Theta^2.$$

A contract is feasible if the decision function is implementable, and the transfers additionally satisfy the participation (or individual rationality) constraint:

$$U(x_1(\theta), x_2(\theta), t(\theta), \theta) \geq 0, \ \forall \theta \in \Theta.$$

[10] In the general case where U is not linear in transfers, we may look for a Pareto optimum such that $\lambda V^1 + (1-\lambda)V^2$ is maximized for some weight, λ. When U is quasi-linear we may consider the simple sum of the principals' payoffs. Here we focus on the latter.

Throughout this paper we will restrict ourselves to continuous decision functions which have piecewise continuous first derivatives (i.e., are *piecewise* C^1). Following the methodology in Mirrlees [1971] we may characterize the set of feasible mechanisms in the following two theorems.[11] Although the results of Theorems 1 and 2 are standard, we present them in the Appendix for completeness and comparison with the proofs used in characterizing implementability and feasibility under common agency.

Theorem 1 *(Necessary Conditions.) A piecewise C^1 decision function is implementable only if*

$$U_t(x_1, x_2, t, \theta)t'(\theta) = -\sum_{i=1}^{2} U_{x_i}(x_1, x_2, t, \theta)x_i'(\theta), \tag{1}$$

and

$$\frac{\partial}{\partial\theta}\left(\frac{U_{x_1}(x_1, x_2, t, \theta)}{U_t(x_1, x_2, t, \theta)}\right)x_1'(\theta) + \frac{\partial}{\partial\theta}\left(\frac{U_{x_2}(x_1, x_2, t, \theta)}{U_t(x_1, x_2, t, \theta)}\right)x_2'(\theta) \geq 0, \tag{2}$$

for any θ such that $x_i = x_i(\theta)$, $t = t(\theta)$ are differentiable at θ, which is the case except at a finite number of points. In addition, an allocation is feasible only if

$$U(x_1(\underline{\theta}), x_2(\underline{\theta}), t(\underline{\theta}), \underline{\theta}) \geq 0. \tag{3}$$

Before proceeding with the sufficiency theorem, we make two assumptions.

Assumption 1 *Constant sign of the marginal rate of substitution. On the relevant domain of x_1, x_2, t, and θ, $\frac{\partial}{\partial\theta}\left(\frac{U_{x_i}(x_1,x_2,t,\theta)}{U_t(x_1,x_2,t,\theta)}\right) > 0$, $i = 1, 2$. Additionally, the agent's utility increases in θ : $U_\theta(x_1, x_2, t, \theta) > 0, \forall x_1, x_2, t, \theta$.*

Assumption 2 *Boundary behavior of $U(\cdot)$. For any $(x_1, x_2, t, \theta) \in \mathcal{X}^2 \times \Re \times \Theta$, there exists a $K > 0$ such that*

$$\left\|\sum_{i=1}^{2}\left[\frac{U_{x_i}(x_1(\theta), x_2(\theta), t, \theta)}{U_t(x_1(\theta), x_2(\theta), t, \theta)} - \frac{U_{x_i}(x_1(\theta), x_2(\theta), t', \theta)}{U_t(x_1(\theta), x_2(\theta), t', \theta)}\right]\frac{dx_i(\theta)}{d\hat{\theta}}\right\| \leq K\|t - t'\|,$$

uniformly in x_1, x_2, and θ, where $\|\varphi\| = \sup_{\theta\in\Theta} |\varphi(\theta)|$.

Assumption A.1 is the well known Spence-Mirrlees single-crossing condition; this partial derivative exists because U is C^2 and strictly increasing in t. Without loss of generality, we assume the signs are positive. The condition that the agent's utility increases in θ is natural in most economic environments where the marginal rate of

[11] This section closely follows the development in Guesnerie and Laffont [1984]. For another exposition, combined with a more recent review of the literature, see Fudenberg and Tirole [1991, chapter 7].

substitution between activity and transfer is positive. We take A.1 as given throughout this paper.

Assumption A.2 is a Lipschitz condition which assures us that the marginal rates of substitution between decisions and transfers do not increase too fast when the transfer increases. With preferences that are linear in transfers, this condition is trivially satisfied. We now state the sufficiency theorem.

Theorem 2 *(Sufficient Conditions.) Given assumptions A.1-A.2, any piecewise C^1 decision profile for which $x_i'(\theta) \geq 0$, $\forall \theta \in \Theta$, $i = 1, 2$, is implementable by a transfer function satisfying (1). Furthermore, given that a piecewise C^1 allocation satisfies condition (3), the allocation is also feasible.*

The traditional approach to mechanism design takes (1) and (3) above and chooses a mechanism which maximizes the principal's utility. It is then checked that the resulting mechanism is monotone. In the event that it is not, an algorithm such as that in Guesnerie and Laffont [1984] is employed which monotonizes the decision functions in an optimal manner. In the present case of cooperative contracts, we may proceed accordingly. First, however, for tractability in the principals' optimization problem, we make additional assumptions regarding the contracting environment.

Assumption 3 *(a) Agent's preferences are quasi-linear: $U(x_1, x_2, t, \theta) = \mathcal{U}(x_1, x_2, \theta) + t$.*
(b) Principals' preferences are quasi-linear: $V^i(x_1, x_2, t_i) = \mathcal{V}^i(x_1, x_2) - t_i$.
(c) The range of allowable decision functions, $\mathcal{X}$, is the interval $[0, \overline{x}]$, where $(\overline{x}, \overline{x})$ is greater than any $(x_1, x_2) \in \arg\max_{x_1, x_2} \{\mathcal{U}(x_1, x_2, \overline{\theta}) + \mathcal{V}^i(x_1, x_2)\}$, for $i = 1, 2$ and greater than any $(x_1, x_2) \in \arg\max_{x_1, x_2} \{\mathcal{U}(x_1, x_2, \overline{\theta}) + \mathcal{V}^1(x_1, x_2) + \mathcal{V}^2(x_1, x_2)\}$.

Assumption 4 *Concavity and monotonicity.*
(a) The following function (the principals' virtual surplus) is globally strictly concave in x_1 and x_2, and for all θ attains an interior maximum over $\mathcal{X}^2$:

$$\mathcal{V}^1(x_1, x_2) + \mathcal{V}^2(x_1, x_2) + \mathcal{U}(x_1, x_2, \theta) - \frac{1-F(\theta)}{f(\theta)} \mathcal{U}_\theta(x_1, x_2, \theta);$$

additionally, $\mathcal{U}_{\theta\theta}(x_1, x_2, \theta) \leq 0$.
(b) For $i = 1, 2$, and for any x_1, x_2, θ,

$$\left[\mathcal{V}^1_{x_1 x_2} + \mathcal{V}^2_{x_1 x_2} + \mathcal{U}_{x_1 x_2} - \frac{1-F(\theta)}{f(\theta)} \mathcal{U}_{x_1 x_2 \theta}\right] \left[\mathcal{U}_{x_j \theta}\left(1 - \frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\right) - \frac{1-F(\theta)}{f(\theta)} \mathcal{U}_{x_j \theta\theta}\right] -$$

$$\left[\mathcal{V}^1_{x_j x_j} + \mathcal{V}^2_{x_j x_j} + \mathcal{U}_{x_j x_j} - \frac{1-F(\theta)}{f(\theta)} \mathcal{U}_{x_j x_j \theta}\right] \left[\mathcal{U}_{x_i \theta}\left(1 - \frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\right) - \frac{1-F(\theta)}{f(\theta)} \mathcal{U}_{x_i \theta\theta}\right] \geq 0.$$

Although assumption A.3(a)-(b) is strong, it allows us to get to the heart of the issues of adverse selection under common agency without introducing additional technical assumptions. Nonetheless, it should be clear to the reader how one proceeds when preferences are not quasi-linear. In our context of two principals buying products from a single supplier (agent), $\mathcal{U}$ represents the costs of production and is negative, while t represents revenues from the principals. A.3(c) additionally requires that the principals are not specifically prevented from implementing the first-best level of activity.

A.4(a) assumes that the principals' incomplete-information problem is well-behaved. This assumption is met whenever the full-information optimum is globally strictly concave (as is the case in many economic problems) and the uncertainty of θ is relatively small. In the absence of A.4(a), it is possible that corner solutions as well as random schemes may be desirable. The condition that $\mathcal{U}_{\theta\theta} \leq 0$ ensures that at the optimum, the expression in A.4(a) is increasing in θ.

Unless a particular economic environment is considered, assumption A.4(b) is not naturally satisfied. A.4(b) (in combination with A.1, A.3, and A.4(a)) requires that the unconstrained solution to the principals' incomplete information problem have increasing decision functions. This simplifies our task considerably, as we do not have to consider such issues as bunching. Sufficient (but not necessary) conditions for A.4(b) to hold are $\mathcal{U}_{x_i\theta\theta} \leq 0$ and $\mathcal{U}_{x_1x_2\theta} \geq 0$. Section 6 provides motivating economic applications that satisfy A.4(b).

Given the additional assumptions A.3-A.4, we can now state the solution to the principals' cooperative contracting problem.

Proposition 1 *Given assumptions A.3 and A.4, the contract which maximizes the sum of the principals' utilities has decision functions which satisfy* $\forall\theta \in [\theta_i^*, \overline{\theta}], i = 1,2$

$$(4) \qquad \mathcal{V}^1_{x_i}(x_1,x_2) + \mathcal{V}^2_{x_i}(x_1,x_2) + \mathcal{U}_{x_i}(x_1,x_2,\theta) = \frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_i\theta}(x_1,x_2,\theta),$$

and $\forall\theta \in [\underline{\theta}, \theta_i^*)$, $x_i(\theta) = 0$, *where* θ_i^* *is defined by*

$$\mathcal{V}^1(x_1(\theta_i^*), x_2(\theta_i^*)) + \mathcal{V}^2(x_1(\theta_i^*), x_2(\theta_i^*)) + \mathcal{U}(x_1(\theta_i^*), x_2(\theta_i^*), \theta_i^*)$$

$$-\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_\theta(x_1(\theta_i^*), x_2(\theta_i^*), \theta_i^*) = 0,$$

if the resulting $\theta_i^* \geq \underline{\theta}$, *and* $\theta_i^* = \underline{\theta}$ *otherwise. Moreover, the transfer function in the optimal contract satisfies* $\forall\theta \in \Theta$.

$$(5) \qquad t(\theta) = \int_{\underline{\theta}}^{\theta} \mathcal{U}_\theta(x_1(s), x_2(s), s)ds - \mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta}).$$

The proof of the proposition is standard and provided in the appendix. Proposition 1 indicates that the contracted levels of x_i are below the efficient level for all $\theta < \overline{\theta}$. The intuition behind the result is straightforward. The principals contract for levels of x_i for a given θ such that the marginal expected efficiency gain from raising the level of x_i, i.e. $(\mathcal{V}^1_{x_i} + \mathcal{V}^2_{x_i} + \mathcal{U}_{x_i})f(\theta)$, is equal to the marginal loss of rents which must be given to agents with types better than θ to induce incentive compatibility, i.e. $\mathcal{U}_{x_i\theta}[1-F(\theta)]$. Of course, when the principals have unaligned preferences (i.e., neither principal cares about maximizing the joint surplus) and the contracts are chosen noncooperatively, this result is fundamentally altered.

In the noncooperative contracting game in which the principals have different preferences for contracting activities, the presence of externalities alters the result in Proposition 1. Two channels exist for the transmission of externalities. First, when $\mathcal{V}^j$ depends on x_i, principal i will not take into account $\mathcal{V}^j$ when maximizing her payoffs and may choose x_i inefficiently from the point of view of maximizing joint surplus. We examine this effect in the following section. The second channel which exists even if $\mathcal{V}^j$ is independent of x_i, is both more interesting and more subtle. To the extent that $\mathcal{U}_{x_1x_2} \neq 0$, the contract of one principal may change the marginal disutility to the agent from the other principal's contracting activity, thereby affecting the equilibrium contracts offered by each principal. The examination of this second channel is undertaken in the remainder of this paper.

2.3 The Noncooperative Benchmark with Contractual Independence

We now depart from the earlier analysis where we assumed that the two principals could coordinate contracts with the agent, and where each principal learned of both reports. Instead we suppose a common agency environment where each principal may condition her contract only upon the report meant for her that is sent by the agent. Each principal's mechanism, $y_i(\cdot) = \{x_i(\cdot), t_i(\cdot)\}$, is a function only of $\hat{\theta}_i$. Such a representation is equivalent to the nonlinear tariff contract where $t_i = t_i(x_i)$, and t_i is independent of x_j.

Under full-information, a set of equilibrium contracts which maximizes the principals' joint surplus exists where each principal makes the agent the residual claimant for her profit, thereby internalizing the externalities the principals would otherwise impose upon one another. When information is private, we must again address the issue of incentive compatibility.

As before, given a pair of contracts and our assumption of quasi-linear payoffs, we can denote the utility of an agent with type θ who makes reports $\hat{\theta}_i$ to principal i as

$$U(\hat{\theta}_1, \hat{\theta}_2, \theta) \equiv \mathcal{U}(x_1(\hat{\theta}_1), x_2(\hat{\theta}_2), \theta) + t_1(\hat{\theta}_1) + t_2(\hat{\theta}_2).$$

With this definition, we can define incentive compatibility for the common agency contracting environment.

Definition 2 *A pair of decision functions, $\{x_1(\cdot), x_2(\cdot)\}$, where $x_i : \Theta \mapsto \mathcal{X}$, is* ***commonly implementable*** *if there exists a transfer function $t_i(\cdot) : \Theta \mapsto \Re$ for each principal such that the pair of contracts satisfies the common incentive compatibility (CIC) constraint:*

$$U(\theta, \theta, \theta) \geq U(\hat{\theta}_1, \hat{\theta}_2, \theta), \ \forall (\hat{\theta}_1, \hat{\theta}_2, \theta) \in \Theta^3.$$

*A pair of contracts, $y : \Theta^2 \mapsto \mathcal{X}^2 \times \Re^2$, is **commonly feasible** if the decision functions are implementable, and the transfers satisfy the participation (or individual rationality) constraint:*

$$U(\theta, \theta, \theta) \geq 0, \ \forall \theta \in \Theta.$$

For completeness we consider the simple case of contractual independence in agent's utility as a benchmark. When the agent's utility from contracting with one principal is independent of the contracting activity with the other (i.e., $\mathcal{U}_{x_1 x_2} = 0$ for all x_1, x_2, θ), the equilibrium of the common agency contracting game is readily calculated. With contractual independence, we abstract away from concerns imposed by global incentive compatibility which manifest themselves whenever the agent can make two different reports – one to each principal. This benchmark, however, is intriguing as it highlights the strategic interactions which result from our assumption of intrinsic agency and the contracting requirement of individual rationality.

Because the activities are independent from the agent's viewpoint when $\mathcal{U}_{x_1 x_2} = 0$ and A.3(a) holds, Theorems 1 and 2 still apply with only slight modifications in their statements.

Theorem 1' *(Necessary Conditions.) Suppose $\mathcal{U}_{x_1 x_2} = 0$. A piecewise C^1 decision function is implementable only if*

$$t_i'(\theta) = -\mathcal{U}_{x_i}(x_1, x_2, \theta) x_i'(\theta),$$

and $x_i'(\theta) \geq 0$, for any θ such that $x_i = x_i(\theta)$, $t = t_i(\theta)$ are differentiable at θ, which is the case except at a finite number of points. In addition, an allocation is feasible only if

$$\mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta}) + t_1(\underline{\theta}) + t_2(\underline{\theta}) \geq 0.$$

Theorem 2' *(Sufficient Conditions.) Suppose that $\mathcal{U}_{x_1 x_2} = 0$. Any piecewise C^1 decision function, x_i, for which $x_i'(\theta) \geq 0$, is implementable by a transfer function, $t_i(\cdot)$, satisfying the differential equation in Theorem 1' above. Furthermore, given that a piecewise C^1 allocation satisfies condition the necessary individual rationality condition in Theorem 1', the allocation is also feasible.*

The proofs follow those from Theorems 1 and 2. Note, however, that the necessary individual rationality condition in Theorem 1' requires principal i's contract to satisfy a *global* participation constraint. This in an artifact of our intrinsic agency framework. With delegated agency, this condition would be replaced with the participation constraint specific to principal i: $\mathcal{U}(x_1(\theta), x_2(\theta), \theta) + t_i(\theta) \geq \mathcal{U}(0, x_2(\theta), \theta)$. With intrinsic agency, however, we have the possibility that one principal may pay less than her implicit share for the agent's production. This will have an affect on the characterization of equilibrium contracts.

To proceed with our examination of the contractual independence equilibrium, we modify A.4 as follows:

Assumption 4' *Concavity.* *(a) In addition to A.4(a) holding, the following function (principal i's virtual surplus) is globally strictly concave in x_i, and for all x_j and θ attains an interior maximum over $\mathcal{X}$:*

$$\mathcal{V}^i(x_1,x_2)+\mathcal{U}(x_1,x_2,\theta)-\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_\theta(x_1,x_2,\theta).$$

(b) For all x_1,x_2,θ, $i=1,2$, $j\neq i$,

$$\begin{aligned}\Phi^j(x_1,x_2,\theta) &\equiv \psi^j(x_1,x_2,\theta)\left[\mathcal{V}^i_{x_ix_i}(x_1,x_2)+\mathcal{U}_{x_ix_i}(x_1,x_2,\theta)-\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_ix_i\theta}(x_1,x_2,\theta)\right]\\ &- \psi^i(x_1,x_2,\theta)\mathcal{V}^j_{x_1x_2}(x_1,x_2)\geq 0,\end{aligned}$$

where

$$\psi^i(x_1,x_2,\theta)\equiv\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_i\theta\theta}(x_1,x_2,\theta)-\left[1-\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\right]\mathcal{U}_{x_i\theta}(x_1,x_2,\theta).$$

(c) For all x_1,x_2,θ and $i=1,2$, $j\neq i$,

$$(\mathcal{V}^i_{x_j}-\frac{1-F}{f}\mathcal{U}_{\theta x_j})\frac{\Phi^j}{\det\Omega}+\left[1-\frac{d}{d\theta}\left(\frac{1-F}{f}\right)\right]\mathcal{U}_\theta-\frac{1-F}{f}\mathcal{U}_{\theta\theta}\geq 0,$$

where Ω is the Hessian of the expression in A.4'(a).

A.4 has been modified in three ways in order to deal with the strategic interactions induced by the externalities inherent in the principal's payoffs. First, concavity is assumed over an individual principal's objective function. Second, in A.4'(b) conditions related to concavity have been assumed to ensure that $x_i'(\theta)\geq 0$. These latter conditions are satisfied if, for example, $\mathcal{U}_{x_i\theta\theta}\leq 0$ and $\mathcal{V}^j_{x_1x_2}$ is not too negative relative to $\mathcal{V}^i_{x_ix_i}+\mathcal{U}_{x_ix_i}$; in this sense, A.4'(b) is akin to sufficient concavity of the full information collective surplus. Third, A.4'(c) effectively requires that principal i's virtual profits be nondecreasing in θ. The condition is satisfied, for example, whenever $\mathcal{U}_{\theta\theta}\leq 0$ and any negative externality born by principal i from x_j is small on the margin compared to the information rents paid to the agent. With A.4' satisfied, we can now state our result.

Proposition 2 *Given assumption A.4' and $\mathcal{U}_{x_1x_2}=0$, $\forall x_1,x_2,\theta$, any pure-strategy Nash equilibrium in the simultaneous contracting game satisfies $\forall\theta\in[\theta_i^*,\overline{\theta}]$*

$$\mathcal{V}^i_{x_i}(x_1,x_2)+\mathcal{U}_{x_i}(x_1,x_2,\theta)=\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_i\theta}(x_1,x_2,\theta),\ i=1,2, \tag{6}$$

and for all $\theta\in[\underline{\theta},\theta_i^*)$, $x_i(\theta)=0$, where θ_i^* is defined by

$$\mathcal{V}^i(x_1(\theta_i^*),x_2(\theta_i^*))+\mathcal{U}(x_1(\theta_i^*),x_2(\theta_i^*),\theta_i^*)-\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_\theta(x_1(\theta_i^*),x_2(\theta_i^*),\theta_i^*)+t_j(\theta_i^*)=0,$$

if the resulting $\theta_i^* \geq \underline{\theta}$, and $\theta_i^* = \underline{\theta}$ otherwise. Moreover, the transfer function in the optimal contract satisfies $\forall\theta \in [\theta_i^*, \overline{\theta}]$

$$(7) \qquad t_i(\theta) = \int_{\theta_i^*}^{\theta} \mathcal{U}_{x_i}(x_1(s), x_2(s), s)x_i'(s)ds - \mathcal{U}(x_1(\theta_i^*), x_2(\theta_i^*), \theta_i^*) - t_j(\theta_i^*),$$

and $t_i(\theta) = 0$ for all $\theta \in [\underline{\theta}, \theta_i^*)$.

The proof is analogous to that of Proposition 1 and is discussed in the Appendix. Two principals simultaneously maximize their payoffs. Depending upon the relationship between the principals' payoffs, the resulting contracts can either require greater or lesser contracting activity. If $\mathcal{V}_{x_j}^i < 0$, a principal's contract introduces a negative externality, and production is greater under common agency than under the cooperative contract. The opposite conclusion holds for $\mathcal{V}_{x_j}^i > 0$. This result is related to the work of Gal-Or [1989], who argues that common agency may impose a cost on the principals in a common marketing relationship. Increased sales of one principal's product by a marketing agent hurts the second principal through reductions in demand.

An additional difference with the result in Proposition 1 involves the nature of the cutoff types, θ_i^*. Because intrinsic agency requires that each principal's contract satisfy global participation, it is possible that multiple equilibria exist. Supposing that principal i pays only a small fraction of $\mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta})$, principal j may find it worthwhile to contract only with $\theta \geq \theta_j^*$. That is, it may be too costly for principal j to pay the difference in order to satisfy the global participation constraint for $\theta < \hat{\theta}_j^*$. Consequently, the equilibrium share, α_i, of $\mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta})$ that principal i pays may be required to lie inside a subinterval of [0,1] in order for all types to be contracted. We can say more about the nature of such shares as the following corollary suggests.

Corollary 1 *Suppose each principal's contribution to the joint surplus is positive at $\underline{\theta}$ for a pair of decision functions x_1, x_2 which satisfy (6) above: i.e.,*

$$\mathcal{V}^i(x_1(\underline{\theta}), x_2(\underline{\theta})) + \mathcal{U}(x_i(\underline{\theta}), 0, \underline{\theta}) - \frac{1 - F(\underline{\theta})}{f(\underline{\theta})}\mathcal{U}_\theta(x_i(\underline{\theta}), 0, \underline{\theta}) \geq 0.$$

Then there exists t_1, t_2 such that x_1, x_2 is a Nash equilibrium and $\theta_i^* = \underline{\theta}$ for $i = 1, 2$.

Proof: For $\theta_i^* = \underline{\theta}$, it must be that

$$\mathcal{V}^i(x_1(\underline{\theta}), x_2(\underline{\theta})) + \alpha_i\mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta}) - \frac{1 - F(\underline{\theta})}{f(\underline{\theta})}\mathcal{U}_\theta(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta}) \geq 0,$$

for $i = 1, 2$. Since $\mathcal{U}(0, 0, \underline{\theta}) = 0$ and $\mathcal{U}_{x_1x_2} = 0$, $\mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta}) = \mathcal{U}(x_1(\underline{\theta}), 0, \underline{\theta}) + \mathcal{U}(0, x_2(\underline{\theta}), \underline{\theta})$. Setting $\alpha_i = \mathcal{U}(x_i(\underline{\theta}), 0, \underline{\theta})/\mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta})$, and

$$t_i(\theta) = \int_{\underline{\theta}}^{\theta} \mathcal{U}_{x_i}(x_1(s), x_2(s), s)x_i'(s)ds - \alpha_i\mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta}),$$

we satisfy the required condition for $i = 1, 2$. $\square$

The proof uses an α_i set equal to the ratio of principal i's production cost to the total production cost so as to obtain full contacting by both principals. In fact, an interval for α_i defined by

$$\left[\frac{\mathcal{U}(x_1,x_2,\theta)+\mathcal{V}^j(x_1,x_2)-\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_\theta(x_1,x_2,\theta)}{\mathcal{U}(x_1,x_2,\theta)}, \frac{\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_\theta(x_1,x_2,\theta)-\mathcal{V}^i(x_1,x_2)}{-\mathcal{U}(x_1,x_2,\theta)}\right]$$

exists at each θ such that, for all α_i contained in the interval, all types of agent greater than θ are contracted with by the principals. Any α_i which lies in the interval defined at $\underline{\theta}$ will support the Nash equilibrium given by (6) and $\theta_i^* = \underline{\theta}$.

In order to more fully understand the ramifications of common agency in contexts of adverse selection, we now focus our attention to the more subtle problem of non-independent contracting activities.

3. Incentive Constraints under Common Agency

3.1 Implementable and Feasible Contracts

As in Section 2.3, we suppose a common agency environment where each principal may condition her contract only upon the report meant for her: a principal's mechanism, $\{y_i(\hat{\theta}_i)\} = \{x_i(\hat{\theta}_i), t_i(\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta}$, may depend only upon $\hat{\theta}_i$. In this Section we characterize a set of necessary and sufficient conditions for common incentive compatibility and participation when $\mathcal{U}_{x_1x_2} \neq 0$, but for simplicity we assume no externalities between the principals' payoffs (i.e., $\mathcal{V}^i_{x_j} = 0$). Because each principal's contract can only depend upon $\hat{\theta}_i$, the necessary and sufficient conditions will be stronger than in (1)-(3) above. With conditions similar to (1)-(3), we can only guarantee that an agent will not make consistent reports, $\hat{\theta}_1 = \hat{\theta}_2$, that differ from θ. Stronger conditions must be satisfied to guarantee in addition that the agent will not gain from making inconsistent lies.

We proceed with two theorems analogous to the necessity and sufficiency theorems presented in Section 2.

Theorem 3 *(Necessary Conditions.) A pair of piecewise C^1 decision functions are commonly implementable only if, for $i = 1, 2$,*

(8) $$U_{\hat{\theta}_i}(\theta,\theta,\theta) = 0,$$

(9) $$U_{\hat{\theta}_i\theta}(\theta,\theta,\theta) + U_{\hat{\theta}_1\hat{\theta}_2}(\theta,\theta,\theta) \geq 0,$$

(10) $$U_{\hat{\theta}_1\theta}(\theta,\theta,\theta)U_{\hat{\theta}_2\theta}(\theta,\theta,\theta) + U_{\hat{\theta}_1\hat{\theta}_2}(\theta,\theta,\theta)\left(U_{\hat{\theta}_1\theta}(\theta,\theta,\theta) + U_{\hat{\theta}_2\theta}(\theta,\theta,\theta)\right) \geq 0,$$

for any $x_i(\theta)$, $t_i(\theta)$, $\theta \in \Theta$ such that x_i is differentiable at θ. In addition, a pair of piecewise C^1 contracts is commonly feasible only if

$$U(\underline{\theta},\underline{\theta},\underline{\theta}) \geq 0. \tag{11}$$

Proof: As in Theorem 1, using a Taylor expansion and revealed preference, it can be shown that piecewise C^1 decision functions imply that transfer functions are also piecewise C^1.

A necessary condition for maximization by the agent is the satisfaction of first-order and local second-order conditions at $\hat{\theta}_1 = \hat{\theta}_2 = \theta$, at all points of differentiability. This implies

$$U_{\hat{\theta}_i}(\theta,\theta,\theta) = 0, \; i = 1,2,$$

$$U_{\hat{\theta}_i\hat{\theta}_i}(\theta,\theta,\theta) \leq 0, \; i = 1,2,$$

$$U_{\hat{\theta}_1\hat{\theta}_1}(\theta,\theta,\theta)U_{\hat{\theta}_2\hat{\theta}_2}(\theta,\theta,\theta) - \left(U_{\hat{\theta}_1\hat{\theta}_2}(\theta,\theta,\theta)\right)^2 \geq 0,$$

$\forall\theta \in (\underline{\theta},\overline{\theta})$. The first expression is (8) above. Totally differentiating this expression with respect to θ yields $U_{\hat{\theta}_i\hat{\theta}_i}(\theta,\theta,\theta) + U_{\hat{\theta}_i\theta}(\theta,\theta,\theta) + U_{\hat{\theta}_1\hat{\theta}_2}(\theta,\theta,\theta) = 0$, $i = 1,2$, which allows us equivalently to express the local second-order conditions (the second and third expressions above) as (9) and (10).[12] Finally, feasibility implies (11) trivially. □

Using the implication of quasi-linearity that $\mathcal{U}_t = 1$, we can equivalently state (8)-(10) in simpler form.

Corollary 2 *A pair of piecewise C^1 decision functions are commonly implementable only if*

$$t_i'(\theta) = -\mathcal{U}_{x_i}(x_1,x_2,\theta)x_i'(\theta), \; i = 1,2, \tag{12}$$

$$\mathcal{U}_{x_1x_2}(x_1,x_2,\theta)x_1'(\theta)x_2'(\theta) + \mathcal{U}_{x_i\theta}(x_1,x_2,\theta)x_i'(\theta) \geq 0, \; i = 1,2, \tag{13}$$

$$\mathcal{U}_{x_1\theta}\mathcal{U}_{x_2\theta}x_1'(\theta)x_2'(\theta) + \mathcal{U}_{x_1x_2}x_1'(\theta)x_2'(\theta)\left[\mathcal{U}_{x_1\theta}x_1'(\theta) + \mathcal{U}_{x_2\theta}x_2'(\theta)\right] \geq 0. \tag{14}$$

for any $x_i(\theta), t_i(\theta), \theta \in \Theta$ such that x_i is differentiable at θ, where the arguments of $\mathcal{U}$ are understood to be $x_1(\theta), x_2(\theta), \theta$.

[12] Because x_i is piecewise C^1, we know that $U_{\hat{\theta}_i\theta}$ exists everywhere but at a finite set of points. Additionally, with A.3, $U_{\hat{\theta}_1\hat{\theta}_2} = \mathcal{U}_{x_1x_2}(x_1,x_2,\theta)x_1'(\theta)x_2'(\theta)$ which also exists everywhere but at a finite set of points. Thus, a Taylor expansion of $U_{\hat{\theta}_i}$ around θ yields the existence of $U_{\hat{\theta}_i\hat{\theta}_i}$ at all but a finite number of points.

In what follows, it will be useful to distinguish between two cases of contractual spillovers: contractual complements and substitutes. $\mathcal{U}_{x_1x_2} > 0$ corresponds to the case where the agent's activities are *contract complements*, while $\mathcal{U}_{x_1x_2} < 0$ corresponds to the case of *contract substitutes*.

Following Theorem 3, we can say something about the characteristics of commonly implementable contracts.

Corollary 3 *If the contracting activities are complements, a pair of piecewise C^1 decision functions are commonly implementable only if each principal's decision function has a nonnegative derivative at all points of differentiability.*

Proof: Suppose otherwise. Suppose without loss of generality that only x_1 is decreasing over some interval of Θ, while x_2 is nondecreasing. By $\mathcal{U}_{x_1x_2} > 0$, (13) is violated. Suppose instead that each x_i is decreasing over some interval of Θ. (9) implies that $\mathcal{U}_{x_1x_2}x_1'x_2' \geq -\mathcal{U}_{x_1\theta}x_1'$. (14) implies that

$$\mathcal{U}_{x_1\theta}\mathcal{U}_{x_2\theta}x_1'x_2' - \mathcal{U}_{x_1\theta}x_1'\left(\mathcal{U}_{x_1\theta}x_1' + \mathcal{U}_{x_2\theta}x_2'\right) \geq 0,$$

which contradicts our assumption that $\mathcal{U}_{x_i\theta} > 0$. $\square$

When the contracting activities are substitutes, the analysis is slightly more complicated. The necessary conditions in Theorem 3 are insufficient to prove that both decision functions are monotonically increasing. Instead, it is possible that one schedule may be decreasing if the other is sufficiently increasing. We can only be certain at this point that both functions may not be decreasing over the same interval. We will find in Section 5, however, that under some simplifying conditions on preferences and the distribution of θ both decision functions will be increasing in equilibrium.

The corollary makes clear that in a common agency environment with complements, a cost may exist from the principals not being able to pool their monotonicity constraints. In the cooperative contract regime, (2) indicates that it is possible that one decision function decreases over a range provided that the other increases sufficiently to compensate. Because of the complexity of analyzing the costs of monotonicity constraints on principals under common agency, we do not consider the issue explicitly in this paper, but instead focus attention on environments where the initial cooperative contract is nondecreasing in each argument over Θ.

In order to prove sufficiency in the common agency setting, we will need a modification of assumption A.2 to hold, or alternatively, we can assume A.3 holds for the remainder of this paper. We choose to do the latter.[13] We are now prepared to provide an equivalent condition for common implementability and feasibility.

[13] Such a modification would require for any $(x_i, t_i, \theta) \in \mathcal{X}^2 \times \Re \times \Theta$, there exists a $K > 0$ such that

$$\left\| \left(\frac{U_{x_i}(x_1(\theta), x_2(\theta), t_1 + t_2, \theta)}{U_{t_i}(x_1(\theta), x_2(\theta), t_1 + t_2, \theta)} - \frac{U_{x_i}(x_1(\theta), x_2(\theta), t_1' + t_2', \theta)}{U_{t_i}(x_1(\theta), x_2(\theta), t_1' + t_2', \theta)} \right) \frac{dx_i(\theta)}{d\hat{\theta}_i} \right\| \leq K \sum_{j=1}^{2} \|t_j - t_j'\|,$$

uniformly in x_1, x_2, and θ, where $\|\varphi\| = \sup_{\theta \in \Theta} |\varphi(\theta)|$.

Theorem 4 *Any pair of piecewise C^1 decision functions is commonly implementable if and only if* $\forall(\hat{\theta}_1, \hat{\theta}_2, \theta) \in \Theta^3$

$$(15) \quad \int_\theta^{\hat{\theta}_2} \int_\theta^{\hat{\theta}_1} U_{\hat{\theta}_1\hat{\theta}_2}(t,s,\theta)dtds + \int_\theta^{\hat{\theta}_2} \int_s^{\theta} \left(U_{\hat{\theta}_1\hat{\theta}_2}(t,s,t) + U_{\hat{\theta}_2\theta}(t,s,t)\right) dtds + \int_\theta^{\hat{\theta}_1} \int_s^{\theta} \left(U_{\hat{\theta}_1\hat{\theta}_2}(s,t,t) + U_{\hat{\theta}_1\theta}(s,t,t)\right) dtds \leq 0,$$

and (8) [equivalently, (12)] is satisfied. In addition, if and only if (11) holds, the contract pair is commonly feasible.

Proof: Following an identical argument to that in the proof of Theorem 2, quasi-linearity guarantees the existence of transfer functions which satisfy (12) at all points where $x_i(\theta)$ is differentiable. See Hurewicz [1958, Ch. 2, Theorem 12].

To prove incentive compatibility, we suppose to the contrary that there exists some $(\hat{\theta}_1, \hat{\theta}_2, \theta) \in \Theta^3$ such that $U(\hat{\theta}_1, \hat{\theta}_2, \theta) - U(\theta, \theta, \theta) > 0$. This implies

$$U(\hat{\theta}_1, \hat{\theta}_2, \theta) - U(\hat{\theta}_1, \theta, \theta) + U(\hat{\theta}_1, \theta, \theta) - U(\theta, \theta, \theta) > 0.$$

Integrating we obtain

$$\int_\theta^{\hat{\theta}_2} U_{\hat{\theta}_2}(\hat{\theta}_1, s, \theta)ds + \int_\theta^{\hat{\theta}_1} U_{\hat{\theta}_1}(s, \theta, \theta)ds > 0.$$

(8) implies that $U_{\hat{\theta}_i}(s,s,s) = 0 \; \forall s \in (\underline{\theta}, \overline{\theta})$, $i = 1, 2$, and so

$$\int_\theta^{\hat{\theta}_2} \left[\left(U_{\hat{\theta}_2}(\hat{\theta}_1, s, \theta) - U_{\hat{\theta}_2}(\theta, s, \theta)\right) + \left(U_{\hat{\theta}_2}(\theta, s, \theta) - U_{\hat{\theta}_2}(s,s,s)\right)\right] ds + \int_\theta^{\hat{\theta}_1} \left(U_{\hat{\theta}_1}(s,\theta,\theta) - U_{\hat{\theta}_1}(s,s,s)\right) ds > 0.$$

Integrating again yields

$$\int_\theta^{\hat{\theta}_2} \int_\theta^{\hat{\theta}_1} U_{\hat{\theta}_1\hat{\theta}_2}(t,s,\theta)dtds + \int_\theta^{\hat{\theta}_2} \int_s^{\theta} \left(U_{\hat{\theta}_1\hat{\theta}_2}(t,s,t) + U_{\hat{\theta}_2\theta}(t,s,t)\right) dtds + \int_\theta^{\hat{\theta}_1} \int_s^{\theta} \left(U_{\hat{\theta}_1\hat{\theta}_2}(s,t,t) + U_{\hat{\theta}_1\theta}(s,t,t)\right) dtds > 0,$$

which contradicts our initial assumption.

Given (8) and A.1, we know the agent's utility is nondecreasing in θ. Together with (11) this implies that the participation constraint of the agent is satisfied and the contract pair is feasible. □

The condition in (15) illustrates the additional problems involved in common agency contract design. Under the cooperative contract, providing the contract functions are monotone, the sufficient condition for incentive compatibility is $\mathcal{U}_{x_i\theta} > 0$. This is the Spence-Mirrlees single-crossing property: better types find it marginally cheaper to provide x_i. Under common agency, our first instinct is to suppose that some generalized form of the single-crossing property is sufficient. For example, taking $x_j(\cdot)$ as given, the single-crossing analog in the common agency setting is $\mathcal{U}_{x_i\theta}+\mathcal{U}_{x_1x_2}x_j'(\theta) > 0$. If principal i can be assured that principal j's contract is always incentive compatible (for example, principal j actually observes θ), then this is sufficient, as (15) indicates. For instance, take $\hat{\theta}_1 = \theta$ and $\hat{\theta}_2 \neq \theta$. Then only the second term of (15) matters, which must be negative if our generalized single-crossing property holds. But even if this general single-crossing property is true for both contracts, the first term in (15) may still be positive when $\hat{\theta}_1 \neq \theta \neq \hat{\theta}_2$. In particular, if $\mathcal{U}_{x_1x_2} < 0$ and $\hat{\theta}_i < \theta < \hat{\theta}_j$, or if $\mathcal{U}_{x_1x_2} > 0$ and either $\hat{\theta}_1, \hat{\theta}_2 > \theta$ or $\hat{\theta}_1, \hat{\theta}_2 < \theta$, the first term may be sufficiently positive to violate the condition in (15).

Unfortunately, unlike the simple monotonicity conditions in the cooperative contracting environment, our global incentive compatibility condition under common agency is complicated. With assumptions restricting the magnitude and sign of various third partial derivatives, however, we can find sufficient conditions for the satisfaction of (15). Technically, by restricting the change of $\mathcal{U}_{x_1x_2}$ when evaluated at different points in the domain of $\Theta \times \mathcal{X}^2$, we can verify (15) by using more convenient limits of integration. In our analysis of common agency, the complements case is the simplest to examine as there is an easily discernible set of conditions which are sufficient for the validity of (15).

Theorem 5 *Let $\mathcal{U}_{x_1x_2} > 0$ and $\mathcal{U}_{x_1x_2\theta} \leq 0$ for all x_1, x_2, θ. Then any pair of piecewise C^1 contracts for which $x_i'(\theta) \geq 0$ and (12) are satisfied is commonly implementable.*

The proof of the theorem is provided in the appendix. Providing that the contracts which we analyze in the complements contracting game have nondecreasing decision functions, the simple condition that $\mathcal{U}_{x_1x_2}$ does not increase in θ is sufficient for incentive compatibility.

Incentive compatibility with substitutes is more difficult to characterize. Here, we shall also make use of restrictions on $\mathcal{U}_{x_1x_2}$, but we shall use slightly stronger restrictions to obtain a characterization theorem.

Theorem 6 *Let $\mathcal{U}_{x_1x_2} < 0$ and suppose the cross-partial derivatives of $\mathcal{U}$ are constant (i.e., $\mathcal{U}_{x_1x_2}(x_1, x_2, \theta) = u_{12}$, $\mathcal{U}_{x_1\theta}(x_1, x_2, \theta) = u_{1\theta}$, and $\mathcal{U}_{x_2\theta}(x_1, x_2, \theta) = u_{2\theta}$). Then the necessary conditions in (12)-(14) are sufficient for common implementability if x_1 and x_2 are nondecreasing.*

The proof for this theorem is also provided in the appendix. Note that the above conditions on $\mathcal{U}_{x_1x_2\theta}$ in both theorems are not necessary for incentive compatibility and are only used for convenience. To the extent that an agent's utility (e.g., production function, etc.) is satisfactorily approximated by a second-order Taylor expansion, we may rest content with the above simplifications. If not, utility functions with higher order terms may be dealt with by a direct check on the integral conditions contained in (15).

3.2 Strategic Revelation Effects

We now turn to an examination of the conditions for Nash equilibrium in contracts in the principal's contracting game. We initially note that each principal will typically attempt to induce the agent to report falsely to her rival and thereby extract a larger share of the agent's information rents. In equilibrium, all contracts are incentive compatible so that such attempts are useless, but their possibility imposes constraints on the set of equilibrium contracts.

If instead of studying direct-revelation mechanisms we analyzed nonlinear (tax) schedules, $t_i : \mathcal{X} \mapsto \Re$, the rent-competition effect can be thought of as follows. Principal 1 may decide to change her nonlinear schedule in such a way so as to induce a type-θ agent to choose a contract pair, $\{x_2', t_2'\}$, from Principal 2 meant for type-θ', – a choice which Principal 2 had not originally intended. In this manner, Principal 1 may act as an accomplice in helping the agent retain additional information rents from Principal 2. Some of these additional rents are, in turn, extracted by Principal 1's new contract.

If we wish to use the direct revelation mechanism design methodology in the common agency setting, we must introduce additional constructions. Suppose that the decision functions are continuous and U is strictly concave in reports so that we may define the following functions:[14]

$$\hat{\theta}_1[\hat{\theta}_2, \theta | x_1(\cdot), x_2(\cdot), t_1(\cdot), t_2(\cdot)] = \arg\max_{\theta'} U(\theta', \hat{\theta}_2, \theta),$$

$$\hat{\theta}_2[\hat{\theta}_1, \theta | x_1(\cdot), x_2(\cdot), t_1(\cdot), t_2(\cdot)] = \arg\max_{\theta'} U(\hat{\theta}_1, \theta', \theta).$$

Note the functional dependence of each $\hat{\theta}_i$ on the mechanisms offered to the agent. Holding Principal 2's contract fixed, a change in Principal 1's contract will affect the report of the agent to Principal 2. For notational ease, we will at times write $\hat{\theta}_1[\theta | x_2(\theta)]$ and $\hat{\theta}_2[\theta | x_1(\theta)]$, since agent preferences are quasi-linear; with such notation it is understood that the offering principal's contract is incentive compatible (i.e., $\hat{\theta}_2 = \theta$ in the first case, and $\hat{\theta}_1 = \theta$ in the second case). Of course, each function depends on all elements of both contracts even though notationally we have only explicitly recognized dependence on the offering firm's decision function.

In our direct-revelation Nash equilibrium contracting environment, each principal chooses her contract offer taking the offer of her rival as fixed. When maximizing over

[14] The continuity of the decision functions is implied by the strict concavity of each principal's pointwise objective function together with a few technical assumptions, which we take up in the next two sections. For now, however, we take continuity as given.

decision functions, the principal also considers the effect of her contract on the agent's choice from her rival's contract.

In equilibrium all contracts are incentive compatible and we can characterize the effect of a change in one principal's contract on the reports of the agent to the other principal.

Theorem 7 *In any pure-strategy differentiable Nash equilibrium, $\forall \hat{\theta}_1, \hat{\theta}_2 \in (\underline{\theta}, \overline{\theta})$ with x_j strictly increasing*

$$\frac{\partial \hat{\theta}_j[\theta | x_i(\theta)]}{\partial x_i} = \mathcal{U}_{x_1 x_2}(x_1(\theta), x_2(\theta), \theta) \,/\, \left[\mathcal{U}_{x_j \theta}(x_i(\theta), x_j(\theta), \theta) + \mathcal{U}_{x_1 x_2}(x_i(\theta), x_j(\theta), \theta) x_i'(\theta)\right].$$

If $x_j'(\theta) = 0$, then $\frac{\partial \hat{\theta}_j[\cdot]}{\partial x_i} = 0$.

Proof: Suppose that $\{x_1, x_2, t_1, t_2\}$ is a piecewise C^1 pure-strategy Nash equilibrium. Then we know that the agent's first-order condition (12) holds for each principal's contract for all but a finite set of θ. Fix firm 1's contract and consider the effect of a change in firm 2's menu. A necessary condition for $\hat{\theta}_1$ to be chosen by the agent given his true type is θ and principal 2 contracts with the type-θ agent for $x_2(\theta)$ is that

$$t_1'(\hat{\theta}_1) = -\mathcal{U}_{x_1}(x_1(\hat{\theta}_1), x_2(\theta), \theta) x_1'(\hat{\theta}_1).$$

From (12), this condition becomes

$$\left(\mathcal{U}_{x_1}(x_1(\hat{\theta}_1), x_2(\theta), \theta) - \mathcal{U}_{x_1}(x_1(\hat{\theta}_1), x_2^e(\hat{\theta}_1), \hat{\theta}_1)\right) x_1'(\hat{\theta}_1) = 0,$$

where principal 1 expects principal 2 to offer $x_2^e(\theta)$ in her contract with the agent. If x_1 strictly increases in θ, the bracketed expression must be equal to zero. Totally differentiating this expression with respect to $x_2(\theta)$ and $\hat{\theta}_1$ yields

$$\begin{aligned}\mathcal{U}_{x_1 x_2}(x_1(\hat{\theta}_1), x_2(\theta), \theta) dx_2 = &\\ &\left[\mathcal{U}_{x_1 x_1}(x_1(\hat{\theta}_1), x_2^e(\hat{\theta}_1), \hat{\theta}_1) x_1'(\hat{\theta}_1) - \mathcal{U}_{x_1 x_1}(x_1(\hat{\theta}_1), x_2(\theta), \theta) x_1'(\hat{\theta}_1) + \right.\\ &\left.\mathcal{U}_{x_1 x_2}(x_1(\hat{\theta}_1), x_2^e(\hat{\theta}_1), \hat{\theta}_1) x_2^{e\prime}(\hat{\theta}_1) + \mathcal{U}_{x_1 \theta}(x_1(\hat{\theta}_1), x_2^e(\hat{\theta}_1), \hat{\theta}_1)\right] d\hat{\theta}_1.\end{aligned}$$

In a pure-strategy Nash equilibrium, $x_2(\theta) = x_2^e(\theta)$ and, without loss of generality, the agent tells the truth to each principal so we evaluate this total differential at $\hat{\theta}_1 = \hat{\theta}_2 = \theta$. Simplification immediately results in the expression of the theorem. When $x_i(\theta)$ is constant, a local change in x_2 can have no effect on $\hat{\theta}_1$, and so $\frac{\partial \hat{\theta}_1[\cdot]}{\partial x_2} = 0$. □

The expressions in Theorem 7 represent the marginal effect that an increase in one principal's contract menu has on the revelation of the agent to the principal's rival. By characterizing the effects of one contract on the incentive compatibility of another, the expression in Theorem 7 will greatly facilitate our search for Nash equilibria in the contract game. One caveat, however, must be made. The validity of Theorem 7 is restricted to the interior of Θ. As a consequence, each principal must additionally consider whether there is a gain to inducing the agent to choose the corner contract from her rival's offer. In a Nash equilibrium, a principal must not find it beneficial to create bunching at the corner of her rival's contract, where the agent's first-order conditions may not hold with equality. With complements, this will not be a concern; in the case of substitutes, we will require an additional assumption.

Theorem 7 implies that if (15) holds and the decision functions are increasing, then the sign of the report function's derivative is the same as the sign of $\mathcal{U}_{x_1x_2}$. In an equilibrium with complementary goods, an increase in the contracted activity by one principal will result in an increase in the activity of the other by inducing the revelation of a higher type. The reverse is true when the goods are substitutes. Consequently, examining the cases of complements and substitutes separately is in order.

4. Analysis of Equilibria with Contract Complements

By decision complements we mean that $\mathcal{U}_{x_1x_2} > 0$ for all values of x_1, x_2 and θ. That is, an increase in x_1 raises the marginal value (or lowers the marginal cost) of an increase in x_2. Situations in which the agent's technology possesses economies of scope or positive spillovers (e.g., learning by doing) are cases where an analysis of contract complements is appropriate. We will need an additional technical requirement before we present a partial characterization of the pure strategy Nash equilibrium contracts set.

Assumption 4" *The following function is globally strictly concave and has an interior maximum over x_i for i=1,2, $\forall\theta \in \Theta$ and for $x_j(\theta), t_j(\theta)$:*

$$\mathcal{V}^i(x_i) + \mathcal{U}(x_i, x_j(\hat{\theta}_j[\theta|x_i]), \theta) - \frac{1-F(\theta)}{f(\theta)} \frac{\partial \mathcal{U}(x_i, x_j(\hat{\theta}_j[\theta|x_i]), \theta)}{\partial \theta} + t_j(\hat{\theta}_j[\theta|x_i]),$$

where $\frac{\partial \hat{\theta}_j[\theta|x_i]}{\partial x_i} = \frac{\mathcal{U}_{x_1x_2}}{\mathcal{U}_{x_j\theta} + \mathcal{U}_{x_1x_2}x_i'}$, *and* $\frac{\partial^2 \hat{\theta}_j[\theta|x_i]}{\partial x_i^2} = \frac{\partial \hat{\theta}_j[\theta|x_i]}{\partial x_i}\left(\frac{\mathcal{U}_{x_1x_2x_i}}{\mathcal{U}_{x_1x_2}} + \mathcal{U}_{x_1x_2x_j}x_j'\right)$. *In addition, we restrict $t_i \geq 0$, and assume that* $\mathcal{V}^i(x_i) + \mathcal{U}(x_1, x_2, \theta) - \frac{1-F(\theta)}{f(\theta)}\mathcal{U}_\theta(x_1, x_2, \theta) \geq 0$ *and* $\mathcal{U}_{\theta\theta}(x_1, x_2, \theta) \leq 0$ *for all $\theta \in \Theta$ and for all $x_1, x_2 \in \mathcal{X}^2$.*

Assumption 4" is a modification of A.4(a) which guarantees us that each principal's maximization program will be pointwise concave in x_i and involve some positive trade with the agent. Consequently, our concerns with θ_i^* in Section 2 will not arise. Even a zero contribution (negative transfers are not allowed) by principal i will not result in principal j refusing to serve some types in Θ.[15] A.4" may have to be checked ex post, as the condition depends upon the signs and magnitudes of third-order partial

[15] As in Bernheim-Whinston [1986], we wish to focus on equilibria in which positive activity by the agent occurs. As a consequence of intrinsic agency, a Nash equilibrium always exists in which both

derivatives and the decision functions' derivative which in turn depend endogenously on the choice of x_i by each principal. The assumption, however, is met whenever the full-information maximization program is sufficiently concave and the degree of uncertainty about θ is small. Alternatively, it is also sufficient if the agent's utility function is quadratic in x_1 and x_2.

We are now prepared to obtain results for equilibrium existence and characterization.

Proposition 3 *Suppose the contracting activities are complements, A.4" is satisfied, and* $\mathcal{U}_{x_1 x_2 \theta} \leq 0$. *Furthermore, suppose a pair of decision functions exists which satisfies the following system of differential equations such that* $x_i'(\cdot) \geq 0$, $\forall x_1, x_2, \theta \in \Theta$*:*

$$(16) \quad \mathcal{V}^i_{x_i}(x_i) + \mathcal{U}_{x_i} = \frac{1-F(\theta)}{f(\theta)} \left[\mathcal{U}_{x_i\theta} + \mathcal{U}_{x_j\theta} x_j'(\theta) \mathcal{U}_{x_1 x_2} \left(\mathcal{U}_{x_j\theta} + \mathcal{U}_{x_1 x_2} x_i'(\theta) \right)^{-1} \right].$$

Given our suppositions, these decision functions constitute a pure-strategy differentiable Nash equilibrium of the common agency contracting game. In such a case, the transfer functions satisfy for $i = 1, 2$,

$$(17) \qquad t_i(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\partial \mathcal{U}(x_1(s), x_2(s), s)}{\partial x_i} x_i'(s) ds + \alpha_i \mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta}),$$

for some α_i *such that* $\alpha_1 + \alpha_2 = 1$.

The proof is presented in the appendix. Unfortunately, we cannot generally show that a nondecreasing solution to the differential equations will exist. Additionally, when such solutions do exist it is quite possible that multiple equilibria arise – differing in both contract levels and transfers– as in Theorem 8 below. We can, however, indicate simple circumstances in which we will indeed have pure strategy differential equilibria.

Theorem 8 *In a symmetric contracting game where* $\mathcal{V}^1 \equiv \mathcal{V}^2$ *and* $\mathcal{U}(s,t,\theta) \equiv \mathcal{U}(t,s,\theta)$, $\forall s, t$, *and where, for all* x_1, x_2, θ, $(\mathcal{U}_{x_i x_i \theta} + \mathcal{U}_{x_1 x_2 \theta}) \leq 0$, $\mathcal{U}_{x_1 x_2 \theta} \leq 0$, *and* $\mathcal{U}_{x_i \theta\theta} \leq 0$, *a continuum of symmetric differentiable Nash equilibria exist.*

The proof is in the appendix. The conditions on the third derivatives of $\mathcal{U}$ are sufficient, but not necessary. They merely simplify the analysis in the proof. In the case of symmetric equilibria in symmetric games, there is one equilibrium which is Pareto superior from the principals' viewpoint.[16] It is the contract whose contractual

principals offer contracts which induce non-participation by the agent. We ignore this equilibria in the analysis which follows.

[16] There is actually an infinite number of such equilibria, but all share identical decision functions; they differ only in transfers via the choice of α_i. Such a distinction is of minimal economic interest, and so we loosely refer to this situation as one with a *unique* equilibrium.

offering for $\underline{\theta}$ is equal to $x_i^{coop}(\underline{\theta})$, the contractual offering to the lowest type under the cooperative solution. This contract introduces the least distortion from the cooperative contracts. As we shall see, this contract is also the one most preferred by the agent.

In general, solving for the differential equations in (16) is not always straightforward and may require the use of numerical methods. Nonetheless, we can say something about the properties of equilibria which satisfy the differential equations in (16) with two corollaries to Proposition 3.

Corollary 4 *Suppose that A.4'(a) holds. The equilibrium contracts in the common agency game with complementary activities have the property that* $\forall\theta$, $x_i(\theta) \leq x_i^{coop}(\theta)$, *where* $x_i^{coop}(\theta)$ *is the contract offered by principal i in the cooperative contracting game.*

Proof: Define $\hat{x}_i^{coop}(x_j(\theta),\theta)$ as the solution to

$$K^i(x_1,x_2,\theta) \equiv \mathcal{V}^i_{x_i}(x_i) + \mathcal{U}_{x_i}(x_1,x_2,\theta) - \frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_i\theta}(x_1,x_2,\theta) = 0,$$

which is uniquely defined given the condition on strict concavity in A.4'(a). Thus $x_i^{coop}(\theta) = \hat{x}_i^{coop}(x_j^{coop}(\theta),\theta)$. With nondecreasing contracts, Theorem 3 implies that the right hand side of (16) is positive, and so $\hat{x}_i^{coop}(x_j(\theta),\theta) \geq x_i(\theta)$. If contracts are symmetric, we are done. Suppose the contracts are not symmetric and that the Corollary is false. That is,

$$\hat{x}_1^{coop}(x_2(\theta),\theta) \geq x_1(\theta) > x_1^{coop}(\theta).$$

Complementarity implies $\frac{\partial \hat{x}_i^{coop}}{\partial x_j} > 0$, and so it is also the case that

$$\hat{x}_2^{coop}(x_1(\theta),\theta) \geq x_2(\theta) \geq x_2^{coop}(\theta).$$

Because $x_i(\theta) > x_i^{coop}(\theta)$, it must be that $K^i(x_i, x_j^{coop}, \theta) < 0$. By (16), we know that $K^i(x_i,x_j,\theta) \geq 0$. Thus, by continuity and the mean-value theorem, there exists an $\tilde{x}_j$ such that $K^i(x_i,\tilde{x}_j,\theta) = 0$, where $\tilde{x}_j \in (x_j^{coop}, x_j]$. Similarly, there exists a $\tilde{x}_i \in (x_i^{coop}, x_i]$ such that $K^j(x_j,\tilde{x}_i,\theta) = 0$. We can as a consequence define continuous mappings, $\phi^i : [x_j^{coop}, x_j] \mapsto (x_i^{coop}, x_i]$, for $i = 1, 2$, and hence by Brouwer's Theorem there exists a fixed point that lies in $(x_1^{coop}, x_1] \times (x_2^{coop}, x_2]$, such that $K^1(\tilde{x}_1,\tilde{x}_2,\theta) = K^2(\tilde{x}_1,\tilde{x}_2,\theta) = 0$. But by A.4', there is a unique fixed point which satisfies the first-order conditions for a cooperative contract, and that fixed point differs from $x^{coop}(\theta)$. □

Corollary 5 *Both principals and the agent weakly prefer the cooperative contract relative to the outcome in the common agency environment.*

Proof: The fact that the two principals are weakly worse off is a trivial implication of the noncooperative setting. To understand the agent's demise, note that the agent's utility is given by

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\partial \mathcal{U}(x_1(s), x_2(s), s)}{\partial \theta} ds + U(\underline{\theta}).$$

Because $\mathcal{U}_{x_i\theta} > 0$, the integrand above must be less under common agency than under the cooperative contract (given our result in Corollary 4). □

Corollary 4 indicates that the distortions introduced by each principal are greater in the common agency environment. The explanation is straightforward. Equation (16) has an additional information rent distortion on the right hand side that is not present in the cooperative contract of Section 2.2. This term represents the rent effect introduced by competition among principals. First, note that there still is no distortion for the agent with type $\overline{\theta}$. Second, since $\mathcal{U}_{x_1x_2} > 0$, the economic activities of the agent are complements, and the distortion introduced by the principals increases. The dependence of the rent effect on the economic nature of the agent's activities is intuitive: In the case of complements, a principal will decrease its exchange of x_i with the agent to attempt to decrease the agent's exchange of x_j with its rival contractor as this allows principal i to elicit truth telling more cheaply from the agent. Of course, in equilibrium each principal attempts to extract as much rent as possible with the result that the competition for the agent's activity decreases the agent's information rents.

The right hand side of (4) in Proposition 1 reflects the effect of x_i on the inframarginal rents which must be paid to all types greater than θ. The right hand side of (16) in Proposition 3 also reflects the effect of x_i on the inframarginal rents, but the existence of a strategic complementarity adds to $\mathcal{U}_{x_i\theta}$ and increases the rents which must be paid for an increase of x_i. That is, an increase in x_i directly increases the agent's inframarginal rents through $\mathcal{U}_{x_i\theta}$, but it also indirectly increases rents by raising the choice of x_j by the agent, which in turn raises $\mathcal{U}_\theta$ still further. Hence, in equilibrium the level of x_i is correspondingly lower than in the cooperative case.

Corollary 5 presents another interesting result under common agency with contract complements – all parties are worse off. Common agency makes information rent reduction by each principal more profitable on the margin, which in turn hurts the agent. The conclusion is analogous to the familiar result with product differentiated duopolists competing in prices: when products are complements, each duopolist charges a price in excess of the monopoly price and consumers are harmed by the presence of competition. In our case, the existence of asymmetric information (together with the possibility of secret contracting) prevents the three parties from eliminating the externalities which they impose upon one another.

The work of Laffont and Tirole [1990] on privitization is related to this point. Their model examines the costs and benefits of government ownership of a firm compared to the regulated environment where both the government and stockholders offer managers noncooperative contracts. The benefits of regulation over public ownership are better incentives for managers to make investments in the firm because the lack of government ownership is a form of commitment not to appropriate managerial inputs. On the other hand, the effect of common agency is to produce less powerful incentive schemes for cost-reducing effort with greater distortions from efficiency. In Laffont-Tirole, however, only one activity by the agent is contractible and there is conflict between the objectives of the government and the stockholders. To understand the intuition of their results regarding the costs of common agency with a single contractible good, consider the case where $\mathcal{U}_{x_1x_2}(x_1, x_2, \theta) \to \infty$ (i.e., $\mathcal{U}(x_1, x_2, \theta)$ is approximated by a Leontief function). In such a case, there is effectively one contractual activity and the right hand side of (16) approaches $2\mathcal{U}_{x\theta}$. With a single activity under common agency, the resulting distortion from the first-best full information case increases two-fold in absence of payoff dependencies between the principals' objectives.

A final remark about the relationship between intrinsic and delegated agency is in order. When contracting activities are complements, in equilibrium it will never be the case that the agent prefers to contract with only one principal rather than both. As a consequence, there is no loss in generality in examining the case of intrinsic agency for this class of models. With decision complements, it is never profitable for one principal to offer a contract that induces the agent to deal exclusively with her, leaving her competitor without any trade. With complements, we do not have to consider the constraints which an induced exclusive dealing contract would impose on the equilibrium contracts.

5. Analysis of Equilibria with Contract Substitutes

Decisions are substitutes when $\mathcal{U}_{x_1x_2} < 0$ for all x_i and θ. As was the case with our discussion of complements, we do not directly prove the general existence of equilibrium decision contracts which satisfy (15). Rather, we make a weaker proposition regarding the characteristics of such functions when they exist in our simple differentiable setting.

Even this is problematic, however, as our previous use of the first-order approach by principal i when considering the effect of her contract on the agent's report to principal j is questionable without further assumptions. Specifically, it is arguable that principal i may find it worthwhile to induce an agent in some interval of Θ to always choose the corner contract from principal j's menu (i.e., report either $\underline{\theta}$ or $\overline{\theta}$ to principal j). If, for example, principal i's ideal offer of x_i for the agent choosing $\underline{x}_j = x_j(\underline{\theta})$ from principal j' menu is such that the first-order condition for the agent choosing $\underline{x}_j$ is slack, principal i might prefer to induce corner choices by the agent. Such an offer by the principal is not discovered using the first-order approach in her maximization program because the set of incentive compatible allocations may not be a subset of those satisfying the agent's first-order condition for $\underline{\theta}$ and $\overline{\theta}$. This was not a concern in the case of complements where the first-order condition of the agent always binds in an optimal contract. With substitutes, the following assumption is sufficient to remove the problem.

Assumption 5 *For all* $x_1, x_2, \bar{x}_1, \bar{x}_2, \theta$,

$$\frac{\left(1 - \frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\right)\mathcal{U}_{x_1\theta}(x_1, x_1, \theta) - \frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_1\theta\theta}(x_1, x_2, \theta)}{\mathcal{V}^1_{x_1x_1}(x_1) + \mathcal{U}_{x_1x_1}(x_1, x_2, \theta) - \frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_1x_1\theta}(x_1, x_2, \theta)} \geq \frac{\mathcal{U}_{x_2\theta}(\bar{x}_1, x_2, \theta)}{\mathcal{U}_{x_1x_2}(\bar{x}_1, x_2, \theta)},$$

$$\frac{\left(1 - \frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\right)\mathcal{U}_{x_2\theta}(x_1, x_1, \theta) - \frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_2\theta\theta}(x_1, x_2, \theta)}{\mathcal{V}^2_{x_2x_2}(x_2) + \mathcal{U}_{x_2x_2}(x_1, x_2, \theta) - \frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_2x_2\theta}(x_1, x_2, \theta)} \geq \frac{\mathcal{U}_{x_1\theta}(x_1, \bar{x}_2, \theta)}{\mathcal{U}_{x_1x_2}(x_1, \bar{x}_2, \theta)}.$$

Roughly speaking, A.5 requires that the joint surplus of principal i and the agent is sufficiently concave relative to the substitution term, $\mathcal{U}_{x_1x_2}$, and third-order terms are not sufficiently large in absolute value. Following the analysis in Section 4, we can now state the following Proposition.

Proposition 4 *Suppose $\mathcal{U}_{x_1x_2} < 0$, A.4" and A.5 are satisfied. Furthermore, suppose suppose $\mathcal{U}$ has constant cross-partials, and suppose that a pair of piecewise C^1 decision functions exists which satisfies (15) and the following system of differential equations, $\forall x_1, x_2, \theta \in \Theta$, $i = 1,2$,*

$$(18)\quad \mathcal{V}^i_{x_i}(x_i) + \mathcal{U}_{x_i} = \frac{1-F(\theta)}{f(\theta)}\left[\mathcal{U}_{x_i\theta} + \mathcal{U}_{x_j\theta}x_j'(\theta)\mathcal{U}_{x_1x_2}\left(\mathcal{U}_{x_j\theta} + \mathcal{U}_{x_1x_2}x_i'(\theta)\right)^{-1}\right].$$

Given our suppositions, these decision functions constitute a pure-strategy Nash equilibrium in the contracting game. Additionally, the transfer functions satisfy for $i = 1,2$,

$$(19)\qquad t_i(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\partial \mathcal{U}(x_1(s), x_2(s), s)}{\partial x_i} x_i'(s)ds + \alpha_i \mathcal{U}(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta}),$$

for some α_i such that $\alpha_1 + \alpha_2 = 1$.

The proof essentially follows Proposition 3 except that we concern ourselves with the possibility that one principal may desire to induce bunching on the corner of her rival's contract. This problem is taken up in the Appendix.

Equation (18) has an additional information rent distortion on the right hand side that is not present in the cooperative contract of Section 2.2, which represents the rent effect introduced by competition among principals. There still is no distortion for the agent with type $\overline{\theta}$, and since the economic activities of the agent are substitutes, the distortion introduced by the principals decreases. A principal will increase her exchange of x_i with the agent to decrease the agent's exchange of x_j with her rival as this allows principal i to elicit truth telling more cheaply from the agent. In equilibrium each principal attempts to extract more rents on the margin with the result that the total sum of the extracted information rents is reduced together with an increase in productive efficiency.

The righthand side of (18) in Proposition 4 reflects the effect of x_i on the inframarginal rents. Additionally, the existence of a strategic substitutability affords principal i the opportunity to reduce the rents which must be paid to the agent by decreasing x_j. An increase in x_i directly increases the agent's inframarginal rents through $\mathcal{U}_{x_i\theta}$, but it also indirectly decreases rents by lowering the choice of x_j by the agent, which in turn lowers $\mathcal{U}_\theta$.

When both the preferences and the equilibrium contracts in Proposition 4 are symmetric, the equilibrium common agency contract lies almost everywhere above the cooperative contract due to the informational externalities which each principal imposes upon the other. Each principal prefers to offer a more powerful incentive structure to the agent to reduce the agent's activity with her rival and thereby reduce information rents. In equilibrium, the principals offer sufficiently efficient contracts so that on the margin nothing is gained by reducing a rival's activity with the agent. When contracts are not symmetric, the nature of the distortions from the efficient level is more difficult to ascertain. Along these lines, we have the following corollary.

Corollary 6 *The commonly implementable contract pair in the pure-strategy contract substitutes equilibrium defined by (18), if it exists, must necessarily have*

$$x_i(\theta) \in [x_i^{coop}(x_j(\theta),\theta), x_i^{eff}(x_j(\theta),\theta)], \forall\theta,$$

where $x_i^{coop}(x_j(\theta),\theta)$ is the cooperative contract solution by principal i when facing $x_j(\theta)$, and where $x_i^{eff}(x_j(\theta),\theta)$ is the efficient full-information contract solution by principal i given $x_j(\theta)$. Furthermore, providing that for all values of $x_1, x_2, \tilde{x}_1, \tilde{x}_2, \theta$ the following conditions hold:

(20)

$$\frac{\mathcal{V}^1_{x_1x_1}(x_1,x_2,\theta)+\mathcal{U}_{x_1x_1}(x_1,x_2,\theta)-\mathcal{U}_{x_1x_1\theta}(x_1,x_2,\theta)\frac{1-F(\theta)}{f(\theta)}}{\mathcal{U}_{x_1x_2}(x_1,x_2,\theta)-\mathcal{U}_{x_1x_2\theta}(x_1,x_2,\theta)\frac{1-F(\theta)}{f(\theta)}} \geq \frac{\mathcal{U}_{x_1\theta}(x_1,\tilde{x}_2,\theta)}{\mathcal{U}_{x_2\theta}(x_1,\tilde{x}_2,\theta)},$$

(21)

$$\frac{\mathcal{U}_{x_1\theta}(\tilde{x}_1,x_2,\theta)}{\mathcal{U}_{x_2\theta}(\tilde{x}_1,x_2,\theta)} \geq \frac{\mathcal{U}_{x_1x_2}(x_1,x_2,\theta)-\mathcal{U}_{x_1x_2\theta}(x_1,x_2,\theta)\frac{1-F(\theta)}{f(\theta)}}{\mathcal{V}^2_{x_2x_2}(x_1,x_2,\theta)+\mathcal{U}_{x_2x_2}(x_1,x_2,\theta)-\mathcal{U}_{x_2x_2\theta}(x_1,x_2,\theta)\frac{1-F(\theta)}{f(\theta)}},$$

then the agent is always weakly better off (and the principals are always weakly worse off) with common agency relative to the cooperative solution.

Proof: The principals are necessarily weakly worse off compared to the cooperative contract. By (9) in Theorem 3, we know that

$$\frac{\mathcal{U}_{x_j\theta}\mathcal{U}_{x_1x_2}x_j'(\theta)}{\mathcal{U}_{x_j\theta}+\mathcal{U}_{x_1x_2}x_i'(\theta)} \leq 0,$$

which in turn implies that $\mathcal{V}^i_{x_i}(x_i)+\mathcal{U}_{x_i}-\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_{x_i\theta} \leq 0$, and so x_i is chosen above the cooperative levels given x_j: $x_i(\theta) \geq x_i^{coop}(x_j(\theta),\theta)$. By (10) in Theorem 3, we know that the righthand side of (18) must be nonnegative for all θ, and so x_i is chosen inefficiently low given the choice of x_j. That is, $x_i(\theta) \leq x_i^{eff}(x_j(\theta),\theta)$.

The agent's rents from the contracting relationship are given by

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\partial\mathcal{U}(x_1(s),x_2(s),s)}{\partial\theta}ds + U(\underline{\theta}).$$

Because $\mathcal{U}_{x_i\theta} > 0$, a higher level of contracting activity leads to a larger integrand and hence greater rents for the agent. To see that the agent is at least weakly worse off with the cooperative contract, consider as a reference point in $\mathcal{X}^2$ the cooperative contract for a given θ: $\{x_1^{coop}(\theta), x_2^{coop}(\theta)\}$. The agent's indifference curve through this point has slope $\frac{dx_2}{dx_1} = -\frac{\mathcal{U}_{x_1\theta}}{\mathcal{U}_{x_2\theta}}$. The functions $x_1^{coop}(x_2,\theta)$ and $x_2^{coop}(x_1,\theta)$ also pass through this point, but with slopes of $-\frac{\mathcal{V}^1_{x_1x_1}+\mathcal{U}_{x_1x_1}-\mathcal{U}_{x_1x_1\theta}\frac{1-F}{f}}{\mathcal{U}_{x_1x_2}-\mathcal{U}_{x_1x_2\theta}\frac{1-F}{f}}$ and $-\frac{\mathcal{U}_{x_1x_2}-\mathcal{U}_{x_1x_2\theta}\frac{1-F}{f}}{\mathcal{V}^2_{x_2x_2}+\mathcal{U}_{x_2x_2}-\mathcal{U}_{x_2x_2\theta}\frac{1-F}{f}}$, respectively. As a consequence of our assumptions, the set of all $\{x_1, x_2\}$ which lie above the curves $x_i^{coop}(x_j,\theta)$ are preferred by the agent compared to the cooperative equilibrium. $\square$

Although the set of all allocations that are Pareto superior (as judged by both principals and the agent) is a convex set supported by the agent's indifference curve and therefore weakly preferred by the agent, we cannot say that the common agency contracts lie in this convex set. It may be that when preferences and equilibria are not symmetric and the degree of substitution between x_1 and x_2 is high, that an equilibrium exists outside this set. As the degree of substitution approaches zero, however, the common agency contracts must become efficient relative to the cooperative equilibrium.

We still have not proven that common agency equilibria as given by (18) actually exist. This is a very difficult undertaking. If, however, we are content with second-order approximations to preferences, and if the underlying uncertainty about type is generated by a process whose hazard rate can be approximated by a linear function, we can make considerably stronger statements about the contracting equilibrium, as we can analytically solve for the contracts given by (18). First, we posit the following definition.

Definition 3 *We say that a random process belongs to the class of linear inverse hazard rate distributions (LIHRD) if* $f(\theta) = \frac{1}{\gamma}(\overline{\theta}-\underline{\theta})^{-\frac{1}{\gamma}}(\overline{\theta}-\theta)^{\frac{1-\gamma}{\gamma}}$.

A probability distribution that belongs to such a class has a hazard rate given by $\frac{1-F(\theta)}{f(\theta)} = \gamma(\overline{\theta}-\theta)$. Such a family of probability functions contains the uniform distribution ($\gamma = 1$), as well as arbitrarily close approximations to any exponential distribution.[17]

We can now state our result for quadratic preferences.

Theorem 9 *Suppose that the distribution of θ belongs to the LIHRD class with $\gamma > 0$, the preferences of the principals and the agent are quadratic, and*

$$\frac{\mathcal{V}^i_{x_ix_i}+\mathcal{U}_{x_ix_i}}{\mathcal{U}_{x_1x_2}} \geq 2(1+\gamma)\frac{\mathcal{U}_{x_i\theta}}{\mathcal{U}_{x_j\theta}}, \qquad i,j = 1,2, \tag{22}$$

then there exists a unique linear pure-strategy Nash equilibrium in the common agency game $\{x_1^{ca}(\cdot), x_2^{ca}(\cdot)\}$ such that the agent is weakly better off and the principals are weakly worse off than under the optimal cooperative contract.

[17] An exponential function defined by parameter β over $[0,\infty)$ can be approximated in the linear inverse hazard rate family by choosing $\underline{\theta} = 0$ and letting $\gamma \to 0$, $\overline{\theta} \to \infty$ while maintaining $\gamma\overline{\theta} = \beta$. The resulting inverse hazard rate is β.

The result is provided in the Appendix.

6. Applications

As we have indicated, many contracting environments are confounded by the presence of common agency. When two or more principals find themselves contracting with the same agent, they generally find themselves worse off because of their failure to cooperate and offer a coordinated set of agency contracts. Understanding the nature of these costs is a requisite first step in our understanding of complex common agency arrangements. We have included here two short analyses of economic problems which involve some form of common agency. The treatments are necessarily incomplete, focusing essentially on the cost aspects of common agency, but they illuminate the broad themes contained in this paper. The first economic problem we address is determining the gains from internalizing transactions to eliminate the costs of common agency in market situations. We examine two manifestations of this concern: the gains to downstream manufacturers from coordination of contracts when dealing with a single input supplier, and the benefits to a firm from using an internal sales force rather than contracting with an independent agency. In a second problem area, we consider the situation of two regulators with imperfectly aligned preferences and ask the welfare question of who gains and who loses from fragmented regulatory authority.

6.1 Internal versus External Organization

For the purposes of discussion, we refer to exclusive-agent contracts as *internal* contracting arrangements; in contrast, we say common agency transactions are market-based or *external* arrangements. Internal arrangements are contracts where the parties to the agreement can prevent external forces (such as other principals) from interfering with their relationship. External arrangements are characterized by the absence of such protections. For exposition, we consider joint ventures between firms for the supply of inputs and in-house employees (as opposed to outside agents) as two examples of relationships designed to overcome the externalities of common agency. We recognize that a joint venture is neither necessary nor sufficient for cooperative contracting, and in-house labor is neither necessary nor sufficient for exclusive dealing contracts. Although cooperation could arguably be accomplished through simple contracts between the principals, the existence of additional legal obligations and duties to one another imposed by a joint venture may provide a more effective organization. Similarly, the employee relationship may be a more effective form of internal contracting. Masten (1988), for instance, has emphasized that the legal treatment of employment contracts by courts provides more authority to employers over their employees than a firm could ordinarily have over an independent contractor.[18]

[18]We do not wish to make too much of these institutional differences between various alternative organizational forms. If one takes the view that any particular organization is simply a set of "standardized contracts" and is distinguished from other organizations only by the terms of those contracts, the interesting questions focus on the costs and benefits of the various possible contractual terms. Our

Much has been written on the question of when firms prefer such restricted internal relationships (joint ventures, exclusive long-term contracting, internal labor markets, etc.) to unrestricted external transactions. Williamson [1985] indicates gains from internal relationships exist when investment is important but capable of being expropriated in a transient market relationship *and* internal arrangements can prevent such opportunism. On the other hand, Williamson notes that such organizational form is plagued with internal contracting costs, bureaucracies, etc., which result in low-powered incentives, in comparison to the market. Williamson concludes that internalization of market activities occurs when the benefits exceed these costs.

As we shall see below, Williamson's claim that internal contracts are less powerful than market schemes is consistent with common agency under contract substitutes. If effort or productivity of an agent is not observable and the agent's activities are partially substitutable between the two principals' projects, market-based (i.e., external) transactions will be associated with high-powered incentives; exclusive-agent contracts will be associated with low-powered incentives. But here the low-powered internal incentives are not the cost of internal organization, but rather the benefit. Without the influence of another principal's contract, the principal will take advantage of low-powered schemes which are more profitable. It is the presence of excessively powerful market-based schemes that drives the choice to internalize transactions. With complements, we find the implication for the power of schemes is reversed. Market-based transactions are low-powered relative to the internal contracts which would be offered. This affords us a test to determine the importance of a *common agency theory* of internal transactions. A comparison of the schemes from internal and external relationships across firms with varying degrees of economies of scope and scale would be telling in this regard.

6.1.1 Economies of Scope and Contract Complements

Consider a very simple model of a vertical supplier relationship where economies of scope exist in input supply. Two downstream manufacturers, $i = 1, 2$, must each contract for a differentiated input, x_i, which has a constant marginal benefit to manufacture i's profit of unity. That is, each firm's (principal's) preferences can be summarized as

$$\mathcal{V}^i(x_i) = x_i - t_i,$$

for $i = 1, 2$, where t_i is the payment to the supplier. The supplier's (agent's) preferences exhibit complementarity in production: there are economies of scope available in the production of x_1 and x_2. For example we suppose

$$\mathcal{U}(x_1, x_2, \theta) = -(\vartheta - \theta)[x_1^2 + x_2^2 - \alpha x_1 x_2],$$

where $\theta \in [\underline{\theta}, \overline{\theta}]$, θ has cumulative distribution function $F(\theta)$, $\vartheta > \overline{\theta}$ and α is a measure for the economies of scope. For concreteness, let $[\underline{\theta}, \overline{\theta}] = [0, 1]$ and $F(\theta) = \theta$ (i.e., θ

analysis can thought of as an examination of the economic costs and benefits of exclusive-agent versus common-agent terms.

is uniformly distributed on [0, 1]), and let $\alpha = 1$ and $\vartheta = 2$. Then following Propositions 1 and 3 we can derive the optimal contracts under full-information, $x_i^{eff}(\theta)$, incomplete information with cooperation, $x_i^{coop}(\theta)$, and incomplete information with common agency, $x_i^{ca}(\theta)$.

Result 1 *The first-best contract and the cooperative contract are given by the following, respectively:*

$$x_i^{eff}(\theta) = \frac{1}{2-\theta},$$
$$x_i^{coop}(\theta) = \frac{1}{3-2\theta}.$$

The Pareto-dominating common agency equilibrium is defined by the following differential equation

$$\frac{dx_i^{ca}(\theta)}{d\theta} = -\frac{x_i^{ca}(\theta)}{2-\theta}\left[\frac{1-(3-2\theta)x_i^{ca}(\theta)}{1-(4-3\theta)x_i^{ca}(\theta)}\right]$$

with $x_i^{ca}(\underline{\theta}) = 1/3$.

These contracts are illustrated in Figure 1. Here, common agency introduces more variation in the decision variables, although the quantity/quality spectrum remains unchanged under the Pareto superior equilibrium contract.

Now consider the decision to internalize the supply transaction. Suppose that principal 1 is already committed to contracting with the agent because of the high cost of alternative arrangements. Principal 2, however, has a choice: she can contract with the same agent, or setup her own input supplier with whom she will exclusively contract. Under the latter internal contracting relationship with an exclusive agent, the agent's preferences are given by

$$U = t_i + \mathcal{U}(x_1, x_2, \theta) = t_i - (\vartheta - \theta)[x_i^2 - \alpha\beta x_1 x_2],$$

where $\beta \in [0, \frac{1}{2}]$ measures the degree to which the principal can capture the scope economies through internal production. If $\beta = \frac{1}{2}$, the principal can convert all of the economies of scope which existed in the common agency framework to economies in producing x_2 alone. Alternatively, one can think of β as the degree to which spillovers continue to occur between two internalized vertical relationships. The unknown marginal cost parameters of each agent are assumed to be independently distributed. We have the following result.

Result 2 *There exists a value of* $\beta^* \in (0, \frac{1}{2})$ *such that the manager will prefer to internalize production whenever* $\beta \geq \beta^*$.

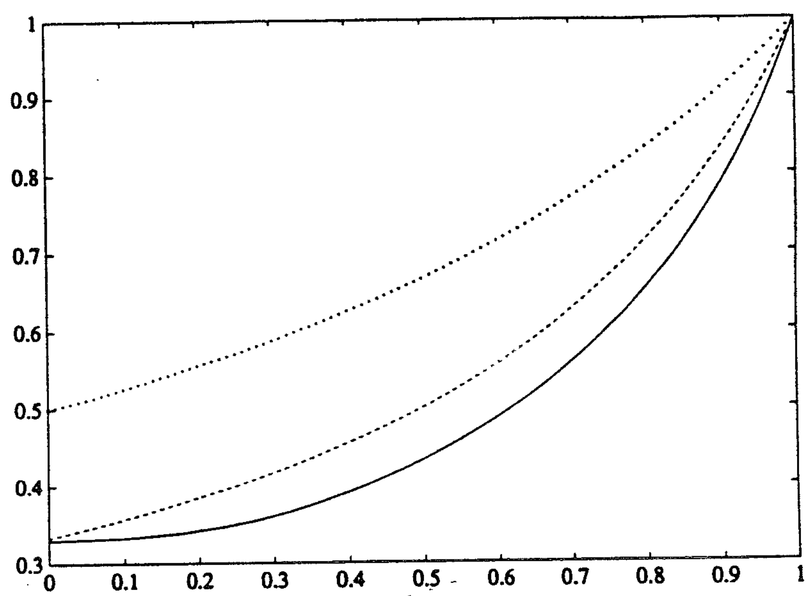
The result follows from Proposition 3. In the symmetric model under study, $\beta = \frac{1}{2}$ corresponds to the principal obtaining the same level of profits as in the cooperative contracting case. Because there are positive losses associated with common agency in our model, profits under the internalized organization must be greater. Because profits are increasing and continuous in β, we have the result.

FIGURE 1: CONTRACTUAL RELATIONSHIPS WITH COMPLEMENTARY PRODUCTION

.... FULL-INFORMATION CONTRACT

---- COOPERATIVE CONTRACT

—— NON-COOPERATIVE CONTRACT



AGENT'S TYPE, θ

6.1.2 Contract Substitutes

Related to this work is that of Holmström and Milgrom [1990] who consider a similar question: When does a manager find it desirable to use an internal sales force rather than an independent contractor to sell her products? Assuming that internal sales forces can be monitored so as to prevent an agent from working for two principals (while an independent agent cannot), they argue that the independent agent's option of exerting effort selling another firm's products may make an internal sales force more desirable.[19] An internal force can be expected to expend a minimal amount of time on the firm's own sales; an independent sales force must be given high-powered incentive schemes to induce the correct level of effort. Their theories regarding the optimal job-task design are closely related to this work on adverse selection. Common agency applied to corporate organization can be thought of as a special case of activity design for an agent; the choice is whether to allow the agent two activities (common agency) or only one (exclusive dealing). With adverse selection and substitutes, the common agency story told here reaches similar conclusions: It may be desirable to exclude the agent from the market in order to allow lower-powered, more profitable contracts. This story, as well as that of Holmström and Milgrom, is consistent with the empirical work by Anderson and Schmittlein [1984]. They find that uncertainty caused by difficulty in equitably measuring individual performance among sales people in the electronics industry is statistically significant in explaining the extent of vertical integration with a firm's sales force.

We now consider a related model which addresses the question: When does a firm find it optimal to use an internal sales force if sales effort is substitutable across the principals' product lines and the productivity of the sales force is private information. Specifically, we consider the case of common agency under adverse selection and moral hazard using a model similar to that in Laffont and Tirole [1986] but with two principals.

Consider a production environment with two risk-neutral firms (principals) and a risk-neutral sales person (agent). The question at issue is the magnitude of the gain that a firm will obtain from internalizing its sales force rather than contracting with a common agent.

The sales force sells units, x_1 and x_2, for each firm, which are a function of an intrinsic productivity parameter of the agent, θ, and the agent's effort, e_i: $x_i = \theta + e_i$. The agent's cost of effort is convex and quadratic, and efforts are substitutes: $\psi(e_1, e_2) \equiv \frac{1}{2}\psi_{11}e_1^2 + \frac{1}{2}\psi_{22}e_2^2 + \psi_{12}e_1e_2$, where $\psi_{11} > 0$, $\psi_{22} > 0$, and $\psi_{12} > 0$. θ is distributed uniformly on $[0, 1]$.

The payoffs of the two firms are $\mathcal{V}^i(x_i, t_i) \equiv v_i x_i - t_i$, $i = 1, 2$, where t_i is the transfer paid to the employee for the sales of x_i, and v_i is the per unit profit a firm earns on each sale. The firms do not compete on the product market. Their only interactions

[19] Again, for exposition we have supposed that a firm cannot write an exclusive-dealing contract with an independent agent. Alternatively, we could define agents with exclusive employment contracts to be internal employees and agents with unrestricted contracts to be independent agents without affecting our analysis.

are through a common agent's marginal costs. Substituting out the agent's effort from his utility function using the sales function results in agent's payoffs that are $U(x_1, x_2, t_1, t_2, \theta) = t_1 + t_2 - \psi(x_1 - \theta, x_2 - \theta)$. With this cost, the full-information efficient contract would set

$$e_i^{eff}(\theta) = x_i^{eff} - \theta_i = \frac{v_i \psi_{jj} - \psi_{12} v_j}{\psi_{11}\psi_{22} - \psi_{12}^2}.$$

In a joint venture, firms can coordinate and offer one contract to the supplier which optimally trades off production distortions against information rent reduction. Following Proposition 1 in Section 2.2, the solution to the joint venture contract is easily derived.

Result 3 *The optimal joint venture contract for a common sales force has* $e_i^{coop}(\theta) = x_i^{coop}(\theta) - \theta = e_i^{eff}(\theta) - (1-\theta)$, $i = 1, 2$, $\forall \theta \in [0,1]$.

In order to compare the costs and benefits between using a common agent and using one's own sales force, we need to be precise about the nature of the substitutes under the internal arrangement where one firm uses its own agent exclusively. There are two possible benefits from exclusive agency. First, if firm 1 hired its own exclusive sales force and the agent's cost function remained unchanged, there would be a reduction in sales costs driven by $x_2 = 0$. Second, and more interestingly, more information rents are extracted from the agent absent common agency. In order to focus on the second point, we assume that when a firm employs its own agency, the costs of selling the principal's product are still negatively affected by the level of sales activity undertaken by the other principal's agent. The only change in the environments is that principal i cannot influence agent j's report to principal j through the choice of her contract. Consequently, the cooperative outcome is identical to the outcome when firms decide to train and employ their own sales force.

When an independent sales force is commonly contracted with by both principals, Theorem 9 provides the following result.[20]

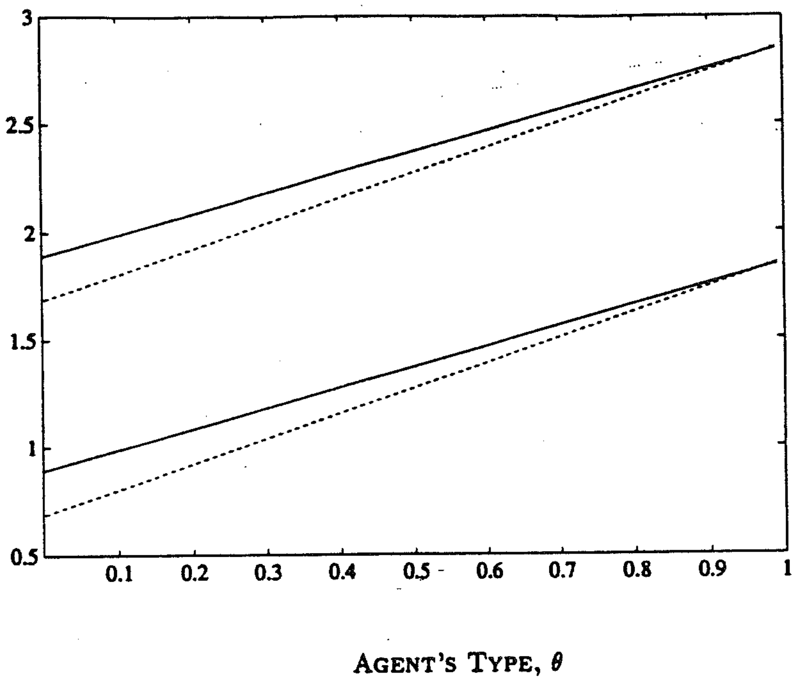
Result 4 *There exists a unique linear pure-strategy Nash equilibrium in the common agency contract game.*

A comparison of the different contracting environments is provided in Figure 2 when parameter values are $\psi_{11} = 3$, $\psi_{22} = 3$, $\psi_{12} = \frac{1}{2}$, $v_1 = 7.5$, and $v_2 = 5$. Firm 1 contracts for a higher level of sales due to its higher per unit profits. Here, because efforts are substitutes, sales are distorted downward more under the internal contracting environment than under common agency. This distortion, however, is profit maximizing for the firms. High-powered contracts are less profitable.

Now suppose more realistically that there are startup costs, I, to training and employing a sales force. Such costs must be completely born by the firm with an

[20] We assume that $\psi_{ii}/\psi_{12} \geq 4(\psi_{ii} + \psi_{12})/(\psi_{jj} + \psi_{12})$ for $i, j = 1, 2$, so as to satisfy the conditions of Theorem 9.

FIGURE 2: INTERNAL VERSUS EXTERNAL SALES FORCE.
.... FULL-INFORMATION CONTRACTS
---- JOINT VENTURE/INTERNAL SALES FORCE CONTRACTS
—— INDEPENDENT SALES FORCE CONTRACTS



internal sales force, but are shared by both cooperative and noncooperative common-agency principals. Let π_i^{ex}, π_i^{coop}, and π_i^{ca} be the profits, excluding setup costs, associated with an exclusive sales force, joint venture (i.e., cooperative arrangement), and common agent (i.e., noncooperative arrangement), respectively. We know that $\pi_i^{coop} = \pi_i^{ex} > \pi_i^{ca}$. When, however, setup costs are such that $0 < I < (\pi_i^{coop} - \pi_i^{ca})$, the cooperative arrangement is preferred, followed by an exclusive sales force, and then the non-cooperative common agency relationship. As a consequence, when costs are low, even though principals prefer to share the fixed cost associated with a sales force, they would prefer to expend the extra costs necessary to isolate their agents from their contracting rival when they cannot cooperate.

6.2 Multiple Regulators

Consider the problem of two regulatory agencies, each having power to regulate some aspect of an agent's (e.g., a public utility's) operation. This environment is the rule rather than the exception when it comes to administrative law in the United States. Nevertheless, this problem has received little study. One noteworthy exception is the work of Baron [1985]. Baron considers the problem of the dual regulators. In his example, the Environmental Protection Agency (EPA) regulates the level of pollution which a public utility produces and a local Public Utility Commission (PUC) sets rates and production levels for, say, electricity. The EPA has the ability to move first, promulgating some regulation before the PUC has an opportunity to set rates. We consider a simplified version of the same problem, but with simultaneous contracts.

Let x_1 be the level of pollution abatement which the firm achieves. The EPA has simple preferences:

$$\mathcal{V}^{EPA}(x_1) = \sqrt{x_1} - t_1.$$

Analogously, the PUC has preferences in accord with local consumers who essentially are unaffected by the utility's pollution (e.g., a coal plant produces acid rain which has no effect on local consumers).

$$\mathcal{V}^{PUC}(x_2) = \sqrt{x_2} - t_2.$$

We could, of course, make the preferences of the EPA and the PUC each a function of the firm's profits as well (i.e., make them partially accountable to industry), but we have not done this so as to keep the preferences completely independent.

It is natural to assume that the marginal cost of reducing pollution increases with the level of output. In this case, the contract activities are substitutes. Specifically, let the agent's preferences be like those of the supplier in Section 6.1.2. The agent's final production of x_i is $e_i + \frac{1}{2}\theta$, where θ is some unknown cost-reducing productivity parameter. We assume that θ is uniformly distributed on $[1, 2]$. The agent's preferences are

$$U = t_1 + t_2 + \mathcal{U}(x_1, x_2, \theta) = t_1 + t_2 - \frac{1}{2}(x_1 + x_2 - \theta)^2.$$

Following Propositions 1 and 4,[21] we have 2 simple results.

Result 5 *In a cooperative regulatory regime, x_i are chosen to satisfy*

$$\frac{1}{2}x_i^{-\frac{1}{2}} - (x_1 + x_2 - \theta) - (1 - \theta) = 0.$$

Result 6 *In a symmetric equilibrium with fragmented regulation, the EPA and the local PUC choose each x_i in excess of what they would choose with coordinated regulation (i.e., with joint preferences of $\mathcal{V}(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2} - t$). They each choose x_i to satisfy*

$$\frac{dx_i(\theta)}{d\theta} = -\frac{\frac{1}{2}x_i^{-\frac{1}{2}} - (x_1 + x_2 - \theta) - (1 - \theta)}{\frac{1}{2}x_i^{-\frac{1}{2}} - (x_1 + x_2 - \theta) - 2(1 - \theta)}.$$

This is illustrated in Figure 3. Common agency reduces the distortion in the decision variables which coordinated regulators would otherwise implement. This has several interesting implications for the problem we are examining. First, local consumers and the national constituency for the EPA are worse off. This is due to the costs of common agency. Second, *both the firm and environmentalists are better off* from the high-powered incentive schemes. The firm enjoys more information rents; environmentalists (who we suppose prefer less pollution than the EPA's constituency, perhaps because they pay less taxes) enjoy a more efficient (i.e. lower) level of pollution. This perverse alliance corresponds to that in Laffont-Tirole [1989] where low-powered incentive schemes result from regulatory capture by environmentalists and the regulated firm. In that case both parties gain from collusive arrangements with the regulator to hide information about the firm's costs.

7. Conclusion and Further Remarks

Common agency is as prevalent as a lay person's reading of the term would imply. The main focus of this paper has been to develop a theory of techniques to study common agency and to consider the economic effects of common agency on contractual relationships. We have shown that in such environments, if the agent has private information regarding his gains from the contracting activity and the contracting activities in each possible principal-agent relationship are substitutable (complementary), the principals will typically extract less (more) information rents in total and induce less (more) productive inefficiency in the contracting equilibrium than if there were a single principal contracting over the same activities.

The underlying theme of the results presented is that common agency entails costs for the principals. These costs, in turn, can help explain many economic phenomena

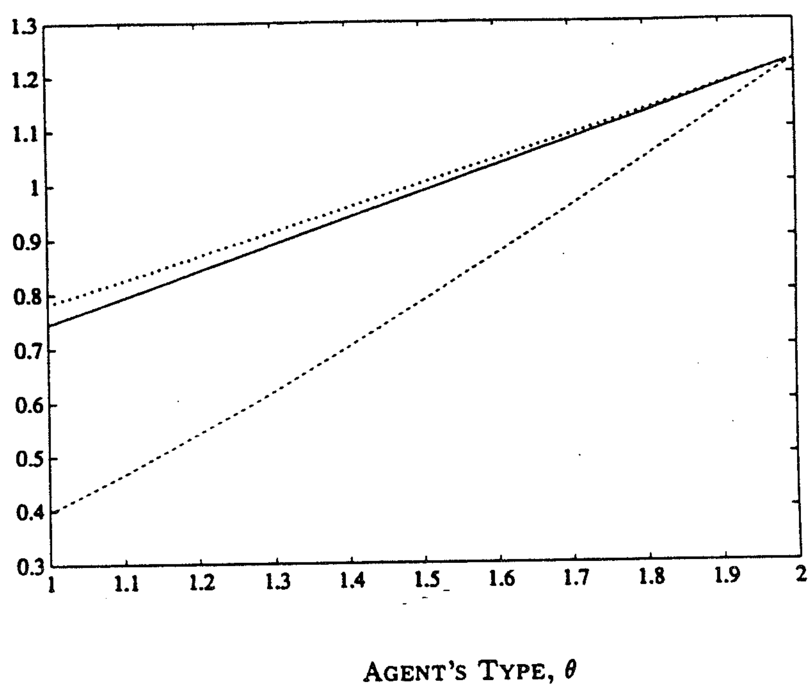
[21] A.5 for Proposition 4 is actually violated in this example. Nonetheless, a numerical examination of the equilibrium contracts reveals that the sufficient condition of $\breve{x}_i(\underline{x}_j, \theta) \geq x_i^{coop}(\underline{x}_j, \theta)$, for all θ, which is used in the proof to Proposition 4, is met. A.5 is merely a simpler sufficient condition (not necessary) to guarantee that the inequality holds.

FIGURE 3: THE EFFECTS OF MULTIPLE REGULATORS

.... FULL INFORMATION REGULATION

---- COORDINATED REGULATION

—— FRAGMENTED REGULATION



which we observe. Additionally, as the analysis on substitutes has suggested, common agency may result in high-powered contracts which extract very little of the agent's information rents. Since typical contracting environments have multiple principals, when the contracting activities are substitutes we should expect to see little use of distortionary contracting to reduce information rents. Consequently, even though an environment might be ideal for selection contracts, such contracts may not be observed due to competition. In identical environments with a single principal (e.g., internal organization of a firm), we would expect such contracting schemes. The fact that we do not see many selection contracts may be evidence of healthy competition rather than an oversight by individuals in the marketplace.

APPENDIX

Proof of Theorem 1: First we show that incentive compatibility implies that $t(\theta)$ is piecewise C^1. By revealed preference

$$U(\theta+\Delta\theta,\theta+\Delta\theta)-U(\theta+\Delta\theta,\theta)\geq U(\theta+\Delta\theta,\theta+\Delta\theta)-U(\theta,\theta)\geq U(\theta,\theta+\Delta\theta)-U(\theta,\theta).$$

Dividing by $\Delta\theta>0$ and taking limits as $\Delta\theta\rightarrow 0$ yields $\frac{dU(\theta,\theta)}{d\theta}=U_\theta(x_1,x_2,t,\theta)$. Thus, incentive compatibility implies that the total differential of $U(\theta,\theta)$ exists everywhere. We can use a Taylor expansion at all but a finite number of points and write

$$\begin{aligned}U(\theta+\Delta\theta,\theta+\Delta\theta)-U(\theta,\theta) &= U_{x_1}(x_1,x_2,t,\theta)x_1'(\theta)\Delta\theta+U_{x_2}(x_1,x_2,t,\theta)x_2'(\theta)\Delta\theta\\ &+ U_t(x_1,x_2,t,\theta)\frac{t(\theta+\Delta\theta)-t(\theta)}{\Delta\theta}\Delta\theta+O(\Delta\theta^2),\end{aligned}$$

for all $\Delta\theta$. Dividing by $\Delta\theta$ we have

$$\frac{t(\theta+\Delta\theta)-t(\theta)}{\Delta\theta}=\frac{U(\theta+\Delta\theta,\theta+\Delta\theta)-U(\theta,\theta)}{\Delta\theta}-U_{x_1}x_1'(\theta)-U_{x_2}x_2'(\theta)-O(\Delta\theta).$$

The limit on the righthand side exists everywhere but at a finite set of points given the piecewise continuity of $x_i'(\theta)$, and thus $t(\cdot)$ is itself piecewise C^1.

A necessary condition for maximization by the agent is the satisfaction of first-order and local second-order conditions at $\hat{\theta}=\theta$, at all points of differentiability:

$$U_{\hat{\theta}}(\theta,\theta)=0,$$

$$U_{\hat{\theta}\hat{\theta}}(\theta,\theta)\leq 0,$$

$\forall\theta\in\Theta$. The first expression immediately gives us (1) above. Totally differentiating this expression with respect to θ yields $U_{\hat{\theta}\hat{\theta}}(\theta,\theta)+U_{\hat{\theta}\theta}(\theta,\theta)=0$, which allows us alternatively to express the local second-order condition as

$$U_{\hat{\theta}\theta}(\theta,\theta)\geq 0,$$

at all points of differentiability. Equivalently,

$$U_{x_1\theta}(x_1,x_2,t,\theta)x_1'(\theta)+U_{x_2\theta}(x_1,x_2,t,\theta)x_2'(\theta)+U_t(x_1,x_2,t,\theta)t'(\theta)\geq 0,$$

for all but a finite set of θ in Θ. Using (1) to eliminate $t'(\theta)$ and simplifying yields (2). Finally, feasibility implies (3) by definition. □

Proof of Theorem 2: We proceed by showing that there exists a function $t(\cdot)$ satisfying (1). Because x_i is piecewise C^1, there exists a finite set of intervals of Θ on which $(U_{x_i}/U_t)x_i'$ is defined and continuous. Piecewise continuity and the boundedness of x_i allows us to take the closure of these intervals and extend the function over each of these compact subsets of Θ. Following Hurewicz [1958, Ch. 1, Theorem 12], A.2 and $U\in C^2$ implies the existence of a solution which satisfies (1) over each of the open intervals, and thus at all points where x_i is differentiable.

To prove that the resulting contracts are globally incentive compatible, suppose otherwise. Let $\hat{\theta}\neq\theta$ be the optimal report for an agent of type θ. By revealed preference, $U(\hat{\theta},\theta)-U(\theta,\theta)>0$. Equivalently,

$$\int_\theta^{\hat{\theta}}U_{\hat{\theta}}(s,\theta)ds>0.$$

Using the fact that (1) holds everywhere except at a finite number of points yields

$$\int_{\theta}^{\hat{\theta}} (U_{\hat{\theta}}(s,\theta) - U_{\hat{\theta}}(s,s))\, ds = \int_{\theta}^{\hat{\theta}} \int_{s}^{\theta} U_{\hat{\theta}\theta}(s,t)dtds > 0.$$

But A.1, together with the assumption of monotonic decision functions, guarantees that $U_{\hat{\theta}\theta}(\hat{\theta},\theta) \geq 0$, which implies that the preference inequality is violated and we obtain a contradiction. Thus, the contracts are commonly implementable.

Given (1) and A.1, the agent's utility is nondecreasing in θ. Therefore, (3) is sufficient for participation by the agent and the contracts are feasible. □

Proof of Proposition 1: Following Mirrlees [1971], we use the agent's indirect utility function: $U(\theta) \equiv \mathcal{U}(x_1(\theta), x_2(\theta), t(\theta))$. Incentive compatibility implies (1) which allows us to write

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \mathcal{U}_\theta(x_1(s), x_2(s), s)ds + U(\underline{\theta}).$$

A.3(a) implies that $t(\theta) = U(\theta) - \mathcal{U}(x_1(\theta), x_2(\theta), \theta)$, and so A.3(b) implies that the sum of the principals' utilities equals

$$\mathcal{V}^1(x_1(\theta), x_2(\theta)) + \mathcal{V}^2(x_1(\theta), x_2(\theta)) + \mathcal{U}(x_1(\theta), x_2(\theta), \theta) - \int_{\underline{\theta}}^{\theta} \mathcal{U}_\theta(x_1(s), x_2(s), s)ds.$$

That is, the principals' joint surplus equals the total gains from trade less information rents which accrue to the agent. Note that partial integration yields

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} \mathcal{U}_\theta(x_1(s), x_2(s), s) f(\theta) ds d\theta = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1-F(\theta)}{f(\theta)} \mathcal{U}_\theta(x_1(\theta), x_2(\theta), \theta) f(\theta) d\theta.$$

From Theorem 2 and A.1, we know that incentive compatibility and agent participation is satisfied if (1), (3) and monotonicity hold. We have already used (1) to substitute out transfers from the maximization problem. Once we obtain the optimal decision functions, we use (1) to determine the transfer function, which exists by A.2. This yields (5) in the Proposition. It is clear that the maximization of principals' utility requires that (3) be binding; it is never profitable to leave information rents to the lowest type agent. Ignoring monotonicity and boundary considerations (i.e., $x_i \in \mathcal{X}$), the principals' relaxed problem reduces to maximizing the expectation of their joint virtual utility

$$\mathcal{V}^1(x_1(\theta), x_2(\theta)) + \mathcal{V}^2(x_1(\theta), x_2(\theta)) + \mathcal{U}(x_1(\theta), x_2(\theta), \theta) - \frac{1-F(\theta)}{f(\theta)} \mathcal{U}_\theta(x_1(\theta), x_2(\theta), \theta).$$

Because the integrand is continuous over a compact set $\mathcal{X}$, a solution exists for each θ. Maximizing the integrand pointwise in θ yields (4) $\forall\theta \in [\theta_i^*, \bar{\theta}]$, and $x_i(\theta) = 0\ \forall\theta \in [\underline{\theta}, \theta_i^*)$. A.4 implies that the integrand is globally concave in x_i, and so the first-order conditions are sufficient. Note that the joint virtual utility evaluated at (4) is increasing in θ because $\mathcal{U}_{\theta\theta} \leq 0$ (A.4).

Suppose $\theta_1^* \leq \theta_2^*$. Then θ_2^* is chosen to satisfy

$$\mathcal{V}^2(x_1, x_2) + \mathcal{U}(x_1, x_2, \theta_2^*) - \frac{1-F(\theta_2^*)}{f(\theta_2^*)} \mathcal{U}_\theta(x_1, x_2, \theta_2^*) = 0.$$

This completely defines x_2 over Θ. We now choose θ_1^* to satisfy

$$\mathcal{V}^1(x_1,0)+\mathcal{U}(x_1,0,\theta_1^*)-\frac{1-F(\theta_1^*)}{f(\theta_1^*)}\mathcal{U}_\theta(x_1,0,\theta_1^*)=0.$$

This completely determines x_1 over Θ. A similar exercise is used when $\theta_1^* > \theta_2^*$. In either case, the choice of θ_i^* satisfy the conditions of the Proposition.

We now check that the monotonicity and boundary conditions are satisfied. Totally differentiating (4), together with A.4, imply that each x_i is nondecreasing in θ, thereby satisfying (2). Because each x_i is nondecreasing in θ, A.4(b) implies that the maximum value of each x_i is in $\mathcal{X}$. □

Proof of Proposition 2: (Sketch) Proposition 2 follows from the arguments used in Proposition 1, with the exception of proving monotonicity of x_i the existence of a single pair of cutoff types, θ_i^*. Supposing that x_1, x_2 satisfy (6) over $[\theta_i^*, \bar{\theta}]$, we need to show that $x_i'(\theta) \geq 0$ and that a cutoff point, θ_i^*, exists such that if and only if $\theta \geq \theta_i^*$ are principal i's profits nonnegative.

(6) provides a system of two equations that define x_1, x_2. Totally differentiating this system with respect to x_1, x_2, and θ, and using Cramer's rule to solve for $x_i'(\theta)$ yields $x_i'(\theta) \geq 0$ in light of A.4'(b). Furthermore, given the condition in A.4'(c) which requires

$$(\mathcal{V}^i_{x_j}-\frac{1-F}{f})\mathcal{U}_{\theta x_2}x_j'(\theta)-\frac{1-F}{f}\mathcal{U}_{\theta\theta}+\left[1-\frac{d}{d\theta}\left(\frac{1-F}{f}\right)\right]\mathcal{U}_\theta \geq 0,$$

principal i's objective function increases in θ. □

Proof of Theorem 5: Following Theorem 4, it is sufficient to show that (15) is satisfied for any pair of nondecreasing decision functions. That is,

$$\begin{aligned}\Lambda(\hat{\theta}_1,\hat{\theta}_2,\theta) \equiv\; & \int_\theta^{\hat{\theta}_2}\int_\theta^{\hat{\theta}_1}\mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dsdt \\ & + \int_\theta^{\hat{\theta}_1}\int_s^{\theta}\left[\mathcal{U}_{x_1x_2}(s,t,t)x_1'(s)x_2'(t)+\mathcal{U}_{x_1\theta}(s,t,t)x_1'(s)\right]dtds \\ & + \int_\theta^{\hat{\theta}_2}\int_t^{\theta}\left[\mathcal{U}_{x_1x_2}(s,t,\bar{s})x_1'(s)x_2'(t)+\mathcal{U}_{x_2\theta}(s,t,s)x_2'(t)\right]dsdt \leq 0,\end{aligned}$$

$\forall(\hat{\theta}_1,\hat{\theta}_2,\theta) \in \Theta^3$. Note that we can decompose the first double integral:

$$\begin{aligned}\int_\theta^{\hat{\theta}_2}\int_\theta^{\hat{\theta}_1}\mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dsdt = & \int_\theta^{\hat{\theta}_1}\int_\theta^{\gamma+\beta s}\mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dtds \\ & + \int_\theta^{\hat{\theta}_2}\int_\theta^{\frac{t-\gamma}{\beta}}\mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dsdt,\end{aligned}$$

where $\beta = \frac{\hat{\theta}_2-\theta}{\hat{\theta}_1-\theta}$, and $\gamma = \theta(1-\beta)$. Thus,

$$\begin{aligned}
\Lambda(\hat{\theta}_1,\hat{\theta}_2,\theta) &= \int_\theta^{\hat{\theta}_1}\int_s^\theta \left[\mathcal{U}_{x_1x_2}(s,t,t)x_2'(t) - \mathcal{U}_{x_1x_2}(s,t,\theta)x_2'(t) + \mathcal{U}_{x_1\theta}(s,t,t)\right] x_1'(s)dtds \\
&+ \int_\theta^{\hat{\theta}_2}\int_t^\theta \left[\mathcal{U}_{x_1x_2}(s,t,s)x_1'(s) - \mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s) + \mathcal{U}_{x_2\theta}(s,t,s)\right] x_2'(t)dsdt \\
&+ \int_\theta^{\hat{\theta}_1}\int_s^{\gamma+\beta s} \mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dtds + \int_\theta^{\hat{\theta}_2}\int_t^{\frac{t-\gamma}{\beta}} \mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dsdt.
\end{aligned}$$

Integrating yields

$$\begin{aligned}
\Lambda(\hat{\theta}_1,\hat{\theta}_2,\theta) &= \int_\theta^{\hat{\theta}_1}\int_s^\theta \left\{\mathcal{U}_{x_1\theta}(s,t,t) + \int_\theta^t \mathcal{U}_{x_1x_2\theta}(s,t,u)x_2'(t)du\right\} x_1'(s)dtds \\
&+ \int_\theta^{\hat{\theta}_2}\int_t^\theta \left\{\mathcal{U}_{x_2\theta}(s,t,s) + \int_\theta^s \mathcal{U}_{x_1x_2\theta}(s,t,u)x_1'(s)du\right\} x_2'(t)dsdt \\
&+ \int_\theta^{\hat{\theta}_1}\int_s^{\gamma+\beta s} \mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dtds + \int_\theta^{\hat{\theta}_2}\int_t^{\frac{t-\gamma}{\beta}} \mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dsdt.
\end{aligned}$$

But note that we can combine the last two terms to obtain

$$\begin{aligned}
\Lambda(\hat{\theta}_1,\hat{\theta}_2,\theta) &= \int_\theta^{\hat{\theta}_1}\int_s^\theta \left\{\mathcal{U}_{x_1\theta}(s,t,t) + \int_\theta^t \mathcal{U}_{x_1x_2\theta}(s,t,u)x_2'(t)du\right\} x_1'(s)dtds \\
&+ \int_\theta^{\hat{\theta}_2}\int_t^\theta \left\{\mathcal{U}_{x_2\theta}(s,t,s) + \int_\theta^s \mathcal{U}_{x_1x_2\theta}(s,t,u)x_1'(s)du\right\} x_2'(t)dsdt \\
&+ \int_{\hat{\theta}_1}^{\hat{\theta}_2}\int_s^{\hat{\theta}_1} \mathcal{U}_{x_1x_2}(s,t,\theta)x_1'(s)x_2'(t)dtds.
\end{aligned}$$

Given our assumptions about monotonicity and $\mathcal{U}_{x_1x_2\theta}$, it is straightforward to verify that each of the three terms in Λ are necessarily nonpositive. Thus (15) is satisfied and the pair of contracts is commonly implementable. □

Proof of Theorem 6: Following Theorem 4, it is sufficient to show that (15) is satisfied under the conditions on $\mathcal{U}$ providing that the necessary conditions in (13)-(14) are satisfied and each x_i is nondecreasing. That is, we take as given for all θ

$$\begin{aligned}
u_{1\theta} + u_{12}x_2'(\theta) &\geq 0, \\
u_{2\theta} + u_{12}x_1'(\theta) &\geq 0, \\
u_{1\theta}u_{2\theta} + u_{12}(u_{1\theta}x_1'(\theta) + u_{2\theta}x_2'(\theta)) &\geq 0.
\end{aligned}$$

First, note that (15) can be simplified under our conditions on monotonicity and $\mathcal{U}$:

$$\begin{aligned}
\int_\theta^{\hat{\theta}_2}\int_\theta^{\hat{\theta}_1} u_{12}x_1'(s)x_2'(t)dtds \quad &+ \quad \int_\theta^{\hat{\theta}_1}\int_s^\theta \left[u_{12}x_1'(s)x_2'(t) + u_{1\theta}x_1'(s)\right] dtds \\
&+ \quad \int_\theta^{\hat{\theta}_2}\int_t^\theta \left[u_{12}x_1'(s)x_2'(t) + u_{2\theta}x_2'(t)\right] dsdt \leq 0.
\end{aligned}$$

Using (14),

$$u_{1\theta}x_1'(s) + u_{12}x_1'(s)x_2'(t) \geq -u_{12}\frac{u_{1\theta}}{u_{2\theta}}x_1'(t)x_1'(s),$$

and

$$u_{2\theta}x_2'(t) + u_{12}x_1'(s)x_2'(t) \geq -u_{12}\frac{u_{2\theta}}{u_{1\theta}}x_2'(t)x_2'(s).$$

Using the fact that $u_{12} < 0$, it is sufficient for (15) that

$$\int_\theta^{\hat\theta_2}\int_\theta^{\hat\theta_1} x_1'(s)x_2'(t)dtds - \int_\theta^{\hat\theta_1}\int_s^{\theta} \frac{u_{1\theta}}{u_{2\theta}}x_1'(t)x_1'(s)dtds - \int_\theta^{\hat\theta_2}\int_t^{\theta} \frac{u_{2\theta}}{u_{1\theta}}x_2'(t)x_2'(s)dsdt \geq 0.$$

Consider the three terms independently. After simplification,

$$\int_\theta^{\hat\theta_2}\int_\theta^{\hat\theta_1} x_1'(s)x_2'(t)dtds = [x_1(\hat\theta_1) - x_1(\theta)][x_2(\hat\theta_2) - x_2(\theta)],$$

$$\int_\theta^{\hat\theta_1}\int_s^{\theta} -\frac{u_{1\theta}}{u_{2\theta}}x_1'(t)x_1'(s)dtds = -\frac{u_{1\theta}}{2u_{2\theta}}[x_1(\hat\theta_1) - x_1(\theta)]^2,$$

$$\int_\theta^{\hat\theta_2}\int_t^{\theta} -\frac{u_{2\theta}}{u_{1\theta}}x_2'(t)x_2'(s)dsdt = -\frac{u_{2\theta}}{2u_{1\theta}}[x_2(\hat\theta_2) - x_2(\theta)]^2.$$

The sum of these expressions forms a binomial which can be simplified to yield

$$\left\{u_{1\theta}[x_1(\hat\theta_1) - x_1(\theta)] - u_{2\theta}[x_2(\hat\theta_2) - x_2(\theta)]\right\}^2 \geq 0.$$

Thus, (15) holds. □

Proof of Proposition 3: Again we use the agent's indirect utility function:

$$U(\theta) \equiv \mathcal{U}(x_1(\theta), x_2(\theta), \theta) + t_1(\theta) + t_2(\theta).$$

Incentive compatibility implies (12), which allows us to write

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\partial \mathcal{U}(x_1(s), x_2(s), s)}{\partial\theta}ds + U(\underline{\theta}).$$

A.3 implies that $t_1(\theta) + t_2(\theta) = U(\theta) - \mathcal{U}(x_1(\theta), x_2(\theta), \theta)$. We first analyze the problem of Principal 1. A.3(b) implies that her gain from an incentive compatible exchange (but not necessarily an exchange which is incentive compatible for her rival's contract) is

$$\mathcal{V}^1(x_1(\theta)) + \mathcal{U}(x_1(\theta), x_2(\hat\theta_2[\theta|x_1(\theta)]), \theta) - \int_{\underline{\theta}}^{\theta} \frac{\partial \mathcal{U}(x_1(s), x_2(\hat\theta_2[s|x_1(s)]), s)}{\partial\theta}ds$$

$$+ t_2(\hat\theta_2[\theta|x_1(\theta)]) - U(\underline{\theta}),$$

providing that $\hat\theta_2 \in (\underline{\theta}, \overline{\theta})$.

Principal 1's surplus equals the total gains from trade in x_1 less information rents which accrue to the agent plus the agent's compensation from principal 2. Partial integration allows us to conclude

$$\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\theta}\frac{\partial\mathcal{U}(x_1(s),x_2(\hat{\theta}_2[s|x_1(s)]),s)}{\partial\theta}f(\theta)dsd\theta =$$
$$\int_{\underline{\theta}}^{\overline{\theta}}\frac{1-F(\theta)}{f(\theta)}\frac{\partial\mathcal{U}(x_1(\theta),x_2(\hat{\theta}_2[\theta|x_1(\theta)]),\theta)}{\partial\theta}f(\theta)d\theta.$$

From Theorem 5 we know that incentive compatibility and agent participation are satisfied provided that (12), and monotonicity hold. We have already used (12) to substitute out transfers from the maximization problem. Once we obtain the optimal decision functions, we use (12) to determine the transfer function.

$$\int_{\underline{\theta}}^{\overline{\theta}}\Bigg\{\mathcal{V}^1(x_1(\theta))+\mathcal{U}(x_1(\theta),x_2(\hat{\theta}_2[\theta|x_1(\theta)]),\theta)-$$
$$\frac{1-F(\theta)}{f(\theta)}\frac{\partial\mathcal{U}(x_1(\theta),x_2(\hat{\theta}_2[\theta|x_1(\theta)]),\theta)}{\partial\theta}+t_2(\hat{\theta}_2[\theta|x_1(\theta)])-U(\underline{\theta})\Bigg\}f(\theta)d\theta.$$

By A.4", the solution to the relaxed program can be found by differentiating the integrand pointwise in θ and setting the result equal to zero yielding (16) if we can be certain that principal 1 finds it optimal that $\hat{\theta}_2\in(\underline{\theta},\overline{\theta})$, for all $\theta\in(\underline{\theta},\overline{\theta})$.

To see that bunching at $\underline{\theta}$ is not optimal for principal 1, consider the functions $x_1^{coop}(\underline{x}_2,\theta)$ and $\check{x}_1(\underline{x}_2,\theta)$. The first function is defined as the value of x_1 which principal 1 would prefer to choose if principal 2 always offered $\underline{x}_2=x_2(\underline{\theta})$. The second is the maximum value of x_1 which principal 1 can offer to agent θ in order to induce the agent to choose the $\{x_1(\theta),\underline{x}_2\}$ allocation. If $x_1^{coop}(\underline{x}_2,\theta)\geq\check{x}_1(\underline{x}_2,\theta)$, then the constraint facing the principal who wishes to induce bunching at $\underline{\theta}$ must be binding. If it binds, the first-order condition of the agent is satisfied, and the program above which uses the first-order approach is valid. To see that the sufficient inequality above holds, note that at $\underline{\theta}$ it must be the case (since $\mathcal{U}_{x_1x_2}>0$) that $x_1^{coop}(\underline{x}_2,\underline{\theta})\geq\check{x}_1(\underline{x}_2,\underline{\theta})$. Furthermore, under our assumptions in A.4", x_1^{coop} is increasing in θ while $\check{x}_1$ is decreasing in θ. Thus, the desired inequality holds. A similar argument establishes that with complements, bunching is never optimal at $\overline{\theta}$.

Given that a nondecreasing solution to (16) exists, our assumption that $\mathcal{U}_{x_1x_2\theta}\leq 0$ implies that the contracts are commonly implementable. A.4" implies that an α_i exists such that it is optimal for all θ to be served by each principal. Providing that the transfers are chosen as in the Proposition, the contracts are globally incentive compatible and individually rational. □

Proof of Theorem 8: The proof follows directly from Proposition 3, except that we must additionally show that a continuum of symmetric, nondecreasing solution to the differential equations in (16) exists. Define $s(x,\theta)\equiv\mathcal{V}_x(x)+\mathcal{U}_x(x,x,\theta)$ and define the surface

$$\mathcal{D}\equiv\{x,\theta|\theta\in[\underline{\theta},\overline{\theta}),N(x,\theta)\geq 0,D(x,\theta)<0\},$$

where $N(x,\theta)\equiv s(x,\theta)-\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_\theta(x,x,\theta)$ and $D(x,\theta)\equiv s(x,\theta)-2\frac{1-F(\theta)}{f(\theta)}\mathcal{U}_\theta(x,x,\theta)$. Our assumptions on $\mathcal{U}$ imply that there is a unique x for each θ such that $N(x,\theta)=0$; this point lies in $\mathcal{D}$, and so the latter is nonempty. Furthermore, our assumptions imply that $\frac{\partial D}{\partial x}<0$, and that along the curve defined by $N(x,\theta)=0$ we have $\frac{dx}{d\theta}>0$. As a consequence, we have the curve given by N lying above the curve given by $D(x,\theta)=0$ over the domain of Θ, with the former having positive slope everywhere.

Manipulating the differential equation given in Proposition 3 and using symmetry implies that

$$x'(\theta)=-\frac{\mathcal{U}_{x\theta}(x,x,\theta)}{\mathcal{U}_{x_1x_2}(x,x,\theta)}\frac{N(x,\theta)}{D(x,\theta)}.$$

Thus, if a differential equation exists in $\mathcal{D}$, it necessarily has the desired monotonicity property. Choose any point in $\mathcal{D}$ and consider its direction of movement. It cannot cross the $N(x,\theta)$ locus from below, as the derivative in the neighborhood of N is 0 and the locus N has strictly positive slope. It cannot cross the D locus from above as $x'\to+\infty$ as x approaches D and the locus of points satisfying D has finite slope. Thus, any point in $\mathcal{D}$ remains in $\mathcal{D}$; and moreover, in any neighborhood, x' locally satisfies a Lipschitz condition. Following Hurewicz [1958, Chapter 2, Theorem 12], a global differential equation exists which satisfies the equation in Proposition 3. Additionally, such an equation exists for any initial point in the half-open interval $\mathcal{D}(\underline{\theta})=\{x,\underline{\theta}|N(x,\underline{\theta})\geq 0, D(x,\underline{\theta})<0\}$. We thus have a continuum of nondecreasing solutions. □

Proof (Sketch) of Proposition 4: Proposition 4 follows from the analysis of Proposition 3, except in so far as we must check that A.5 is sufficient for corner bunching to be suboptimal.

First, note that bunching will never occur at $\overline{\theta}$. If principal 1 chooses to induce bunching by the agent on Principal 2's contract, the higher level of induced x_2 will result in both more information rents being paid to the agent by Principal 1, as well as reduced profits from lower purchases from the agent.

Second, consider bunching at $\underline{\theta}$. As in the proof to Proposition 3, it is sufficient to show that $x_1^{coop}(\underline{x}_2,\theta)\leq\check{x}_1(\underline{x}_2,\theta)$, where $\check{x}_1(\underline{x}_2,\theta)$ is now the *minimum* value of x_1 which principal 1 can offer to agent θ in order to induce the agent to choose the $\{x_1(\theta),\underline{x}_2\}$ allocation. In such a case the constraint facing the principal who wishes to induce bunching at $\underline{\theta}$ must be binding. If it binds, the first-order condition of the agent is satisfied, and the program above which uses the first-order approach is valid. To see that the sufficient inequality above holds, note that at $\underline{\theta}$ it must be that (since $\mathcal{U}_{x_1x_2}<0$) $x_1^{coop}(\underline{x}_2,\underline{\theta})<\check{x}_1(\underline{x}_2,\underline{\theta})$. Furthermore, under our assumptions in A.5, x_1^{coop} is increasing at a slower rate in θ than is $\check{x}_1$. Thus, the desired inequality holds. □

Proof of Theorem 9: Define the quadratic preferences as follows:

$$\mathcal{V}^i(x_i)\equiv v_0^i+v_ix_i+\frac{1}{2}v_{ii}x_i^2,\ i=1,2,$$

$$\mathcal{U}(x_1,x_2,\theta)\equiv u_0+(u_1+u_{1\theta}\theta)x_1+(u_2+u_{2\theta}\theta)x_2+u_{12}x_1x_2+\frac{u_{11}}{2}x_1^2+\frac{u_{22}}{2}x_2^2.$$

We look for linear solutions of the form $x_i=\overline{x}_i^{eff}-\lambda_i(\underline{\theta}-\theta)$, where $\overline{x}_i^{eff}$ is the efficient allocation given that $\theta=\overline{\theta}$. From Theorem 6 and Proposition 4, we need only show that of the linear solutions to (18), there is a unique pair $\{\lambda_1,\lambda_2\}$ such that each $\lambda_i\geq 0$ and (13)-(14) are satisfied (i.e., $\lambda_i\leq-\frac{u_{j\theta}}{u_{12}}$ and $u_{1\theta}u_{2\theta}+u_{12}(u_{1\theta}\lambda_1+u_{2\theta}\lambda_2)\geq 0$).

Substituting the candidate linear solutions into (18) and simplifying yields, for $i=1,2$,

$$v_i+v_{ii}(\overline{x}_i^{eff}-\lambda_i(\overline{\theta}-\theta))+u_i+u_{i\theta}\theta+u_{12}(\overline{x}_j^{eff}-\lambda_j(\overline{\theta}-\theta))+u_{ii}(\overline{x}_i^{eff}-\lambda_i(\overline{\theta}-\theta))$$

$$= \gamma(\overline{\theta} - \theta)\left(u_{i\theta} + \frac{u_{j\theta}\lambda_j u_{12}}{u_{j\theta} + u_{12}\lambda_i}\right).$$

This expression must hold for any θ.

First, note that if some $\lambda_i = 0$, the above expression cannot be true. For example, $\lambda_1 = 0$ implies that $\lambda_2 = -\frac{u_{1\theta}}{u_{12}}$, but then the optimal choice for $\lambda_1 \neq 0$. Hence, no fixed point can contain a zero component and we can treat λ_i as a nonzero number. Second, note that since the above expression must hold true for all θ, a necessary and sufficient condition for $\{\lambda_1, \lambda_2\}$ is that the coefficients of θ sum to zero. That is, for $i = 1, 2$,

$$(23) \qquad -(u_{j\theta} + u_{12}\lambda_i)(u_{i\theta} + u_{12}\lambda_j + (v_{ii} + u_{ii})\lambda_i) = \gamma(u_{1\theta}u_{2\theta} + u_{12}(u_{1\theta}\lambda_1 + u_{2\theta}\lambda_2)).$$

(23) provides a system of 2 quadratic equations in 2 unknowns. The solution to such a problem, if one exists, may have up to four possible roots. Solving (23) for λ_1 as a function of λ_2, we obtain two functions representing the two roots from the quadratic formula: $\lambda_1^-(\lambda_2)$ and $\lambda_1^+(\lambda_2)$. We can obtain similar functions for λ_2. The four possible roots correspond to the four possible fixed points which may exist with these functions. Two of these solutions have zero components, $\{(0, -\frac{u_{1\theta}}{u_{12}}), (-\frac{u_{2\theta}}{u_{12}}, 0)\}$, and result because we rightly assumed $\lambda_i > 0$ when we simplified (18). The two remaining candidates consist of the fixed points in $(\lambda_1^-, \lambda_2^-)$ and $(\lambda_1^+, \lambda_2^+)$. It is straightforward to verify that the latter pair of functions map to a set which violates (13)-(14). We must show that $(\lambda_1^-, \lambda_2^-)$ has a fixed point with the desired properties.

An examination of $(\lambda_1^-, \lambda_2^-)$ indicates that

$$(\lambda_1^-(\cdot), \lambda_2^-(\cdot)) : \quad [0, -\frac{u_{1\theta}}{u_{12}}] \times [0, -\frac{u_{2\theta}}{u_{12}}] \quad \mapsto \quad [0, -(1+\gamma)\frac{u_{1\theta}}{v_{11} + u_{11}}] \times [0, -(1+\gamma)\frac{u_{2\theta}}{v_{22} + u_{22}}],$$

and such a function is continuous. By assumption, the range is contained in the domain, and so we may apply Brouwer's theorem to establish the existence of a fixed point. Such a solution satisfies (13)-(14) and so it is incentive compatible. Moreover, it is straightforward to check that the fixed point consists of a strictly positive solution.

Next, we must check that a principal does not find it desirable to induce bunching at the corner of her rival's contract. By assumption, the conditions of A.5 are met, so bunching at a corner is not optimal.

Finally, we must show that the agent prefers the common agency environment, and the principals prefer the cooperative outcome. This follows from Corollary 6. □

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