Do Short-Term Objectives Lead to Under- or Overinvestment in Long-Term Projects?

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ABSTRACT

We examine managerial investment decisions in the presence of imperfect information and short-term managerial objectives. Prior research has argued that such an environment induces managers to underinvest in long-run projects. We show that short-term objectives and imperfect information may also lead to overinvestment, and we identify how the direction of the distortion depends upon the type of informational imperfection present. When investors cannot observe the level of investment in the long-run project, suboptimal investment will be induced. When investors can observe investment but not its productivity, however, overinvestment will occur.

There has been in recent years substantial public debate on the question of whether the long-run investment decisions of the managers of publicly traded companies may be distorted by market pressures. Recent work by economists has tried to identify the potential source and nature of such distortions.\(^1\) Research has naturally focused on situations that are characterized by (i) short-term managerial objectives—the managers are concerned not only with the firm’s long-run stock price but also with the firm’s short-run stock price (due to incentive schemes or the fear of losing control), and (ii) imperfect information—the market has less information than the firm’s managers about the firm’s long-run projects. The results of this research have indicated that in some situations, short-term objectives and imperfect information may lead to underinvestment in long-run projects.

This paper seeks to extend prior work on the effects of short-term objectives and imperfect information on long-run investment decisions. We demonstrate that imperfect information, together with an emphasis on a firm’s short-run valuation, may lead not only to underinvestment, but also to overinvestment in long-run projects. The direction of the distortion depends

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\(^1\) See, for example, Narayanan (1985a, 1985b), Stein (1988, 1989), Shleifer and Vishny (1990), Bresnahan et al. (1990), and Paul (1991a).
upon the nature of the manager’s information advantage over uninformed investors. As will be shown, two common types of informational imperfections produce different types of distortions. Although the existence of short-term objectives may imply investment inefficiencies, whether there is underinvestment or overinvestment depends critically upon the observable parameters in the model.

In modeling the long-run investment decisions of managers, we consider two different situations. In the first situation, which has been the focus of past research, investors cannot observe the level of investment in long-run projects. In this case, underinvestment may result. The manager underinvests in future projects because increased investment capital affects the current stock market price less than the increase in net present value. On the margin, this distorts investment below efficient profit-maximizing levels.

The contribution of this paper lies, however, in its analysis of the second situation. In this alternative setting, investors observe the level of investment in a given long-run project. We show that in this case overinvestment may result. A manager with a highly productive investment opportunity may signal to the market that the long-run outlook of the firm is good by overinvesting in the future—an action that a manager with a lesser long-run project would by unwilling to choose. Therefore, when the market has incomplete information regarding the returns to investment from the long-term project but can actually observe investment levels, asymmetries of information may induce overinvestment in the future.²

It appears that each of the two cases analyzed is likely to occur with some frequency. Clearly, there are many situations in which managers are likely to have some private information about the amount invested in a long-run project. An obvious example is the amount of managerial time and attention devoted to such a project.

It is equally clear, however, that there are other situations in which investors have adequate information about the level of investment in a long-run project, but the managers have private information regarding the project’s returns on investment. The amount of money invested by a company in a given project is often either observable or at least disclosed by the firm (and verified by the firm’s auditors). This is likely to be the case with respect to many “hard” investments in plants and equipment. Additionally, this may be typical for investments in research and development, which are a type of long-run investment that has received much attention in the policy debate. Both the standards of the Financial Accounting Standards Board and the

²A recent paper by Paul (1991b) suggests another reason for overinvestment in long-term projects. He focuses on a situation in which the market makes an imperfect assessment of a firm’s profits. Whenever the firm’s long-term cash flows provide a better predictor of the firm’s profits than the short-term flows, the firm will overinvest in long-term projects (the firm will underinvest when the opposite is true). Hirshleifer and Chordia (1991) consider a related signaling model in which resolution-advancing decisions serve as signals of managerial ability. In their model, managers with high ability may make excessive expenditures to advance the resolution of long-term project uncertainty.
rules of the Securities Exchange Commission require companies to disclose and provide certain details about any material research and development (R & D) expenditures (see, e.g., Anthony and Reece (1989), p. 67).

Thus, given that both of the cases we analyze are plausible, no general conclusion can be established about the direction in which short-term objectives distort long-run investment. As the model below illustrates, in any given situation an examination of the nature of managers' private information is necessary to determine the likely consequences of short-term objectives.

I. The Model

A. Framework of Analysis

For simplicity, we consider a two-period time horizon—the short-term and the long-term. Managers make an investment decision among two projects. The short-run project will realize a return after one period; the long-run project will realize a return in the second period. We denote the stock market’s valuation in period $t$ of a firm’s total value over both periods by $\mathcal{V}_t$.

We take as our starting point, as other authors have done, the assumption that the managers are concerned not only about the long-term value of the firm, $\mathcal{V}_2$, but also the market’s immediate valuation, $\mathcal{V}_1$.\footnote{See, for example, Narayanan (1985a, 1985b), Stein (1989), Shleifer and Vishny (1990).} This phenomenon of short-term objectives is now commonly accepted, for two reasons. First, managers commonly receive compensation packages that are partly tied to $\mathcal{V}_1$. Second, a higher $\mathcal{V}_1$ makes it less likely that the managers will lose their position at $t = 1$ as a result of a takeover or proxy contest. Consistent with this emphasis by managers on the current stock market price of their firm, Abegglen and Stalk (1985) find survey evidence that suggests that U.S. managers have a narrower focus on immediate stock market performance than their Japanese counterparts.

Following the above arguments about managerial objectives, we can model managers as having utility that depends upon both first-period firm valuation as well as second-period valuation, $\mathcal{U}(\mathcal{V}_1, \mathcal{V}_2)$. We consider the case of linear preferences of the form

$$
\mathcal{U}(\mathcal{V}_1, \mathcal{V}_2) = \gamma + \alpha_1 \mathcal{V}_1 + \alpha_2 \mathcal{V}_2, \tag{1}
$$

for some $\alpha_1, \alpha_2 > 0$.\footnote{Like Stein (1989), and Shleifer and Vishny (1990), we take managerial preferences as exogenous to our model. Bebchuk (1990) considers the determination of $\alpha_1$ and $\alpha_2$ and, in particular, examines whether the shortening of investors’ horizons should be expected to lead them to favor incentive schemes with more weight on short-term objectives (i.e., a higher $\alpha_1/\alpha_2$).}

Two projects exist in which the manager can invest a fixed amount of capital. The short-term project realizes a return in the first period; the long-term project yields a return in the second. The realization of the short-term project’s return is $\hat{S} = S(k_1) + \epsilon$, where $k_1$ is the level of short-term
investment, $S'(k_1) > 0$, $S''(k_1) < 0$, and $\varepsilon$ is a random disturbance with mean zero and unbounded support. The long-term project yields a return of $\bar{L} = \theta L(k_2) + \eta$ where $k_2$ is the level of long-term investment, $\theta$ is a measure of the productivity of the long-run project, $L'(k_2) > 0$, $L''(k_2) < 0$, and $\eta$ is another independently distributed mean-zero disturbance.

It is assumed that except for $k_1$, $k_2$, and $\theta$, everything is common knowledge between the market and the manager. Lastly, throughout this paper we assume that the market forms rational expectations about the firm’s value given the information available to it.

The manager has a limited amount of capital of which to allocate among the two investment projects: $K$.  Consequently, the investment decision can be reduced to a single variable, $x$, the level of long-run investment ($K - x$ is invested in the short-run project). As a benchmark, let $x^*$ represent the value-maximizing level of investment in the long-term project; that is, $x^*$ solves

$$\max_{x \in [0,K]} W(x) = S(K - x) + \theta L(x).$$

In period 1, the output of the short-run project will be known by the manager and the market. The expected output of the long-run project, however, will be known by the manager but unobserved by the market. The manager knows $x$ and $\theta$ in all cases, but two alternative assumptions regarding the market’s information present themselves. In Section I.B, we assume that the market only observes $\theta$; in Section I.C we assume that the market only observes $x$.

### B. Unobservable Investment

Let us first assume that the market does not observe $x$, but that $\theta$ is common knowledge (and can be assumed to equal one without loss of general-

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5 We additionally assume as a technical convenience that $S'(0) = L'(0) = \infty$, so that it is never optimal to invest all available capital in one project.

6 Dybvig and Zender (1991) have articulated an important criticism regarding signaling models which are based upon exogenously given preferences of the managers: When managerial contracts can be designed as an arbitrary function of observables which the financial markets also use in determining their rational forecasts, inefficient signaling will not take place. A similar criticism can be leveled against the model of this paper, but there are several reasons why we choose to proceed as we do. First, we wish to follow the literature on managerial myopia as closely as possible in order to show that overinvestment is possible with only slightly different informational assumptions. Second, it may be necessary to provide a certain level of income to the manager in every period. Thus, arbitrary contracts in $\mathcal{F}_1$ and $\mathcal{F}_2$ are not available to undo any short-term managerial preference. (For example, it is not always feasible to pay the manager a bonus equal to $\alpha_1(\mathcal{F}_2 - \mathcal{F}_1)$ in period two.) Third, for standard incentive reasons (left unmodelled), it may be desirable to provide the manager with compensation in the first period that depends upon stock price rather than a fixed wage. In addition, the threat of takeovers, as in Stein (1988), may also make the manager concerned about the short-run stock price. Although this effect of the takeover can be undone by adopting sufficiently generous golden parachutes, such responses would also destroy the disciplinary role of takeovers as well. Fourth, casual empiricism suggests that, for whatever the reason, managers are in fact often concerned about present as well as future stock prices.
Do Short-Term Objectives Lead to Under- or Overinvestment?

ity). In this case, there will be underinvestment in the long-run project. This conclusion is already a familiar one; we establish it quickly below for contrast with our main result in the next section.

Once the market learns at $t = 1$ the value of $\hat{S}$, it can use $\hat{S}$ to form expectations regarding both $x$ and $\hat{L}$, which will in turn determine the period 1 valuation of the firm. Let $G(\hat{\bar{x}}|\hat{S}) = G(\hat{\bar{x}}|S(K - x) + \bar{\bar{x}})$ be the market’s conditional probability distribution over long-run investment, where $x$ influences $G$ through its effect on $\hat{S}$. The manager’s expected utility is given by

$$U = \gamma + \alpha_1 [S(K - x) + E_{G, \epsilon}[L(\hat{\bar{x}})|\hat{S}]] + \alpha_2 [S(K - x) + L(x)].$$

Providing that $G(\hat{\bar{x}}|\hat{S})$ is differentiable, it is straightforward to see that $x$ will be chosen to satisfy the manager’s first-order condition, which yields

$$S'(K - x) - L'(x) = \frac{\alpha_1}{\alpha_1 + \alpha_2} \left[ \frac{\partial E_{G, \epsilon}[L(\hat{\bar{x}})|\hat{S}]}{\partial x} - L'(x) \right].$$

Note that the full-information incentive contract requires $S'(K - x) - L'(x) = 0$. Here, however, there is a distortion from the first-best incentive contract to the extent that $\frac{\partial E[L]}{\partial x} \neq L'(x)$.

Unfortunately, a pure-strategy Nash equilibrium necessarily has such properties. Because the strategies of the managers are deterministic and the support of $\epsilon$ is sufficiently large, the market does not consider $\hat{S}$ informative:

$$\frac{\partial E[L]}{\partial x'} = 0.$$

Knowing that investment has no effect on the stock market’s valuation of the firm, the manager will choose to underinvest with $x$ such that

$$L'(x) - S'(K - x) = \frac{\alpha_1}{\alpha_1 + \alpha_2} L'(x).$$

Thus, we have the following proposition.

**Proposition 1:** A unique pure-strategy Nash equilibrium exists in which the manager underinvests in the long-run project relative to the first best.

As a consequence, a manager will always choose to underinvest in future projects when the market does not observe the apportionment of the firms’ capital between short-run and long-run projects. Intuitively, the manager knows that regardless of the first-period outcome, the market will not change its expectation of investment levels. The first-period stock price can therefore be increased at the expense of second-period profits by shifting investment towards the short-term project and away from the long-term. The manager will choose to trade some loss in second-period stock price for an increase in the first-period price since $\alpha_1 > 0$, and the market will correctly anticipate this distortion. Note that as the above first-order condition indicates, the underinvestment distortion increases as the importance of the current period stock price intensifies (i.e., $\alpha_1/\alpha_2$ increases).
Because nothing that the manager can do will change the market's perceptions of investment levels, the considered equilibrium is one which exhibits "signal jamming" and is closely related to the equilibria discussed in Holmström (1982) and Fudenberg and Tirole (1986). In equilibrium, no information is transmitted by the manager to the market and underinvestment results. This result, however, depends crucially upon our informational assumptions as the following section indicates.

C. Unobservable Productivity

We now turn to show that imperfect information and short-term managerial objectives may lead to excessive investment in long-run projects. Previously, we assumed that the market could not observe long-term investment by the manager. In this section, we assume instead that the market can observe the level of long-term investment, but cannot observe \( \theta \), the parameter representing the profitability of such investment; \( \theta \) is known only by the manager. That is, our model was previously one of hidden action; now, our model is one of hidden information. As we will see, this plausible change of assumptions radically affects the results.

It is common knowledge by both the market and the manager that \( \theta \) is distributed according to the continuous probability function \( f(\theta) \) on \([\tilde{\theta}, \tilde{\theta}]\). Clearly a manager with a highly profitable project would prefer to demonstrate to the market today that the firm's \( \theta \) is high and increase current market valuation rather than wait for the market to react to the realization of \( \theta L(x) \) in the future. Given our assumptions about the manager's preferences, it is less costly for a manager to overinvest in a highly productive (high \( \theta \)) long-term project than to overinvest in a less productive (lower \( \theta \)) long-term project. This condition will imply that a signaling equilibrium has managers with profitable long-run projects signaling the profitability of their projects by overinvesting in them.

We search for a separating equilibrium in which managers signal the productivity of long-term projects through their levels of investment. A pure-strategy equilibrium will have the form that a manager of type \( \theta \) chooses \( x \), and so we can represent an equilibrium by the function \( x(\theta) \). Let \( \Theta(x) \) be the set of all \( \theta \) that choose \( x \). That is, \( \Theta(x) \) is the inverse of \( x(\theta) \) which may be a multivalued function. The market will have expectations of \( \theta \) which will be a function of \( x \). These expectations, by Bayes' rule, are given by

\[
\theta^*(x) = E[\theta|x] = \frac{\int_{\Theta(x)} \theta f(\theta) d\theta}{\int_{\Theta(x)} f(\theta) d\theta}.
\]

When \( x(\theta) \) is strictly increasing in \( \theta \), these expectations are merely the inverse of \( x(\theta) \).

Given these expectations, the manager's utility is given by

\[
U(x, \theta) = \gamma + \alpha_1[S(K - x) + \theta^*(x)L(x)] + \alpha_2[S(K - x) + \theta L(x)]. \tag{2}
\]
We can immediately state the following:

**LEMMA:** In any Nash equilibrium of the signaling game, $x(\theta)$ will be a nondecreasing function of $\theta$.

This result is proved using the standard revealed preference argument and is in the Appendix. Because $x(\theta)$ is monotonic, it follows that it is differentiable almost everywhere. Consequently, we can use simple differential arguments to characterize the equilibrium in the investment game. Our results are stated in the following proposition, which is proved in the Appendix.

**PROPOSITION 2:** A unique fully separating Perfect Bayesian Equilibrium exists which involves overinvestment with probability one and where the equilibrium choice of investment, $x(\theta)$, is such that $x(\theta) = x^*(\theta)$ and for all $\theta \in (\underline{\theta}, \overline{\theta}]$

$$
\frac{dx}{d\theta} = \frac{\alpha_1 L(x)}{\alpha_1 + \alpha_2 S'(K - x) - \theta L'(x)},
$$

and consequently, $x(\theta) > x^*(\theta)$.

The proposition indicates that in the separating equilibrium managers of every firm but the worst overinvest to signal to the market that the firm’s productivity of its long-term project is high, and thereby increase the current valuation of the firm. In this sense, the equilibrium is similar to the results in Spence (1973), where the fully separating equilibrium in his job market model has talented employees overinvesting in education so as to signal their product of labor to employers. Unlike the signal-jamming equilibrium in Proposition 1, here the manager successfully communicates information to the market via observable investment choices.

Intuitively, the manager with a highly productive project will want to invest more in that project to signal to the market that the project is valuable. Managers with a less productive project prefer to invest efficiently rather than mimic the highly productive firms and overinvest. This occurs because overinvestment is more costly to low productivity firms: $U_{xp}(x, \theta) > 0$. As a consequence, the lowest type firm’s manager chooses the efficient level of investment $x(\theta) = x^*(\theta)$, while all other firms ($\theta > \underline{\theta}$) choose inefficiently high levels of investment, $x(\theta) > x^*(\theta)$. The distortion evident in Proposition 2 arises from short-term managerial objectives; i.e., $\alpha_1 > 0$. In the case where $\alpha_1 = 0$, no gain from deceiving the market exists, and so no signaling via overinvestment occurs: The absence of short-term objectives results in the first-best level of investment.

**II. Conclusion**

We have shown that the existence of short-term managerial objectives (coupled with incomplete information) may lead not only to underinvestment but also to overinvestment in long-run projects. Whether underinvestment or overinvestment results depends critically on the nature of the managers’
informational advantage over the stock market. Our model enables us to predict the likely direction of the distortion in a given situation.

Underinvestment will occur when the market has incomplete information about the level of investment undertaken. This is likely to be the case with respect to many types of "soft" investment. Some important examples include managerial effort and time, entrepreneurial talent, internal personnel resources, and investments which must be kept secret from competitors such as new product designs and developments. Thus, for example, managers can be expected to underinvest in the amount of managerial time and attention devoted to their decision making about the future. Similarly, when expenses made to boost the company's long-run reputation or worker morale are unobservable by the market, managers can be expected to underinvest in such expenditures.

On the other hand, overinvestment will occur when the market observes the level of investment but not its productivity. This is likely to be the case with respect to many types of "hard" investment and perhaps some "soft" investments which are credibly disclosed by the company. Thus, for example, the amount invested in modernizing plants, installing new equipment, constructing new buildings, beginning new product lines, and undertaking R & D is often observable to the market (either directly or because it is disclosed by the company), and in such cases our model predicts that managers will make excessive investments to signal that the firm's present value is high.\(^7\)

Consider R & D expenditures and the predictions of the above propositions. Investment in R & D has received much attention in the literature. Our research points out that it is crucial to determine whether the level of investment in R & D is observable to the market and whether or not the managers have private information concerning the expected profitability of such investment. As noted in the introduction, R & D expenditures are commonly disclosed. But to the extent that managers may camouflage some regular expenses as R & D, the level of investment may be unobservable. When R & D expenditures are "soft" and unobservable, but the market has good information about the productivity of such expenditures, we should expect to see underinvestment. When the level of investment in R & D is observable, but the productivity of the investment is unknown by the market, overinvestment is expected.

Our theoretical finding concerning the possibility of overinvestment is consistent with the empirical work of Meulbroek et al. (1991). In their work they find that the presence of antitakeover provisions (which according to

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\(^7\) Assuming that unobservability is especially likely to occur with respect to "soft" investments, one implication of our model is that, as the weight of short-term objectives decreases in managers' objective functions, their investment in long-run projects is likely to be characterized by a higher ratio of soft/hard investment. As a consequence, if the common claim that Japanese managers give less weight to short-term objectives is correct, then the investments in long-run projects by Japanese companies should be characterized by a greater soft/hard investment ratio. These implications would be difficult to test, however, because some "soft" investments may well be unobservable not only to the market but also to the researcher.
Stein (1988) should reduce the weight accorded to short-term objectives) reduces long-term investments. It should be emphasized, however, that our work suggests that the identified reduction may lower long-term investment from an excessive level to a more efficient level. As a consequence, the reduction may be welfare improving—an insight generally overlooked by the literature.

Finally, it is worth noting one main direction in which the theoretical analysis of this paper calls for extension. While our model takes as given whether the amount spent on a certain type of investment is observable, observability may often be affected by managerial action. In many situations, an investment would be unobservable unless the managers, possibly at a cost, take action to make it observable (say, by making expenditures to secure separate and verifiable reporting). Conversely, in many other situations, an investment would be observable unless the managers take actions, possibly at some cost, to make it unobservable. Once the implications of observability for the direction of divergence from optimal long-run investment are recognized, the question naturally arises as to when managers will take actions to make an unobservable investment observable, and, finally, when such actions will increase or decrease firm value.

Appendix

Proof of Lemma: Suppose otherwise. Let \((x, \theta)\) and \((x', \theta')\) be two investment-type pairs used in equilibrium by managers where \(\theta > \theta'\) but \(x' > x\). It must be the case, by revealed preference, that

\[
U(x, \theta) \geq U(x', \theta),
\]

\[
U(x', \theta') \geq U(x, \theta').
\]

Adding these inequalities yields (after simplification)

\[
\theta L(x) + \theta' L(x') \geq \theta L(x') + \theta' L(x),
\]

or equivalently,

\[
(\theta - \theta')(L(x) - L(x')) \geq 0,
\]

which contradicts our initial hypothesis. \(\Box\)

Proof of Proposition 2: We know that any Perfect Bayesian Equilibrium in the investment game must have beliefs which are a nondecreasing function of investment. Because \(\lim_{x \to 0} \frac{\partial U(x, \theta)}{\partial x} = +\infty\) and \(\lim_{x \to k} \frac{\partial U(x, \theta)}{\partial x} = -\infty\), necessary conditions for the manager's choice of investment are

\[
\frac{\partial U(x, \theta)}{\partial x} = 0, \quad \frac{\partial^2 U(x, \theta)}{\partial x^2} \leq 0,
\]
∀x, θ. The first-order condition gives us an identity in x and θ which we can totally differentiate to obtain

\[ \frac{\partial^2 U(x, \theta)}{\partial \theta \partial x} + \frac{\partial^2 U(x, \theta)}{\partial x^2} \frac{dx}{d\theta} = 0. \]

Thus, the necessary local second-order condition above can be restated as \( U_{x\theta} \geq 0 \), which is true by our assumption that \( L'(x) > 0 \).

Furthermore, if \( x(\theta) \) is nondecreasing, the local conditions for a maximum are sufficient. To see that the monotonicity of \( x(\theta) \) and the first-order condition are sufficient for a separating equilibrium, suppose \( x(\theta) \) satisfies these conditions but the manager prefers to choose otherwise. Suppose that \( x' \) (where \( x' = x(\theta') \)) rather than \( x \) is the chosen investment by a manager with productivity \( \theta \). Then, revealed preference implies \( U(x(\theta'), \theta) - U(x(\theta), \theta) > 0 \). Integrating, we obtain

\[ \int_0^{\theta'} U_x(x(s), \theta) \frac{dx(s)}{ds} ds > 0. \]

But by hypothesis, \( U_x(x(\theta), \theta) = 0 \) for all \( \theta \). Thus,

\[ \int_0^{\theta'} [U_x(x(s), \theta) - U_x(x(s), s)] \frac{dx(s)}{ds} ds > 0. \]

Integrating again,

\[ \int_0^{\theta'} \int_s^\theta U_{x\theta}(s, t) \frac{dx(s)}{ds} dt \, ds > 0. \]

But by assumption, the above double integral is always nonpositive, which contradicts our hypothesis.

Consequently, if our solution satisfies the local first-order condition and the manager's investment function is nondecreasing in \( \theta \), we have characterized the equilibrium path of a Perfect Bayesian-Nash equilibrium. The first-order condition is

\[ \frac{\partial U(x, \theta)}{\partial x} = (\alpha_1 + \alpha_2)[\theta L'(x) - S'(K - x)] + \alpha_1 \frac{d\theta}{dx} L(x) = 0. \]

Rearranging the terms,

\[ \frac{d\theta}{dx} = \frac{\alpha_1 + \alpha_2 S'(K - x) - \theta L'(x)}{\alpha_1 L(x)}. \]

Let \( x^*(\theta) \) be the efficient level of investment for a given productivity, \( \theta \). In a fully separating equilibrium, the worst inference which the market can place on a manager is that the productivity of the long-term project is \( \theta \) and, consequently, the worst firm's manager must earn at least \( U(x^*(\theta), \theta) \). Thus, in a fully separating equilibrium, \( x(\theta) = x^*(\theta) \), and we have an initial condition to the ordinary differential equation above in (3).
Lastly, we must specify beliefs off the equilibrium path. Let \( \chi = [x, \bar{x}] \) be the set of all investment levels which arise with positive probability in the proposed equilibrium of the signaling game. One set of arbitrary beliefs which holds together the equilibrium has the market believing that for any \( x \in [0, \bar{x}] \) the firm's type is \( \theta \), and for any \( x > \bar{x} \) the firm's type is \( \bar{\theta} \).

The above differential equation implies that \( dx/d\theta = \infty \) at \( \theta \) and at any other points where \( x(\theta) = x^*(\theta) \). Because the signaling condition requires that \( x(\theta) \) is monotonic in \( \theta \), and by construction \( x^*(\theta) \) has finite slope, \( x(\theta) \) must remain above \( x^*(\theta) \) for all \( \theta \in (\theta, \bar{\theta}) \). Hence, we have a uniquely defined (up to arbitrary off-the-equilibrium-path beliefs) fully separating equilibrium which exhibits overinvestment with probability one.

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