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THE ECONOMICS OF LIQUIDATED DAMAGE CLAUSES IN CONTRACTUAL ENVIRONMENTS WITH PRIVATE INFORMATION

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Abstract

This paper considers the potential use of liquidated damage clauses under asymmetric information. Courts typically allow parties to stipulate the damages each will pay the other in event of breach, providing that such liquidated damage terms *do not* greatly *exceed* actual losses. This restriction acts as a ceiling, however, as courts generally enforce terms that are equal to or *below* actual losses. This anomaly can be explained when bargaining occurs under asymmetric information. Here, the liquidated damage clause serves a dual role, both promoting efficient breach and signaling a party's valuation of trade. As a result it is always optimal for parties to set damages at or below valuation, thereby providing a consistent theory for the courts' asymmetric treatment of contractual damages: When damages are significantly below actual losses, courts may plausibly maintain the presumption that the contract is the result of rational bargaining.

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ABSTRACT. This paper considers the potential use of liquidated damage clauses under asymmetric information. Courts typically allow parties to stipulate the damages each will pay the other in event of breach, providing that such liquidated damage terms *do not* greatly *exceed* actual losses. This restriction acts as a ceiling, however, as courts generally enforce terms that are equal to or *below* actual losses. This anomaly can be explained when bargaining occurs under asymmetric information. Here, the liquidated damage clause serves a dual role, both promoting efficient breach and signaling a party's valuation of trade. As a result it is always optimal for parties to set damages at or below valuation, thereby providing a consistent theory for the courts' asymmetric treatment of contractual damages: When damages are significantly below actual losses, courts may plausibly maintain the presumption that the contract is the result of rational bargaining.

1. INTRODUCTION

Economists have long recognized that agreements freely entered into by all effected parties with full information and cognizance of the terms of trade necessarily improve social welfare in the traditional Pareto sense. It comes as no surprise that economists look at the law with skepticism whenever courts invalidate mutually

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agreed upon terms within a contract. Nonetheless, courts have routinely decided to invalidate contractually stipulated damages for breach of contract (commonly known as liquidated damages) when such damages are "unreasonably large" relative to actual or expected losses, but not those that are unreasonably small.[1] Even the courts themselves often do not know why they do what they do.

> [T]he ablest of judges have declared that they felt themselves embarrassed in ascertaining the principle on which the decisions [distinguishing penalties from liquidated damages] were founded. Cotheal v. Talmadge, 9 N.Y. 551, 553 (1854).

The invalidation of excessive stipulated damage clauses is difficult to justify economically. Liquidated damage clauses promote efficiency in contractual relationships by reducing the litigation and judicial costs which accompany breach, by providing the correct incentives for a breaching party, and by optimally allocating risk.[2] Most importantly, stipulation of damages by the parties rather than by judicial determination allows parties to efficiently utilize their superior information which typically courts can only imperfectly access.

The courts have had difficulty motivating the invalidation of excessive stipulated damage clauses as penalties. One theory often presented by legal scholars posits that legal remedies for breach of contract serve only to compensate and n-

[1]See, *Uniform Commercial Code*, §§2-302(1), 2-718(1), and the *Restatement of Contracts (Second)*, §§208, 356. The *U.C.C.*, §2-718(1) maintains:

> Damages for breach by either party may be liquidated in the agreement but only at an amount which is reasonable in the light of the anticipated or actual harm caused by the breach, the difficulties of proof of loss, and the inconvenience or nonfeasibility of otherwise obtaining an adequate remedy. A term fixing unreasonably large liquidated damages is void as a penalty.

The *Restatement of Contracts (Second)*, §356(1), similarly maintains:

> Damages for breach by either party may be liquidated in the agreement but only for an amount that is reasonable in the light of the anticipated or actual loss caused by the breach and the difficulties of the proof of loss. A term fixing unreasonably large liquidated damages is unenforceable on grounds of public policy as a penalty.

[2]Shavell [1980] analyzes the use of damage remedies to provide incentives for efficient breach. Although Shavell does not explicitly entertain the idea of stipulated damages, his analysis is closely related. In complementary work, Polinsky [1983] has shown that in some instances it is efficient from a risk-allocation viewpoint to contract for stipulated damages in excess of the actual loss from breach. Such conditions require, among other things, that the buyer should bear some of the price risk introduced from third-party, breach-inducing offers. Rea [1984, p.154], however, has argued that these conditions are rare.

ever to punish.[3] Such a principle has economic merit. We ordinarily want parties to breach contracts when it is economically efficient that they do so. By making the promisor more than compensate the loss incurred from his nonperformance, the contract induces a suboptimal level of breach.

Unfortunately, this simple explanation falls short on two points. First, why would rational individuals agree to such a contract when there exists another contract that sets damages at the value of performance and which makes both parties better off? Second, why do the courts fail to extend this operating principle to situations of under-compensatory stipulated damage agreements which produce a super-optimal level of breach?

Many courts and legal scholars answer the first question by arguing that excessive liquidated damages are presumptive evidence of a contractual failure such as fraud or mutual mistake. Arguably, courts view excessive damages as evidence that at least one party has wrongly agreed to a contract that is not Pareto improving,[4] and respond by striking such clauses. But this does not explain why the court does not also strike extremely low liquidated damage clauses which presumably are also the product of contractual failures. We are left with an apparent asymmetry in legal principles.

This paper provides a partial explanation for the lack of legal symmetry: Undercompensatory damages are the likely result of the rational decision of two individuals bargaining in an environment where each possesses private information about the exchange. The same cannot be said for supercompensatory damages. Because the low damages do not necessarily represent a contractual failure but can

[3]Farnsworth [1982, p.896] has indicated in his treatise on contract law such a principle of compensation:

> If ... the stipulated sum is significantly larger than the amount required to compensate the injured party for his loss, the stipulation may have a quite different advantage to him – an *in terrorem* effect on the other party that will deter breach by compelling him to perform. Enforcement of such a provision would allow the parties to depart from the fundamental principle that the law's goal on breach of contract is not to deter breach by compelling the promisor to perform, but rather to redress breach by compensating the promisee. It is this departure that is proscribed when a court characterizes such a provision as a penalty.

[4]Aghion-Bolton [1987] provide an additional story. Two individuals may desire to sign a contract which assigns excessive liquidated damages for breach so as to foreclose entry by another supplier. Of course, these damages are socially inefficient. In a related paper, Diamond-Maskin [1979] consider the joint problems of breach and search for new trading partners. They find that because an individual who breaches can get his new partner to share the burden of the liquidated damage he pays to his old partner, a pair of partners in a contract exerts some monopoly power over potential partners, thereby making liquidated damages supercompensatory.

realistically reflect a jointly beneficial contract arrived at under the constraints of asymmetric information, legal institutions are arguably consistent in enforcing such terms. This essay's principle thesis maintains that when each party to a contract possesses private information whose disclosure would adversely affect its position in the contractual bargaining, rationally calculated liquidated damages will be set at *under-compensatory* levels. Thus, although this paper does not explain why the court would want to invalidate excessive damages (or why parties would ever want to write such contracts), it does partially answer the riddle of why courts enforce liquidated damage clauses set below the level of actual loss from breach.

Current economic analysis of liquidated damage clauses has been generally limited to symmetrically informed parties. In many contractual situations, however, the assumption that parties entered into the contract without private information is not palatable. When such asymmetries in information are present, the liquidated damage clause takes on dual roles: (i) providing incentives for efficient breach, and (ii) efficiently screening among different types of buyers and sellers. Specifically, this paper demonstrates that when parties have asymmetric information, stipulated damages may be used to communicate valuable information at the pre-contractual stage. As such, the loss from insufficient or excessive breach may be offset by informational gains. In fact, in the typical buyer-seller contract where each party has private information, stipulated damages will almost always fall short of actual losses from the breach.[5]

This paper examines the buyer-seller relationship, although its results appear much more general. We assume that the buyer has private information regarding the value of the product to herself, and that the seller has private information regarding alternative markets where the product may be sold absent a sale to the present buyer. The contractual framework is modeled in Section 2. In Section 3 we examine various bargaining situations. In section 3.1, we analyze the consequences of placing all of the bargaining power in the hands of the buyer; in Section 3.2, we assume all of the bargaining power resides with the seller.[6] Later, in Section 3.3, we examine what an efficient broker would assign as stipulated damages, thereby providing an upper bound on the joint gains from exchange under any mechanism and giving us insight into the nature of more general efficient contracting mecha-

[5] Schwartz [1990] has established a weaker version of this result for two-type distributions when private information exists only on the buyer's side and the seller has the bargaining power; this case is similar to the contracts developed in Section 3.2 of this paper. This result was pointed out to the author after the present research was completed.

[6] Placing all of the bargaining power in the hands of one party manifests itself as the opportunity of the party to write a contract and make a take-it-or-leave-it offer to the other.

nisms. In all of the contracting scenarios, we find that there is no role for excessive stipulations, but there is a positive role for under-compensatory terms. Indeed, under-compensatory terms occur with probability one. These terms provide a valuable method for both parties to signal to each other their private information, increasing the gains from trade.

Section 4 examines the policy question of whether a perfectly informed court would generally improve matters by requiring that all stipulated damages be exactly compensating. We find that under plausible conditions, even a perfectly informed court can be a menace to the parties' contract and to social welfare if it naively imposes a requirement that liquidated damages equal actual value, ex post. There is a direct benefit and a direct cost from judicial intervention. Eliminating the agents' abilities to set liquidated damages below valuation reduces inefficient breach of contract. If $\ell = v$, the seller will breach only if it is efficient to do so. Unfortunately, such a restriction on liquidated damages also restricts the offerer to pooling contracts which set a single price. This restriction may lead to buyer-designed contracts which only induce trade with low-opportunity sellers, and seller-designed contracts which only induce trade with high-value buyers. Consequently, some individuals may be foreclosed from trade, leaving unrealized gains from exchange. These results are analogous to the social planner's decision of whether to allow second-degree price discrimination in the context of monopoly pricing. Section 5 summarizes and concludes.

2. The Contractual Framework

We examine the contractual relationship between a buyer and a seller, where third-party offers for the seller's services may induce breach after an agreement has been reached. The buyer and seller recognize this possibility and bargain both over price and a damage stipulation which the seller agrees to pay the buyer in event of non-performance.[7]

The buyer (she) and a seller (he) contract to trade a single good at date 1. After contracting, the buyer cannot find other sellers (e.g., the buyer makes relation-specific investments or her outside opportunities disappear) but the seller's opportunism is constrained by the non-performance damage terms of the contract. At date 2 a third-party offer is made to the seller for his wares. The seller can either

[7] The character of the results remain unchanged if one considers instead that the buyer may breach after finding an alternative product. In this alternative, both parties negotiate damages which the buyer will compensate the seller for the lost transaction. The issue of robustness is briefly discussed in Section 7.

accept the third-party's offer and pay the buyer the stipulated damages, or deliver the product to the buyer as promised.

The buyer has valuation, v, distributed according to the continuous, positive density function, $f(\cdot)$, on $[\underline{v}, \overline{v}]$, with cumulative distribution, $F(\cdot)$. Only the buyer knows v, although its distribution is common knowledge. The third-party's offer for the seller's product is equal to $\theta+\epsilon$, where θ is known by the seller at date 1, and ϵ is an unknown outside valuation shock at date 2. θ is distributed according to the continuous, positive density function, $g(\theta)$ on $[\underline{\theta}, \overline{\theta}]$, with cumulative distribution, $G(\theta)$, such that $\overline{v} > \underline{\theta}$ (i.e., there are gains from trade with some probability). Only the seller knows θ, although its distribution is common knowledge. ϵ is distributed according to the continuous, positive density function, $h(\epsilon)$, on $[\underline{\epsilon}, \overline{\epsilon}]$, with cumulative distribution, $H(\epsilon)$.[8] Neither party observes ϵ at date 1, and only the seller observes ϵ at date 2. The expected value of ϵ is zero, thereby making θ an unbiased estimate at date 1 of the alternative market at date 2. Additionally, it is common knowledge that the seller's costs are to be zero, although zero costs is without loss of generality. Absent any contract offer, the seller expects to make θ. The buyer's outside opportunities have been normalized to zero. Finally, we assume that $\frac{1-F(v)}{f(v)}$ is nonincreasing in v, $\frac{G(\theta)}{g(\theta)}$ is nondecreasing in θ, and payoffs are not discounted.

The contract consists of a binary decision to agree to trade, δ ($\delta = 1$ if there is an agreement to trade, $\delta = 0$, otherwise), a price, p, paid at the time of signing, and a stipulated damage payment of ℓ to be paid at date 2 in the event of the supplier's breach after a decision to trade has been made. Thus, a contract outcome is given by $\{\delta, p, \ell\}$. As is standard, we restrict our attention to deterministic, piecewise C^1

[8] To ensure the global concavity of each of the several maximization programs in this paper, we assume that for all θ and for all $\ell \in [\theta + \underline{\epsilon}, \theta + \overline{\epsilon}]$,

$$h(\ell-\theta) > h'(\ell-\theta)[v-\ell],$$

$$h(\ell-\theta) > h'(\ell-\theta)[v-\ell-\frac{G(\theta)}{g(\theta)}],$$

$$h(\ell-\theta) > h'(\ell-\theta)[v-\ell-\frac{1-F(v)}{f(v)}],$$

$$h(\ell-\theta) > h'(\ell-\theta)[v-\ell-\frac{G(\theta)}{g(\theta)}-\frac{1-F(v)}{f(v)}].$$

These conditions are made for tractability; a weaker (but more complicated) set of conditions would also suffice. In any case, if h is a uniform distribution, these conditions are trivially satisfied. Additionally, we further assume that the support of ϵ is sufficiently large so as to eliminate corner solution problems. The latter condition is also for simplicity and would be satisfied, for example, if $F(v), G(\theta)$, and $H(\epsilon)$ are uniform distributions and $\underline{\epsilon} \leq \min\{\underline{\theta}, \underline{v}+\underline{\theta}-2\overline{\theta}, 2\underline{v}-\overline{\theta}-\overline{v}\}$ and $\overline{\epsilon} \geq \overline{v}-\underline{\theta}$.

contracts. For now, we assume that only the contract and the existence of breach is observable by the court. Later in Section 4 we relax this assumption to determine if a perfectly informed, but myopic, court could always improve contracting among the parties.

Given ℓ, the supplier will breach whenever $\theta + \epsilon > \ell$, and perform otherwise. Thus, the probability of performance is $H(\ell - \theta)$ and the probability of breach is $1 - H(\ell - \theta)$.

The *net* profit of the supplier from a contract, $\{\delta, p, \ell\}$, is

$$\pi^s(\theta) = \delta \left(p + \int_{\ell-\theta}^{\bar{\epsilon}} [\theta + \epsilon - \ell] dH(\epsilon) - \theta \right),$$

where the second term in the parentheses represents the expected gain from breach when the outside opportunity is lucrative, and the third term in parentheses is the seller's opportunity cost in agreeing to a contract (i.e., the lost expected profit, θ). The profit of the buyer is

$$\pi^b(v, \theta) = \delta \left[vH(\ell - \theta) + \ell[1 - H(\ell - \theta)] - p \right].$$

We consider three different contracting scenarios to provide a diverse range of environments for analysis. First, the buyer may propose the contract to the seller, and the seller may accept or reject it. Second, the seller may propose the contract, and the buyer may accept or reject it. Finally, an uninformed broker may design a contract which maximizes the joint surplus from trade between the parties. Interestingly, these three different contractual environments share common characteristics in ℓ.

Before considering each case in the following sections, we consider the full-information benchmark solutions for comparisons: In all three cases, the optimal full-information contract involves trade (i.e., $\delta = 1$) if and only if

$$(v - \theta)H(v - \theta) + \int_{v-\theta}^{\bar{\epsilon}} (\theta + \epsilon - v) dH(\epsilon) \geq 0,$$

and in such case, $\ell = v$. The above expression represents the expected gains from trade given that $\ell = v$. The first term is the expected gain under performance of the exchange; the second term is the option value of the outside opportunity that is available to the seller whenever $\theta + \epsilon \geq v$.

If the buyer has all of the bargaining power, the buyer's optimal strategy is to maximize her profits subject to the seller's acceptance of the conditions (i.e.,

$\pi^s \geq 0$). Substituting for p and simplifying yields the following program for the solution of δ and ℓ:

$$\max_{\delta,\ell} \delta \left(vH(\ell-\theta) + \ell[1 - H(\ell-\theta)] - \theta + \int_{\ell-\theta}^{\overline{\epsilon}} (\theta + \epsilon - \ell) dH(\epsilon) \right). \tag{2.1}$$

The necessary first-order condition is $\ell = v$. Given our assumptions on $h(\epsilon)$, this is also sufficient. $\delta = 1$ whenever (2.1) is positive at $\ell = v$. The contract price offered by the buyer is

$$p = \theta - \int_{v-\theta}^{\overline{\epsilon}} (\theta + \epsilon - v) dH(\epsilon). \tag{2.2}$$

Similarly, if the seller has all of the bargaining power, the seller's optimal strategy is to maximize his profits, subject to the buyer's acceptance of terms (i.e., $\pi \geq 0$). Substituting for p and simplifying yields

$$\max_{\delta,\ell} \delta \left(vH(\ell-\theta) + \ell[1 - H(\ell-\theta)] - \theta + \int_{\ell-\theta}^{\overline{\epsilon}} (\theta + \epsilon - \ell) dH(\epsilon) \right).$$

which is identical to the buyer's program above for the choice of δ and ℓ. Thus, we again find $\ell = v$. Note, however, the price paid by the buyer to the seller under this scheme is $p = v$, which extracts all of the buyer's rent. Finally, if an uninformed broker proposes a contract for the parties, the broker will maximize the expected gains from trade by choosing ℓ to maximize the collective surplus

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} \left\{ (v-\theta) H(\ell-\theta) + \int_{\ell-\theta}^{\overline{\epsilon}} \epsilon dH(\epsilon) \right\} dF(v) dG(\theta). \tag{2.3}$$

Again, the solution is to set $\ell = v$. The broker then chooses a price to allocate the gains from trade with $p \in \left[\theta - \int_{v-\theta}^{\overline{\epsilon}} (\theta + \epsilon - v) dH(\epsilon), v\right]$. It is not surprising that the optimal full-information contract under trade specifies $\ell = v$ for each contracting environment, since this condition guarantees that breach occurs if and only if it is efficient.

When information is not public, the resulting contract typically has $\ell \neq v$. Instead, ℓ will depend upon v and θ in a manner which will elicit a party's private information by creating distortions from efficient breach. The precise relationship between ℓ, v, and θ will depend on the contractual context: Buyer power, Seller power, or Brokered Contracts.

3. Optimal Contracts in Various Environments

3.1 The Buyer's Optimal Contract

Because the buyer does not know the seller's expected outside opportunity, θ, she must take into account the effect of the liquidated damage clause on the seller's gains from trade. If ℓ is set arbitrarily high, it will effectively lock the seller out of the alternative market; a low ℓ preserves the option value of breach, which in turn is an increasing function of θ. For seller's with high θ's, this will require a higher price to offset the loss in opportunity. Recognizing this relationship, the buyer can effectively use the damage clause to screen among different types of sellers much in the same way that a price-discriminating monopolist screens among different consumers by offering multiple quantity-price packages. The buyer will offer a menu of contracts, from which the seller chooses the one most profitable given his θ.[9] That is, the buyer may offer a continuum of contracts to the seller represented by a function $p(\ell)$. Following the Revelation Principle, we reparameterize according to the seller's outside opportunity, θ, as $x(\theta) \equiv \{\delta(\theta), p(\theta), \ell(\theta)\}$. Accordingly, we may solve for the buyer's optimal choice of x for every θ, subject to each seller type finding it optimal to choose the contract designed for his type.

The buyer's expected profit from any mechanism $x(\cdot)$ is

$$\pi^b(v) = \int_{\underline{\theta}}^{\overline{\theta}} \delta(\theta)\left\{vH(\ell(\theta)-\theta) + \ell(\theta)[1-H(\ell(\theta)-\theta)] - p(\theta)\right\} dG(\theta). \tag{3.1}$$

She maximizes this subject to two sets of constraints: the seller must be willing to sign the contract (i.e, not make a loss from trade) and the seller must select the contract designed for his type.

Define $\pi^s(\tilde{\theta}|\theta)$ as the profit to a seller with outside opportunity θ, who selects the contract designed for a seller of type $\tilde{\theta}$. That is,

$$\pi^s(\tilde{\theta}|\theta) \equiv \delta(\tilde{\theta})\left(p(\tilde{\theta}) + \int_{\ell(\tilde{\theta})-\theta}^{\overline{\epsilon}} (\theta + \epsilon - \ell(\tilde{\theta}))dH(\epsilon) - \theta\right).$$

The buyer's first constraint requires that every seller's truthful selection must yield nonnegative profits. Thus,

$$\pi^s(\theta|\theta) \geq 0$$

[9] The choice of contract by the buyer may possibly reveal information about the buyer's type, v, to the seller. There is no problem with mechanism design by informed principal in this case, however, as the seller's utility is independent of v, and the buyer has no action to take which could indirectly affect the seller's utility.

for all θ, which represents the *individual rationality* (IR) or participation constraint. Second, every seller must select the correct contract from the menu. These *incentive compatibility* (IC) constraints require

$$\pi^s(\theta|\theta) \geq \pi^s(\tilde{\theta}|\theta), \forall \theta, \tilde{\theta}.$$

These IR and IC constraints are intractable in their present form so we follow the standard procedure of replacing them with the significantly simpler representation in Lemma 1.

Lemma 1 *The mechanism* $\{p(\theta), \ell(\theta)\}$ *satisfies the IR and IC constraints if*

$$\pi^s(\theta) = \pi^s(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \delta(t) H(\ell(t) - t) dt, \tag{3.2}$$

$$\pi^s(\overline{\theta}) \geq 0, \tag{3.3}$$

$$\delta(\theta) H(\ell(\theta) - \theta) \geq \delta(\theta') H(\ell(\theta') - \theta), \forall \theta' > \theta. \tag{3.4}$$

In addition, (3.2) and (3.3) are necessary conditions for IR and IC.

The proof is standard and provided in the Appendix. Intuitively, the lemma follows from the envelope theorem: Assuming truthful selection is optimal for the seller, totally differentiating $\pi^s(\theta|\theta)$ results in $\frac{d\pi^s(\theta|\theta)}{d\theta} = -H(\ell(\theta) - \theta)$. Integrating this derivative produces (3.2) and (3.3). The condition in (3.4) is a second-order condition for truthful selection.

With this simplification, we proceed by substituting (3.2)-(3.3) into the buyer's expected profits function. Equating (3.2) with $\pi^s(\theta|\theta)$ and solving for $\delta(\theta)p(\theta)$ yields

$$\delta(\theta)p(\theta) = \theta - \int_{\ell(\theta)-\theta}^{\overline{\epsilon}} \delta(\theta)(\theta + \epsilon - \ell(\theta)) dH(\epsilon) + \pi^s(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \delta(t) H(\ell(t) - t) dt. \tag{3.5}$$

Recognizing that the buyer will optimally set $\pi^s(\overline{\theta}) = 0$, taking the expectation of $\delta(\theta)p(\theta)$ over θ, and integrating by parts yields

$$\int_{\underline{\theta}}^{\overline{\theta}} \delta(\theta)p(\theta)dG(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \delta(\theta) \left\{ \theta - \int_{\ell(\theta)-\theta}^{\overline{\epsilon}} (\theta + \epsilon - \ell(\theta)) dH(\epsilon) + H(\ell(\theta) - \theta) \frac{G(\theta)}{g(\theta)} \right\} dG(\theta).$$

Substituting this expression into the buyer's objective function yields the unconstrained problem

$$\text{(3.6)}\quad \max_{\delta,\ell} \int_{\underline{\theta}}^{\overline{\theta}} \delta(\theta) \left\{ \left(v - \theta - \frac{G(\theta)}{g(\theta)} \right) H(\ell(\theta) - \theta) + \int_{\ell(\theta)-\theta}^{\overline{\epsilon}} \epsilon dH(\epsilon) \right\} dG(\theta),$$

which may be solved by maximizing ℓ pointwise over θ and checking that the solution satisfies (3.4). This program yields the following Proposition which is proved in the Appendix.

Proposition 1 *The optimal menu of contracts, $\{\delta(\theta), p(\theta), \ell(\theta)\}$, for the buyer consists of a contract with*

$$\ell(\theta) = v - \frac{G(\theta)}{g(\theta)},$$

$p(\theta)$ such that (3.5) is satisfied, and $\delta(\theta)$ such that

$$\delta(\theta) = \begin{cases} 1 & \forall \theta \in [\underline{\theta}, \theta^*) \\ 0 & \forall \theta \in [\theta^*, \overline{\theta}] \end{cases}$$

where θ^* is either the unique value of $\theta \in [\underline{\theta}, \overline{\theta}]$ such that the integrand in (3.6) is zero, if such a value exists, or $\theta^* = \overline{\theta}$ otherwise.

As the Proposition indicates, the actual buyer loss from breach, v, almost always exceeds the amount of stipulated damages in the optimal contract when the buyer has all of the bargaining power. Specifically, the liquidated damage term is set equal to the valuation of the buyer, less an amount which represents the seller's rents from his private information. This additional rent term, $\frac{G(\theta)}{g(\theta)}$ increases in θ. Note that the lowest type seller, $\underline{\theta}$, chooses damages $\ell = v$; all sellers with types above $\underline{\theta}$ will choose undercompensatory liquidated damage terms in exchange for a lower contract price, p.

3.2 The Seller's Optimal Contract

Because the seller does not know the buyer's valuation of the good, he must take into account the effect of the liquidated damage clause on protecting the buyer's value. If ℓ is set arbitrarily low, it will allow the seller to breach and use the alternative market whenever ϵ is favorable, thereby imposing a loss of v on the buyer. For buyers with high v's, this will produce a lower reserve price due to the lower likelihood that the value of the bargain will accrue. Recognizing this, the seller can effectively use the stipulated damage clause to select among the different types of buyers just as the buyer was previously shown to select among sellers.

The seller may offer a menu of contracts like that in the previous section and allow the buyer to choose the one most profitable given her v. In this case, the menu can be represented by either the function $p(\ell)$ or the parametric triplet $\{\delta(v), p(v), \ell(v)\}$.

The seller's expected profit from a contract is

$$\pi^s(\theta) = \int_{\underline{v}}^{\overline{v}} \delta(v) \left\{ p(v) + \int_{\ell(v)-\theta}^{\overline{\epsilon}} (\theta + \epsilon - \ell(v)) dH(\epsilon) - \theta \right\} dF(v). \tag{3.7}$$

He maximizes this subject to the buyer's IR and IC constraints, analogous to the Buyer's problem above. In this section for simplicity we also assume that θ is observed by the buyer.[10]

Define $\pi^b(\tilde{v}|v)$ by

$$\pi^b(\tilde{v}|v) \equiv vH(\ell(\tilde{v}) - \theta) + \ell(\tilde{v})[1 - H(\ell(\tilde{v}) - \theta)] - p(\tilde{v}),$$

which represents the profit to a seller with outside opportunity v who selects the contract designed for a seller of type $\tilde{v}$. Analogously to the buyer-contract case, the IR and IC constraints are, respectively,

$$\pi^b(v|v) \geq 0,$$

$$\pi^b(v|v) \geq \pi^b(\tilde{v}|v),$$

for all v and $\tilde{v}$. Again, as in the buyer-designed contract, the above two sets of constraints are difficult to work with but can be greatly simplified, as in Lemma 2.

[10] Because the buyer's expected returns from trade depend negatively on θ, the seller might otherwise attempt to signal to the buyer that θ is in fact low by making a contractual offer which only low-θ sellers would find profitable to make. The general problem of mechanism design by an informed principal has been studied by Maskin-Tirole [1990]. Rather than assuming θ is known by the buyer, an alternative way to avoid the informed-principal problem is by assuming that all sellers offer the same contract (i.e., they collectively pool), $\underline{\theta}$ is sufficiently large that all types of sellers find it optimal to contract with the buyer (i.e., in terms of Proposition 2, $v^* = \underline{v}$), and any seller who offers a different contract than the expected pooled contract is assumed to be a high-type θ by the buyer. Because the equilibrium contracts derived in Proposition 2 are independent of θ, this forms a Bayesian-Nash equilibrium.

Lemma 2 *The mechanism $\{\delta(v), p(v), \ell(v)\}$ satisfies the buyer's IR and IC constraints if*

(3.8)
$$\pi^b(v) = \pi^b(\underline{v}) + \int_{\underline{v}}^{v} \delta(v) H(\ell(t) - \theta) dt,$$

(3.9)
$$\pi^b(\underline{v}) \geq 0,$$

(3.10)
$$\delta(v) H(\ell(v) - \theta) \geq \delta(v') H(\ell(v') - \theta), \forall v, v'.$$

In addition, (3.8)-(3.9) are necessary conditions for IR and IC.

The proof is provided in the Appendix and again is an application of a the envelope theorem. With this simplification, we proceed by substituting (3.8)-(3.9) into the seller's objective, (3.7). Equating (3.8) with $\pi^b(v|v)$ and solving for $\delta(v)p(v)$ yields

$$\delta(v)p(v) = vH(\ell(v) - \theta) + \ell(v)[1 - H(\ell(v) - \theta)] - \pi^b(\underline{v}) - \int_{\underline{v}}^{v} \delta(v) H(\ell(t) - \theta) dt.$$

Recognizing that the seller will optimally set $\pi^b(\underline{v}) = 0$, taking the expectation of $\delta(v)p(v)$ over v, and integrating by parts, produces

$$\int_{\underline{v}}^{\overline{v}} \delta(v) p(v) dF(v) =$$

$$\int_{\underline{v}}^{\overline{v}} \delta(v) \left\{ vH(\ell(v) - \theta) + \ell(v)[1 - H(\ell(v) - \theta)] - H(\ell(v) - \theta) \frac{1 - F(v)}{f(v)} \right\} dF(v).$$

Substituting this expression into the buyer's objective function yields the unconstrained problem

(3.11)
$$\max_{\delta,\ell} \int_{\underline{v}}^{\overline{v}} \delta(v) \left\{ \left(v - \theta - \frac{1 - F(v)}{f(v)} \right) H(\ell(v) - \theta) + \int_{\ell(v)-\theta}^{\overline{\epsilon}} s dH(\epsilon) \right\} dF(v).$$

This problem may be solved by maximizing ℓ pointwise over v, and checking that the resulting solution satisfies (3.10). The resulting expressions provide Proposition 2 which is proved in the Appendix. As the Proposition indicates, ℓ is again almost always below actual loss.

Proposition 2 *The optimal menu of contracts, $\{\delta(v), p(v), \ell(v)\}$, for the seller sets*

$$\ell(v) = v - \frac{1 - F(v)}{f(v)},$$

$p(v)$ such that (3.8) holds, and $\delta(v)$ such that

$$\delta(v) = \begin{cases} 1 & \forall v \in (v^*, \overline{v}] \\ 0 & \forall v \in [\underline{v}, v^*] \end{cases}$$

where v^* is either the unique value of $v \in [\underline{v}, \overline{v}]$ such that the integrand in (3.11) is zero, if such a value exists, or $v^* = \underline{v}$ otherwise.

As Proposition 2 demonstrates, the actual buyer loss from breach, v, almost always exceeds the amount of stipulated damages in the optimal contract when the seller behaves as a monopolist. Specifically, the liquidated damage term is set equal to the valuation of the buyer, less an amount which represents the buyer's information rents (or consumer surplus, to use the monopoly analogy) from her private information. This additional rent term, $\frac{1-F(v)}{f(v)}$ decreases in v. Note that the highest type buyer, $\overline{v}$, chooses damages $\ell = v$ and all buyers with lower types choose undercompensatory liquidated damage terms in exchange for a lower contract price, p.

3.3 Brokered Contracts

Rather than place all of the bargaining power in the hands of one agent, we now consider the resulting contract where both agents delegate the contractual terms to a outside party (e.g., a broker) who knows neither v nor θ.[11] This broker is concerned only with maximizing the total gains from trade when each party knows only its own private information. As a motivation, one might suppose that certain institutions evolve which maximize the joint gains from trade between agents from an ex ante point of view (which is equivalent to an uninformed broker designing a mechanism for exchange); agents, in turn, use these institutions in order to avoiding signaling adverse information to one another, although this motivation is admittedly very loose. An alternative motivation has the buyer and seller contracting ex ante, before they learn their private information, but subject to a limited-liability constraint where either party can legally walk away from the contract once private information is learned if losses are sufficiently great. This latter

[11] In this section, we return to our assumption that the buyer does not observe θ.

explanation may be realistic in the requirements contracting context. Before, when one party had full contractual power, that party traded off breach inefficiencies against increased rent extraction. Under such a skewed bargaining environment, ℓ never exceeds v in the optimal contract. We now find that even when a broker is employed, the optimal contract never involves excessive stipulated damages.

The problem facing the broker is to maximize the joint gains from trade by designing a menu of contracts. The contracts may depend upon both θ and v. We can think of the menu as an offer of a menu of menus to the buyer, one of which the buyer selects. From the selected menu, the seller is allowed to choose the final contract. Alternatively, we may parameterize this family of contracts by (θ, v), and envision the contract as a direct revelation mechanism where each party announces his or her private information and the broker selects the appropriate contract according to $z(\theta, v) \equiv \{\delta(\theta, v), p(\theta, v), \ell(\theta, v)\}$.

Because there is two-sided asymmetric information, the traditional techniques need to be be augmented slightly; we follow Myerson-Satterthwaite [1983] in this regard.[12] Lemma 3 is a direct extension of Lemmas 1 and 2 and characterizes the set of all contracts which are incentive compatible and individually rational for both parties. Proposition 3 provides the solution of the broker's problem.

Lemma 3 *There exists a function $p(\theta, v)$ such that $\ell(\theta, v)$ is IC and IR if and only if*

(3.12)
$$\int_{\underline{v}}^{\overline{v}} \delta(\theta, v) H(\ell(\theta, v) - \theta) dF(v) \leq \int_{\underline{v}}^{\overline{v}} \delta(\theta', v) H(\ell(\theta', v) - \theta') dF(v), \forall \theta > \theta',$$

(3.13)
$$\int_{\underline{\theta}}^{\overline{\theta}} \delta(\theta, v) H(\ell(\theta, v) - \theta) dG(\theta) \geq \int_{\underline{\theta}}^{\overline{\theta}} \delta(\theta, v') H(\ell(\theta, v') - \theta) dG(\theta), \forall v > v'.$$

hold, and

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} \left\{ \left[\left(v - \frac{1 - F(v)}{f(v)} \right) - \left(\theta - \frac{G(\theta)}{g(\theta)} \right) \right] H(\ell(\theta, v) - \theta) \right.$$

$$\left. + \int_{\ell(\theta, v) - \theta}^{\overline{\epsilon}} \epsilon dH(\epsilon) \right\} dF(v) dG(\theta) \geq 0. \tag{3.14}$$

[12] Also see Williams [1987] for a fuller treatment and extension of Myerson-Satterthwaite's model. Williams characterizes the efficient locus of contracts, depending upon the weights attached to the buyer and seller's utility.

The proof of Lemma 3 follows from Lemmas 1 and 2 and is contained in the Appendix.

With this Lemma, we may write the broker's problem as maximizing the expected profit of each party subject to (3.14) above. That is

$$\max_{\delta,\ell} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} \delta(\theta, v) \left\{ (v-\theta)H(\ell(\theta,v)-\theta) + \int_{\ell(\theta,v)-\theta}^{\overline{\epsilon}} \epsilon dH(\epsilon) \right\} dF(v)dG(\theta),$$

subject to (3.14). Let $\mu\delta(\theta, v)$ be the Lagrange multiplier for (3.14). We multiply μ by δ without loss of generality as the IC and IR constraints do not bind when $\delta = 0$. Bringing the constraint into the integral above and simplifying yields the third party's objective function:

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} (1+\mu)\delta(\theta,v) \left\{ \left[(v-\theta) - \frac{\mu}{1+\mu} \left(\frac{1-F(v)}{f(v)} + \frac{G(\theta)}{g(\theta)} \right) \right] H(\ell(\theta,v)-\theta) \right.$$

$$\left. + \int_{\ell(\theta,v)-\theta}^{\overline{\epsilon}} \epsilon dH(\epsilon) \right\} dF(v)dG(\theta). \tag{3.15}$$

The functions δ and ℓ which maximize this integral may be found by maximizing the expression for δ and ℓ pointwise in θ and v, and checking that the solution satisfies (3.12) and (3.13). The solution results in the following Proposition which is proved in the Appendix.

Proposition 3 *The optimal contracts for the brokered buyer-seller relationship consists of*

$$\ell(\theta, v) = v - \frac{\mu}{1+\mu} \left(\frac{1-F(v)}{f(v)} + \frac{G(\theta)}{g(\theta)} \right),$$

where $\mu \geq 0$, and with

$$\delta(v) = \begin{cases} 1 & \forall \theta, v, \textit{ s.t. (3.15) is nonnegative} \\ 0 & \textit{otherwise.} \end{cases}$$

Additionally, $\mu > 0$ if

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} \left\{ \left[\left(v - \frac{1-F(v)}{f(v)} \right) - \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \right] H(v-\theta) + \int_{v-\theta}^{\overline{\epsilon}} \epsilon dH(\epsilon) \right\} dF(v)dG(\theta) > 0.$$

Proposition 3 demonstrates that stipulated damages do not exceed the actual loss from breach of contract and are strictly less than actual loss whenever the expected gains from trade are less than the expected information rents for almost all θ and v. As in the monopsony and monopoly contexts above, the liquidated damage term is set equal to the buyer's valuation less some information rents term. Now, however, this rent is the sum of both $\frac{G(\theta)}{g(\theta)}$ and $\frac{1-F(v)}{f(v)}$ rather than just one or the other, but weighted with a coefficient less than unity. It is thus quite analogous to the previous contractual outcomes. Note that except for when $(\theta, v) = (\underline{\theta}, \overline{v})$, it is the case that $\ell < v$. Because the brokered contract yields greater combined gains from trade than either the buyer-contract or the seller-contract but still utilizes undercompensatory liquidated damage contracts, our previous results appear quite robust.

4. Welfare Implications and Policy Conclusions

We have seen that the existence of private information by contracting parties in a wide range of bargaining environments introduces the likelihood that liquidated damages will be below the actual losses caused by breach. Using liquidated damages to select among different types of economic agents is, in a sense, second-degree price discrimination where the monopolist offers price-damage, rather than price-quantity, bundles. The question then arises as to whether public policy should require that all damages for breach of contract equal the true losses incurred, providing such information about losses is available to the court after the breach. It is arguable that any intervention would be precarious at best, especially given our limitations of knowledge about the actual contracting conditions between parties.

To consider the issue of judicial intervention, we posit the strongest possible assumption in favor of activism to determine the most optimistic assessment: assume that courts can perfectly determine actual losses from breach ex post. That is, assume v becomes known to the court and to both parties at date 2 in the event of breach. With this assumption, we seek to answer the question of whether the court should require $\ell = v$ in all breached contracts. For realism, further assume that parties cannot base their contract price on the judicial determination of v; otherwise, the court would become nothing more than an auditing agency for private contracts. That is, observed v can only be used to determine ℓ.

There is a benefit and a cost from judicial intervention: Eliminating the agents' abilities to set liquidated damages below valuation reduces inefficient breach of contract, but may foreclose some buyers and sellers from efficient trade.

If there were only two possible types of buyers and sellers, and assuming the offerer would chose to serve only part of the market if it were permissible to choose

$\ell \neq v$, then judicial intervention always produces inefficiencies. To see this, note that when the offeree is of a good type (i.e., high-value buyer or low-opportunity seller), liquidated damages are set at actual value. Consequently, for good types there is no inefficiency with or without judicial intervention. When the offerees are of bad type (i.e., low-value buyers or high-opportunity sellers), offerers will set inefficient damage levels when given the option. But while the terms are inefficient, individual rationality implies both parties are better off trading than not trading. Judicial intervention that prevents the use of under-compensatory damages must therefore decrease social welfare.

When a continuum of types of buyers and sellers exist, the analysis is more difficult. Consider the problem facing the buyer with all of the bargaining power who is constrained to set $\ell = v$, ex post, in all offered contracts. If she sets the contract price low, only very low θ-type sellers will accept the terms, but she will make a larger profit on those contracts where such a sale is made. If she sets the price high, her terms will be accepted by most sellers, but her gains from actual trade will be lower. The problem facing her is much the same as that facing a monopolist setting one price: higher prices result in fewer sales but greater profits per sale. Her maximization problem is simply,

$$\max_p (v-p)G(\theta(p)),$$

where $\theta(p)$ is defined as the highest θ-type seller who would be willing to buy at price p, and is given implicitly by (2.2) above, which is

$$p = \theta - \int_{v-\theta}^{\bar{\epsilon}} (\theta + \epsilon - v)dH(\epsilon).$$

Each buyer will set a different price depending on her type v, just as a monopolist's prices vary with marginal cost. Because the relation above implies that $1/\theta'(p) = H(v - \theta(p))$, maximization reveals that the optimal constrained-contract price is implicitly given by $p = v - H(v-\theta(p))\frac{G(\theta(p))}{g(\theta(p))}$, which defines $p^*(v)$. Using (2.2) again allows us to define $\theta^*(v)$ as the threshold type of seller who chooses not to sell at the buyer's offered price of $p^*(v)$. All sellers with types lower than $\theta^*(v)$ sell to the buyer at buyer's asking price of $p^*(v)$. That is, $\theta^*(v)$ is defined by

$$\left(v - \theta^* - \frac{G(\theta^*)}{g(\theta^*)}\right) H(v-\theta^*) + \int_{v-\theta^*}^{\bar{\epsilon}} \epsilon dH(\epsilon) = 0.$$

Under the policy of requiring $\ell = v$, when the buyer has the bargaining power (i.e., behaves as a monopsonist) and is of type v, only seller types lower than $\theta^*(v)$

will sell the good to the buyer, even though the socially efficient cutoff level is in fact higher than $\theta^*(v)$.

Consider now the seller's problem when the seller has all of the bargaining power. Because of the constraint that $\ell = v$, the seller's maximization problem is to choose p to solve

$$\max_p \int_p^{v} \left(p + \int_{v-\theta}^{\overline{\epsilon}} (\theta + \epsilon - v) dH(\epsilon) \right) dF(v) + \int_{\underline{v}}^{p} \theta dF(v).$$

Maximization reveals that the seller's optimal constrained-contract price , satisfies

$$p = \theta + \frac{1 - F(p)}{f(p)} - \int_{p-\theta}^{\overline{\epsilon}} (\theta + \epsilon - p) dH(\epsilon).$$

We can analogously define $v^*(\theta)$ as the threshold valuation by a buyer such that no purchase is made. Any buyer with a higher value buys; a buyer with lower value sells. Thus, $v^*(\theta)$ is defined by

$$v^* = \theta + \frac{1 - F(v^*)}{f(v^*)} - \int_{v^*-\theta}^{\overline{\epsilon}} (\theta + \epsilon - v^*) dH(\epsilon).$$

Under a policy of $\ell = v$, when the seller has the bargaining power (i.e., behaves as a monopolist) and is of type θ, only buyers with valuations above $v^*(\theta)$ will buy the good even though the socially efficient cutoff level is in fact lower than $v^*(\theta)$.

With the above definitions of the marginal buyers and sellers under policies of judicially mandated actual-damage policy, we can define the social welfare changes. These changes are represented by the following two expressions where $\delta(\theta)$ and $\delta(v)$ are the socially efficient trading functions.

$$\Delta W^{bd} = - \int_{\theta^*(v)}^{\overline{\theta}} \delta(\theta) \left[(v-\theta) H\left(v - \theta - \frac{G(\theta)}{g(\theta)}\right) + \int_{v-\theta-\frac{G}{g}}^{\overline{\epsilon}} \epsilon dH(\epsilon) \right] dG(\theta) dF(v)$$

(4.1)

$$+ \int_{\underline{v}}^{\overline{v}} \int_{\theta}^{\theta^*(v)} \left[(v-\theta) \left(H(v-\theta) - H\left(v - \theta - \frac{G(\theta)}{g(\theta)}\right)\right) - \int_{v-\theta-\frac{G}{g}}^{v-\theta} \epsilon dH(\epsilon) \right] dG(\theta) dF(v).$$

$$\Delta W^{sd} = - \int_{\underline{v}}^{v^*(\theta)} \delta(v) \left[(v-\theta) H\left(v - \theta - \frac{1-F(v)}{f(v)}\right) + \int_{v-\theta-\frac{1-F(v)}{f(v)}}^{\overline{\epsilon}} \epsilon dH(\epsilon) \right] dF(v) dG(\theta)$$

(4.2)

$$+ \int_{\underline{\theta}}^{\overline{\theta}} \int_{v^*(\theta)}^{\overline{v}} \left[(v-\theta) \left(H(v-\theta) - H\left(v - \theta - \frac{1-F}{f}\right)\right) - \int_{v-\theta-\frac{1-F(v)}{f(v)}}^{v-\theta} \epsilon dH(\epsilon) \right] dF(v) dG(\theta).$$

The former equation represents the welfare gain from judicial intervention under buyer-designed contracts. The latter equation represents the gain under seller-designed contracts. In each equation, the first term in brackets represents the loss from reduced trade; the second term represents the gain from more efficient breach. The first terms consist of the expected loss from inefficient trade induced by monopoly or monopsony distortions due to the inability to effectively screen buyers or sellers by type. The second terms consist of the expected efficiency gain from more efficient breach of contract; the judicial restriction has the effect of decreasing the occurrence of breach to more efficient levels.

The central question is under what general conditions are these equations either positive (i.e., judicial intervention is good) or negative (i.e., judicial intervention is bad). Unfortunately, there are no clear general conditions. Rather, the sign of the equations depends fundamentally on the distributions of v, θ, and ϵ. But, as in the two-type example mentioned above, when the distributions are such that there is a sufficiently large mass of high valuation buyers (low opportunity-cost sellers) in the distribution function, the net costs of judical intervention in the seller-designed (buyer-designed) contracting scenario is positive. Furthermore, it should be noted that these results depend upon the optimistic assumption that the courts know perfectly the value of loss. If courts make errors, the above loss in welfare may be even greater.[13]

5. Extensions and Conclusions

The modeling approach taken in this paper was to assume that the seller may breach with some probability and that the buyer's valuation needed protection from such behavior. Alternatively, we could have chosen an alternative framework where the buyer breaches with some probability and the seller's sunk production costs need to be protected. In this case, we obtain similar results: liquidated damages never exceed the seller's production costs and frequently fail to protect the seller's investment fully. In this sense our explanation regarding the asymmetric treatment of liquidated damage clauses by the courts is robust.

This paper has demonstrated that when information of contracting parties is private, liquidated damage clauses serve a dual role of promoting efficient breach and increasing the likelihood of trade. Furthermore, even if the judicial system had perfect information, intervention in the form of prohibiting under-compensatory

[13]The courts determination of value does not enter the expressions linearly, and so even an unbiased estimate by the court introduces additional nonlinear effects.

damages does not necessarily improve social welfare. This may explain why courts have not found it necessary to invalidate under-compensatory damage clauses, but have continued to strike over-compensatory clauses. The former may be the result of a belief that bargaining parties made rational choices, while the latter may be best explained as a belief that excessive damage clauses are symptomatic of contractual failure.

Appendix

Proof of Lemma 1:

Necessity of (3.2) and (3.3):
Incentive compatibility and the definition of $\pi^s(\theta|\theta)$ implies

$$\begin{aligned}\pi^s(\theta|\theta) \geq \pi^s(\tilde{\theta}|\tilde{\theta}) \quad &+ \quad \delta(\tilde{\theta})\left[\int_{\ell(\tilde{\theta})-\theta}^{\ell(\tilde{\theta})-\tilde{\theta}} \epsilon dH(\epsilon)\right. \\ &+ \quad \left.[\ell(\tilde{\theta})-\theta]H(\ell(\tilde{\theta})-\theta)-[\ell(\tilde{\theta})-\tilde{\theta}]H(\ell(\tilde{\theta})-\tilde{\theta})\right].\end{aligned}$$

Integrating by parts and simplifying yields

$$\begin{aligned}\pi^s(\theta|\theta) \geq \pi^s(\tilde{\theta}|\theta) \quad &= \quad \pi^s(\tilde{\theta}|\tilde{\theta}) - \int_{\ell(\tilde{\theta})-\theta}^{\ell(\tilde{\theta})-\tilde{\theta}} \delta(\tilde{\theta})dH(\epsilon), \\ &= \quad \pi^s(\tilde{\theta}|\tilde{\theta}) - \int_{\tilde{\theta}}^{\theta} \delta(\tilde{\theta})H(\ell(\tilde{\theta})-t)dt.\end{aligned}$$

Similarly, $\pi^s(\tilde{\theta}|\tilde{\theta}) \geq \pi^s(\theta|\theta) + \int_{\tilde{\theta}}^{\theta} \delta(\theta)H(\ell(\theta)-t)dt$. Thus, combining the inequalities, we obtain

$$-\int_{\tilde{\theta}}^{\theta} \delta(\theta)H(\ell(\theta)-t)dt \geq \pi^s(\theta|\theta) - \pi^s(\tilde{\theta}|\tilde{\theta}) \geq -\int_{\tilde{\theta}}^{\theta} \delta(\tilde{\theta})H(\ell(\tilde{\theta})-t)dt.$$

Take $\theta > \tilde{\theta}$, without loss of generality, divide by $(\theta - \tilde{\theta})$, and take the limit as $\theta \to \tilde{\theta}$. This yields

$$\frac{\pi^s(\theta|\theta)}{d\theta} = -\delta(\theta)H(\ell(\theta)-\theta).$$

By the inequalities above, $\pi^s(\theta)$ is monotonic, and therefore Riemann-integrable, and so we may characterize the seller's profits as

$$\pi^s(\theta|\theta) \equiv \pi^s(\theta) = \pi^s(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \delta(t)H(\ell(t)-t)dt, \tag{5.1}$$

which is (3.2).

Sufficiency of (3.2)-(3.4):
Substituting (3.2) into (IC) implies $\pi^s(\overline{\theta}) \geq 0$ for all θ; this is individual rationality. To prove incentive compatibility, note that the definition of $\pi^s(\tilde{\theta}|\theta)$ implies

$$\pi^s(\tilde{\theta}|\tilde{\theta}) = \pi^s(\tilde{\theta}|\theta) + \int_{\tilde{\theta}}^{\theta} \delta(\tilde{\theta})H(\ell(\tilde{\theta})-t)dt.$$

But (3.2) implies that

$$\pi^s(\overline{\theta})+\int_{\theta}^{\overline{\theta}} \delta(t)H(\ell(t)-t)dt+\int_{\tilde{\theta}}^{\theta} \delta(t)H(\ell(t)-t)dt = \pi^s(\tilde{\theta}|\theta)+\int_{\tilde{\theta}}^{\theta} \delta(\tilde{\theta})H(\ell(\tilde{\theta})-t)dt.$$

Applying (3.2) again yields

$$\pi^s(\theta|\theta) = \pi^s(\tilde{\theta}|\theta) + \int_{\tilde{\theta}}^{\theta} [\delta(\tilde{\theta})H(\ell(\tilde{\theta}) - t) - \delta(t)H(\ell(t) - t)]dt.$$

By (3.4), the integral above integral is nonnegative which implies $\pi^s(\theta|\theta) \geq \pi^s(\tilde{\theta}|\theta)$. Hence, $\{\delta(\theta), p(\theta), \ell(\theta)\}$ is incentive compatible. □

Proof of Proposition 1: Ignoring the decision to trade, pointwise maximization of (3.6) yields the expression for $\ell(\theta)$, which is monotonically decreasing. (Given our assumptions on H, F, and G, this pointwise optimization program is concave in ℓ. (3.5) provides us with $p(\theta)$ such that IC and IR are satisfied if the monotonicity condition in (3.4) is satisfied. Pointwise maximization of $\delta(\theta)$ yields $\delta = 1$ whenever the integrand in (3.6) is nonnegative. Given our assumptions on the inverse hazard rate, the integrand in (3.6) is strictly decreasing in θ, which implies that $\delta(\theta)$ is decreasing in θ and that there is at most one value of θ such that the integrand in (3.6) is exactly zero; thus θ^* is unique. Consequently, the monotonicity condition in (3.3) are satisfied and so the mechanism is IC and IR. □

Proof of Lemma 2:

Necessity of (3.8)-(3.9):

Incentive compatibility and the definition of $\pi^b(v|v)$ implies

$$\pi^b(v|v) \geq \pi^b(\tilde{v}|v) = \pi^b(\tilde{v}|\tilde{v}) + \int_{\underline{\theta}}^{\overline{\theta}} \delta(\tilde{v})(v - \tilde{v})H(\ell(\tilde{v}) - \theta)dG(\theta).$$

Similarly,

$$\pi^b(\tilde{v}|\tilde{v}) = \pi^b(v|v) + \int_{\underline{\theta}}^{\overline{\theta}} \delta(v)(\tilde{v} - v)H(\ell(v) - \theta)dG(\theta).$$

Thus, combining the inequalities, we obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} \delta(v)(v - \tilde{v})H(\ell(v) - \theta)dG(\theta) \geq \pi^b(v|v) - \pi^b(\tilde{v}|\tilde{v}) \geq \int_{\underline{\theta}}^{\overline{\theta}} \delta(\tilde{v})(v - \tilde{v})H(\ell(\tilde{v}) - \theta)dG(\theta).$$

Take $v > \tilde{v}$, without loss of generality, divide by $(v - \tilde{v})$, and take the limit as $v \to \tilde{v}$. This yields

$$\frac{d\pi^b(v|v)}{dv} = \int_{\underline{\theta}}^{\overline{\theta}} \delta(v)H(\ell(v) - \theta)dG(\theta).$$

By the inequalities above, $\pi^b(v)$ is monotonic and therefore Riemann-integrable. Hence, we may characterize the buyer's profits as

$$\pi^b(v|v) \equiv \pi^b(v) = \pi^b(\underline{v}) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} \delta(t)H(\ell(t) - \theta)dtdG(\theta),$$

which is (3.8).

Sufficiency of (3.8)-(3.10):
Substituting (3.9) into (3.8) implies $\pi^b(\underline{v}) \geq 0$ for all v; this is individual rationality. To prove incentive compatibility, note that the definition of $\pi^b(\tilde{v}|v)$ implies

$$\pi^b(\tilde{v}|\tilde{v}) \equiv \pi^b(\tilde{v}) = \pi^b(\tilde{v}|v) - \int_{\underline{\theta}}^{\overline{\theta}} \delta(\tilde{v})(v - \tilde{v})H(\ell(\tilde{v}) - \theta)dG(\theta).$$

But this implies that

$$\pi^b(\underline{v}) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} \delta(t)H(\ell(t) - \theta)dtdG(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{v}^{\tilde{v}} \delta(t)H(\ell(t) - \theta)dtdG(\theta),$$

$$= \pi^b(\tilde{v}|v) + \int_{\underline{\theta}}^{\overline{\theta}} \delta(\tilde{v})(v - \tilde{v})H(\ell(\tilde{v}) - \theta)dG(/th).$$

Applying (3.8) again yields

$$\pi^b(v) = \pi^b(\tilde{v}|v) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\tilde{v}}^{v} [\delta(\tilde{v})(H(\ell(\tilde{v}) - \theta) - \delta(t)H(\ell(t) - \theta]dtdG(\theta).$$

By (3.10), the double integral above is nonnegative, which implies $\pi^b(v|v) \geq \pi^b(\tilde{v}|v)$. Hence, $\{\delta(v), p(v), \ell(v)\}$ is incentive compatible. □

Proof of Proposition 2: Ignoring the decision to trade, pointwise maximization of (3.11) yields the expression for $\ell(v)$, which is monotonically increasing. (Given our assumptions on F, G, and H, the pointwise optimization program is concave in ℓ.) (3.8) provides us with $p(v)$ such that IC and IR are satisfied if the monotonicity condition in (3.10) is satisfied. Pointwise maximization of $\delta(v)$ yields $\delta = 1$ whenever the integrand in (3.11) is nonnegative. Given our assumptions on the inverse hazard rate, the integrand in (3.11) is strictly increasing in v, which implies that $\delta(v)$ is decreasing in v and that there is at most one value of v such that the integrand in (3.11) is exactly zero; thus v is unique. Consequently, the monotonicity conditions in (3.10) are satisfied and so the mechanism is IC and IR. □

Proof of Lemma 3:

Necessity follows from Lemmas 1 and 2. To see that (3.12)-(3.14) are sufficient for incentive compatibility and individual rationality, we construct $p(\theta, v)$ such that $\delta(\theta, v)$ and $\ell(\theta, v)$ are IC and IR.

First note that because we have restricted ourselves to piecewise C^1 contracts, $\delta(\theta, v)$ is well defined and $\delta_\theta(\theta, v) = 0$ at all but at a finite number of points. From the envelope theorem, incentive compatibility requires that p must satisfy

$$p_\theta(\theta, v) = [1 - H(\ell(\theta, v) - \theta)]\ell_\theta(\theta, v), \tag{5.2}$$

$$p_v(\theta, v) = [(v - \ell(\theta, v))h(\ell(\theta, v) - \theta) + [1 - H(\ell(\theta, v) - \theta)]]\,\ell_v(\theta, v), \tag{5.3}$$

whenever $\delta(\theta, v) = 1$. If the constructed price function satisfies these two partial differential equations, we know from Lemmas 1 and 2 that the monotonicity conditions expressed in (3.12)-(3.13) are sufficient for incentive compatibility. One possible construction of p has $p(\theta, v)$ such that $\pi^b(\underline{v}) = 0$. That is,

$$\underline{v}H(\ell(\theta, \underline{v}) - \theta) + \ell(\theta, \underline{v})[1 - H(\ell(\theta, \underline{v}) - \theta)] = p(\theta, \underline{v}).$$

Define,

$$\begin{aligned}
p(\theta, v) \;\equiv\; & \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{v} \{[t - \ell(s,t)]h(\ell(s,t) - s) + [1 - H(\ell(s,t) - s)]\}\,\ell_v(s,t)\,dt\,dG(s) \\
+ & \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{v} \{\underline{v}H(\ell(s,\underline{v}) - s) + [1 - H(\ell(s,\underline{v}) - s)]\ell(s,\underline{v})\}\,dG(s) \\
- & \int_{\underline{v}}^{\overline{v}} \int_{\theta}^{\overline{\theta}} [1 - H(\ell(s,t) - s)]\ell_\theta(s,t)\,ds\,dF(t) \\
- & \int_{\underline{v}}^{\overline{v}} \int_{\underline{\theta}}^{\overline{\theta}} \left\{[1 - H(\ell(s,t) - s)]\ell_\theta(s,t)\frac{G(s)}{g(s)}\right\} dG(s)\,dF(t).
\end{aligned}$$

The first two expressions represent the expectation over θ of the integral $p_v(\theta, v)$ and the endpoint $p(\theta, \underline{v})$. The second pair of expressions are zero in expectation. It is straightforward to check that (5.2)-(5.3) above are satisfied by this price function, and so the mechanism is incentive compatible. Moreover, (3.14) implies that $\pi^b(\underline{v}) + \pi^s(\overline{\theta}) \geq 0$. And since p was constructed so that $\pi^b(\underline{v} = 0)$, the mechanism is individually rational. $\square$

Proof of Proposition 3: Maximizing the above expression over δ and ℓ pointwise in θ and v yields $\delta(\theta, v)$ and $\ell(\theta, v)$. These in turn provide for the construction of $p(\theta, v)$ that satisfies

$$\pi^s(\theta) = \int_{\underline{v}}^{\overline{v}} \left\{ \pi^s(\overline{\theta}, v) + \int_{\theta}^{\overline{\theta}} \delta(\theta, v) H(\ell(s, v) - s) ds \right\} dF(v),$$

$$\pi^b(v) = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \pi^b(\theta, \underline{v}) + \int_{\underline{v}}^{v} \delta(\theta, v) H(\ell(\theta, t) - \theta) dt \right\} dG(\theta).$$

ℓ is nonnegative, and therefore $\ell(\theta, v)$ is nondecreasing in v and nonincreasing in θ. Moreover, at the optimum, (3.15) is increasing in v and decreasing in θ, implying that $\delta(\theta, v)$ is nondecreasing in v and nonincreasing in θ. Hence, (3.12)-(3.13) are satisfied and the mechanism is IC and IR. To prove that $\mu > 0$ under the integral condition above, suppose to the contrary that $\mu = 0$. Then $\ell(\theta, v) = v$, and by our hypothesis, (3.14) must fail, indicating that $\mu > 0$. $\square$

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